THE PRACTICAL MODEL CALCULATOR,

FOR THE

ENGINEER, MECHANIC, MACHINIST,
MANUFACTURER OF ENGINE-WORK, NAVAL ARCHITECT,
MINER, AND MILLWRIGHT.

BY

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THE
PRACTICAL MODEL CALCULATOR.

WEIGHTS AND MEASURES.

THE UNIT OF LENGTH.

THE YARD.—If a pendulum vibrating seconds in vacuo, in Philadelphia, be divided into 2509 equal parts, 2310 of such equal parts is the length of the standard yard; the measures are taken on brass rods at the temperature of 32° Fahrenheit. This yard will not be in error the ten-millionth part of an inch.

2310 : 2509 as 1· to 1·086142 nearly.

THE UNIT OF WEIGHT.

The Pound, avoirdupois, is 27·7015 cubic inches of distilled water, weighed in air, at the temperature of maximum density, 39°·82; the barometer at 30 inches.

THE LIQUID UNIT.

The Gallon, 231 cubic inches, contains 8·3388822 pounds avoirdupois, equal 58372·1754 grains troy of distilled water, at 39°·82 Fah.; the barometer at 30 inches.

UNIT OF DRY CAPACITY.

The Bushel contains 2150·42 cubic inches, 77·627412 pounds avoirdupois, 543391·89 grains of distilled water, at the temperature of maximum density; the barometer at 30 inches.

The French unit of length or distance is the metre, and is the ten-millionth of the quadrant of the globe, measured from the equator to the pole.

The French *Metre* = 3·2808992 English *feet* linear measure = 39·3707904 *inches*.

For Multiples the following Greek words are used:	*For Divisors the following Latin words are used:*
Deca for .... 10 times.	*Deci* for the 10*th* part.
Hecto — .... 100 times.	*Centi* — 100*th* part.
Kilo — .... 1000 times.	*Milli* — 1000*th* part.
Myria — .... 10000 times.	Thus a *Kilometre* = 1000 metres.

$$\textit{Millimetre} = \frac{\text{metre}}{1000}$$

The square *Deca Metre*, called the *Are*, is the element of land measure in France, which = 1076·42996 *square feet* English.

The *Stere* is a cubic metre = 35·316582 *cubic feet* English.

The *Litre* for liquid measure is a cubic decimetre = 1·76077 *imperial pints* English, at the temperature of melting ice; a *litre* of distilled water weighs 15434 *grains troy.*

The unit of weight is the *gramme:* it is the weight of a cubic centimetre of distilled water, or of a millilitre, and therefore equal to 15·434 *grains troy.*

The kilogramme is the weight of a cubic decimetre of distilled water, at the temperature of maximum density, 4° centigrade.

The pound troy contains 5760 grains.

The pound avoirdupois contains 7000 grains.

The English imperial gallon contains 277·274 cubic inches; and the English corn bushel contains eight such gallons, or 2218·192 cubic inches.

APOTHECARIES' WEIGHT.

Grains			marked	gr.
20 Grains	make	1 Scruple	—	sc. or ℈
3 Scruples	—	1 Dram	—	dr. or ʒ
8 Drams	—	1 Ounce	—	oz. or ℥
12 Ounces	—	1 Pound	—	lb. or ℔.

gr. sc.
20 = 1 dr.
60 = 3 = 1 oz.
480 = 24 = 8 = 1 lb.
5760 = 288 = 96 = 12 = 1

This is the same as troy weight, only having some different divisions. Apothecaries make use of this weight in compounding their medicines; but they buy and sell their drugs by avoirdupois weight.

AVOIRDUPOIS WEIGHT.

Drams			marked	dr.
16 Drams	make	1 Ounce	—	oz.
16 Ounces	—	1 Pound	—	lb.
28 Pounds	—	1 Quarter	—	qr.
4 Quarters	—	1 Hundred Weight	—	cwt.
20 Hundred Weight	—	1 Ton	—	ton.

dr. oz.
16 = 1 lb.
256 = 16 = 1 qr.
7168 = 448 = 28 = 1 cwt.
28672 = 1792 = 112 = 4 = 1 ton.
573440 = 35840 = 2240 = 80 = 20 = 1

By this weight are weighed all things of a coarse or drossy nature, as Corn, Bread, Butter, Cheese, Flesh, Grocery Wares, and some Liquids; also all Metals except Silver and Gold.

		Oz.	Dwt.	Gr.	
Note, that 1 lb. avoirdupois	=	14	11	15½	troy.
1 oz. —	=	0	18	5½	—
1 dr. —	=	0	1	3½	—

TROY WEIGHT.

Grains		marked	Gr.
24 Grains	make 1 Pennyweight		Dwt.
20 Pennyweights	1 Ounce		Oz.
12 Ounces	1 Pound		Lb.

Gr.	Dwt.		
24 =	1	Oz.	
480 =	20 =	1	Lb.
5760 =	240 =	12 =	1

By this weight are weighed Gold, Silver, and Jewels.

LONG MEASURE.

3	Barley-corns	make	1 Inch	marked	In.
12	Inches	—	1 Foot	—	Ft.
3	Feet	—	1 Yard	—	Yd.
6	Feet	—	1 Fathom	—	Fth.
5	Yards and a half	—	1 Pole or Rod	—	Pl.
40	Poles	—	1 Furlong	—	Fur.
8	Furlongs	—	1 Mile	—	Mile.
3	Miles	—	1 League	—	Lea.
69⅙	Miles nearly	—	1 Degree	—	Deg. or °.

In.	Ft.				
12 =	1	Yd.			
36 =	3 =	1	Pl.		
198 =	16½ =	5½ =	1	Fur.	
7920 =	660 =	220 =	40 =	1	Mile.
63360 =	5280 =	1760 =	320 =	8 =	1

CLOTH MEASURE.

2 Inches and a quarter	make	1 Nail	marked	Nl.
4 Nails	—	1 Quarter of a Yard	—	Qr.
3 Quarters	—	1 Ell Flemish	—	E F.
4 Quarters	—	1 Yard	—	Yd.
5 Quarters	—	1 Ell English	—	E E.
4 Qrs. 1⅕ Inch	—	1 Ell Scotch	—	E S.

SQUARE MEASURE.

144	Square Inches	make	1 Sq. Foot	marked	Ft.
9	Square Feet	—	1 Sq. Yard	—	Yd.
30¼	Square Yards	—	1 Sq. Pole	—	Pole.
40	Square Poles	—	1 Rood	—	Rd.
4	Roods	—	1 Acre	—	Acr.

Sq. Inc.	Sq. Ft.				
144 =	1	Sq. Yd.			
1296 =	9 =	1	Sq. Pl.		
39204 =	272¼ =	30¼ =	1	Rd.	
1568160 =	10890 =	1210 =	40 =	1	Acr.
6272640 =	43560 =	4840 =	160 =	4 =	1

When three dimensions are concerned, namely, length, breadth, and depth or thickness, it is called cubic or solid measure, which is used to measure Timber, Stone, &c.

The cubic or solid Foot, which is 12 inches in length, and breadth, and thickness, contains 1728 cubic or solid inches, and 27 solid feet make one solid yard.

DRY, OR CORN MEASURE.

2 Pints	make	1 Quart	marked	Qt.
2 Quarts	—	1 Pottle	—	Pot.
2 Pottles	—	1 Gallon	—	Gal.
2 Gallons	—	1 Peck	—	Pec.
4 Pecks	—	1 Bushel	—	Bu.
8 Bushels	—	1 Quarter	—	Qr.
5 Quarters	—	1 Weigh or Load...	—	Wey.
2 Weys	—	1 Last	—	Last.

Pts. Gal.
8 = 1 Pec.
16 = 2 = 1 Bu.
64 = 8 = 4 = 1 Qr.
512 = 64 = 32 = 8 = 1 Wey.
2560 = 320 = 160 = 40 = 5 = 1 Last.
5120 = 640 = 320 = 80 = 10 = 2 = 1

WINE MEASURE.

2 Pints	make	1 Quart	marked	Qt.
2 Quarts	—	1 Gallon	—	Gal.
42 Gallons	—	1 Tierce	—	Tier.
63 Gallons or 1½ Tier..	—	1 Hogshead	—	Hhd.
2 Tierces	—	1 Puncheon	—	Pun.
2 Hogsheads	—	1 Pipe or Butt....	—	Pi.
2 Pipes	—	1 Tun	—	Tun.

Pts. Qts.
2 = 1 Gal.
8 = 4 = 1 Tier.
336 = 168 = 42 = 1 Hhd.
504 = 252 = 63 = 1½ = 1 Pun.
672 = 336 = 84 = 2 = 1½ = 1 Pi.
1008 = 504 = 126 = 3 = 2 = 1½ = 1 Tun.
2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1.

ALE AND BEER MEASURE.

2 Pints	make	1 Quart	marked	Qt.
4 Quarts	—	1 Gallon	—	Gal.
36 Gallons	—	1 Barrel	—	Bar.
1 Barrel and a half....	—	1 Hogshead	—	Hhd.
2 Barrels	—	1 Puncheon	—	Pun.
2 Hogsheads	—	1 Butt	—	Butt.
2 Butts	—	1 Tun	—	Tun.

Pts. Qt.
2 = 1 Gal.
8 = 4 = 1 Bar.
288 = 144 = 36 = 1 Hhd.
432 = 216 = 54 = 1½ = 1 Butt.
864 = 432 = 108 = 3 = 2 = 1

OF TIME.

60 Seconds	make	1 Minute	marked	M. or ′.
60 Minutes	—	1 Hour	—	Hr.
24 Hours	—	1 Day	—	Day.
7 Days	—	1 Week	—	Wk.
4 Weeks	—	1 Month	—	Mo.
13 Months, 1 Day, 6 Hours, or 365 Days, 6 Hours.	—	1 Julian Year	—	Yr.

Sec.	Min.	Hr.	Day.	Wk.	Mo.
60 =	1				
3600 =	60 =	1			
86400 =	1440 =	24 =	1		
604800 =	10080 =	168 =	7 =	1	
2419200 =	40320 =	672 =	28 =	4 =	1
31557600 =	525960 =	8766 =	365¼ =	1 Year.	

	Wk.	Da.	Hr.		Mo.	Da.	Hr.	
Or	52	1	6	=	13	1	6	= 1 Julian Year.

	Da.	Hr.	M.	Sec.	
But	365	5	48	48	= 1 Solar Year.

The time of rotation of the earth on its axis is called a *sidereal day*, for the following reason: If a permanent object be placed on the surface of the earth, always retaining the same position, it may be so located as to be posited in the same plane with the observer and some selected fixed star at the same instant of time; although this coincidence may be but momentary, still this coincidence continually recurs, and the interval elapsed between two consecutive coincidences has always throughout all ages appeared the same.

It is this interval that is called a sidereal day.

The sidereal day increased in a certain ratio, and called the *mean solar day*, has been adopted as the standard of time.

Thus, 366·256365160 *sidereal days* = 366·256365160 − 1 or 365·256365160 *mean solar days*, whence sidereal day : mean solar day : : 365·256365160 : 366·256365160 : : 0·997269672 : 1 or as 1 : 1·002737803, when 23 *hours*, 56 *minutes* 4·0996608 *sec.* of mean solar time = 1 *sidereal day;* and 24 *hours*, 3 *minutes*, 56·5461797 *sec.* of sidereal time = 1 *mean solar day.*

The *true solar day* is the interval between two successive coincidences of the sun with a fixed object on the earth's surface, bringing the sun, the fixed object, and the observer in the same plane.

This interval is variable, but is susceptible of a maximum and minimum, and oscillates about that mean period which is called a mean solar day.

Apparent or *true time* is that which is denoted by the sun-dial, from the apparent motion of the sun in its diurnal revolution, and differs several minutes in certain parts of the ecliptic from the mean time, or that shown by the clock. The difference is called the equation of time, and is set down in the almanac, in order to ascertain the true time.

ARITHMETIC.

Arithmetic is the art or science of numbering; being that branch of Mathematics which treats of the nature and properties of numbers. When it treats of whole numbers, it is called *Common Arithmetic;* but when of broken numbers, or parts of numbers, it is called *Fractions.*

Unity, or a *Unit*, is that by which every thing is called one; being the beginning of number; as one man, one ball, one gun.

Number is either simply one, or a compound of several units; as one man, three men, ten men.

An *Integer* or *Whole Number*, is some certain precise quantity of units; as one, three, ten. These are so called as distinguished from *Fractions*, which are broken numbers, or parts of numbers; as one-half, two-thirds, or three-fourths.

NOTATION AND NUMERATION.

Notation, or Numeration, teaches to denote or express any proposed number, either by words or characters; or to read and write down any sum or number.

The numbers in Arithmetic are expressed by the following ten digits, or Arabic numeral figures, which were introduced into Europe by the Moors about eight or nine hundred years since: viz. 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cipher or nothing. These characters or figures were formerly all called by the general name of *Ciphers;* whence it came to pass that the art of Arithmetic was then often called *Ciphering.* Also, the first nine are called *Significant Figures*, as distinguished from the cipher, which is quite insignificant of itself.

Besides this value of those figures, they have also another, which depends upon the place they stand in when joined together; as in the following Table:

&c.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
&c.	9	8	7	6	5	4	3	2	1
		9	8	7	6	5	4	3	2
			9	8	7	6	5	4	3
				9	8	7	6	5	4
					9	8	7	6	5
						9	8	7	6
							9	8	7
								9	8
									9

Here any figure in the first place, reckoning from right to left, denotes only its own simple value; but that in the second place denotes ten times its simple value; and that in the third place a hundred times its simple value; and so on; the value of any figure, in each successive place, being always ten times its former value.

Thus, in the number 1796, the 6 in the first place denotes only six units, or simply six; 9 in the second place signifies nine tens, or ninety; 7 in the third place, seven hundred; and the 1 in the fourth place, one thousand; so that the whole number is read thus—one thousand seven hundred and ninety-six.

As to the cipher 0, it stands for nothing of itself, but being joined on the right-hand side to other figures, it increases their value in the same tenfold proportion: thus, 5 signifies only five; but 50 denotes 5 tens, or fifty; and 500 is five hundred; and so on.

For the more easily reading of large numbers, they are divided into periods and half-periods, each half-period consisting of three figures; the name of the first period being units; of the second, millions; of the third, millions of millions, or bi-millions, contracted to billions; of the fourth, millions of millions of millions, or tri-millions, contracted to trillions; and so on. Also, the first part of any period is so many units of it, and the latter part so many thousands.

The following Table contains a summary of the whole doctrine:

Periods.	Quadrill.;	Trillions;	Billions;	Millions;	Units.
Half-per.	th. un.	th. un.	th. un.	th. un.	th. un.
Figures.	123,456;	789,098;	765,432;	101,234;	567,890.

Numeration is the reading of any number in words that is proposed or set down in figures.

Notation is the setting down in figures any number proposed in words.

OF THE ROMAN NOTATION.

The Romans, like several other nations, expressed their numbers by certain letters of the alphabet. The Romans only used seven numeral letters, being the seven following capitals: viz. I for *one;* V for *five;* X for *ten;* L for *fifty;* C for a *hundred;* D for *five hundred;* M for a *thousand.* The other numbers they expressed by various repetitions and combinations of these, after the following manner:

1 = I.
2 = II.
3 = III.
4 = IIII. or IV.
5 = V.
6 = VI.
7 = VII.
8 = VIII.
9 = IX.
10 = X.
50 = L.
100 = C.
500 = D or IƆ.

As often as any character is repeated, so many times is its value repeated.

A less character before a greater diminishes its value.

A less character after a greater increases its value.

For every Ɔ annexed, this becomes ten times as many.

1000 = M or CIƆ.
2000 = MM.
5000 = $\overline{\text{V}}$ or IƆƆ.
6000 = $\overline{\text{VI}}$.
10000 = $\overline{\text{X}}$ or CCIƆƆ.
50000 = $\overline{\text{L}}$ or IƆƆƆ.
60000 = $\overline{\text{LX}}$.
100000 = $\overline{\text{C}}$ or CCCIƆƆƆ.
1000000 = $\overline{\text{M}}$ or CCCCIƆƆƆƆ.
2000000 = $\overline{\text{MM}}$.
&c. &c.

For every C and Ɔ, placed one at each end, it becomes ten times as much.

A bar over any number increases it 1000 fold.

EXPLANATION OF CERTAIN CHARACTERS.

There are various characters or marks used in Arithmetic and Algebra, to denote several of the operations and propositions; the chief of which are as follow:

+	signifies *plus*, or addition.	: :: :	.... proportion.
—	.... *minus*, or subtraction.	=	.... equality.
×	.... multiplication.	$\sqrt{\ }$	.... square root.
÷	.... division.	$\sqrt[3]{\ }$	.... cube root, &c.

Thus,

5 + 3, denotes that 3 is to be added to 5 = 8.
6 − 2, denotes that 2 is to be taken from 6 = 4.
7 × 3, denotes that 7 is to be multiplied by 3 = 21.
8 ÷ 4, denotes that 8 is to be divided by 4 = 2.
2 : 3 :: 4 : 6, shows that 2 is to 3 as 4 is to 6, and thus, 2×6=3×4.
6 + 4 = 10, shows that the sum of 6 and 4 is equal to 10.
$\sqrt{3}$, or $3^{\frac{1}{2}}$, denotes the square root of the number 3 = 1·7320508.
$\sqrt[3]{5}$, or $5^{\frac{1}{3}}$, denotes the cube root of the number 5 = 1·709976.
7^2, denotes that the number 7 is to be squared = 49.
8^3, denotes that the number 8 is to be cubed = 512.
&c.

RULE OF THREE.

THE RULE OF THREE teaches how to find a fourth proportional to three numbers given. Whence it is also sometimes called the Rule of Proportion. It is called the Rule of Three, because three terms or numbers are given to find the fourth; and because of its great and extensive usefulness, it is often called the Golden Rule.

This Rule is usually considered as of two kinds, namely, Direct and Inverse.

The Rule of Three Direct is that in which more requires more, or less requires less. As in this: if 3 men dig 21 yards of trench in a certain time, how much will 6 men dig in the same time? Here more requires more, that is, 6 men, which are more than 3 men, will also perform more work in the same time. Or when it is thus: if 6 men dig 42 yards, how much will 3 men dig in the same time? Here, then, less requires less, or 3 men will perform proportionally less work than 6 men in the same time. In both these cases, then, the Rule, or the Proportion, is Direct; and the stating must be

thus, As 3 : 21 : : 6 : 42,
or thus, As 6 : 42 : : 3 : 21.

But, the Rule of Three Inverse is when more requires less, or less requires more. As in this: if 3 men dig a certain quantity of trench in 14 hours, in how many hours will 6 men dig the like quantity? Here it is evident that 6 men, being more than 3, will perform an equal quantity of work in less time, or fewer hours. Or thus: if 6 men perform a certain quantity of work in 7 hours, in how many hours will 3 men perform the same? Here less requires more, for 3 men will take more hours than 6 to perform the same work. In both these cases, then, the Rule, or the Proportion, is Inverse; and the stating must be

thus, As 6 : 14 : : 3 : 7,
or thus, As 3 : 7 : : 6 : 14.

And in all these statings the fourth term is found, by multiplying the 2d and 3d terms together, and dividing the product by the 1st term.

Of the three given numbers, two of them contain the supposition, and the third a demand. And for stating and working questions of these kinds observe the following general Rule:

RULE.—State the question by setting down in a straight line the three given numbers, in the following manner, viz. so that the 2d term be that number of supposition which is of the same kind that the answer or 4th term is to be; making the other number of supposition the 1st term, and the demanding number the 3d term, when the question is in direct proportion; but contrariwise, the other number of supposition the third term, and the demanding number the 1st term, when the question has inverse proportion.

Then, in both cases, multiply the 2d and 3d terms together, and divide the product by the first, which will give the answer, or 4th term sought, of the same denomination as the second term.

Note, If the first and third terms consist of different denominations, reduce them both to the same; and if the second term be a compound number, it is mostly convenient to reduce it to the lowest denomination mentioned. If, after division, there be any remainder, reduce it to the next lower denomination, and divide by the same divisor as before, and the quotient will be of this last denomination. Proceed in the same manner with all the remainders, till they be reduced to the lowest denomination which the second term admits of, and the several quotients taken together will be the answer required.

Note also, The reason for the foregoing Rules will appear when we come to treat of the nature of Proportions. Sometimes also two or more statings are necessary, which may always be known from the nature of the question.

An engineer having raised 100 yards of a certain work in 24 days with 5 men, how many men must he employ to finish a like quantity of work in 15 days?

$$\begin{array}{rcccl} & \text{da. men.} & & \text{da. men.} & \\ \text{As } 15 & : 5 & :: & 24 & : 8 \text{ Ans.} \\ & & & \underline{5} & \\ & & 15\,) & \overline{120} & (\,8 \text{ Answer.} \\ & & & \underline{120} & \end{array}$$

COMPOUND PROPORTION.

Compound Proportion teaches how to resolve such questions as require two or more statings by Simple Proportion; and that, whether they be Direct or Inverse.

In these questions, there is always given an odd number of terms, either five, or seven, or nine, &c. These are distinguished into terms of supposition and terms of demand, there being always one term more of the former than of the latter, which is of the same kind with the answer sought.

Rule.—Set down in the middle place that term of supposition which is of the same kind with the answer sought. Take one of the other terms of supposition, and one of the demanding terms which is of the same kind with it; then place one of them for a first term, and the other for a third, according to the directions given in the Rule of Three. Do the same with another term of supposition, and its corresponding demanding term; and so on if there be more terms of each kind; setting the numbers under each other which fall all on the left-hand side of the middle term, and the same for the others on the right-hand side. Then to work.

By several Operations.—Take the two upper terms and the middle term, in the same order as they stand, for the first Rule of Three question to be worked, whence will be found a fourth term. Then take this fourth number, so found, for the middle term of a second Rule of Three question, and the next two under terms in the general stating, in the same order as they stand, finding a fourth term from them; and so on, as far as there are any numbers in the general stating, making always the fourth number resulting from each simple stating to be the second term of the next following one. So shall the last resulting number be the answer to the question.

By one Operation.—Multiply together all the terms standing under each other, on the left-hand side of the middle term; and, in like manner, multiply together all those on the right-hand side of it. Then multiply the middle term by the latter product, and divide the result by the former product, so shall the quotient be the answer sought.

How many men can complete a trench of 135 yards long in 8 days, when 16 men can dig 54 yards in 6 days?

General stating.

yds. 54 : 16 men :: 135 yds.
days 8 6 days
432 810
16
4860
810
432) 12960 (30 Ans. by one operation.
1296
0

The same by two operations.

1st.	2d.
As 54 : 16 :: 135 : 40	As 8 : 40 :: 6 : 30
16	6
810	8) 240 (30 Ans.
135	24
54) 2160 (40	0
216	
0	

OF COMMON FRACTIONS.

A Fraction, or broken number, is an expression of a part, or some parts, of something considered as a whole.

It is denoted by two numbers, placed one below the other, with a line between them:

thus, $\frac{3}{4}$ $\left.\begin{matrix}\text{numerator}\\\text{denominator}\end{matrix}\right\}$ which is named three-fourths.

The Denominator, or number placed below the line, shows how many equal parts the whole quantity is divided into; and represents the Divisor in Division. And the Numerator, or number set above the line, shows how many of those parts are expressed by the Fraction; being the remainder after division. Also, both these numbers are, in general, named the Terms of the Fractions.

Fractions are either Proper, Improper, Simple, Compound, or Mixed.

A Proper Fraction is when the numerator is less than the denominator; as $\frac{1}{2}$, or $\frac{2}{3}$, or $\frac{3}{4}$, &c.

An Improper Fraction is when the numerator is equal to, or exceeds, the denominator; as $\frac{3}{3}$, or $\frac{5}{4}$, or $\frac{7}{5}$, &c.

A Simple Fraction is a single expression denoting any number of parts of the integer; as $\frac{2}{3}$, or $\frac{3}{2}$.

A Compound Fraction is the fraction of a fraction, or several fractions connected with the word *of* between them; as $\frac{1}{2}$ of $\frac{2}{3}$, or $\frac{3}{5}$ of $\frac{5}{6}$ of 3, &c.

A Mixed Number is composed of a whole number and a fraction together; as $3\frac{1}{4}$, or $12\frac{4}{6}$, &c.

A whole or integer number may be expressed like a fraction, by writing 1 below it, as a denominator; so 3 is $\frac{3}{1}$, or 4 is $\frac{4}{1}$, &c.

A fraction denotes division; and its value is equal to the quotient obtained by dividing the numerator by the denominator; so $\frac{12}{4}$ is equal to 3, and $\frac{20}{5}$ is equal to 4.

Hence, then, if the numerator be less than the denominator, the value of the fraction is less than 1. If the numerator be the same as the denominator, the fraction is just equal to 1. And if the numerator be greater than the denominator, the fraction is greater than 1.

REDUCTION OF FRACTIONS.

REDUCTION OF FRACTIONS is the bringing them out of one form or denomination into another, commonly to prepare them for the operations of Addition, Subtraction, &c., of which there are several cases.

To find the greatest common measure of two or more numbers.

The Common Measure of two or more numbers is that number which will divide them both without a remainder: so 3 is a common measure of 18 and 24; the quotient of the former being 6, and of the latter 8. And the greatest number that will do this, is the greatest common measure: so 6 is the greatest common measure of 18 and 24; the quotient of the former being 3, and of the latter 4, which will not both divide farther.

RULE.—If there be two numbers only, divide the greater by the less; then divide the divisor by the remainder; and so on, dividing always the last divisor by the last remainder, till nothing remains; then shall the last divisor of all be the greatest common measure sought.

When there are more than two numbers, find the greatest common measure of two of them, as before; then do the same for that common measure and another of the numbers; and so on, through all the numbers; then will the greatest common measure last found be the answer.

If it happen that the common measure thus found is 1, then the numbers are said to be incommensurable, or to have no common measure.

To find the greatest common measure of 1998, 918, and 522.

918) 1998 (2
1836
162) 918 (5
810
108) 162 (1
108
54) 108 (2
108

So 54 is the greatest common measure of 1998 and 918.

Hence 54) 522 (9
486
36) 54 (1
36
18) 36 (2
36

So that 18 is the answer required.

To abbreviate or reduce fractions to their lowest terms.

Rule.—Divide the terms of the given fraction by any number that will divide them without a remainder; then divide these quotients again in the same manner; and so on, till it appears that there is no number greater than 1 which will divide them; then the fraction will be in its lowest terms.

Or, divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required, of the same value as at first.

That dividing both the terms of the fraction by the same number, whatever it be, will give another fraction equal to the former, is evident. And when those divisions are performed as often as can be done, or when the common divisor is the greatest possible, the terms of the resulting fraction must be the least possible.

1. Any number ending with an even number, or a cipher, is divisible, or can be divided by 2.

2. Any number ending with 5, or 0, is divisible by 5.

3. If the right-hand place of any number be 0, the whole is divisible by 10; if there be 2 ciphers, it is divisible by 100; if 3 ciphers, by 1000; and so on, which is only cutting off those ciphers.

4. If the two right-hand figures of any number be divisible by 4, the whole is divisible by 4. And if the three right-hand figures be divisible by 8, the whole is divisible by 8; and so on.

5. If the sum of the digits in any number be divisible by 3, or by 9, the whole is divisible by 3, or by 9.

6. If the right-hand digit be even, and the sum of all the digits be divisible by 6, then the whole will be divisible by 6.

7. A number is divisible by 11 when the sum of the 1st, 3d, 5th, &c., or of all the odd places, is equal to the sum of the 2d, 4th, 6th, &c., or of all the even places of digits.

8. If a number cannot be divided by some quantity less than the square of the same, that number is a prime, or cannot be divided by any number whatever.

9. All prime numbers, except 2 and 5, have either 1, 3, 7, or 9, in the place of units; and all other numbers are composite, or can be divided.

10. When numbers, with a sign of addition or subtraction between them, are to be divided by any number, then each of those numbers must be divided by it. Thus, $\frac{10 + 8 - 4}{2} = 5 + 4 - 2 = 7$.

11. But if the numbers have the sign of multiplication between them, only one of them must be divided. Thus, $\frac{10 \times 8 \times 3}{6 \times 2} =$

$$\frac{10 \times 4 \times 3}{6 \times 1} = \frac{10 \times 4 \times 1}{2 \times 1} = \frac{10 \times 2 \times 1}{1 \times 1} = \frac{20}{1} = 20.$$

Reduce $\frac{144}{240}$ to its least terms.

$\frac{144}{240} = \frac{72}{120} = \frac{36}{60} = \frac{18}{30} = \frac{9}{15} = \frac{3}{5}$, the answer.

Or thus:

```
144 ) 240 ( 1   Therefore 48 is the greatest common measure, and
      144             48 ) 144/240 = 3/5 the answer, the same as before.
       96 ) 144 (1
             96
             48 ) 96 ( 2
                  96
```

To reduce a mixed number to its equivalent improper fraction.

Rule.—Multiply the whole number by the denominator of the fraction, and add the numerator to the product; then set that sum above the denominator for the fraction required.

Reduce $23\frac{2}{5}$ to a fraction.

```
                  Or,
 23
  5      (23 × 5) + 2   117
 115     ------------ = ---.
  2           5          5
 117
 ---
  5
```

To reduce an improper fraction to its equivalent whole or mixed number.

Rule.—Divide the numerator by the denominator, and the quotient will be the whole or mixed number sought.

Reduce $\frac{12}{3}$ to its equivalent number.

Here $\frac{12}{3}$ or $12 \div 3 = 4$.

Reduce $\frac{15}{7}$ to its equivalent number.

Here $\frac{15}{7}$ or $15 \div 7 = 2\frac{1}{7}$.

Reduce $\frac{749}{17}$ to its equivalent number.

```
Thus, 17 ) 749 ( 44 1/17
           68
           69           So that 749/17 = 44 1/17
           68
            1
```

To reduce a whole number to an equivalent fraction, having a given denominator.

Rule.—Multiply the whole number by the given denominator, then set the product over the said denominator, and it will form the fraction required.

Reduce 9 to a fraction whose denominator shall be 7.

Here $9 \times 7 = 63$, then $\frac{63}{7}$ is the answer.

For $\frac{63}{7} = 63 \div 7 = 9$, the proof.

To reduce a compound fraction to an equivalent simple one.

Rule.—Multiply all the numerators together for a numerator, and all the denominators together for the denominator, and they will form the simple fraction sought.

When part of the compound fraction is a whole or mixed number, it must first be reduced to a fraction by one of the former cases.

And, when it can be done, any two terms of the fraction may be divided by the same number, and the quotients used instead of them. Or, when there are terms that are common, they may be omitted.

Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction.

Here $\frac{1 \times 2 \times 3}{2 \times 3 \times 4} = \frac{6}{24} = \frac{1}{4}$.

Or, $\frac{1 \times 2 \times 3}{2 \times 3 \times 4} = \frac{1}{4}$, by omitting the twos and threes.

Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{10}{11}$ to a simple fraction.

Here $\frac{2 \times 3 \times 10}{3 \times 5 \times 11} = \frac{60}{165} = \frac{12}{33} = \frac{4}{11}$.

Or, $\frac{2 \times 3 \times 10}{3 \times 5 \times 11} = \frac{4}{11}$, the same as before.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

Rule.—Multiply each numerator into all the denominators except its own for the new numerators; and multiply all the denominators together for a common denominator.

It is evident, that in this and several other operations, when any of the proposed quantities are integers, or mixed numbers, or compound fractions, they must be reduced, by their proper rules, to the form of simple fractions.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

$1 \times 3 \times 4 = 12$ the new numerator for $\frac{1}{2}$.
$2 \times 2 \times 4 = 16$........................ for $\frac{2}{3}$.
$3 \times 2 \times 3 = 18$........................ for $\frac{3}{4}$.
$2 \times 3 \times 4 = 24$ the common denominator.

Therefore, the equivalent fractions are $\frac{12}{24}$, $\frac{16}{24}$, and $\frac{18}{24}$.

Or, the whole operation of multiplying may be very well performed mentally, and only set down the results and given fractions thus: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4} = \frac{12}{24}, \frac{16}{24}, \frac{18}{24} = \frac{6}{12}, \frac{8}{12}, \frac{9}{12}$, by abbreviation.

When the denominators of two given fractions have a common measure, let them be divided by it; then multiply the terms of each given fraction by the quotient arising from the other's denominator.

When the less denominator of two fractions exactly divides the greater, multiply the terms of that which hath the less denominator by the quotient.

When more than two fractions are proposed, it is sometimes convenient first to reduce two of them to a common denominator, then these and a third; and so on, till they be all reduced to their least common denominator.

To find the value of a fraction in parts of the integer.

Rule.—Multiply the integer by the numerator, and divide the product by the denominator, by Compound Multiplication and Division, if the integer be a compound quantity.

Or, if it be a single integer, multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator. Then, if any thing remains, multiply it by the parts in the next inferior denomination, and divide by the denominator as before; and so on, as far as necessary; so shall the quotients, placed in order, be the value of the fraction required.

What is the value of $\frac{3}{5}$ of a pound troy?	7 oz. 4 dwts.
What is the value of $\frac{5}{16}$ of a cwt.?	1 qr. 7 lb.
What is the value of $\frac{5}{8}$ of an acre?	2 ro. 20 po.
What is the value of $\frac{3}{10}$ of a day?	7 hrs. 12 min.

To reduce a fraction from one denomination to another.

Rule.—Consider how many of the less denomination make one of the greater; then multiply the numerator by that number, if the reduction be to a less name, or the denominator, if to a greater.

Reduce $\frac{2}{7}$ of a cwt. to the fraction of a pound.

$$\frac{2}{7} \times \frac{4}{1} \times \frac{28}{1} = \frac{32}{1}.$$

ADDITION OF FRACTIONS.

To add fractions together that have a common denominator.

Rule.—Add all the numerators together, and place the sum over the common denominator, and that will be the sum of the fractions required.

If the fractions proposed have not a common denominator, they must be reduced to one. Also, compound fractions must be reduced to simple ones, and mixed numbers to improper fractions; also, fractions of different denominations to those of the same denomination.

To add $\frac{3}{5}$ and $\frac{4}{5}$ together. Here $\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$.

To add $\frac{3}{5}$ and $\frac{5}{6}$ together. $\frac{3}{5} + \frac{5}{6} = \frac{18}{30} + \frac{25}{30} = \frac{43}{30} = 1\frac{13}{30}$.

To add $\frac{5}{8}$ and $7\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{3}{4}$ together.

$$\frac{5}{8} + 7\frac{1}{2} + \frac{1}{3} \text{ of } \frac{3}{4} = \frac{5}{8} + \frac{15}{2} + \frac{1}{4} = \frac{5}{8} + \frac{60}{8} + \frac{2}{8} = \frac{67}{8} = 8\frac{3}{8}.$$

SUBTRACTION OF FRACTIONS.

Rule.—Prepare the fractions the same as for Addition; then subtract the one numerator from the other, and set the remainder over the common denominator, for the difference of the fractions sought.

To find the difference between $\frac{5}{6}$ and $\frac{1}{6}$.

Here $\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

To find the difference between $\frac{3}{4}$ and $\frac{5}{7}$.

$\frac{3}{4} - \frac{5}{7} = \frac{21}{28} - \frac{20}{28} = \frac{1}{28}$.

MULTIPLICATION OF FRACTIONS.

Multiplication of any thing by a fraction implies the taking some part or parts of the thing; it may therefore be truly expressed by a compound fraction; which is resolved by multiplying together the numerators and the denominators.

Rule.—Reduce mixed numbers, if there be any, to equivalent fractions; then multiply all the numerators together for a numerator, and all the denominators together for a denominator, which will give the product required.

Required the product of $\frac{3}{4}$ and $\frac{2}{9}$.

Here $\frac{3}{4} \times \frac{2}{9} = \frac{6}{36} = \frac{1}{6}$.

Or, $\frac{3}{4} \times \frac{2}{9} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Required the continued product of $\frac{2}{3}$, $3\frac{1}{4}$, 5, and $\frac{3}{4}$ of $\frac{3}{5}$.

$$\text{Here } \frac{2}{3} \times \frac{13}{4} \times \frac{5}{1} \times \frac{3}{4} \times \frac{3}{5} = \frac{13 \times 3}{4 \times 2} = \frac{39}{8} = 4\tfrac{7}{8}.$$

DIVISION OF FRACTIONS.

Rule.—Prepare the fractions as before in Multiplication; then divide the numerator by the numerator, and the denominator by the denominator, if they will exactly divide; but if not, then invert the terms of the divisor, and multiply the dividend by it, as in Multiplication.

Divide $\frac{25}{9}$ by $\frac{5}{3}$.

Here $\frac{25}{9} \div \frac{5}{3} = \frac{5}{3} = 1\frac{2}{3}$, by the first method.

Divide $\frac{5}{9}$ by $\frac{2}{15}$.

Here $\frac{5}{9} \div \frac{2}{15} = \frac{5}{9} \times \frac{15}{2} = \frac{5}{3} \times \frac{5}{2} = \frac{25}{6} = 4\frac{1}{6}$, by the latter.

RULE OF THREE IN FRACTIONS.

Rule.—Make the necessary preparations as before directed; then multiply continually together the second and third terms, and the first with its terms inverted as in Division, for the answer. This is only multiplying the second and third terms together, and dividing the product by the first, as in the Rule of Three in whole numbers.

If $\frac{3}{8}$ of a yard of velvet cost $\frac{2}{5}$ of a dollar, what will $\frac{5}{16}$ of a yard cost?

$$\text{Here } \frac{3}{8} : \frac{2}{5} :: \frac{5}{16} : \frac{8}{3} \times \frac{2}{5} \times \frac{5}{16} = \tfrac{1}{3} \text{ of a dollar.}$$

DECIMAL FRACTIONS.

A Decimal Fraction is that which has for its denominator a unit (1) with as many ciphers annexed as the numerator has places; and it is usually expressed by setting down the numerator only, with a point before it on the left hand. Thus, $\frac{5}{10}$ is ·5, and $\frac{25}{100}$ is ·25, and $\frac{75}{1000}$ is ·075, and $\frac{124}{100000}$ is ·00124; where ciphers are prefixed to make up as many places as are in the numerator, when there is a deficiency of figures.

A mixed number is made up of a whole number with some decimal fraction, the one being separated from the other by a point. Thus, 3·25 is the same as $3\frac{25}{100}$, or $\frac{325}{100}$.

Ciphers on the right hand of decimals make no alteration in their value; for ·5, or ·50, or ·500, are decimals having all the same value, being each = $\frac{5}{10}$ or $\frac{1}{2}$. But if they are placed on the left hand, they decrease the value in a tenfold proportion. Thus, ·5 is $\frac{5}{10}$ or 5 tenths, but ·05 is only $\frac{5}{100}$ or 5 hundreths, and ·005 is but $\frac{5}{1000}$ or 5 thousandths.

The first place of decimals, counted from the left hand towards the right, is called the place of primes, or 10ths; the second is the place of seconds, or 100ths; the third is the place of thirds, or 1000ths; and so on. For, in decimals, as well as in whole numbers, the values of the places increase towards the left hand, and decrease towards the right, both in the same tenfold proportion; as in the following Scale or Table of Notation:

millions.	hundred thousands.	ten thousands.	thousands.	hundreds.	tens.	units.	tenth parts.	hundredth parts.	thousandths parts.	ten thousandths parts.	hundred thousandth parts.	millionth parts.
3	3	3	3	3	3	3	3	3	3	3	3	3

ADDITION OF DECIMALS.

Rule.—Set the numbers under each other according to the value of their places, like as in whole numbers; in which state the decimal separating points will stand all exactly under each other. Then, beginning at the right hand, add up all the columns of number as in integers, and point off as many places for decimals as are in the greatest number of decimal places in any of the lines that are added; or, place the point directly below all the other points.

To add together 29·0146, and 3146·5, and 2109, and 62417, and 14·16.

29·0146
3146·5
2109·
·62417
14·16

5299·29877, the sum.

The sum of 376·25 + 86·125 + 637·4725 + 6·5 + 41·02 + 358·865 = 1506.2325.

The sum of 3·5 + 47·25 + 2.0073 + 927·01 + 1·5 = 981.2673.

The sum of 276 + 54·321 + 112 + 0.65 + 12·5 + ·0463 = 455·5173.

SUBTRACTION OF DECIMALS.

Rule.—Place the numbers under each other according to the value of their places, as in the last rule. Then, beginning at the right hand, subtract as in whole numbers, and point off the decimals as in Addition.

To find the difference between 91.73 and 2.138.

$$\begin{array}{r} 91\cdot73 \\ 2\cdot138 \\ \hline 89\cdot592 \end{array}\ \text{the difference.}$$

The difference between 1·9185 and 2·73 = 0·8115.
The difference between 214·81 and 4·90142 = 209·90858.
The difference between 2714 and ·916 = 2713·084.

MULTIPLICATION OF DECIMALS.

Rule.—Place the factors, and multiply them together the same as if they were whole numbers. Then point off in the product just as many places of decimals as there are decimals in both the factors. But if there be not so many figures in the product, then supply the defect by prefixing ciphers.

$$\begin{array}{lr} \text{Multiply} & \cdot321096 \\ \text{by} & \cdot2465 \\ \hline & 1605480 \\ & 1926576 \\ & 1284384 \\ & 642192 \\ \hline & \cdot0791501640 \end{array}\ \text{the product.}$$

Multiply 79·347 by 23·15, and we have 1836·88305.
Multiply ·63478 by ·8204, and we have ·520773512.
Multiply ·385746 by ·00464, and we have ·00178986144.

CONTRACTION I.

To multiply decimals by 1 *with any number of ciphers, as* 10, *or* 100, *or* 1000, *&c.*

This is done by only removing the decimal point so many places farther to the right hand as there are ciphers in the multiplier; and subjoining ciphers if need be.

The product of 51·3 and 1000 is 51300.
The product of 2·714 and 100 is 271·4.
The product of ·916 and 1000 is 916.
The product of 21·31 and 10000 is 213100.

CONTRACTION II.

To contract the operation, so as to retain only as many decimals in the product as may be thought necessary, when the product would naturally contain several more places.

Set the units' place of the multiplier under that figure of the multiplicand whose place is the same as is to be retained for the last in the product; and dispose of the rest of the figures in the inverted or contrary order to what they are usually placed in. Then, in multiplying, reject all the figures that are more to the right than each multiplying figure; and set down the products, so that their right hand figures may fall in a column straight below each other; but observing to increase the first figure of every line with what would arise from the figures omitted, in this manner, namely, 1 from 5 to 14, 2 from 15 to 24, 3 from 25 to 34, &c.; and the sum of all the lines will be the product as required, commonly to the nearest unit in the last figure.

To multiply 27·14986 by 92·41035, so as to retain only four places of decimals in the product.

Contracted way.	Common way.
27·14986	27·14986
53014·29	92·41035
24434874	13 574930
542997	81 44958
108599	2714 986
2715	108599 44
81	542997 2
14	24434874
2508·9280	2508·9280 650510

DIVISION OF DECIMALS.

Rule.—Divide as in whole numbers; and point off in the quotient as many places for decimals, as the decimal places in the dividend exceed those in the divisor.

When the places of the quotient are not so many as the rule requires, let the defect be supplied by prefixing ciphers.

When there happens to be a remainder after the division; or when the decimal places in the divisor are more than those in the dividend; then ciphers may be annexed to the dividend, and the quotient carried on as far as required.

179) ·48624097 (·00271643	·2685) 27·00000 (100·55865
1282	15000
294	15750
1150	23250
769	17700
537	15900
000	24750

Divide 234·70525 by 64·25.	3·653.
Divide 14 by ·7854.	17·825.
Divide 2175·68 by 100.	21·7568.
Divide ·8727587 by ·162.	5·38739.

CONTRACTION I.

When the divisor is an integer, with any number of ciphers annexed; cut off those ciphers, and remove the decimal point in the dividend as many places farther to the left as there are ciphers cut off, prefixing ciphers if need be; then proceed as before.

Divide 45·5 by 2100.

21·00) ·455 (·0216, &c.
35
140
14

CONTRACTION II.

Hence, if the divisor be 1 with ciphers, as 10, or 100, or 1000, &c.; then the quotient will be found by merely moving the decimal point in the dividend so many places farther to the left as the divisor has ciphers; prefixing ciphers if need be.

So, 217·3 ÷ 100 = 2·173, and 419 ÷ 10 = 41·9.
And 5·16 ÷ 100 = ·0516, and ·21 ÷ 1000 = ·00021.

CONTRACTION III.

When there are many figures in the divisor; or only a certain number of decimals are necessary to be retained in the quotient, then take only as many figures of the divisor as will be equal to the number of figures, both integers and decimals, to be in the quotient, and find how many times they may be contained in the first figures of the dividend, as usual.

Let each remainder be a new dividend; and for every such dividend, leave out one figure more on the right hand side of the divisor; remembering to carry for the increase of the figures cut off, as in the 2d contraction in Multiplication.

When there are not so many figures in the divisor as are required to be in the quotient, begin the operation with all the figures, and continue it as usual till the number of figures in the divisor be equal to those remaining to be found in the quotient, after which begin the contraction.

Divide 2508·92806 by 92·41035, so as to have only four decimals in the quotient, in which case the quotient will contain six figures.

Contracted.	Common way.
92·4103,5) 2508·928,06 (27·1498	92·4103,5) 2508·928,06 (27·1498
660721	66072106
13849	13848610
4608	46075750
912	91116100
80	79467850
6	5539570

REDUCTION OF DECIMALS.

To reduce a common fraction to its equivalent decimal.

RULE.—Divide the numerator by the denominator as in Division of Decimals, annexing ciphers to the numerator as far as necessary; so shall the quotient be the decimal required.

Reduce $\frac{7}{24}$ to a decimal.

$24 = 4 \times 6$. Then $4\,)\,7\cdot$
$6\,)\,1\cdot750000$
$\cdot291666$, &c.

$\frac{3}{8}$ reduced to a decimal, is ·375.
$\frac{1}{25}$ reduced to a decimal, is ·04.
$\frac{3}{192}$ reduced to a decimal, is ·015625.
$\frac{275}{3842}$ reduced to a decimal, is ·071577, &c.

CASE II.

To find the value of a decimal in terms of the inferior denominations.

Rule.—Multiply the decimal by the number of parts in the next lower denomination; and cut off as many places for a remainder, to the right hand, as there are places in the given decimal.

Multiply that remainder by the parts in the next lower denomination again, cutting off for another remainder as before.

Proceed in the same manner through all the parts of the integer; then the several denominations, separated on the left hand, will make up the value required.

What is the value of ·0125 lb. troy:— 3 dwts.
What is the value of ·4694 lb. troy:— 5 oz. 12 dwt. 15·744 gr.
What is the value of ·625 cwt.:— 2 qr. 14 lb.
What is the value of ·009943 miles:— 17 yd. 1 ft. 5·98848 in.
What is the value of ·6875 yd.:— 2 qr. 3 nls.
What is the value of ·3375 ac.:— 1 rd. 14 poles.
What is the value of ·2083 hhd. of wine:— 13·1229 gal.

CASE III.

To reduce integers or decimals to equivalent decimals of higher denominations.

Rule.—Divide by the number of parts in the next higher denomination; continuing the operation to as many higher denominations as may be necessary, the same as in Reduction Ascending of whole numbers.

Reduce 1 dwt. to the decimal of a pound troy.

20 | 1 dwt.
12 | 0·05 oz.
| 0·004166, &c. lb.

Reduce 7 dr. to the decimal of a pound avoird.:— ·02734375 lb.
Reduce 2·15 lb. to the decimal of a cwt.:— ·019196 cwt.
Reduce 24 yards to the decimal of a mile:— ·013636, &c. miles.
Reduce ·056 poles to the decimal of an acre:— ·00035 ac.
Reduce 1·2 pints of wine to the decimal of a hhd.:— ·00238 hhd.
Reduce 14 minutes to the decimal of a day:— ·009722, &c. da.
Reduce ·21 pints to the decimal of a peck:— ·013125 pec.

When there are several numbers, to be reduced all to the decimal of the highest.

Set the given numbers directly under each other, for dividends, proceeding orderly from the lowest denomination to the highest.

Opposite to each dividend, on the left hand, set such a number for a divisor as will bring it to the next higher name; drawing a perpendicular line between all the divisors and dividends.

Begin at the uppermost, and perform all the divisions; only observing to set the quotient of each division, as decimal parts, on the right hand of the dividend next below it; so shall the last quotient be the decimal required.

Reduce 5 oz. 12 dwts. 16 gr. to lbs.:— ·46944, &c. lb.

RULE OF THREE IN DECIMALS.

Rule.—Prepare the terms by reducing the vulgar fractions to decimals, any compound numbers either to decimals of the higher denominations, or to integers of the lower, also the first and third terms to the same name: then multiply and divide as in whole numbers.

Any of the convenient examples in the Rule of Three or Rule of Five in Integers, or Common Fractions, may be taken as proper examples to the same rules in Decimals.—The following example, which is the first in Common Fractions, is wrought here to show the method.

If $\frac{3}{8}$ of a yard of velvet cost $\frac{2}{5}$ of a dollar, what will $\frac{5}{16}$ yd. cost?

yd. \$ yd. \$

$\frac{3}{8} = \cdot375$ ·375 : ·4 : : ·3125 : ·333, &c.

·4

$\frac{2}{5} = \cdot4$ ·375) ·12500 (·333333, $33\frac{1}{3}$ cts.

1250

125

$\frac{5}{16} = \cdot3125.$

DUODECIMALS.

Duodecimals, or Cross Multiplication, is a rule made use of by workmen and artificers, in computing the contents of their works.

Dimensions are usually taken in feet, inches, and quarters; any parts smaller than these being neglected as of no consequence. And the same in multiplying them together, or casting up the contents.

Rule.—Set down the two dimensions, to be multiplied together, one under the other, so that feet stand under feet, inches under inches, &c.

Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and set the result of each straight under its corresponding term, observing to carry 1 for every 12, from the inches to the feet.

In like manner, multiply all the multiplicand by the inches and parts of the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand; omitting, however, what is below parts of inches, only carrying to these the proper number of units from the lowest denomination.

Or, instead of multiplying by the inches, take such parts of the multiplicand as these are of a foot.

Then add the two lines together, after the manner of Compound Addition, carrying 1 to the feet for 12 inches, when these come to so many.

Multiply 4 f. 7 inc.
by 6 4
27 6
1 6⅓
29 0⅓

Multiply 14 f. 9 inc.
by 4 6
59 0
7 4½
66 4½

INVOLUTION.

INVOLUTION is the raising of Powers from any given number, as a root.

A Power is a quantity produced by multiplying any given number, called the Root, a certain number of times continually by itself. Thus,

$2 = 2$ is the root, or first power of 2.
$2 \times 2 = 4$ is the 2d power, or square of 2.
$2 \times 2 \times 2 = 8$ is the 3d power, or cube of 2.
$2 \times 2 \times 2 \times 2 = 16$ is the 4th power of 2, &c.

And in this manner may be calculated the following Table of the first nine powers of the first nine numbers.

TABLE OF THE FIRST NINE POWERS OF NUMBERS.

1st	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

The Index or Exponent of a Power is the number denoting the height or degree of that power; and it is 1 more than the number of multiplications used in producing the same. So 1 is the index or exponent of the 1st power or root, 2 of the 2d power or square, 3 of the 3d power or cube, 4 of the 4th power, and so on.

Powers, that are to be raised, are usually denoted by placing the index above the root or first power.

So $2^2 = 4$, is the 2d power of 2.
$2^3 = 8$, is the 3d power of 2.
$2^4 = 16$, is the 4th power of 2.
540^4, is the 4th power of $540 = 85030560000$.

When two or more powers are multiplied together, their product will be that power whose index is the sum of the exponents of the factors or powers multiplied. Or, the multiplication of the powers answers to the addition of the indices. Thus, in the following powers of 2.

	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	10th.
	2	4	8	16	32	64	128	256	512	1024
or,	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}

Here, $4 \times 4 = 16$, and $2 + 2 = 4$ its index;
and $8 \times 16 = 128$, and $3 + 4 = 7$ its index;
also $16 \times 64 = 1024$, and $4 + 6 = 10$ its index.

The 2d power of 45 is 2025.
The square of 4·16 is 17·3056.
The 3d power of 3·5 is 42·875.
The 5th power of ·029 is ·000000020511149.
The square of $\frac{2}{3}$ is $\frac{4}{9}$.
The 3d power of $\frac{5}{9}$ is $\frac{125}{729}$.
The 4th power of $\frac{3}{4}$ is $\frac{81}{256}$.

EVOLUTION.

EVOLUTION, or the reverse of Involution, is the extracting or finding the roots of any given powers.

The root of any number, or power, is such a number as, being multiplied into itself a certain number of times, will produce that power. Thus, 2 is the square root or 2d root of 4, because $2^2 = 2 \times 2 = 4$; and 3 is the cube root or 3d root of 27, because $3^3 = 3 \times 3 \times 3 = 27$.

Any power of a given number or root may be found exactly, namely, by multiplying the number continually into itself. But there are many numbers of which a proposed root can never be exactly found. Yet, by means of decimals we may approximate or approach towards the root to any degree of exactness.

These roots, which only approximate, are called Surd roots; but those which can be found quite exact, are called Rational roots. Thus, the square root of 3 is a surd root; but the square root of 4 is a rational root, being equal to 2: also, the cube root of 8 is rational, being equal to 2; but the cube root of 9 is surd, or irrational.

Roots are sometimes denoted by writing the character $\surd$ before the power, with the index of the root against it. Thus, the third root of 20 is expressed by $\sqrt[3]{20}$; and the square root or 2d root of it is $\sqrt{20}$, the index 2 being always omitted when the square root is designed.

When the power is expressed by several numbers, with the sign + or − between them, a line is drawn from the top of the sign over all the parts of it; thus, the third root of 45 − 12 is $\sqrt[3]{45 - 12}$, or thus, $\sqrt[3]{}(45 - 12)$, enclosing the numbers in parentheses.

But all roots are now often designed like powers, with fractional indices: thus, the square root of 8 is $8^{\frac{1}{2}}$, the cube root of 25 is $25^{\frac{1}{3}}$, and the 4th root of 45 − 18 is $\overline{45-18}|^{\frac{1}{4}}$, or, $(45-18)^{\frac{1}{4}}$.

TO EXTRACT THE SQUARE ROOT.

RULE.—Divide the given number into periods of two figures each, by setting a point over the place of units, another over the place of hundreds, and so on, over every second figure, both to the left hand in integers, and to the right in decimals.

Find the greatest square in the first period on the left hand, and set its root on the right hand of the given number, after the manner of a quotient figure in Division.

Subtract the square thus found from the said period, and to the remainder annex the two figures of the next following period for a dividend.

Double the root above mentioned for a divisor, and find how often it is contained in the said dividend, exclusive of its right-hand figure; and set that quotient figure both in the quotient and divisor.

Multiply the whole augmented divisor by this last quotient figure, and subtract the product from the said dividend, bringing down to the next period of the given number, for a new dividend.

Repeat the same process over again, namely, find another new divisor, by doubling all the figures now found in the root; from which, and the last dividend, find the next figure of the root as before, and so on through all the periods to the last.

The best way of doubling the root to form the new divisor is by adding the last figure always to the last divisor, as appears in the following examples. Also, after the figures belonging to the given number are all exhausted, the operation may be continued into decimals at pleasure, by adding any number of periods of ciphers, two in each period.

To find the square root of 29506624.

$$
\begin{array}{r|l}
\multicolumn{2}{l}{2\dot{9}5\dot{0}6\dot{6}2\dot{4}\,(\,5432 \text{ the root.}} \\
\multicolumn{2}{l}{25} \\ \hline
104 & 450 \\
4 & 416 \\ \hline
1083 & 3466 \\
3 & 3249 \\ \hline
10862 & 21724 \\
2 & 21724 \\ \hline
\end{array}
$$

When the root is to be extracted to many places of figures, the work may be considerably shortened, thus:

Having proceeded in the extraction after the common method till there be found half the required number of figures in the root, or one figure more; then, for the rest, divide the last remainder by its corresponding divisor, after the manner of the third contraction in Division of Decimals; thus,

To find the root of 2 to nine places of figures.

```
            2(1·4142
            1
       ---------
       24 | 100
        4 |  96
      281 | 400
        1 | 281
      ------------
     2824 | 11900
        4 | 11296
     -------------
    28282 | 60400
        2 | 56564
    -----------------
    28284)  3836(1356
            1008
             160
              19
               2
```

$1{\cdot}41421\overline{356}$ the root required.

The square root of ·000729 is ·027.
The square root of 3 is 1·732050.
The square root of 5 is 2·236068.
The square root of 6 is 2·449489.

RULES FOR THE SQUARE ROOTS OF COMMON FRACTIONS AND MIXED NUMBERS.

First, prepare all common fractions by reducing them to their least terms, both for this and all other roots. Then,

1. Take the root of the numerator and of the denominator for the respective terms of the root required. And this is the best way if the denominator be a complete power; but if it be not, then,

2. Multiply the numerator and denominator together; take the root of the product: this root being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional root required.

$$\text{That is, } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}.$$

And this rule will serve whether the root be finite or infinite.

3. Or reduce the common fraction to a decimal, and extract its root.

4. Mixed numbers may be either reduced to improper fractions, and extracted by the first or second rule; or the common fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

The root of $\frac{25}{36}$ is $\frac{5}{6}$.
The root of $\frac{9}{49}$ is $\frac{3}{7}$.
The root of $\frac{9}{12}$ is 0·866025.
The root of $\frac{5}{12}$ is 0·645497.
The root of $17\frac{3}{8}$ is 4·168333.

By means of the square root, also, may readily be found the 4th root, or the 8th root, or the 16th root, &c.; that is, the root of any power whose index is some power of the number 2; namely, by extracting so often the square root as is denoted by that power of 2; that is, two extractions for the 4th root, three for the 8th root, and so on.

So, to find the 4th root of the number 21035·8, extract the square root twice as follows:

```
       .  .  .  .  .            .  .  .  .  .
       21035·8000              (145·037237 ( 12·0431407, the 4th root.
       1                         1
  24 | 110                  22 |  45
   4 |  96                   2 |  44
  285 | 1435              2404 | 10372
    5 | 1425                 4 |  9616
  29003 | 108000         24083 |  75637
      6 |  87009             6 |  72249
           20991 ( 7237            3388 ( 1407
             687                    980
             107                     17
              20
```

TO EXTRACT THE CUBE ROOT.

1. Divide the page into three columns (I), (II), (III), in order, from left to right, so that the breadth of the columns may increase in the same order. In column (III) write the given number, and divide it into periods of three figures each, by putting a point over the place of units, and also over every third figure, from thence to the left in whole numbers, and to the right in decimals.

2. Find the nearest less cube number to the first or left-hand period; set its root in column (III), separating it from the right of the given number by a curve line, and also in column (I); then multiply the number in (I) by the root figure, thus giving the square of the first root figure, and write the result in (II); multiply the number in (II) by the root figure, thus giving the cube of the first root figure, and write the result below the first or left-hand period in (III); subtract it therefrom, and annex the next period to the remainder for a dividend.

3. In (I) write the root figure below the former, and multiply the sum of these by the root figure; place the product in (II), and add the two numbers together for a trial divisor. Again, write the root figure in (I), and add it to the former sum.

4. With the number in (II) as a trial divisor of the dividend, omitting the two figures to the right of it, find the next figure of the root, and annex it to the former, and also to the number in (I). Multiply the number now in (I) by the new figure of the root, and write the product as it arises in (II), but extended two places of figures more to the right, and the sum of these two numbers will be the corrected divisor; then multiply the corrected divisor by the

last root figure, placing the product as it arises below the dividend; subtract it therefrom, annex another period, and proceed precisely as described in (3), for correcting the columns (I) and (II). Then with the new trial divisor in (II), and the new dividend in (III), proceed as before.

When the trial divisor is not contained in the dividend, after two figures are omitted on the right, the next root figure is 0, and therefore one cipher must be annexed to the number in (I); two ciphers to the number in (II); and another period to the dividend in (III).

When the root is interminable, we may contract the work very considerably, after obtaining a few figures in the decimal part of the root, if we omit to annex another period to the remainder in (III); cut off one figure from the right of (II), and two figures from (I), which will evidently have the effect of cutting off *three* figures from each column; and then work with the numbers on the left, as in contracted multiplication and division of decimals.

Find the cube root of 21035·8 to ten places of decimals.

(I)	(II)	(III)
2	4	21035·8 (27·60491055944
2	8	8
4	12..	13035
2	469	11683
67	1669	1352800
7	518	1341576
74	2187..	11224......
7	4896	9142444864
816	223596	2081555136
6	4932	2057415281
822	228528....	24139855
6	331216 ·	22860923
82804	2285611216	1278932
4	331232	1143046
82808	228594244|8	135886
4	7453|1	114305
|·8|28|12	228601697|9	21581
	7453|1	20575
	22860915|1	1006
	8|3	914
	22860923|4	92
	8|3	91
	2|2|8|6|0|9|3|2	1

Required the cube roots of the following numbers:—

48228544, 46656, and 15069223.	364, 36, and 247.
64481·201, and 28991029248.	40·1, and 3072.
12821119155125, and ·000076765625.	23405, and ·0425.
$\frac{13824}{42875}$, and 16.	$\frac{24}{35}$, and 2·519842.
91$\frac{1}{8}$, and 7$\frac{6}{7}$.	4·5, and 1·98802366.

TO EXTRACT ANY ROOT WHATEVER.

Let N be the given power or number, n the index of the power, A the assumed power, r its root, R the required root of N.

Then, as the sum of $n + 1$ times A and $n - 1$ times N, is to the sum of $n + 1$ times N and $n - 1$ times A, so is the assumed root r, to the required root R.

Or, as half the said sum of $n + 1$ times A and $n - 1$ times N, is to the difference between the given and assumed powers, so is the assumed root r, to the difference between the true and assumed roots; which difference, added or subtracted, as the case requires, gives the true root nearly.

That is, $(n+1).A + (n-1).N : (n+1).N + (n-1) \cdot A :: r : R.$

Or, $(n+1).\frac{1}{2}A + (n-1).\frac{1}{2}N : A \backsim N :: r : R \backsim r.$

And the operation may be repeated as often as we please, by using always the last found root for the assumed root, and its nth power for the assumed power A.

To extract the 5th root of 21035·8.

Here it appears that the 5th root is between 7·3 and 7·4. Taking 7·3, its 5th power is 20730·71593. Hence then we have,

$N = 21035{\cdot}8;\ r = 7{\cdot}3;\ n = 5;\ \frac{1}{2}.(n+1) = 3;\ \frac{1}{2}.(n-1) = 2.$

A = 20730·716
N − A = 305·084

A = 20730·716 N = 21035·8
3 2
3 A = 62192.148 42071·6
2 N = 42071·6

As 104263·7 : 305·084 : : 7·3 : ·0213605
7·3
915252
2135588
104263·7) 2227·1132 (·0213605, the difference.
14184 7·3 = r add
3758
630 7·321360 = R, the root, true to
5 the last figure.

The 6th root of 21035.8	is 5·254037.
The 6th root of 2	is 1·122462.
The 7th root of 21035·8	is 4·145392.
The 7th root of 2	is 1·104089.
The 9th root of 2	is 1·080059.

OF RATIOS, PROPORTIONS, AND PROGRESSIONS.

Numbers are compared to each other in two different ways: the one comparison considers the difference of the two numbers, and is named Arithmetical Relation, and the difference sometimes Arithmetical Ratio: the other considers their quotient, and is called

Geometrical Relation, and the quotient the Geometrical Ratio. So, of these two numbers 6 and 3, the difference or arithmetical ratio is $6 - 3$ or 3; but the geometrical ratio is $\frac{6}{3}$ or 2.

There must be two numbers to form a comparison: the number which is compared, being placed first, is called the Antecedent; and that to which it is compared the Consequent. So, in the two numbers above, 6 is the antecedent, and 3 is the consequent.

If two or more couplets of numbers have equal ratios, or equal differences, the equality is named Proportion, and the terms of the ratios Proportionals. So, the two couplets, 4, 2 and 8, 6 are arithmetical proportionals, because $4 - 2 = 8 - 6 = 2$; and the two couplets 4, 2 and 6, 3 are geometrical proportionals, because $\frac{4}{2} = \frac{6}{3} = 2$, the same ratio.

To denote numbers as being geometrically proportional, a colon is set between the terms of each couplet to denote their ratio; and a double colon, or else a mark of equality between the couplets or ratios. So, the four proportionals, 4, 2, 6, 3, are set thus, $4 : 2 :: 6 : 3$, which means that 4 is to 2 as 6 is to 3; or thus, $4 : 2 = 6 : 3$; or thus, $\frac{4}{2} = \frac{6}{3}$, both which mean that the ratio of 4 to 2 is equal to the ratio of 6 to 3.

Proportion is distinguished into Continued and Discontinued. When the difference or ratio of the consequent of one couplet and the antecedent of the next couplet is not the same as the common difference or ratio of the couplets, the proportion is discontinued. So, 4, 2, 8, 6 are in discontinued arithmetical proportion, because $4 - 2 = 8 - 6 = 2$, whereas, $2 - 8 = -6$; and 4, 2, 6, 3 are in discontinued geometrical proportion, because $\frac{4}{2} = \frac{6}{3} = 2$, but $\frac{2}{6} = \frac{1}{3}$, which is not the same.

But when the difference or ratio of every two succeeding terms is the same quantity, the proportion is said to be continued, and the numbers themselves a series of continued proportionals, or a progression. So, 2, 4, 6, 8 form an arithmetical progression, because $4 - 2 = 6 - 4 = 8 - 6 = 2$, all the same common difference; and 2, 4, 8, 16, a geometrical progression, because $\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$, all the same ratio.

When the following terms of a Progression exceed each other, it is called an Ascending Progression or Series; but if the terms decrease, it is a Descending one.

So, 0, 1, 2, 3, 4, &c., is an ascending arithmetical progression,
but 9, 7, 5, 3, 1, &c., is a descending arithmetical progression:
Also, 1, 2, 4, 8, 16, &c., is an ascending geometrical progression,
and 16, 8, 4, 2, 1, &c., is a descending geometrical progression.

ARITHMETICAL PROPORTION AND PROGRESSION.

The first and last terms of a Progression are called the Extremes; and the other terms lying between them, the Means.

The most useful part of arithmetical proportions is contained in the following theorems:

THEOREM 1.—If four quantities be in arithmetical proportion, the sum of the two extremes will be equal to the sum of the two means.

Thus, of the four 2, 4, 6, 8, here $2 + 8 = 4 + 6 = 10$.

Theorem 2.—In any continued arithmetical progression, the sum of the two extremes is equal to the sum of any two means that are equally distant from them, or equal to double the middle term when there is an uneven number of terms.

Thus, in the terms 1, 3, 5, it is $1 + 5 = 3 + 3 = 6$.

And in the series 2, 4, 6, 8, 10, 12, 14, it is $2 + 14 = 4 + 12 = 6 + 10 = 8 + 8 = 16$.

Theorem 3.—The difference between the extreme terms of an arithmetical progression, is equal to the common difference of the series multiplied by one less than the number of the terms.

So, of the ten terms, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, the common difference is 2, and one less than the number of terms 9; then the difference of the extremes is $20 - 2 = 18$, and $2 \times 9 = 18$ also.

Consequently, the greatest term is equal to the least term added to the product of the common difference multiplied by 1 less than the number of terms.

Theorem 4.—The sum of all the terms of any arithmetical progression is equal to the sum of the two extremes multiplied by the number of terms, and divided by 2; or the sum of the two extremes multiplied by the number of the terms gives double the sum of all the terms in the series.

This is made evident by setting the terms of the series in an inverted order under the same series in a direct order, and adding the corresponding terms together in that order. Thus,

$$\begin{array}{lrrrrrrrr} \text{in the series,} & 1, & 3, & 5, & 7, & 9, & 11, & 13, & 15; \\ \text{...... inverted,} & 15, & 13, & 11, & 9, & 7, & 5, & 3, & 1; \\ \cline{2-9} \text{the sums are,} & 16 + & 16 + & 16 + & 16 + & 16 + & 16 + & 16 + & 16, \end{array}$$

which must be double the sum of the single series, and is equal to the sum of the extremes repeated so often as are the number of the terms.

From these theorems may readily be found any one of these five parts; the two extremes, the number of terms, the common difference, and the sum of all the terms, when any three of them are given, as in the following Problems:

PROBLEM I.

Given the extremes and the number of terms, to find the sum of all the terms.

Rule.—Add the extremes together, multiply the sum by the number of terms, and divide by 2.

The extremes being 3 and 19, and the number of terms 9; required the sum of the terms?

$$\begin{array}{r} 19 \\ \underline{3} \\ 22 \\ \underline{9} \\ 2\,)\overline{198} \\ \underline{99} \end{array} = \text{the sum.} \qquad \text{Or, } \frac{19 + 3}{2} \times 9 = \frac{22}{2} \times 9 = 11 \times 9 = 99.$$

The strokes a clock strikes in one whole revolution of the index, or in 12 hours, is 78.

PROBLEM II.

Given the extremes, and the number of terms; to find the common difference.

Rule.—Subtract the less extreme from the greater, and divide the remainder by 1 less than the number of terms, for the common difference.

The extremes being 3 and 19, and the number of terms 9; required the common difference?

$$\begin{array}{r} 19 \\ \underline{3} \\ 8)16 \\ \underline{2} \end{array} \qquad \text{Or, } \frac{19-3}{9-1}=\frac{16}{8}=2.$$

If the extremes be 10 and 70, and the number of terms 21; what is the common difference, and the sum of the series?

The com. diff. is 3, and the sum is 840.

PROBLEM III.

Given one of the extremes, the common difference, and the number of terms; to find the other extreme, and the sum of the series.

Rule.—Multiply the common difference by 1 less than the number of terms, and the product will be the difference of the extremes: therefore add the product to the less extreme, to give the greater; or subtract it from the greater, to give the less.

Given the least term 3, the common difference 2, of an arithmetical series of 9 terms; to find the greatest term, and the sum of the series?

$$\begin{array}{rl} 2 & \\ \underline{8} & \\ 16 & \\ \underline{3} & \\ 19 & \text{the greatest term.} \\ \underline{3} & \text{the least.} \\ 22 & \text{sum.} \\ \underline{9} & \text{number of terms.} \\ 2)198 & \\ \underline{99} & \text{the sum of the series.} \end{array}$$

If the greatest term be 70, the common difference 3, and the number of terms 21; what is the least term and the sum of the series? The least term is 10, and the sum is 840.

PROBLEM IV.

To find an arithmetical mean proportional between two given terms.

Rule.—Add the two given extremes or terms together, and take half their sum for the arithmetical mean required. Or, subtract the less extreme from the greater, and half the remainder will be the common difference; which, being added to the less extreme, or subtracted from the greater, will give the mean required.

To find an arithmetical mean between the two numbers 4 and 14.

$$\begin{array}{r} \text{Here, } 14 \\ 4 \\ 2\,)\overline{18} \\ \underline{\overline{9}} \end{array} \qquad \begin{array}{rl} \text{Or, } 14 & \\ 4 & \\ 2\,)\overline{10} & \\ \overline{5} & \text{the com. dif.} \\ 4 & \text{the less extreme.} \\ \underline{\overline{9}} & \end{array} \qquad \begin{array}{r} \text{Or, } 14 \\ 5 \\ \underline{\overline{9}} \end{array}$$

So that 9 is the mean required by both methods.

PROBLEM V.

To find two arithmetical means between two given extremes.

Rule.—Subtract the less extreme from the greater, and divide the difference by 3, so will the quotient be the common difference; which, being continually added to the less extreme, or taken from the greater, gives the means.

To find two arithmetical means between 2 and 8.

$$\begin{array}{r} \text{Here } 8 \\ 2 \\ 3\,)\overline{6} \\ \text{com. dif. } \underline{\overline{2}} \end{array} \qquad \begin{array}{l} \text{Then } 2 + 2 = 4 \text{ the one mean,} \\ \text{and } 4 + 2 = 6 \text{ the other mean.} \end{array}$$

PROBLEM VI.

To find any number of arithmetical means between two given terms or extremes.

Rule.—Subtract the less extreme from the greater, and divide the difference by 1 more than the number of means required to be found, which will give the common difference; then this being added continually to the least term, or subtracted from the greatest, will give the mean terms required.

To find five arithmetical means between 2 and 14.

$$\begin{array}{r} \text{Here } 14 \\ 2 \\ 6\,)\overline{12} \\ \text{com. dif. } \underline{\overline{2}} \end{array} \qquad \begin{array}{l} \text{Then, by adding this com. dif. continually,} \\ \text{the means are found, 4, 6, 8, 10, 12.} \end{array}$$

GEOMETRICAL PROPORTION AND PROGRESSION.

The most useful part of Geometrical Proportion is contained in the following theorems:

Theorem 1.—If four quantities be in geometrical proportion, the product of the two extremes will be equal to the product of the two means.

Thus, in the four 2, 4, 3, 6 it is $2 \times 6 = 3 \times 4 = 12$.

And hence, if the product of the two means be divided by one of the extremes, the quotient will give the other extreme. So, of the above numbers, the product of the means 12 ÷ 2 = 6 the one extreme, and 12 ÷ 6 = 2 the other extreme; and this is the foundation and reason of the practice in the Rule of Three.

THEOREM 2.—In any continued geometrical progression, the product of the two extremes is equal to the product of any two means that are equally distant from them, or equal to the square of the middle term when there is an uneven number of terms.

Thus, in the terms 2, 4, 8, it is $2 \times 8 = 4 \times 4 = 16$.
And in the series 2, 4, 8, 16, 32, 64, 128,
it is $2 \times 128 = 4 \times 64 = 8 \times 32 = 16 \times 16 = 256$.

THEOREM 3.—The quotient of the extreme terms of a geometrical progression is equal to the common ratio of the series raised to the power denoted by one less than the number of the terms.

So, of the ten terms 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, the common ratio is 2, one less than the number of terms 9; then the quotient of the extremes is $\frac{1024}{2} = 512$, and $2^9 = 512$ also.

Consequently, the greatest term is equal to the least term multiplied by the said power of the ratio whose index is one less than the number of terms.

THEOREM 4.—The sum of all the terms of any geometrical progression is found by adding the greatest term to the difference of the extremes divided by one less than the ratio.

So, the sum 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, (whose ratio is 2,) is $1024 + \frac{1024 - 2}{2 - 1} = 1024 + 1022 = 2046$.

The foregoing, and several other properties of geometrical proportion, are demonstrated more at large in Byrne's Doctrine of Proportion. A few examples may here be added to the theorems just delivered, with some problems concerning mean proportionals.

The least of ten terms in geometrical progression being 1, and the ratio 2, what is the greatest term, and the sum of all the terms?

The greatest term is 512, and the sum 1023.

PROBLEM I.

To find one geometrical mean proportional between any two numbers.

RULE.—Multiply the two numbers together, and extract the square root of the product, which will give the mean proportional sought.

Or, divide the greater term by the less, and extract the square root of the quotient, which will give the common ratio of the three terms: then multiply the less term by the ratio, or divide the greater term by it, either of these will give the middle term required.

To find a geometrical mean between the two numbers 3 and 12.

First way.	*Second way.*
12	3)12(4, its root, is 2, the ratio.
3	
36(6 the mean.	Then, 3 × 2 = 6 the mean.
36	Or, 12 ÷ 2 = 6 also.

PROBLEM II.

To find two geometrical mean proportionals between any two numbers.

RULE.—Divide the greater number by the less, and extract the cube root of the quotient, which will give the common ratio of the terms. Then multiply the least given term by the ratio for the first mean, and this mean again by the ratio for the second mean; or, divide the greater of the two given terms by the ratio for the greater mean, and divide this again by the ratio for the less mean.

To find two geometrical mean proportionals between 3 and 24.

Here, 3) 24 (8, its cube root, 2 is the ratio.
Then, 3 × 2 = 6, and 6 × 2 = 12, the two means.
Or, 24 ÷ 2 = 12, and 12 ÷ 2 = 6, the same.
That is, the two means between 3 and 24, are 6 and 12.

PROBLEM III.

To find any number of geometrical mean proportionals between two numbers.

RULE.—Divide the greater number by the less, and extract such root of the quotient whose index is one more than the number of means required, that is, the 2d root for 1 mean, the 3d root for 2 means, the 4th root for 3 means, and so on; and that root will be the common ratio of all the terms. Then with the ratio multiply continually from the first term, or divide continually from the last or greatest term.

To find four geometrical mean proportionals between 3 and 96.

Here, 3) 96 (32, the 5th root of which is 2, the ratio.
Then, 3 × 2 = 6, and 6 × 2 = 12, and 12 × 2 = 24, and 24 × 2 = 48.
Or, 96 ÷ 2 = 48, and 48 ÷ 2 = 24, and 24 ÷ 2 = 12, and 12 ÷ 2 = 6.
That is, 6, 12, 24, 48 are the four means between 3 and 96.

OF MUSICAL PROPORTION.

THERE is also a third kind of proportion, called Musical, which, being but of little or no common use, a very short account of it may here suffice.

Musical proportion is when, of three numbers, the first has the same proportion to the third, as the difference between the first and second has to the difference between the second and third.

As in these three, 6, 8, 12;
where, 6 : 12 : : 8 − 6 : 12 − 8,
that is, 6 : 12 : : 2 : 4.

When four numbers are in Musical Proportion; then the first has the same proportion to the fourth, as the difference between the first and second has to the difference between the third and fourth.

As in these, 6, 8, 12, 18;
where, 6 : 18 : : 8 − 6 : 18 − 12,
that is, 6 : 18 : : 2 : 6.

When numbers are in Musical Progression, their reciprocals are in Arithmetical Progression; and the converse, that is, when numbers are in Arithmetical Progression, their reciprocals are in Musical Progression.

So, in these Musicals 6, 8, 12, their reciprocals $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, are in arithmetical progression; for $\frac{1}{6} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$; and $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$; that is, the sum of the extremes is equal to double the mean, which is the property of arithmeticals.

FELLOWSHIP, OR PARTNERSHIP.

Fellowship is a rule by which any sum or quantity may be divided into any number of parts, which shall be in any given proportion to one another.

By this rule are adjusted the gains, or losses, or charges of partners in company; or the effects of bankrupts, or legacies in case of a deficiency of assets or effects; or the shares of prizes, or the numbers of men to form certain detachments; or the division of waste lands among a number of proprietors.

Fellowship is either Single or Double. It is Single, when the shares or portions are to be proportional each to one single given number only; as when the stocks of partners are all employed for the same time: and Double, when each portion is to be proportional to two or more numbers; as when the stocks of partners are employed for different times.

SINGLE FELLOWSHIP.

General Rule.—Add together the numbers that denote the proportion of the shares. Then,

As the sum of the said proportional numbers
Is to the whole sum to be parted or divided,
So is each several proportional number
To the corresponding share or part.

Or, As the whole stock is to the whole gain or loss,
So is each man's particular stock to his particular share of the gain or loss.

To prove the work.—Add all the shares or parts together, and the sum will be equal to the whole number to be shared, when the work is right.

To divide the number 240 into three such parts, as shall be in proportion to each other as the three numbers, 1, 2, and 3.

Here $1 + 2 + 3 = 6$ the sum of the proportional numbers.

Then, as 6 : 240 : : 1 : 40 the 1st part,
and, as 6 : 240 : : 2 : 80 the 2d part,
also as 6 : 240 : : 3 : 120 the 3d part.
Sum of all 240, the proof.

Three persons, A, B, C, freighted a ship with 340 tuns of wine; of which, A loaded 110 tuns, B 97, and C the rest: in a storm, the seamen were obliged to throw overboard 85 tuns; how much must each person sustain of the loss?

Here, 110 + 97 = 207 tuns, loaded by A and B;
theref., 340 − 207 = 133 tuns, loaded by C.
hence, as 340 : 85 :: 110
or, as 4 : 1 :: 110 : 27½ tuns = A's loss;
and, as 4 : 1 :: 97 : 24¼ tuns = B's loss;
also, as 4 : 1 :: 133 : 33¼ tuns = C's loss.
Sum 85 tuns, the proof.

DOUBLE FELLOWSHIP.

Double Fellowship, as has been said, is concerned in cases in which the stocks of partners are employed or continued for different times.

Rule.—Multiply each person's stock by the time of its continuance; then divide the quantity, as in Single Fellowship, into shares in proportion to these products, by saying:

As the total sum of all the said products
Is to the whole gain or loss, or quantity to be parted,
So is each particular product
To the corresponding share of the gain or loss.

SIMPLE INTEREST.

Interest is the premium or sum allowed for the loan, or forbearance of money.

The money lent, or forborne, is called the Principal.

The sum of the principal and its interest, added together, is called the Amount.

Interest is allowed at so much per cent. per annum, which premium per cent. per annum, or interest of a $100 for a year, is called the Rate of Interest. So,

When interest is at 3 per cent. the rate is 3;
.... 4 per cent. 4;
.... 5 per cent. 5;
.... 6 per cent. .... 6.

Interest is of two sorts: Simple and Compound.

Simple Interest is that which is allowed for the principal lent or forborne only, for the whole time of forbearance.

As the interest of any sum, for any time, is directly proportional to the principal sum, and also to the time of continuance; hence arises the following general rule of calculation.

General Rule.—As $100 is to the rate of interest, so is any given principal to its interest for one year. And again,

As one year is to any given time, so is the interest for a year just found to the interest of the given sum for that time.

Otherwise.—Take the interest of one dollar for a year, which, multiply by the given principal, and this product again by the time of loan or forbearance, in years and parts, for the interest of the proposed sum for that time.

When there are certain parts or years in the time, as quarters, or months, or days, they may be worked for either by taking the aliquot, or like parts of the interest of a year, or by the Rule of Three, in the usual way. Also, to divide by 100, is done by only pointing off two figures for decimals.

COMPOUND INTEREST.

Compound Interest, called also Interest upon Interest, is that which arises from the principal and interest, taken together, as it becomes due at the end of each stated time of payment.

Rules.—1. Find the amount of the given principal, for the time of the first payment, by Simple Interest. Then consider this amount as a new principal for the second payment, whose amount calculate as before; and so on, through all the payments to the last, always accounting the last amount as a new principal for the next payment. The reason of which is evident from the definition of Compound Interest. Or else,

2. Find the amount of one dollar for the time of the first payment, and raise or involve it to the power whose index is denoted by the number of payments. Then that power multiplied by the given principal will produce the whole amount. From which the said principal being subtracted, leaves the Compound Interest of the same; as is evident from the first rule.

POSITION.

Position is a method of performing certain questions which cannot be resolved by the common direct rules. It is sometimes called False Position, or False Supposition, because it makes a supposition of false numbers to work with, the same as if they were the true ones, and by their means discovers the true numbers sought. It is sometimes also called Trial and Error, because it proceeds by *trials* of false numbers, and thence finds out the true ones by a comparison of the *errors*.

Position is either Single or Double.

SINGLE POSITION.

Single Position is that by which a question is resolved by means of one supposition only.

Questions which have their results proportional to their suppositions belong to Single Position; such as those which require the multiplication or division of the number sought by any proposed number; or, when it is to be increased or diminished by itself, or any parts of itself, a certain proposed number of times.

Rule.—Take or assume any number for that required, and perform the same operations with it as are described or performed in the question.

Then say, as the result of the said operation is to the position or number assumed, so is the result in the question to the number sought.

A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining \$60, what had he at first?

Suppose he had at first \$120		Proof.	
Now $\frac{1}{3}$ of 120 is	40	$\frac{1}{3}$ of 144 is	48
$\frac{1}{4}$ of it is	30	$\frac{1}{4}$ of 144 is	36
their sum is	70	their sum	84
which taken from	120	taken from	144
leaves	50	leaves	60 as per question.

Then, 50 : 120 :: 60 : 144.

What number is that, which multiplied by 7, and the product divided by 6, the quotient may be 14? 12.

PERMUTATIONS AND COMBINATIONS.

THE *Permutations* of any number of quantities signify the changes which these quantities may undergo with respect to their order.

Thus, if we take the quantities *a*, *b*, *c*; then, *a b c*, *a c b*, *b a c*, *b c a*, *c a b*, *c b a*, are the permutations of these three quantities taken *all together*; *a b*, *a c*, *b a*, *b c*, *c a*, *c b*, are the permutations of these quantities taken *two* and *two*; *a*, *b*, *c*, are the permutation of these quantities taken singly, or *one* and *one*, &c.

The number of the permutations of the eight letters, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, is 40320; becomes,

$$1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 = 40320.$$

DOUBLE POSITION.

DOUBLE POSITION is the method of resolving certain questions by means of two suppositions of false numbers.

To the Double Rule of Position belong such questions as have their results not proportional to their positions: such are those, in which the numbers sought, or their parts, or their multiples, are increased or diminished by some given absolute number, which is no known part of the number sought.

Take or assume any two convenient numbers, and proceed with each of them separately, according to the conditions of the question, as in Single Position; and find how much each result is different from the result mentioned in the question, noting also whether the results are too great or too little.

Then multiply each of the said errors by the contrary supposition, namely, the first position by the second error, and the second position by the first error.

If the errors are alike, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

But if the errors are unlike, divide the sum of the products by the sum of the errors, for the answer.

The errors are said to be alike, when they are either both too great, or both too little; and unlike, when one is too great and the other too little.

What number is that, which, being multiplied by 6, the product increased by 18, and the sum divided by 9, the quotient shall be 20.

Suppose the two numbers, 18 and 30. Then

First position.	Second position.	Proof.
18	30	27
6 mult.	6	6
108	180	162
18 add.	18	18
9)126	9)198	9)180
14 results.	22	20
20 true res.	20	
+ 6 errors unlike.	— 2	
2d pos. 30 mult.	18 1st pos.	
Errors { 2 180	36	
6 36		
Sum 8)216 sum of products.		
27 answer sought.		

Find, by trial, two numbers, as near the true number as possible, and operate with them as in the question; marking the errors which arise from each of them.

Multiply the difference of the two numbers, found by trial, by the least error, and divide the product by the difference of the errors, when they are alike, but by their sum when they are unlike.

Add the quotient, last found, to the number belonging to the least error, when that number is too little, but subtract it when too great, and the result will give the true quantity sought.

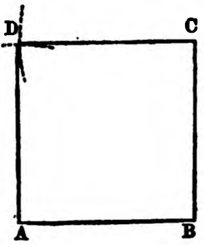
MENSURATION OF SUPERFICIES.

THE *area* of any figure is the measure of its surface, or the space contained within the bounds of that surface, without any regard to thickness.

A square whose side is one inch, one foot, or one yard, &c. is called the *measuring unit*, and the area or content of any figure is computed by the number of those squares contained in that figure.

To find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboides.—Multiply the length by the perpendicular height, and the product will be the area.

The perpendicular height of the parallelogram is equal to the area divided by the base.

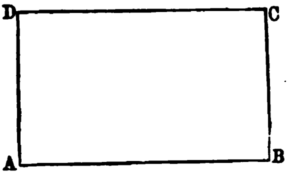


Required the area of the square ABCD whose side is 5 feet 9 inches.

Here 5 *ft.* 9 *in.* = 5·75 : *and* $\overline{5{\cdot}75}|^2$ = 5·75 × 5·75 = 33·0625 *feet* = 33 *fe.* 0 *in.* 9 *pa.* = *area required.*

Required the area of the rectangle ABCD, whose length AB is 13·75 chains, and breadth BC 9·5 chains.

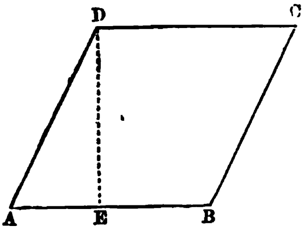
Here 13·75 × 9·5 = 130·625; *and* $\frac{130\cdot625}{10}$ = 13·0625 *ac.* = 13 *ac.* 0 *ro.* 10 *po. = area required.*



Required the area of the rhombus ABCD, whose length AB is 12 feet 6 inches, and its height DE 9 feet 3 inches.

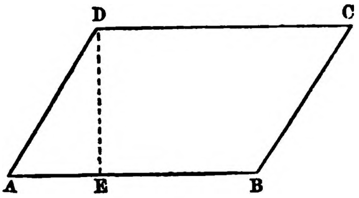
Here 12 *fe.* 6 *in.* = 12·5, *and* 9 *fe.* 3 *in.* = 9·25.

Whence, 12·5 × 9·25 = 115·625 *fe.* = 115 *fe.* 7 *in.* 6 *pa. = area required.*



What is the area of the rhomboides ABCD, whose length AB is 10·52 chains, and height DE 7·63 chains.

Here 10·52 × 7·63 = 80·2676; *and* $\frac{80\cdot2676}{10}$ = 8·02676 *acres* = 8 *ac.* 0 *ro.* 4 *po. area required.*



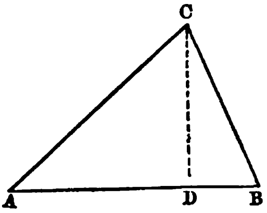
To find the area of a triangle.—Multiply the base by the perpendicular height, and half the product will be the area.

The perpendicular height of the triangle is equal to twice the area divided by the base.

Required the area of the triangle ABC, whose base AB is 10 feet 9 inches, and height DC 7 feet 3 inches.

Here 10 *fe.* 9 *in.* = 10·75, *and* 7 *fe.* 3 *in.* = 7·25.

Whence, 10·75 × 7·25 = 77·9375, *and* $\frac{77\cdot9375}{2}$ = 38·96875 *feet* = 38 *fe.* 11 *in.* 7½ *pa. = area required.*



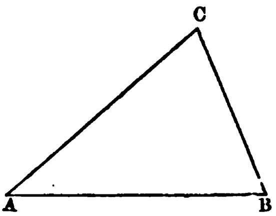
To find the area of a triangle whose three sides only are given.—From half the sum of the three sides subtract each side severally.

Multiply the half sum and the three remainders continually together, and the square root of the product will be the area required

Required the area of the triangle ABC, whose three sides BC, CA, and AB are 24, 36, and 48 chains respectively.

Here $\frac{24 + 36 + 48}{2} = \frac{108}{2} = 54 =$ ½ *sum of the sides.*

Also, 54 − 24 = 30 *first diff.*; 54 − 36 = 18 *second diff.*; *and* 54 − 48 = 6 *third diff.*



Whence, $\sqrt{54 \times 30 \times 18 \times 6} = \sqrt{174960} = 418{\cdot}282$ = *area required.*

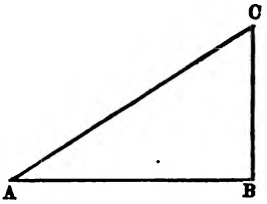
Any two sides of a right angled triangle being given to find the third side.—When the two legs are given to find the hypothenuse, add the square of one of the legs to the square of the other, and the square root of the sum will be equal to the hypothenuse.

When the hypothenuse and one of the legs are given to find the other leg.—From the square of the hypothenuse take the square of the given leg, and the square root of the remainder will be equal to the other leg.

In the right angled triangle ABC, the base AB is 56, and the perpendicular BC 33, what is the hypothenuse?

Here $56^2 + 33^2 = 3136 + 1089 = 4225$, *and* $\sqrt{4225} = 65$ = *hypothenuse* AC.

If the hypothenuse AC be 53, and the base AB 45, what is the perpendicular BC?

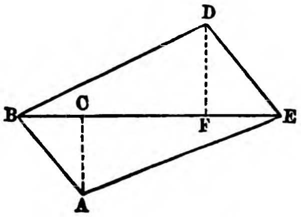


Here $53^2 - 45^2 = 2809 - 2025 = 784$, *and* $\sqrt{784} = 28$ = *perpendicular* BC.

To find the area of a trapezium.—Multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles, and half the product will be the area.

Required the area of the trapezium BAED, whose diagonal BE is 84, the perpendicular AC 21, and DF 28.

Here $\overline{28 + 21} \times 84 = 49 \times 84 = 4116$, *and* $\frac{4116}{2} = 2058$ *the area required.*

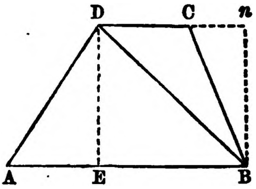


To find the area of a trapezoid, or a quadrangle, two of whose opposite sides are parallel.—Multiply the sum of the parallel sides by the perpendicular distance between them, and half the product will be the area.

Required the area of the trapezoid ABCD, whose sides AB and DC are 321·51 and 214·24, and perpendicular DE 171·16.

Here 321·51 + 214·24 = 535·75 = *sum of the parallel sides* AB, DC.

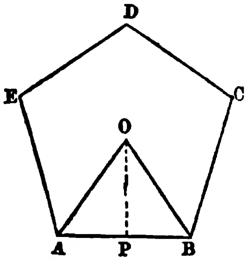
Whence, 535·75 × 171·16 (*the perp.* DE) = 91698·9700, *and* $\frac{91698{\cdot}9700}{2} = 45849{\cdot}485$ *the area required.*



To find the area of a regular polygon.—Multiply half the perimeter of the figure by the perpendicular falling from its centre upon one of the sides, and the product will be the area.

The perimeter of any figure is the sum of all its sides.

Required the area of the regular pentagon ABCDE, whose side AB, or BC, &c., is 25 feet, and the perpendicular OP 17·2 feet.



Here $\frac{25 \times 5}{2} = 62·5 =$ *half perimeter, and* $62·5 \times 17·2 = 1075$ *square feet = area required.*

To find the area of a regular polygon, when the side only is given.—Multiply the square of the side of the polygon by the number standing opposite to its name in the following table, and the product will be the area.

No. of sides.	Names.	Multipliers.	No. of sides.	Names.	Multipliers.
3	Trigon or equil. △	0·433013	8	Octagon	4·828427
4	Tetragon or square	1·000000	9	Nonagon	6·181824
5	Pentagon	1·720477	10	Decagon	7·694209
6	Hexagon	2·598076	11	Undecagon	9·365640
7	Heptagon	3·633912	12	Duodecagon	11·196152

The angle OBP, together with its tangent, for any polygon of not more than 12 sides, is shown in the following table:

No. of sides.	Names.	Angle OBP.	Tangents.
3	Trigon	30°	$·57735 = \frac{1}{3}\sqrt{3}$
4	Tetragon	45°	$1·00000 = 1 \times 1$
5	Pentagon	54°	$1·37638 = \sqrt{1 + \frac{2}{5}\sqrt{5}}$
6	Hexagon	60°	$1·73205 = \sqrt{3}$
7	Heptagon	$64°\frac{2}{7}$	2·07652
8	Octagon	$67°\frac{1}{2}$	$2·41421 = 1 + \sqrt{2}$
9	Nonagon	70°	2·74747
10	Decagon	72°	$3·07768 = \sqrt{5 + 2\sqrt{5}}$
11	Undecagon	$73°\frac{7}{11}$	3·40568
12	Duodecagon	75°	$3·73205 = 2 + \sqrt{3}$

Required the area of a pentagon whose side is 15.

The number opposite pentagon in the table is 1·720477.

Hence $1·720477 \times 15^2 = 1·720477 \times 225 = 387·107325 =$ *area required.*

The diameter of a circle being given to find the circumference, or the circumference being given to find the diameter.—Multiply the diameter by 3·1416, and the product will be the circumference, or

Divide the circumference by 3·1416, and the quotient will be the diameter.

As 7 is to 22, so is the diameter to the circumference; or as 22 is to 7, so is the circumference to the diameter.

As 113 is to 355, so is the diameter to the circumference; or, as 352 is to 115, so is the circumference to the diameter.

If the diameter of a circle be 17, what is the circumference?

Here $3 \cdot 1416 \times 17 = 53 \cdot 4072 =$ *circumference.*

If the circumference of a circle be 354, what is the diameter?

$$\text{Here } \frac{354 \cdot 000}{3 \cdot 1416} = 112 \cdot 681 = \text{diameter.}$$

To find the length of any arc of a circle.—When the chord of the arc and the versed sine of half the arc are given:

To 15 times the square of the chord, add 33 times the square of the versed sine, and reserve the number.

To the square of the chord, add 4 times the square of the versed sine, and the square root of the sum will be *twice the chord of half the arc.*

Multiply twice the chord of half the arc by 10 times the square of the versed sine, divide the product by the reserved number, and add the quotient to twice the chord of half the arc: the sum will be the length of the arc very nearly.

When the chord of the arc, and the chord of half the arc are given.—From the square of the chord of half the arc subtract the square of half the chord of the arc, the remainder will be the square of the versed sine: then proceed as above.

When the diameter and the versed sine of half the arc are given:

From 60 times the diameter subtract 27 times the versed sine, and reserve the number.

Multiply the diameter by the versed sine, and the square root of the product will be the *chord of half the arc.*

Multiply twice the chord of half the arc by 10 times the versed sine, divide the product by the reserved number, and add the quotient to twice the chord of half the arc; the sum will be the length of the arc very nearly.

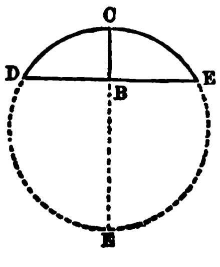
When the diameter and chord of the arc are given, the versed sine may be found thus: From the square of the diameter subtract the square of the chord, and extract the square root of the remainder. Subtract this root from the diameter, and half the remainder will give the versed sine of half the arc.

The square of the chord of half the arc being divided by the diameter will give the versed sine, or being divided by the versed sine will give the diameter.

The length of the arc may also be found by multiplying together the number of degrees it contains, the radius and the number ·01745329.

Or, as 180 is to the number of degrees in the arc, so is 3·1416 times the radius, to the length of the arc.

Or, as 3 is to the number of degrees in the arc, so is ·05236 times the radius to the length of the arc.



If the chord DE be 48, and the versed sine CB 18, what is the length of the arc?

$$\begin{aligned} \text{Here } 48^2 \times 15 &= 34560 \\ 18^2 \times 33 &= \underline{10692} \\ &\ 45252 \text{ reserved number.} \end{aligned}$$

$48^2 = 2304$ = *the square of the chord.*
$18^2 \times 4 = 1296$ = *4 times the square of the versed sine.*
$\sqrt{3600} = 60$ = *twice the chord of half the arc.*

Now $\frac{60 \times 18^2 \times 10}{45252} = \frac{194400}{45252} = 4{\cdot}2959$, *which added to twice the chord of half the arc gives* 64·2959 = *the length of the arc.*

$$\begin{array}{r}
50 \times 60 = 3000 \\
18 \times 27 = 486 \\
\hline
2514 \text{ \emph{reserved number.}}
\end{array}$$

$AC = \sqrt{50 \times 18} = 30$ = *the chord of half the arc.*

$\frac{\overline{30 \times 2} \times \overline{18 \times 10}}{2514} = \frac{10800}{2514} = 4{\cdot}2959$, *which added to twice the chord of half the arc gives* 64·2959 = *the length of the arc.*

To find the area of a circle.—Multiply half the circumference by half the diameter, and the product will be the area.

Or take $\frac{1}{4}$ of the product of the whole circumference and diameter.

What is the area of a circle whose diameter is 42, and circumference 131·946?

$$\begin{array}{r l}
2\,)\,\underline{131{\cdot}946} & \\
65{\cdot}973 & = \tfrac{1}{2} \text{ \emph{circumference.}} \\
21 & = \tfrac{1}{2} \text{ \emph{diameter.}} \\
\hline
65973 & \\
131946 & \\
\hline
1385{\cdot}433 & = \text{\emph{area required.}}
\end{array}$$

What is the area of a circle whose diameter is 10 feet 6 inches, and circumference 31 feet 6 inches?

$$\begin{array}{r r l}
\text{\emph{fe.}} & \text{\emph{in.}} & \\
15 & 9 & = 15{\cdot}75 = \tfrac{1}{2} \text{ \emph{circumference.}} \\
5 & 3 & = 5{\cdot}25 = \tfrac{1}{2} \text{ \emph{diameter.}} \\
\hline
 & & 7875 \\
 & & 3150 \\
 & & 7875 \\
\hline
 & & 82{\cdot}6875 \\
 & & 12 \\
\hline
 & & 8{\cdot}2500
\end{array}$$

82 *feet* 8 *inches.*

Multiply the square of the diameter by ·7854, and the product will be the area; or,

Multiply the square of the circumference by ·07958, and the product will be the area.

The following table will also show most of the useful problems relating to the circle and its equal or inscribed square.

Diameter × ·8862 = side of an equal square.
Circumf. × ·2821 = side of an equal square.
Diameter × ·7071 = side of the inscribed square.

Circumf. × ·2251 = side of the inscribed square.
Area × ·6366 = side of the inscribed square.
Side of a square × 1·4142 = diam. of its circums. circle.
Side of a square × 4·443 = circumf. of its circums. circle.
Side of a square × 1·128 = diameter of an equal circle.
Side of a square × 3·545 = circumf. of an equal circle.

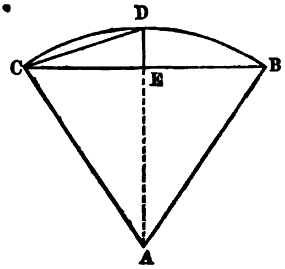
What is the area of a circle whose diameter is 5?

$$\begin{array}{r l} 7854 & \\ 25 & = \textit{square of the diameter.} \\ \hline 39270 & \\ 15708 & \\ \hline 19{\cdot}6350 & = \textit{the answer.} \end{array}$$

To find the area of a sector, or that part of a circle which is bounded by any two radii and their included arc.—Find the length of the arc, then multiply the radius, or half the diameter, by the length of the arc of the sector, and half the product will be the area.

If the diameter or radius is not given, add the square of half the chord of the arc, to the square of the versed sine of half the arc; this sum being divided by the versed sine, will give the diameter.

The radius AB is 40, and the chord BC of the whole arc 50, required the area of the sector.



$\dfrac{80 - \sqrt{80^2 - 50^2}}{2} = 8{\cdot}7750$ = *the versed sine of half the arc.*

$\overline{80 \times 60} - \overline{8{\cdot}7750 \times 27} = 4563{\cdot}0750$ = *the reserved number.*

$2 \times \sqrt{8{\cdot}7750 \times 80} = 52{\cdot}9906$ = *twice the chord of half the arc.*

$\dfrac{52{\cdot}9906 \times 8{\cdot}7750 \times 10}{4563{\cdot}0750} = 1{\cdot}0190$, *which added to twice the chord of half the arc gives* 54·0096 *the length of the arc.*

And $\dfrac{54{\cdot}0096 \times 40}{2} = 1080{\cdot}1920$ = *area of the sector required.*

As 360 is to the degrees in the arc of a sector, so is the area of the whole circle, whose radius is equal to that of the sector, to the area of the sector required.

For a semicircle, a quadrant, &c. take one half, one quarter, &c. of the whole area.

The radius of a sector of a circle is 20, and the degrees in its arc 22; what is the area of the sector?

Here the diameter is 40.

Hence, the area of the circle = $40^2 \times {\cdot}7854 = 1600 \times {\cdot}7854 = 1256{\cdot}64$.

Now, 360° : 22° :: 1256·64 : 76·7947 = *area of the sector.*

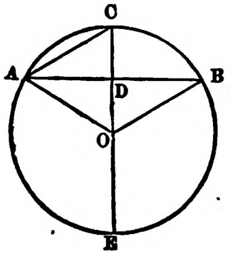
To find the area of a segment of a circle.—Find the area of the sector, having the same arc with the segment, by the last problem.

Find the area of the triangle formed by the chord of the segment, and the radii of the sector.

Then the sum, or difference, of these areas, according as the segment is greater or less than a semicircle, will be the area required.

The difference between the versed sine and radius, multiplied by half the chord of the arc, will give the area of the triangle.

The radius OB is 10, and the chord AC 10; what is the area of the segment ABC?



$CD = \frac{AC^2}{CE} = \frac{100}{20} = 5 =$ *the versed sine of half the arc.*

$\overline{20 \times 60} - \overline{5 \times 27} = 1065 =$ *the reserved number.*

$\frac{\overline{10 \times 2} \times \overline{5 \times 10}}{1065} = {\cdot}9390$, *and this added to twice the chord of half the arc gives* $20{\cdot}9390 =$ *the length of the arc.*

$\frac{20{\cdot}9390 \times 10}{2} = 104{\cdot}6950 =$ *area of the sector* OACB.

$OD = OC = CD = 5$ *the perpendicular height of the triangle.*

$AD = \sqrt{AO^2 - OD^2} = \sqrt{75} = 8{\cdot}6603 = \frac{1}{2}$ *the chord of the arc.*

$8{\cdot}6603 \times 5 = 43{\cdot}3015 =$ *the area of the triangle* AOB.

$104{\cdot}6950 - 43{\cdot}3015 = 61{\cdot}3935 =$ *area of the segment required;* it being in this case less than a semicircle.

Divide the height, or versed sine, by the diameter, and find the quotient in the table of versed sines.

Multiply the number on the right hand of the versed sine by the square of the diameter, and the product will be the area.

When the quotient arising from the versed sine divided by the diameter, has a remainder or fraction after the third place of decimals; having taken the area answering to the first three figures, subtract it from the next following area, multiply the remainder by the said fraction, and add the product to the first area, then the sum will be the area for the whole quotient.

If the chord of a circular segment be 40, its versed sine 10, and the diameter of the circle 50, what is the area?

$5{\cdot}0\,)\,\underline{1{\cdot}0}$

${\cdot}2 =$ *tabular versed sine.*

${\cdot}111823 =$ *tabular segment.*

$2500 =$ *square of* 50.

55911500

223646

$279{\cdot}557500 =$ *area required.*

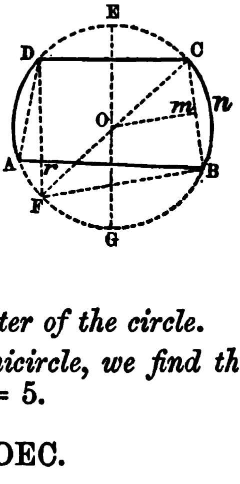
To find the area of a circular zone, or the space included between any two parallel chords and their intercepted arcs.—From the greater chord subtract half the difference between the two, multiply the remainder by the said half difference, divide the product by the breadth of the zone, and add the quotient to the breadth. To the square of this number add the square of the less chord, and the square root of the sum will be the diameter of the circle.

Now, having the diameter EG, and the two chords AB and DC, find the areas of the segments ABEA, and DCED, the difference of which will be the area of the zone required.

The difference of the tabular segments multiplied by the square of the circle's diameter will give the area of the zone.

When the larger segment AEB is greater than a semicircle, find the areas of the segments AGB, and DCE, and subtract their sum from the area of the whole circle: the remainder will be the area of the zone.

The greater chord AB is 20, the less DC 15, and their distance D*r* $17\frac{1}{2}$: required the area of the zone ABCD.



$\frac{20-15}{2} = 2{\cdot}5 = \frac{1}{2} =$ *the difference between the chords.*

$17{\cdot}5 + \frac{(20-2{\cdot}5)\times 2{\cdot}5}{17{\cdot}5} = 17{\cdot}5 + 2{\cdot}5 = 20 =$ DF.

And $\sqrt{20^2+15^2} = \sqrt{625} = 25 =$ *the diameter of the circle.*

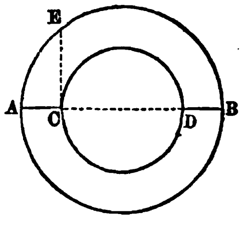
The segment AEB *being greater than a semicircle, we find the versed sine of* DCE $= 2{\cdot}5$, *and that of* AGB $= 5$.

Hence $\frac{2{\cdot}5}{25} = {\cdot}100 =$ *tabular versed sine of* DEC.

And $\frac{5}{25} = {\cdot}200 =$ *tabular versed sine of* AGB.

Now ·040875 × 25² = *area of seg.* DEC = 25·546875
And ·111823 × 25² = *area of seg.* AGB = 69·889375
sum 95·43625
·7854 × 25² = *area of the whole circle,* = 490·87500
Difference = *area of the zone* ABCD = 395·43875

To find the area of a circular ring, or the space included between the circumference of two concentric circles.—The difference between the areas of the two circles will be the area of the ring.



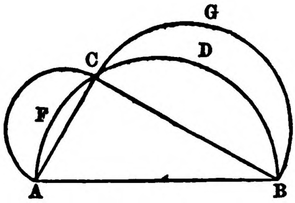
Or, multiply the sum of diameters by their difference, and this product again by ·7854, and it will give the area required.

The diameters AB and CD are 20 and 15: required the area of the circular ring, or the space included between the circumferences of those circles.

Here $\overline{AB+CD}\times\overline{AB-CD}=35\times5=175$, *and* $175\times{}\cdot7854=137\cdot4450$ = *area of the ring required.*

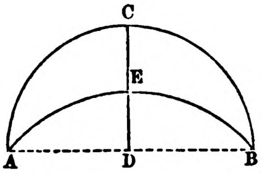
To find the areas of lunes, or the spaces between the intersecting arcs of two eccentric circles.—Find the areas of the two segments from which the lune is formed, and their difference will be the area required.

The following property is one of the most curious:



If ABC be a right angled triangle, and semicircles be described on the three sides as diameters, then will the said triangle be equal to the two lunes D and F taken together.

For the semicircles described on AC and BC = the one described on AB, from each take the segments cut off by AC and BC, then will the lunes AFCE and BDCG = the triangle ACB.



The length of the chord AB is 40, the height DC 10, and DE 4: required the area of the lune ACBEA.

The diameter of the circle of which ACB *is a part* $=\dfrac{20^2+10^2}{10}=50$.

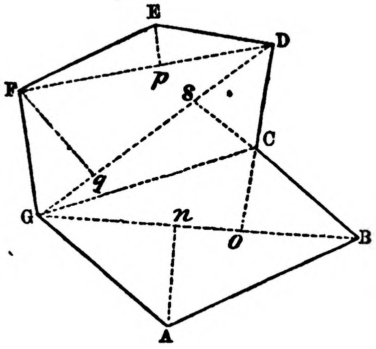
And the diameter of the circle of which AEB *is a part* $=\dfrac{20^2+4^2}{4}=104$.

Now having the diameter and versed sines, we find,

The area of seg. ACB = ·111823 × 50^2 = 279·5575
And area of seg. AEB = ·009955 × 104^2 = 107·6733
Their difference is the area of the lune AEBCA *required,* } = 171·8842

To find the area of an irregular polygon, or a figure of any number of sides.—Divide the figure into triangles and trapeziums, and find the area of each separately.

Add these areas together, and the sum will be equal to the area of the whole polygon.



Required the area of the irregular figure ABCDEFGA, the following lines being given:

GB = 30·5   An = 11·2,   CO = 6
GD = 29   Fq = 11   Cs = 6·6
FD = 24·8   Ep = 4   ......

Here $\dfrac{An+Co}{2}\times GB=\dfrac{11\cdot2+6}{2}\times30\cdot5+8\cdot6\times30\cdot5=262\cdot3$ = *area of the trapezium* ABCG.

And $\frac{Fq + Cs}{2} \times GD = \frac{11 + 6·6}{2} \times 29 = 8·8 \times 29 = 255·2 =$ *area of the trapezium* GCDF.

Also, $\frac{FD \times Ep}{2} = \frac{24·8 \times 4}{2} = \frac{99·2}{2} = 49·6 =$ *area of the triangle* FDE.

Whence 262·3 + 255·2 + 49·6 = 567·1 = *area of the whole figure required.*

DECIMAL APPROXIMATIONS FOR FACILITATING CALCULATIONS IN MENSURATION.

Lineal feet	multiplied by	·00019	= miles.
— yards	—	·000568	= —
Square inches	—	·007	= square feet.
— yards	—	·0002067	= acres.
Circular inches	—	·00546	= square feet.
Cylindrical inches	—	·0004546	= cubic feet.
— feet	—	·02909	= cubic yards.
Cubic inches	—	·00058	= cubic feet.
— feet	—	·03704	= cubic yards.
— —	—	6·232	= imperial gallons.
— inches	—	·003607	= —
Cylindrical feet	—	4·895	= —
— inches	—	·002832	= —
Cubic inches	—	·263	= ℔s. avs. of cast iron.
—	—	·281	= — wrought do.
—	—	·283	= — steel.
—	—	·3225	= — copper.
—	—	·3037	= — brass.
—	—	·26	= — zinc.
—	—	·4103	= — lead.
—	—	·2636	= — tin.
—	—	·4908	= — mercury.
Cylindrical inches	—	·2065	= — cast iron.
—	—	·2168	= — wrought iron.
—	—	·2223	= — steel.
—	—	·2533	= — copper.
—	—	·2385	= — brass.
—	—	·2042	= — zinc.
—	—	·3223	= — lead.
—	—	·207	= — tin.
—	—	·3854	= — mercury.
Avoirdupois ℔s.	—	·009	= cwts.
—	—	·00045	= tons.
183·346 circular inches			= 1 square foot.
2200 cylindrical inches			= 1 cubic foot.
French metres × 3·281			= feet.
— kilogrammes × 2·205			= avoirdupois lb.
— grammes × ·002205			= avoirdupois lbs.

Diameter of a sphere × ·806 = dimensions of equal cube.
Diameter of a sphere × ·6667 = length of equal cylinder.
Lineal inches × ·0000158 = miles.
A French cubic foot = 2093·47 cubic inches.
Imperial gallons × ·7977 = New York gallons.

The average quantity of water that falls in rain and snow at Philadelphia is 36 inches.

At West Point the variation of the magnetic needle, Nov. 16th, 1839, was 7° 58′ 27″ West, and the dip 73° 26′ 28″.

DECIMAL EQUIVALENTS TO FRACTIONAL PARTS OF LINEAL MEASURES.

One inch, the integer or whole number.								
·96875	are equal to	$\frac{7}{8} + \frac{3}{32}$	·625	are equal to	$\frac{5}{8}$	·28125	are equal to	$\frac{1}{4} + \frac{1}{32}$
·9375		$\frac{7}{8} + \frac{1}{16}$	·59375		$\frac{1}{2} + \frac{3}{32}$	·25		$\frac{1}{4}$
·90625		$\frac{7}{8} + \frac{1}{32}$	·5625		$\frac{1}{2} + \frac{1}{16}$	·21875		$\frac{1}{8} + \frac{3}{32}$
·875		$\frac{7}{8}$	·53125		$\frac{1}{2} + \frac{1}{32}$	·1875		$\frac{1}{8} + \frac{1}{16}$
·84375		$\frac{3}{4} + \frac{3}{32}$	·5		$\frac{1}{2}$	·15625		$\frac{1}{8} + \frac{1}{32}$
·8125		$\frac{3}{4} + \frac{1}{16}$	·46875		$\frac{3}{8} + \frac{3}{32}$	·125		$\frac{1}{8}$
·78125		$\frac{3}{4} + \frac{1}{32}$	·4375		$\frac{3}{8} + \frac{1}{16}$	·09375		$\frac{3}{32}$
·75		$\frac{3}{4}$	·40625		$\frac{3}{8} + \frac{1}{32}$	·0625		$\frac{1}{16}$
·71875		$\frac{5}{8} + \frac{3}{32}$	·375		$\frac{3}{8}$	·03125		$\frac{1}{32}$
·6875		$\frac{5}{8} + \frac{1}{16}$	·34375		$\frac{1}{4} + \frac{3}{32}$			
·65625		$\frac{5}{8} + \frac{1}{32}$	·3125		$\frac{1}{4} + \frac{1}{16}$			
One foot, or 12 inches, the integer.								
·9166	are equal to	11 inches.	·4166	are equal to	5 inches.	·0625	are equal to	$\frac{3}{4}$ of in.
·6338		10 —	·3333		4 —	·05208		$\frac{5}{8}$ —
·75		9 —	·25		3 —	·04166		$\frac{1}{2}$ —
·6666		8 —	·1666		2 —	·03125		$\frac{3}{8}$ —
·5833		7 —	·0833		1 —	·02083		$\frac{1}{4}$ —
·5		6 —	·07291		$\frac{7}{8}$ —	·01041		$\frac{1}{8}$ —
One yard, or 36 inches, the integer.								
·9722	are equal to	35 inches.	·6389	are equal to	23 inches.	·3055	are equal to	11 inches.
·9444		34 —	·6111		22 —	·2778		10 —
·9167		33 —	·5833		21 —	·25		9 —
·8889		32 —	·5556		20 —	·2222		8 —
·8611		31 —	·5278		19 —	·1944		7 —
·8333		30 —	·5		18 —	·1667		6 —
·8056		29 —	·4722		17 —	·1389		5 —
·7778		28 —	·4444		16 —	·1111		4 —
·75		27 —	·4167		15 —	·0833		3 —
·7222		26 —	·3889		14 —	·0555		2 —
·6944		25 —	·3611		13 —	·0278		1 —
·6667		24 —	·3333		12 —			

Table containing the Circumferences, Squares, Cubes, and Areas of Circles, from 1 *to* 100, *advancing by a tenth.*

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
1	3·1416	1	1	·7854	9	28·2744	81	729	63·6174
·1	3·4557	1·21	1·331	·9503	·1	28·5885	82·81	753·571	65·0389
·2	3·7699	1·44	1·728	1·1309	·2	28·9027	84·64	778·688	66·4762
·3	4·0840	1·69	2·197	1·3273	·3	29·2168	86·49	804·357	67·9292
·4	4·3982	1·96	2·744	1·5393	·4	29·5310	88·36	830·584	69·3979
·5	4·7124	2·25	3·375	1·7671	·5	29·8452	90·25	857·375	70·8823
·6	5·0265	2·56	4·096	2·0106	·6	30·1593	92·16	884·736	72·3824
·7	5·3407	2·89	4·913	2·2698	·7	30·4735	94·09	912·673	73·8982
·8	5·6548	3·24	5·832	2·5446	·8	30·7876	96·04	941·192	75·4298
·9	5·9690	3·61	6·859	2·8352	·9	31·1018	98·01	970·299	76·9770
2	6·2832	4	8	3·1416	10	31·4160	100	1000	78·5400
·1	6·5973	4·41	9·261	3·4636	·1	31·7301	102·01	1030·301	80·1186
·2	6·9115	4·84	10·648	3·8013	·2	32·0443	104·04	1061·208	81·7130
·3	7·2256	5·29	12·167	4·1547	·3	32·3580	106·09	1092·727	83·3230
·4	7·5398	5·76	13·824	4·5239	·4	32·6726	108·16	1124·864	84·9488
·5	7·8540	6·25	15·625	4·9087	·5	32·9868	110·25	1157·625	86·5903
·6	8·1681	6·76	17·576	5·3093	·6	33·3009	112·36	1191·016	88·2475
·7	8·4823	7·29	19·683	5·7255	·7	33·6151	114·49	1225·043	89·9204
·8	8·7964	7·84	21·952	6·1575	·8	33·9292	116·64	1259·712	91·6090
·9	9·1106	8·41	24·389	6·6052	·9	34·2434	118·81	1295·029	93·3133
3	9·4248	9	27	7·0686	11	34·5576	121	1331	95·0334
·1	9·7389	9·61	29·791	7·5476	·1	34·8717	123·21	1367·631	96·7691
·2	10·0531	10·24	32·768	8·0424	·2	35·1859	125·44	1404·928	98·5205
·3	10·3672	10·89	35·937	8·5530	·3	35·5010	127·69	1442·897	100·2877
·4	10·6814	11·56	39·304	9·0792	·4	35·8142	129·96	1481·544	102·0705
·5	10·9956	12·25	42·875	9·6211	·5	36·1284	132·25	1520·875	103·8691
·6	11·3097	12·96	46·656	10·1787	·6	36·4425	134·56	1560·896	105·6834
·7	11·6239	13·69	50·653	10·7521	·7	36·7567	136·89	1601·613	107·5134
·8	11·9380	14·44	54·872	11·3411	·8	37·0708	139·24	1643·032	109·3590
·9	12·2522	15·21	59·319	11·9459	·9	37·3840	141·61	1685·159	111·2204
4	12·5664	16	64	12·5664	12	37·6992	144	1728	113·0976
·1	12·8805	16·81	68·921	13·2025	·1	38·0133	146·41	1771·561	114·9904
·2	13·1947	17·64	74·088	13·8544	·2	38·3275	148·84	1815·848	116·8989
·3	13·5088	18·49	79·507	14·5220	·3	38·6416	151·29	1860·867	118·8231
·4	13·8230	19·36	85·184	15·2053	·4	38·9558	153·76	1906·624	120·7631
·5	14·1372	20·25	91·125	15·9043	·5	39·2700	156·25	1953·125	122·7187
·6	14·4513	21·16	97·336	16·6190	·6	39·5841	158.76	2000·376	124·6901
·7	14·7655	22·09	103·823	17·3494	·7	39·8983	161·29	2048·383	126·6771
·8	15·0796	23·04	110·592	18·0956	·8	40·2124	163·84	2097·152	128·6799
·9	15·3938	24·01	117·649	18·8574	·9	40·5266	166·41	2146·689	130·6984
5	15·7080	25	125	19·6350	13	40·8408	169	2197	132·7326
·1	16·0221	26·01	132·651	20·4282	·1	41·1549	171·61	2248·091	134·7824
·2	16·3363	27·04	140·608	21·2372	·2	41·4691	174·24	2299·968	136·8480
·3	16·6504	28·09	148·877	22·0618	·3	41·7832	176·89	2352·637	138·9294
·4	16·9646	29·16	157·464	22·9022	·4	42·0974	179·56	2406 104	141·0264
·5	17·2788	30·25	166·375	23·7583	·5	42·4116	182·25	2460·375	143·1391
·6	17·5929	31·36	175·616	24·6301	·6	42·7257	184·96	2515·456	145·2675
·7	17·9071	32·49	185·193	25·5176	·7	43·0399	187·69	2571·353	147·4117
·8	18·2212	33·64	195·112	26·4208	·8	43·3540	190·44	2628·072	149·5715
·9	18·5354	34·81	205·379	27·3397	·9	43·6682	193·21	2685·619	151·7471
6	18·8496	36	216	28·2744	14	43·9824	196	2744	153·9384
·1	19·1637	37·21	226·981	29·2247	·1	44·2965	198·81	2803·221	156·1453
·2	19·4779	38·44	238·328	30·1907	·2	44·6107	201·64	2863·288	158·3680
·3	19·7920	39·69	250·047	31·1725	·3	44·9248	204·49	2924·207	160·6064
·4	20·1062	40·96	262·144	32·1699	·4	45·2390	207·36	2985·984	162·8605
·5	20·4204	42·25	274·625	33·1831	·5	45·5532	210·25	3048·625	165·1303
·6	20·7345	43·56	287·496	34·2120	·6	45·8673	213·16	3112·136	167·4158
·7	21·0487	44·89	300·763	35·2566	·7	46·1815	216·09	3176·523	169·7179
·8	21·3628	46·24	314·432	36·3168	·8	46·4956	219·04	3241·792	172·0340
·9	21·6770	47·61	328·509	37·3928	·9	46·8098	222·01	3307·949	174·3666
7	21·9912	49	343	38·4846	15	47·1240	225	3375	176·7150
·1	22·3053	50·41	357·911	39·5920	·1	47·4381	228·01	3442·951	179·0790
·2	22·6195	51·84	373·248	40·7151	·2	47·7523	231·04	3511·808	181·4588
·3	22·9336	53·29	389·017	41·8539	·3	48·0664	234·09	3581·577	183·8542
·4	23·2478	54·76	405·224	43·0085	·4	48·3806	237·16	3652·264	186·2654
·5	23·5620	56·25	421·875	44·1787	·5	48·6948	240·25	3723·875	188·6923
·6	23·8761	57·76	438·976	45·3647	·6	49·0089	243·36	3796·416	191·1349
·7	24·1903	59·29	456·533	46·5663	·7	49·3231	246·49	3869·893	193·5932
·8	24·5044	60·84	474·552	47·7837	·8	49·6372	249·64	3944·312	196·0672
·9	24·8186	62·41	493·039	49·0168	·9	49·9514	252·81	4019·679	198·5569
8	25·1328	64	512	50·2656	16	50·2656	256	4096	201·0624
·1	25·4469	65·61	531·441	51·5300	·1	50·5797	259·21	4173·281	203·5835
·2	25·7611	67·24	551·368	52·8102	·2	50·8939	262·44	4251·528	206·1209
·3	26·0752	68·89	571·787	54·1062	·3	51·2080	265·69	4330·747	208·6723
·4	26·3894	70·56	592·704	55·4178	·4	51·5224	268·96	4410·944	211·1411
·5	26·7036	72·25	614·125	56·7451	·5	51·8364	272·25	4492·125	213·8251
·6	27·0177	73·96	636·056	58·0881	·6	52·1505	275·56	4574·296	216·4248
·7	27·3319	75·69	658·503	59·4469	·7	52·4647	278·89	4657·463	219·0402
·8	27·6460	77·44	681·472	60·8213	·8	52·7788	282·24	4741·632	221·6712
·9	27·9602	79·21	704·969	62·2115	·9	53·0930	285·61	4826·809	224·3180

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
17	53·4072	289	4913	226·9806	25	78·5400	625	15625	490·8750
·1	53·7213	292·41	5000·211	229·6588	·1	78·8541	630·01	15813·251	494·8098
·2	54·0355	295·84	5088·448	232·3527	·2	79·1683	635·04	16003·008	498·7604
·3	54·3496	299·29	5177·717	235·0623	·3	79·4824	640·09	16194·277	502·7266
·4	54·6038	302·76	5268·024	237·7877	·4	79·7966	645·16	16387·064	506·7086
·5	54·9780	306·25	5359·375	240·5287	·5	80·8108	650·25	16581·375	510·7063
·6	55·2921	309·76	5451·776	243·2855	·6	80·4249	655·36	16777·216	514·7196
·7	55·6063	313·29	5545·233	246·0579	·7	80·7391	660·49	16974·593	518·7488
·8	55·9204	316·84	5639·752	248·8461	·8	81·0532	665·64	17173·512	522·7936
·9	56·2346	320·41	5735·339	251·6500	·9	81·3674	670·81	17373·979	526·8541
18	56·5488	324	5832	254·4696	26	81·6816	676	17576	530·9304
·1	56·8629	327·61	5929·741	257·3048	·1	81·9976	681·21	17779·581	535·0223
·2	57·1771	331·24	6028·568	260·1558	·2	82·3099	686·44	17984·728	539·1299
·3	57·4912	334·89	6128·487	263·0226	·3	82·6240	691·69	18191·447	543·2533
·4	57·8054	338·56	6229·504	265.9050	·4	82·9382	696·96	18399·744	547·3923
·5	58·1196	342·25	6331·625	268·8031	·5	83·2524	702·25	18609·625	551·5471
·6	58·4337	345·96	6434·856	271·7169	·6	83·5665	707·56	18821·096	555·7176
·7	58·7479	349·69	6539·203	274·6465	·7	83·8807	712·89	19034·163	559·9038
·8	59·0620	353·44	6644·672	277·5917	·8	84·1948	718·24	19248·832	564·1056
9	59·3762	357·21	6751·269	280·5527	·9	84·5090	723·61	19465·109	568·3232
19	59·6904	361	6859	283·5294	27	84·8232	729	19683	572·5566
·1	60·0045	364·81	6967·871	286·5217	·1	85·1373	734·41	19902·511	576·8056
·2	60·3187	368·64	7077·888	289·5298	·2	85·4515	739·84	20123·648	581·0703
·3	60·6328	372.49	7189·057	292·5536	·3	85·7656	745·29	20346·417	585·3507
·4	60·9470	376.36	7301·384	295·5931	·4	86·0798	750·76	20570·824	589·6469
·5	61·2612	380·25	7414·875	298·6483	·5	86·3940	756·25	20796·875	593·9587
·6	61·5753	384·16	7529·536	301·7192	·6	86·7081	761·76	21024·576	598·2863
·7	61·8895	388·09	7645·373	304·8060	·7	87·0223	767·29	21253·933	602·0295
·8	62·2036	392·04	7762·392	307·9082	·8	87·3364	772·84	21484·952	606·9885
·9	62·5178	396·01	7880·599	311·0252	·9	87·6506	778·41	21717·639	611·3632
20	62·8320	400	8000	314·1600	28	87·9648	784	21952	615·7536
·1	63·1461	404·01	8120·601	317·3094	·1	88·2789	789·61	22188·041	620·1596
·2	63·4603	408·04	8242·408	320·4746	·2	88·5931	795·24	22425·768	624·5814
·3	63·7744	412·09	8365·427	323·6554	·3	88·9072	800·89	22665·187	629·0190
·4	64·0886	416·16	8489·664	326·8520	·4	89·2214	806·56	22906·304	633·4722
·5	64·4028	420·25	8615·125	330·0643	·5	89·5356	812·25	23149·125	637·9411
·6	64·7161	424·36	8741·816	333·2923	·6	89·8497	817·96	23393·656	642·4257
·7	65·0311	428·49	8869·743	336·5360	·7	90·1639	823·69	23639·903	646·9261
·8	65·3452	432·64	8998·912	339·7954	·8	90·4780	829·44	23887·872	651·4421
·9	65·6594	436·81	9129·329	343·0705	·9	90·7922	835·21	24137·569	655·9739
21	65·9736	441	9261	346·3614	29	91·1064	841	24389	660·5214
·1	66·2870	445·21	9393·931	349·6679	·1	91·4205	846·81	24642·171	665·0845
·2	66·6012	449·44	9528·128	352·9901	·2	91·7347	852·64	24897·088	669·6634
·3	66·7916	453·69	9663·597	356·3281	·3	92·0488	858·49	25153·757	674·2580
·4	67·2930	457·96	9800·344	359·6817	·4	92·3630	864·36	25412·184	678·8683
·5	67·5444	462·25	9938·375	363·0511	·5	92·6772	870·25	25672·375	683·4943
·6	67·8585	466·56	10077·696	366·4362	·6	92·9913	876·16	25934·336	688·1360
·7	68·1727	470·89	10218·313	369·8370	·7	93·3055	882·09	26198·073	692·7934
·8	68·4868	475·24	10360·232	373·2534	·8	93·6196	888·04	26463·592	697·4666
·9	68·8010	479·61	10503·459	376·6856	·9	93·9338	894·01	26730·899	702·1554
22	69·1152	484	10648	380·1336	30	94·2480	900	27000	706·8600
·1	69·4293	488·41	10793·861	383·5972	·1	94·5621	906·01	27270·901	711·5802
·2	69·7435	492·84	10941·048	387·0765	·2	94·8763	912·04	27543·608	716·3162
·3	70·0576	497·29	11089·567	390·5751	·3	95·1904	918·09	27818·127	721·0678
·4	70·3718	501·76	11239·424	394·0823	·4	95·5046	924·16	28094·464	725·8352
·5	70·6860	506·25	11390·625	397·6087	·5	95·8188	930·25	28372·625	730·6183
·6	71·0001	510·76	11543·176	401·1509	·6	96·1329	936·36	28652·616	735·4171
·7	71·3143	515·29	11697·083	404·7087	·7	96·4471	942·49	28934·443	740·2316
·8	71·6284	519·84	11852·352	408·2823	·8	96·7612	948·64	29218·112	745·0618
·9	71·9426	524·41	12008·989	411·8716	·9	97·0754	954·81	29503·629	749·9077
23	72·2568	529	12167	415·4766	31	97·3896	961	29791	754·7694
·1	72·5709	533·61	12326·391	419·0972	·1	97·7037	967·21	30080·231	759·6467
·2	72·8851	538·24	12487·168	422·7336	·2	98·0179	973·44	30371·328	764·5397
·3	73·1992	542·89	12649·337	426·3858	·3	98·3320	979·69	30664·297	769·4485
·4	73·5134	547·56	12812·904	430·0536	·4	98·6452	985·96	30959·144	774·3729
·5	73·8276	552·25	12977·875	433·7371	·5	98·9604	992·25	31255·875	779·3131
·6	74·1417	556·96	13144·256	437·4363	·6	99·2745	998·56	31554·496	784·2689
·7	74·4559	561·69	13312·053	441·1511	·7	99·5887	1004·89	31855·013	789·2406
·8	74·7680	566·44	13481·272	444·8819	·8	99·9028	1011·24	32157·432	794·2278
·9	75·0882	571·21	13651·919	448·6283	·9	100·2170	1017·61	32461·759	799·2308
24	75·3984	576	13824	452·3904	32	100·5312	1024	32768	804·2496
·1	75·7125	580·81	13997·541	456·1681	·1	100·8453	1030·41	33076·161	809·2840
·2	76·0267	585·64	14172·488	459·9616	·2	101·1595	1036·84	38386·248	814·3341
·3	76·3408	590·49	14348·907	463·7708	·3	101·4736	1043·29	33698·267	819·3999
·4	76·6523	595·36	14526·784	467·5957	·4	101·7478	1049·76	34012·224	824·4815
·5	76·9692	600·25	14706·125	471·4363	·5	102·1020	1056·25	34328·125	829·5787
·6	77·2833	605·16	14886·936	475·2926	·6	102·4161	1062·76	34645·976	834·6917
·7	77·5975	610·09	15069·223	479·1646	·7	102·7303	1069·29	34965·783	839·8203
·8	77·9116	615·04	15252·992	483·0524	·8	103·0444	1075·84	35287·552	844·9647
·9	78·2258	620·01	15438·249	486·9558	·9	103·3586	1082·41	35611·289	850·1248

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
33	103·6728	1089	35937	855·3006	41	128·8056	1681	68921	1320·2574
·1	103·9869	1095·61	36264·691	860·4920	·1	129·1197	1689·21	69426·531	1326·7055
·2	104·3011	1102·24	36594·368	865·6992	·2	129·4339	1697·44	69934·528	1333·1693
·3	104·6151	1108·89	36926·037	870·9222	·3	129·7480	1705·69	70444·997	1339·6489
·4	104·9294	1115·56	37259·704	876·1608	·4	130·0622	1713·96	70957·944	1346·1441
·5	105·2436	1122·25	37595·375	881·4151	·5	130·3764	1722·25	71473·375	1352·6551
·6	105·5577	1128·96	37933·056	886·6851	·6	130·6905	1730·56	71991·296	1359·1818
·7	105·8719	1135·69	38272·753	891·9709	·7	131·0047	1738·89	72511·713	1365·7242
·8	106·1860	1142·44	38614·472	897·2723	·8	131·3188	1747·24	73034·632	1372·2822
·9	106·5002	1149·21	38958·219	902·5895	·9	131·6330	1755·61	73560·059	1378·8560
34	106·8144	1156	39304	907·9224	42	131·9472	1764	74088	1385·4456
·1	107·1285	1162·81	39651·821	913·2709	·1	132·2613	1772·41	74618·461	1392·0508
·2	107·4427	1169·64	40001·688	918·6352	·2	132·5755	1780·84	75151·448	1398·6717
·3	107·7568	1176·49	40353·607	924·0115	·3	132·8896	1789·29	75686·967	1405·3083
·4	108·0710	1183·36	40707·584	929·4109	·4	133·2038	1797·76	76225·024	1411·9607
·5	108·3852	1190·25	41063·625	934·8223	·5	133·5180	1806·25	76765·625	1418·6287
·6	108·6993	1197·16	41421·736	940·2494	·6	133·8321	1814·76	77308·776	1425·3125
·7	109·0135	1204·09	41781·923	945·6922	·7	134·1463	1823·29	77854·483	1432·0119
·8	109·3276	1211·04	42144·192	951·1508	·8	134·4604	1831·84	78402·752	1438·7271
·9	109·6418	1218·01	42508·549	956·6250	·9	134·7746	1840·41	78953·589	1445·4580
35	109·9560	1225	42875	962·1150	43	135·0888	1849	79507	1452·2046
·1	110·2701	1232·01	43243·551	967·6206	·1	135·4029	1857·61	80062·991	1458·9668
·2	110·5843	1239·04	43614·208	973·1420	·2	135·7171	1866·24	80621·568	1465·7448
·3	110·8984	1246·09	43986·977	978·6790	·3	136·0332	1874·89	81182·737	1472·5385
·4	111·2126	1253·16	44361·864	984·2318	·4	136·3454	1883·56	81746·504	1479·3460
·5	111·5268	1260·25	44738·875	989·8003	·5	136·6596	1892·25	82312·875	1486·1731
·6	111·8409	1267·36	45118·016	995·3845	·6	136·9737	1900·96	82881·856	1493·0139
·7	112·1551	1274·49	45499·293	1000·9843	·7	137·2879	1909·69	83453·453	1499·8705
·8	112·4692	1281·64	45882·712	1006·6000	·8	137·6020	1918·44	84027·672	1506·7427
·9	112·7834	1288·81	46268·279	1012·2313	·9	137·9162	1927·21	84604·519	1513·6287
36	113·0976	1296	46656	1017·8784	44	138·2304	1936	85184	1520·5344
·1	113·4117	1303·21	47045·881	1023·5411	·1	138·5445	1944·81	85766·121	1527·4537
·2	113·7259	1310·44	47437·928	1029·2195	·2	138·8587	1953·64	86350·888	1534·3888
·3	114·0400	1317·69	47832·147	1034·9131	·3	139·1728	1962·49	86938·307	1541·3396
·4	114·3542	1324·96	48228·544	1040·6235	·4	139·4870	1971·36	87528·384	1548·3061
·5	114·6684	1332·25	48627·125	1046·3491	·5	139·8012	1980·25	88121·125	1555·2883
·6	114·9825	1339·56	49027·896	1052·0904	·6	140·1153	1989·16	88716·536	1562·2862
·7	115·2967	1346·89	49430·863	1057·8474	·7	140·4295	1998·09	89314·623	1569·2998
·8	115·6108	1354·24	49836·032	1063·6200	·8	140·7436	2007·04	89915·392	1576·3292
·9	115·9250	1361·61	50243·409	1069·4084	·9	141·0578	2016·01	90518·849	1583·3742
37	116·2392	1369	50653	1075·2126	45	141·3720	2025	91125	1590·4350
·1	116·5533	1376·41	51064·811	1081·0324	·1	141·6861	2034·01	91733·851	1597·5114
·2	116·8675	1383·84	51478·848	1086·8679	·2	142·0003	2043·04	92345·408	1604·6036
·3	117·1816	1391·29	51895·117	1092·7191	·3	142·3144	2052·09	92959·677	1611·7114
·4	117·4958	1398·76	52313·624	1098·5862	·4	142·6286	2061·16	93576·664	1618·8350
·5	117·8100	1406·25	52734·375	1104·4687	·5	142·9428	2070·25	94196·375	1625·9743
·6	118·1241	1413·76	53157·376	1110·3671	·6	143·2569	2079·36	94818·816	1633·1293
·7	118·4383	1421·29	53582·633	1116·2811	·7	143·5711	2088·49	95443·993	1640·3020
·8	118·7524	1428·84	54010·152	1122·2109	·8	143·8852	2097·64	96071·912	1647·4864
·9	119·0666	1436·41	54439·939	1128·1564	·9	144·1994	2106·81	96702·579	1654·6885
38	119·3808	1444	54872	1134·1176	46	144·5136	2116	97336	1661·9064
·1	119·6949	1451·61	55306·341	1140·0946	·1	144·8277	2125·21	97972·181	1669·1399
·2	120·0091	1459·24	55742·968	1146·0870	·2	145·1419	2134·44	98611·128	1676·3891
·3	120·3232	1466·89	56181·887	1152·0954	·3	145·4560	2143·69	99252·847	1683·6541
·4	120·6374	1474·56	56623·104	1158·1194	·4	145·7702	2152·96	99897·344	1690·9347
·5	120·9516	1482·25	57066·625	1164·1591	·5	146·0844	2162·25	100544·625	1698·2311
·6	121·2657	1489·96	57512·456	1170·2146	·6	146·3985	2171·56	101194·696	1705·5432
·7	121·5799	1497·69	57960·603	1176·2857	·7	146·7127	2180·89	101847·563	1712·8710
·8	121·8940	1505·44	58411·072	1182·3725	·8	147·0268	2190·24	102503·232	1720·2144
·9	122·2082	1513·21	58863·869	1188·4651	·9	147·3410	2199·61	103161·709	1727·5736
39	122·5224	1521	59319	1194·5906	47	147·6552	2209	103823	1734·9486
·1	122·8365	1528·81	59776·471	1200·7273	·1	147·9693	2218·41	104487·111	1742·3392
·2	123·1507	1536·64	60236·288	1206·8770	·2	148·2835	2227·84	105154·048	1749·7455
·3	123·4648	1544·49	60698·457	1213·0424	·3	148·5976	2237·29	105823·817	1757·1675
·4	123·7790	1552·36	61162·984	1219·2243	·4	148·9118	2246·76	106496·424	1764·6045
·5	124·0932	1560·25	61629·875	1225·4203	·5	149·2260	2256·25	107171·875	1772·0587
·6	124·4073	1568·16	62099·136	1231·6328	·6	149·5401	2265·76	107850·176	1779·5279
·7	124·7215	1576·09	62570·773	1237·8610	·7	149·8543	2275·29	108531·333	1787·0127
·8	125·0356	1584·04	63044·792	1244·1210	·8	150·1684	2284·84	109215·352	1794·5133
·9	125·3498	1592·01	63521·199	1250·3646	·9	150·4826	2294·41	109902·239	1802·0296
40	125·6640	1600	64000	1256·6400	48	150·7968	2304	110592	1809·5616
·1	125·9781	1608·01	64481·201	1262·9310	·1	151·1109	2313·61	111284·641	1817·1092
·2	126·2923	1616·04	64964·808	1269·2388	·2	151·4251	2323·24	111980·168	1824·6726
·3	126·6064	1624·09	65450·827	1275·5602	·3	151·7392	2332·89	112678·587	1832·2518
·4	126·9206	1632·16	65939·264	1281·8984	·4	152·0534	2342·56	113379·904	1839·8466
·5	127·2348	1640·25	66430·125	1288·2523	·5	152·3676	2352·25	114084·125	1847·4571
·6	127·5489	1648·36	66923·416	1294·6219	·6	152·6817	2361·96	114791·256	1855·0833
·7	127·8631	1656·49	67419·143	1301·0071	·7	152·9959	2371·69	115501·303	1862·7253
·8	128·1772	1664·64	67917·312	1307·4082	·8	153·3100	2381·44	116214·272	1870·3829
·9	128·4914	1672·81	68417·929	1313·8249	·9	153·6242	2391·21	116930·169	1878·0563

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
49	153·9384	2401	117649	1885·7454	57	179·0712	3249	185193	2551·7646
·1	154·2525	2410·81	118370·771	1893·4501	·1	179·3853	3260·41	186169·411	2560·7260
·2	154·5667	2420·64	119095·488	1901·1706	·2	179·6995	3271·84	187149·248	2569·7031
·3	154·8808	2430·49	119823·157	1908·9068	·3	180·0136	3283·29	188132·517	2578·6959
·4	155·1950	2440·36	120553·784	1916·6587	·4	180·3278	3294·76	189119·224	2587·7045
·5	155·5092	2450·25	121287·375	1924·4263	·5	180·6420	3306·25	190109·375	2596·7287
·6	155·8233	2460·16	122023·936	1932·2096	·6	180·9561	3317·76	191102·976	2605·7687
·7	156·1375	2470·09	122763·473	1940·0086	·7	181·2703	3329·29	192100·033	2614·8243
·8	156·4516	2480·04	123505·992	1947·8234	·8	181·5844	3340·84	193100·552	2623·8957
·9	156·7658	2490·01	124251·499	1955·6538	·9	181·8986	3352·41	194104·539	2632·9828
50	157·0800	2500	125000	1963·5000	58	182·2128	3364	195112	2642·0856
·1	157·3941	2510·01	125751·501	1971·3618	·1	182·5269	3375·61	196122·941	2651·2046
·2	157·7083	2520·04	126506·008	1979·2394	·2	182·8411	3387·24	197137·368	2660·3382
·3	158·0224	2530·09	127263·527	1987·1326	·3	183·1552	3398·89	198155·287	2669·4882
·4	158·3366	2540·16	128024·064	1995·0416	·4	183·4694	3410·56	199176·704	2678·6538
·5	158·6508	2550·25	128787·625	2002·9663	·5	183·7836	3422·25	200201·625	2687·8351
·6	158·9649	2560·36	129554·216	2010·9067	·6	184·0977	3433·96	201230·056	2697·0321
·7	159·2791	2570·49	130323·843	2018·8628	·7	184·4119	3445·69	202262·003	2706·2449
·8	159·5932	2580·64	131096·512	2026·8346	·8	184·7260	3457·44	203297·472	2715·4733
·9	159·9074	2590·81	131872·229	2034·8770	·9	185·0402	3469·21	204336·469	2724·7175
51	160 2216	2601	132651	2042·8254	59	185·3544	3481	205379	2733·9774
·1	160·5357	2611·21	133432·831	2050·8443	·1	185·6685	3492·81	206425·071	2743·2529
·2	160·8499	2621·44	134217·728	2058·8784	·2	185·9827	3504·64	207474·688	2752·5442
·3	161·1640	2631·69	135005·697	2066·9293	·3	186·2696	3516·49	208527·857	2761·8512
·4	161·4782	2641·96	135796·744	2074·9953	·4	186·6110	3528·36	209584·584	2771·1739
·5	161·7924	2652·25	136590·875	2083·0771	·5	186·9252	3540·25	210644·875	2780·5123
·6	162·1065	2662·56	137388·096	2091·1746	·6	187·2393	3552·16	211708·736	2789·8664
·7	162·4207	2672·89	138188·413	2099·2878	·7	187·5535	3564·09	212776·173	2799·2362
·8	162·7348	2683·24	138991·832	2107·4166	·8	187·8676	3576·04	213847·192	2808·6218
·9	163·0490	2693·61	139798·359	2115·5612	·9	188·1818	3588·01	214921·799	2818·0230
52	163·3632	2704	140608	2123·7216	60	188·4960	3600	216000	2827·4400
·1	163·6773	2714·41	141420·761	2131·8976	·1	188·8101	3612·01	217081·801	2836·8726
·2	163·9915	2724·84	142236·648	2140·0893	·2	189·1243	3624·04	218167·208	2846·3210
·3	164·3056	2735·29	143055·667	2148·2967	·3	189·4384	3636·09	219256·227	2855·7850
·4	164·6198	2745·76	143877·824	2156·5199	·4	189·7526	3648·16	220348·864	2865·2648
·5	164·9340	2756·25	144703·125	2164·7587	·5	190·0668	3660·25	221445·125	2874·7603
·6	165·2481	2766·76	145531·576	2173·0133	·6	190·3809	3672·36	222545·016	2884·2615
·7	165·5623	2777·29	146363·183	2181·2835	·7	190·6951	3684·49	223648·543	2893·7984
·8	165·8764	2787·84	147197·952	2189·5695	·8	191·0092	3696·64	224755·712	2903·3410
·9	166·1906	2798·41	148035·889	2197·8712	·9	191·3234	3708·81	225866·529	2912·8993
53	166·5048	2809	148877	2206·1886	61	191·6376	3721	226981	2922·4784
·1	166·8189	2819·61	149721·291	2214·5216	·1	191·9517	3733·21	228099·131	2932·0631
·2	167·1331	2830·24	150568·768	2222·8704	·2	192·2659	3745·44	229220·928	2941·6685
·3	167·4472	2940·89	151419·437	2231·2350	·3	192·5800	3757·69	230346·397	2951·2897
·4	167·7614	2851·56	152273·304	2239·6152	·4	192·8942	3769·96	231475·544	2960·9265
·5	168·0756	2862·25	153130·375	2248·0111	·5	193·2084	3782·25	232608·375	2970·5791
·6	168·3897	2872·96	153990·656	2256·4227	·6	193·5225	3794·56	233744·896	2980·2474
·7	168·7039	2883·69	154854·153	2264·8701	·7	193·8367	3806·89	234885·113	2989·9314
·8	169·0180	2894·44	155720·872	2273·2931	·8	194·1508	3819·24	236029·032	2999·6300
·9	169·3322	2905·21	156590·819	2281·7519	·9	194·4650	3831·61	237176·659	3009·3464
54	169·6464	2916	157464	2290·2264	62	194·7792	3844	238328	3019·0776
·1	169·9605	2926·81	158340·421	2298·7165	·1	195·0933	3856·41	239483·061	3028·8244
·2	170·2747	2937·64	159220·088	2307·2224	·2	195·4075	3868·84	240641·848	3038·5869
·3	170·5888	2948·49	160103·007	2315·7440	·3	195·7216	3881·29	241804·367	3048·3651
·4	170·9030	2959·36	160989·184	2324·2813	·4	196·0358	3893·76	242970·624	3058·1591
·5	171·2172	2970·25	161878·625	2332·8343	·5	196·3500	3906·25	244140·625	3067·9687
·6	171·5313	2981·16	162771·336	2341·4030	·6	196·6641	3918·76	245314·376	3077·7941
·7	171·8455	2992·09	163667·323	2349·9874	·7	196·9783	3931·29	246491·883	3087·6341
·8	172·1596	3003·04	164566·592	2358·5876	·8	197·2924	3943·84	247673·152	3097·4919
·9	172·4738	3014·01	165469·149	2367·2034	·9	197·6066	3956·41	248858·189	3107·3644
55	172·7880	3025	166375	2375·8350	63	197·9208	3969	250047	3117·2526
·1	173·1021	3036·01	167284·151	2384·4822	·1	198·2349	3981·61	251239·591	3127·1564
·2	173·4163	3047·04	168196·608	2393·1452	·2	198·5491	3994·24	252435·968	3137·0758
·3	173·7304	3058·09	169112·377	2401·8238	·3	198·8632	4006·89	253636·137	3147·0114
·4	174·0446	3069·16	170031·464	2410·5182	·4	199·1774	4019·56	254840·104	3156·9664
·5	174·3588	3080·25	170953·875	2419·2283	·5	199·4916	4032·25	256047·875	3166·9291
·6	174·6729	3091·36	171879·616	2427·9541	·6	199·8057	4044·96	257259·456	3176·9115
·7	174·9871	3102·49	172808·693	2436·6956	·7	200·1199	4057·69	258474·853	3186·9097
·8	175·3012	3113·64	173741·112	2445·4528	·8	200·4340	4070·44	259694·072	3196·9235
·9	175·6154	3124·81	174676·879	2454·2257	·9	200·7482	4083·21	260917·119	3206·9531
56	175·9296	3136	175616	2463·0144	64	201·0624	4096	262144	3216·9984
·1	176·2437	3147·21	176558·481	2471·8187	·1	201·3765	4108·81	263374·721	3227·0593
·2	176·5579	3158·44	177504·328	2480·6387	·2	201·6907	4121·64	264609·288	3237·1360
·3	176·8720	3169·69	178453·547	2489·4745	·3	202·0048	4134·49	265847·707	3247·2284
·4	177·1862	3180·96	179406·144	2498·3259	·4	202·3190	4147·36	267089·984	3257·3365
·5	177·5004	3192·25	180362·125	2507·1931	·5	202·6332	4160·25	268336·125	3267·4603
·6	177·8145	3203·56	181321·496	2516·0760	·6	202·9473	4173·16	269586·136	3277·5998
·7	178·1287	3214·89	182284·263	2524·9736	·7	203·2615	4186·09	270840·023	3287·7550
·8	178·4428	3226·24	183250·432	2533·8868	·8	203·5756	4199·04	272097·792	3297·9260
·9	178·7570	3237·61	184220·009	2542·8158	·9	203·8898	4212·01	273359·449	3308·1126

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
65	204·2040	4225	274625	3318·3150	73	229·3368	5329	389017	4185·3966
·1	204·5181	4238·01	275894·451	3328·5340	·1	229·6509	5343·61	390617·891	4196·8712
·2	204·8323	4251·04	277167·808	3338·7668	·2	229·9651	5358·24	392223·168	4208·3614
·3	205·1464	4264·09	278445·077	3349·0162	·3	230·2792	5372·89	393832·837	4219·8678
·4	205·4606	4277·16	279726·264	3359·2814	·4	230·5934	5387·56	395446·904	4231·3896
·5	205·7748	4290·25	281011·375	3369·5623	·5	230·9076	5402·25	397065·375	4242·9271
·6	206·0889	4303·36	282300·416	3379·8589	·6	231·2217	5416·96	398688·256	4254·4803
·7	206·4031	4316·49	283593·393	3390·1712	·7	231·5359	5431·69	400315·553	4266·0493
·8	206·7172	4329·64	284890·312	3400·4992	·8	231·8500	5446·44	401947·272	4277·6339
·9	207·0314	4342·81	286191·179	3410·8429	·9	232·1642	5461·21	403583·419	4289·2343
66	207·3456	4356	287496	3421·2024	74	232·4784	5476	405224	4300·8504
·1	207·6597	4369·21	288804·781	3431·5775	·1	232·7925	5490·81	406869·021	4312·4821
·2	207·9739	4382·44	290117·528	3441·9633	·2	233·1067	5505·64	408518·488	4324·1296
·3	208·2880	4395·69	291434·247	3452·3749	·3	233·4208	5520·49	410172·407	4335·7928
·4	208·6022	4408·96	292754·944	3462·7971	·4	233·7350	5535·36	411830·784	4347·4717
·5	208·9164	4422·25	294079·625	3473·2351	·5	234·0492	5550·25	413493·625	4359·1663
·6	209·2305	4435·56	295408·296	3483·6888	·6	234·3633	5565·16	415160·936	4370·8766
·7	209·5447	4448·89	296740·963	3494·1640	·7	234·6775	5580·09	416832·723	4382·6026
·8	209·8588	4462·24	298077·632	3504·6432	·8	234·9916	5595·04	418508·992	4394·3448
·9	210·1730	4475·61	299418·309	3515·1430	·9	235·3058	5610·01	420189·749	4406·1018
67	210·4872	4489	300763	3525·6606	75	235·6200	5625	421875	4417·8750
·1	210·8013	4502·41	302111·711	3536·1928	·1	235·9341	5640·01	423564·751	4429·6638
·2	211·1155	4515·84	303464·448	3546·7407	·2	236·2483	5655·04	425259·008	4441·4684
·3	211·4296	4529·29	304821·217	3557·3043	·3	236·5624	5670·09	426957·777	4453·2886
·4	211·7438	4542·76	306182·024	3567·8837	·4	236·8766	5685·16	428661·064	4465·1246
·5	212·0580	4556·25	307546·875	3578·4787	·5	237·1908	5700·25	430368·875	4476·9763
·6	212·3721	4569·76	308915·776	3589·0895	·6	237·5049	5715·36	432081·216	4488·8437
·7	212·6863	4583·29	310288·733	3599·7159	·7	237·8191	5730·49	433798·093	4500·7268
·8	213·0004	4596·84	311665·752	3610·3581	·8	238·1332	5745·64	435519·512	4512·6256
·9	213·3146	4610·41	313046·839	3621·0160	·9	238·4474	5760·81	437245·479	4524·5401
68	213·6288	4624	314432	3631·6896	76	238·7616	5776	438976	4536·4704
·1	213·9429	4637·61	315821·241	3642·3788	·1	239·0757	5791·21	440711·081	4548·4163
·2	214·2571	4651·24	317214·568	3653·0838	·2	239·3899	5806·44	442450·728	4560·3787
·3	214·5712	4664·89	318611·987	3663·8040	·3	239·7040	5821·69	444194·947	4572·3553
·4	214·8854	4678·56	320013·504	3674·5410	·4	240·0182	5836·96	445943·744	4584·3583
·5	215·1996	4692·25	321419·125	3685·2931	·5	240·3324	5852·25	447697·125	4596·3571
·6	215·5137	4705·96	322828·856	3696·0060	·6	240·6465	5867·56	449455·096	4608·3816
·7	215·8279	4719·69	324242·703	3706·8445	·7	240·9607	5882·89	451217·663	4620·4218
·8	216·1420	4733·44	325660·672	3717·6437	·8	241·2748	5898·24	452984·832	4632·4776
·9	216·4562	4747·21	327082·769	3728·4587	·9	241·5887	5913·61	454756·609	4644·5492
69	216·7704	4761	328509	3739·2894	77	241·9032	5929	456533	4656·6366
·1	217·0845	4774·81	329939·371	3750·1357	·1	242·2173	5944·41	458314·011	4668·7396
·2	217·3987	4788·64	331373·888	3760·9978	·2	242·5315	5959·84	460099·648	4680·8583
·3	217·7128	4802·49	332812·557	3771·8756	·3	242·8456	5975·29	461889·917	4692·9927
·4	218·0270	4816·36	334255·384	3782·7691	·4	243·1598	5990·76	463684·824	4705·1429
·5	218·3412	4830·25	335702·375	3793·6783	·5	243·4740	6006·25	465484·375	4717·3087
·6	218·6553	4844·16	337153·536	3804·6032	·6	243·7881	6021·76	467288·576	4729·4903
·7	218·9695	4858·09	338608·873	3815·5438	·7	244·1023	6037·29	469097·433	4741·6875
·8	219·2836	4872·04	340068·392	3826·5002	·8	244·4164	6052·84	470910·952	4753·9605
·9	219·5978	4886·01	341532·099	3837·4722	·9	244·7306	6068·41	472729·139	4766·1292
70	219·9120	4900	343000	3848·4600	78	245·0448	6084	474552	4778·3736
·1	220·2261	4914·01	344472·101	3859·4952	·1	245·3589	6099·61	476379·541	4790·6336
·2	220·5403	4928·04	345948·408	3870·4826	·2	245·6731	6115·24	478211·768	4802·9094
·3	220·8544	4942·09	347428·927	3881·5174	·3	245·9872	6130·89	480048·687	4815·2010
·4	221·1686	4956·16	348913·664	3892·5680	·4	246·3014	6146·56	481890·304	4827·5082
·5	221·4828	4970·25	350402·625	3903·6343	·5	246·6156	6162·25	483736·625	4839·8311
·6	221·7969	4984·36	351895·816	3914·7163	·6	246·9297	6177·96	485587·656	4852·1697
·7	222·1111	4998·49	353393·243	3925·8140	·7	247·2439	6193·69	487443·403	4864·5241
·8	222·4252	5012·64	354894·912	3936·9274	·8	247·5580	6209·44	489303·872	4876·8973
·9	222·7394	5026·81	356400·829	3948·0565	·9	247·8722	6225·21	491169·069	4889·2799
71	223·0536	5041	357911	3959·2014	79	248·1864	6241	493039	4901·6814
·1	223·3677	5055·21	359425·431	3970·3619	·1	248·5005	6256·81	494913·671	4914·0985
·2	223·6819	5069·44	360944·128	3981·5381	·2	248·8147	6272·64	496793·088	4926·5314
·3	223·9960	5083·69	362467·097	3992·7301	·3	249·1288	6288·49	498677·257	4938·9820
·4	224·3102	5097·96	363994·344	4003·9378	·4	249·4430	6304·36	500566·184	4951·4443
·5	224·6244	5112·25	365525·875	4015·1611	·5	249·7572	6320·25	502459·875	4963·9243
·6	224·9385	5126·56	367061·696	4026·4002	·6	250·0713	6336·16	504358·336	4976·4840
·7	225·2527	5140·89	368601·813	4037·6550	·7	250·3855	6352·09	506261·573	4988·9314
·8	225·5668	5155·24	370146·232	4048·9254	·8	250·6996	6368·04	508169·592	5001·4586
·9	225·8810	5169·61	371694·959	4060·2116	·9	251·0138	6384·01	510082·399	5014·0014
72	226·1952	5184	373248	4071·5136	80	251·3280	6400	512000	5026·5600
·1	226·5093	5198·41	374805·361	4082·8332	·1	251·6421	6416·01	513922·401	5039·1342
·2	226·8235	5212·84	376367·048	4094·1645	·2	251·9563	6432·04	515849·608	5051·7242
·3	227·1376	5227·29	377933·067	4105·5125	·3	252·2704	6448·09	517781·627	5064·3298
·4	227·4518	5241·76	379503·424	4116·8783	·4	252·5846	6464·16	519718·464	5076·9552
·5	227·7660	5256·25	381078·125	4128·2587	·5	252·8988	6480·25	521660·125	5089·5883
·6	228·0801	5270·76	382657·176	4139·6524	·6	253·2129	6496·36	523606·616	5102·2411
·7	228·3943	5285·29	384240·583	4151·0667	·7	253·5271	6512·49	525557·943	5114·9096
·8	228·7084	5299·84	385828·352	4162·4943	·8	253·8412	6528·64	527514·112	5127·5938
·9	229·0226	5314·41	387420·489	4173·9376	·9	254·1554	6544·81	529475·129	5140·2937

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
81	254·4696	6561	531441	5153·0094	89	279·6024	7921	704969	6221·1534
·1	254·7837	6577·21	533411·731	5165·7407	·1	279·9165	7938·81	707347·971	6235·1413
·2	255·0979	6593·44	535387·328	5178·4877	·2	280·2307	7956·64	709732·288	6249·1450
·3	255·4120	6609·69	537367·797	5191·2505	·3	280·5448	7974·49	712121·957	6263·1044
·4	255·7262	6625·96	539353·144	5204·0285	·4	280·8590	7992·36	714516·984	6277·1995
·5	256·0404	6642·25	541343·375	5216·8231	·5	281·1732	8010·25	716917·375	6291·2035
·6	256·3545	6658·56	543338·496	5229·6330	·6	281·4873	8028·16	719323·136	6305·3168
·7	256·6687	6674·89	545338·513	5242·4586	·7	281·8025	8046·09	721734·273	6319·3990
·8	256·9828	6691·24	547343·432	5255·2998	·8	282·1156	8064·04	724150·792	6333·4970
·9	257·2970	6707·61	549353·259	5268·1568	·9	282·4298	8082·01	726572·699	6347·6813
82	257·6112	6724	551368	5281·0206	90	282·7440	8100	729000	6361·7400
·1	257·9253	6740·41	553387·661	5293·9180	·1	283·0581	8118·01	731432·701	6375·8850
·2	258·2395	6756·84	555412·248	5306·8221	·2	283·3723	8136·04	733870·808	6390·0458
·3	258·5536	6773·29	557441·767	5319·7439	·3	283·6864	8154·09	736314·327	6404·2222
·4	258·8040	6789·76	559476·224	5332·6775	·4	284·0006	8172·16	738763·264	6418·4144
·5	259·1820	6806·25	561515·625	5345·6287	·5	584·3148	8190·25	741217·625	6432·6223
·6	259·4961	6822·76	563559·976	5358·5957	·6	284·6289	8208·36	743677·416	6446·8474
·7	259·8103	6839·29	565609·283	5371·5983	·7	284·9431	8226·49	746142·643	6461·0852
·8	260·1244	6855·84	567663·552	5384·5762	·8	285·2572	8244·64	748613·312	6475·3402
·9	260·4386	6872·41	569722·789	5397·5908	·9	285·5714	8262·81	751089·429	6489·6109
83	260·7528	6889	571787	5410·6206	91	285·8856	8281	753571	6503·8974
·1	261·0669	6905·61	573856·191	5423·6060	·1	286·1997	8299·21	756058·031	6518·1995
·2	261·3811	6922·24	575930·368	5436·7272	·2	286·5139	8317·44	758550·528	6532·5173
·3	261·6952	6938·89	578009·537	5449·8042	·3	286·8290	8335·69	761048·497	6546·8909
·4	262 0094	6955·56	580093·704	5462·8068	·4	287·1422	8353·96	763551·944	6561·2081
·5	262·3236	6972·25	582182·875	5476·0051	·5	287·4564	8372·25	766060·875	6575·6651
·6	262·6376	6988·96	584277·056	5489·1291	·6	287·7705	8390·56	768575·296	6589·9458
·7	262·9519	7005·69	586376·253	5502·2689	·7	288·0847	8408·89	771095·213	6604·3222
·8	263·2640	7022·44	588480·472	5515·4243	·8	288·3988	8427·24	773620·632	6618·7542
·9	263·5802	7039·21	590589·719	5528·5958	·9	288·7130	8445·61	776151·559	6633·1820
84	263·8944	7056	592704	5541·7824	92	289·0272	8464	778688	6647·6256
·1	264·2085	7072·81	594823·321	5554·9849	·1	289·3413	8482·41	781229·961	6662·0848
·2	264·5227	7089·64	596947·688	5568·2032	·2	289·6555	8500·84	783777·448	6676·5597
·3	264·8368	7106·49	599077·107	5581·4372	·3	289·9696	8519·29	786330·467	6691·0161
·4	265·1510	7123·36	601211·584	5594·6869	·4	290·2838	8537·76	788889·024	6705·5567
·5	265·4652	7140·25	603351·125	5607·9523	·5	290·5980	8556·25	791453·125	6720·0787
·6	265·7793	7157·16	605495·736	5621·2334	·6	290·9121	8574·76	794022·776	6734·6165
·7	266·0935	7174·09	607645·423	5634·5682	·7	291·2263	8593·29	796597·983	6749·1699
·8	266·4076	7191·04	609800·192	5647·8428	·8	291·5404	8611·84	799178·752	6763·7391
·9	266·7218	7208·01	611960·049	5661·1710	·9	291·8546	8630·41	801765·089	6778·3240
85	267·0360	7225	614125	5674·5150	93	292·1688	8649	804357	6792·9246
·1	267·3501	7242·01	616295·051	5687·8746	·1	292·4829	8667·61	806954·491	6807·5408
·2	267·6643	7259·04	618470·208	5701·2500	·2	292·7971	8686·24	809557·568	6822·1730
·3	267·9784	7276·09	620650·477	5714·6410	·3	293·1112	8704·89	812166·237	6836·8206
·4	268·2926	7293·16	622835·864	5728·0478	·4	293·4254	8723·56	814780·504	6851·4840
·5	268·6068	7310·25	625026·375	5741·4703	·5	293·7396	8742·25	817400·375	6866·1631
·6	268·9209	7327·36	627222·016	5754·9085	·6	294·0537	8760·96	820025·856	6880·8579
·7	269·2351	7344·49	629422·793	5768·3624	·7	294·3679	8779·69	822656·953	6895·5685
·8	269·5492	7361·64	631628·712	5781·8320	·8	294·6820	8798·44	825293·672	6910·2947
·9	269·8634	7378·81	633839·779	5795·3173	·9	294·9962	8817·21	827936·019	6925·0367
86	270·1776	7396	636056	5808·8184	94	295·3104	8836	830584	6939·7944
·1	270·4917	7413·21	638277·381	5822·3351	·1	295·6245	8854·81	833237·621	6954·5677
·2	270·8059	7430·44	640503·928	5835·8675	·2	295·9387	8873·64	835896·888	6969·3568
·3	271·1200	7447·69	642735·647	5849·4157	·3	296·2436	8892·49	838561·807	6984·1614
·4	271·4342	7464·96	644972·544	5862·9795	·4	296·5670	8911·36	841232·384	6998·0827
·5	271·7484	7482·25	647214·625	5876·5591	·5	296·8812	8930·25	843908·625	7013·8183
·6	272·0665	7499·56	649461·896	5890·1541	·6	297·1953	8949·16	846590·536	7028·6702
·7	272·3767	7516·89	651714·363	5903·7654	·7	297·5095	8968·09	849278·123	7043·5025
·8	272·6908	7534·24	653972·032	5917·3920	·8	297·8236	8987·04	851971·392	7058·4180
·9	273·0050	7551·61	656234·909	5931·0344	·9	298·1378	9006·01	854670·349	7073·3202
87	273·3192	7569	658503	5944·6926	95	298·4520	9025	857375	7088·2350
·1	273·6333	7586·41	660776·311	5958·3644	·1	298·7661	9044·01	860085·351	7103·1654
·2	273·9875	7603·84	663054·848	5972·0559	·2	299·0723	9063·04	862801·408	7118·1116
·3	274·2616	7621·29	665338·617	5985·7691	·3	299·3944	9082·09	865523·177	7133·0734
·4	274·5758	7638·76	667627·624	5999·4821	·4	299·7086	9101·16	868250·664	7148·0510
·5	274·8900	7656·25	669921·875	6013·2187	·5	300·0228	9120·25	870983·875	7163·0443
·6	275·2041	7673·76	672221·376	6026·9711	·6	300·3369	9139·36	873722·816	7178·0533
·7	275·5186	7691·29	674526·133	6040·7391	·7	300·6511	9158·49	876467·493	7193·0780
·8	275·8324	7708·84	676836·152	6054·5149	·8	300·9652	9177·64	879217·912	7208·1184
·9	276·1466	7726·41	679151·439	6068·3224	·9	301·2794	9196·81	881974·079	7223·1745
88	276·4608	7744	681472	6082·1376	96	301·5936	9216	884736	7238·2464
·1	276·7749	7761·61	683797·841	6095·9684	·1	301·9077	9235·21	887503·681	7253·3339
·2	277·0891	7779·24	686128·968	6109·8150	·2	302·2219	9254·44	890277·128	7268·4371
·3	277·4032	7796·89	688465·387	6123·6774	·3	302·5360	9273·69	893056·347	7283·5561
·4	277·7174	7814·56	690807·104	6137·5554	·4	302·8502	9292·96	895841·344	7298·6907
·5	278·0316	7832·25	693154·122	6151·4491	·5	303·1644	9312·25	898632·125	7313·8411
·6	278·3457	7849·96	695506·456	6165·3585	·6	303·4785	9331·56	901428·696	7329·0072
·7	278·6599	7867·69	697864·103	6179·2837	·7	303·7927	9350·89	904231·063	7344·1890
·8	278·9750	7885·44	700227·072	6193·2245	·8	304·1068	9370·24	907039·232	7359·3864
·9	279·2882	7903·21	702595·369	6207·1811	·9	304·4210	9389·61	909853·209	7374·5996

Diam.	Circum.	Square.	Cube.	Area.	Diam.	Circum.	Square.	Cube.	Area.
97	304·7352	9409	912673	7389·8286	·6	309·7617	9721·96	958585·256	7635·6273
·1	305·0493	9428·41	915498·611	7405·0732	·7	310·0759	9741·69	961504·803	7651·1933
·2	305·3635	9447·84	918330·048	7420·3335	·8	310·3960	9761·44	964430·272	7666·6349
·3	305·6776	9467·29	921167·317	7435·6095	·9	310·7042	9781·21	967361·669	7682·1623
·4	305·9918	9486·76	924010·424	7450·9013	99	311·0184	9801	970299	7697·7054
·5	306·3060	9506·25	926859·375	7466·2087	·1	311·3325	9820·81	973242·271	7713·2641
·6	306·6201	9525·76	929714·176	7481·5319	·2	311·6467	9840·64	976191·488	7728·8386
·7	306·9363	9545·29	932574·833	7496·8707	·3	311·9608	9860·49	979146·657	7744·4288
·8	307·2484	9564·84	935441·352	7512·2253	·4	312·2750	9880·36	982107·784	7760·0347
·9	307·5626	9584·41	938313·739	7527·5956	·5	312·5892	9900·25	985074·875	7775·6563
98	307·8768	9604	941192	7542·9816	·6	312·9033	9920·16	988047·936	7791·2936
·1	308·1909	9623·61	944076·141	7558·3832	·7	313·2175	9940·09	991026·973	7806·9466
·2	308·5051	9643·24	946966·168	7573·8006	·8	313·5116	9960·04	994011·992	7822·6154
·3	308·8192	9662·89	949862·087	7589·2338	·9	313·8458	9980·01	997002·999	7838·2998
·4	309·1334	9682·56	952763·904	7604·6826	100	314·1600	10000	1000000	7854·0000
·5	309·4476	9702·25	955671·625	7620·1471					

A Table *of the Length of Circular Arcs, radius being unity.*

Degree.	Length.	Degree.	Length.	Min.	Length.	Sec.	Length.
1	0·0174533	60	1·0471976	1	0·0002909	1	0·000048
2	0·0349066	70	1·2217305	2	0·0005818	2	0·000097
3	0·0523599	80	1·3962634	3	0·0008727	3	0·0000145
4	0·0698132	90	1·5707963	4	0·0011636	4	0·0000194
5	0·0872665	100	1·7453293	5	0·0014544	5	0·0000242
6	0·1047198	120	2·0943951	6	0·0017453	6	0·0000291
7	0·1221730	150	2·6179939	7	0·0020362	7	0·0000339
8	0·1396263	180	3·1415927	8	0·0023271	8	0·0000388
9	0·1570796	210	3·6651914	9	0·0026180	9	0·0000436
10	0·1745329	240	4·1887902	10	0·0029089	10	0·0000485
20	0·3490659	270	4·7123890	20	0·0058178	20	0·0000970
30	0·5235988	300	5·2359878	30	0·0087266	30	0·0001454
40	0·6981317	330	5·7595865	40	0·0116355	40	0·0001939
50	0·8726646	360	6·2831853	50	0·0145444	50	0·0002424

Required the length of a circular arc of 37° 42′ 58″?

30° = 0·5235988
7° = 0·1221730
40′ = 0·0116355
2′ = 0·0020368
50″ = 0·0002424
8″ = 0·0000388
The length 0·6582703 required in terms of the radius.

1207° Fahrenheit = 1° of Wedgewood's pyrometer. Iron melts at about 166° Wedgewood; 200362° Fahrenheit.

Sound passes in air at a velocity of 1142 feet a second, and in water at a velocity of 4700 feet.

Freezing water gives out 140° of heat, and may be cooled as low as 20°. All solids absorb heat when becoming a fluid, and the quantity of heat that renders a substance fluid is termed its caloric of fluidity, or latent heat. Fluids in vacuo boil with 124° less heat, than when under the pressure of the atmosphere.

AREAS *of the Segments and Zones of a Circle of which the* DIAMETER *is Unity, and supposed to be divided into* 1000 *equal parts.*

Height.	Area of Segment.	Area of Zone.	Height.	Area of Segment.	Area of Zone.	Height.	Area of Segment.	Area of Zone.
·001	·000042	·001000	·051	·015119	·050912	·101	·041476	·100309
·002	·000119	·002000	·052	·015561	·051906	·102	·042080	·101288
·003	·000219	·003000	·053	·016007	·052901	·103	·052687	·102267
·004	·000337	·004000	·054	·016457	·053895	·104	·043296	·103246
·005	·000470	·005000	·055	·016911	·054890	·105	·043908	·104223
·006	·000618	·006000	·056	·017369	·055883	·106	·044522	·105201
·007	·000779	·007000	·057	·017831	·056877	·107	·045139	·106178
·008	·000951	·008000	·058	·018296	·057870	·108	·045759	·107155
·009	·001135	·009000	·059	·018766	·058863	·109	·046381	·108131
·010	·001329	·010000	·060	·019239	·059856	·110	·047005	·109107
·011	·001533	·011000	·061	·019716	·060849	·111	·047632	·110082
·012	·001746	·011999	·062	·020196	·061841	·112	·048262	·111057
·013	·001968	·012999	·063	·020680	·062833	·113	·048894	·112031
·014	·002199	·013998	·064	·021168	·063825	·114	·049528	·113004
·015	·002438	·014998	·065	·021659	·064817	·115	·050165	·113978
·016	·002685	·015997	·066	·022154	·065807	·116	·050804	·114951
·017	·002940	·016997	·067	·022652	·066799	·117	·051446	·115924
·018	·003202	·017996	·068	·023154	·067790	·118	·052090	·116896
·019	·003471	·018996	·069	·023659	·068782	·119	·052736	·117867
·020	·003748	·019995	·070	·024168	·069771	·120	·053385	·118838
·021	·004031	·020994	·071	·024680	·070761	·121	·054036	·119809
·022	·004322	·021993	·072	·025195	·071751	·122	·054689	·120779
·023	·004618	·022992	·073	·025714	·072740	·123	·055345	·121748
·024	·004921	023991	·074	·026236	·073729	·124	·056003	·122717
·025	·005230	·024990	·075	·026761	·074718	·125	·056663	·123686
·026	·005546	·025989	·076	·027289	·075707	·126	·057326	·124654
·027	·005867	·026987	·077	·027821	·076695	·127	·057991	·125621
·028	·006194	·027986	·078	·028356	·077683	·128	·058658	·126588
·029	·006527	·028984	·079	·028894	·078670	·129	·059327	·127555
·030	·006865	·029982	·080	·029435	·079658	·130	·059999	·128521
·031	·007209	·030980	·081	·029979	·080645	·131	·060672	·129486
·032	·007558	·031978	·082	·030526	·081631	·132	·061348	·130451
·083	·007913	·032976	·083	·031076	·082618	·133	·062026	·131415
·034	·008273	·033974	·084	·031629	·083604	·134	·062707	·132379
·035	·008638	·034972	·085	·032186	·084589	·135	·063389	·133342
·036	·009008	·035969	·086	·032745	·085574	·136	·064074	·134304
·037	·009383	·036967	·087	·033307	·086559	·137	·064760	·135266
·038	·009763	·037965	·088	·033872	·087544	·138	·065449	·136228
·039	·010148	·038962	·089	·034441	·088528	·139	·066140	·137189
·040	·010537	·039958	·090	·035011	·089512	·140	·066833	·138149
·041	·010931	·040954	·091	·035585	·090496	·141	·067528	·139109
·042	·011330	·041951	·092	·036162	·091479	·142	·068225	·140068
·043	·011734	·042947	·093	·036741	·092461	·143	·068924	·141026
·044	·012142	·043944	·094	·037323	·093444	·144	·069625	·141984
·045	·012554	·044940	·095	·037909	·094426	·145	·070328	·142942
·046	·012971	·045935	·096	·038496	·095407	·146	·071033	·143898
·047	·018392	·046931	·097	.039087	·096388	·147	·071741	·144854
·048	·013818	·047927	·098	.039680	·097369	·148	·072450	·145810
·049	·014247	·048922	·099	.040276	·098350	·149	·073161	·146765
·050	·014681	·049917	·100	.040875	·099330	·150	·073874	·147719

Height.	Area of Seg.	Area of Zone.	Height.	Area of Seg.	Area of Zone.	Height.	Area of Seg.	Area of Zone.
·151	·074589	·148674	·206	·116650	·200915	·261	·163140	·248608
·152	·075306	·149625	·207	·117460	·200924	·262	·164019	·249461
·153	·076026	·150578	·208	·118271	·201835	·263	·164899	·250212
·154	·076747	·151530	·209	·119083	·202744	·264	·165780	·251162
·155	·077469	·152481	·210	·119897	·203652	·265	·166663	·252011
·156	·078194	·153431	·211	·120712	·204559	·266	·167546	·252851
·157	·078921	·154381	·212	·121529	·205465	·267	·168430	·253704
·158	·079649	·155330	·213	·122347	·206370	·268	·169315	·254549
·159	·080380	·156278	·214	·123167	·207274	·269	·170202	·255392
·160	·081112	·157226	·215	·123988	·208178	·270	·171080	·256235
·161	·081846	·158173	·216	·124810	·209080	·271	·171978	·257075
·162	·082582	·159119	·217	·125634	·209981	·272	·172867	·257915
·163	·083320	·160065	·218	·126459	·210882	·273	·173758	·258754
·164	·084059	·161010	·219	·127285	·211782	·274	·174649	·259591
·165	·084801	·161954	·220	·128113	·212680	·275	·175542	·260427
·166	·085544	·162898	·221	·128942	·213577	·276	·176435	·261261
·167	·086289	·163841	·222	·129773	·214474	·277	·177330	·262094
·168	·087036	·165784	·223	·130605	·215369	·278	·178225	·262926
·169	·087785	·165725	·224	·131438	·216264	·279	·179122	·263757
·170	·088535	·166666	·225	·132272	·217157	·280	·180019	·264586
·171	·089287	·167606	·226	·133108	·218050	·281	·180918	·265414
·172	·090041	·168549	·227	·133945	·218941	·282	·181817	·266240
·173	·090797	·160484	·228	·134784	·219832	·283	·182718	·267065
·174	·091554	·170422	·229	·135624	·220721	·284	·183619	·267889
·175	·092313	·171359	·230	·136465	·221610	·285	·184521	·268711
·176	·093074	·172295	·231	·137307	·222497	·286	·185425	·269532
·177	·093836	·173231	·232	·138150	·223354	·287	·186329	·270352
·178	·094601	·174166	·233	·138995	·224269	·288	·187234	·271170
·179	·095366	·175100	·234	·139841	·225153	·289	·188140	·271987
·180	·096134	·176033	·235	·140688	·226036	·290	·189047	·272802
·181	·096908	·176966	·236	·141537	·226919	·291	·189955	·273616
·182	·097674	·177897	·237	·142387	·227800	·292	·190864	·274428
·183	·098447	·178828	·238	·143238	·228680	·293	·191775	·275239
·184	·099221	·179759	·239	·144091	·229559	·294	·192684	·276049
·185	·099997	·180688	·240	·144944	·230439	·295	·193596	·276857
·186	·100774	·181617	·241	·145799	·231313	·296	·194509	·277664
·187	·101553	·182545	·242	·146655	·232189	·297	·195422	·278469
·188	·102334	·183472	·243	·147512	·233063	·298	·196337	·279273
·189	·103116	·184398	·244	·148371	·233937	·299	·197252	·280075
·190	·103900	·185323	·245	·149230	·234809	·300	·198168	·280876
·191	·104685	·186248	·246	·150091	·235680	·301	·199085	·281675
·192	·105472	·187172	·257	·150953	·236550	·302	·200003	·282473
·193	·106261	·188094	·248	·151816	·237419	·303	·200922	·283269
·194	·107051	·189016	·249	·152680	·238287	·304	·201841	·284063
·195	·107842	·189938	·250	·153546	·239153	·305	·202761	·284857
·196	·108636	·190858	·251	·154412	·240019	·306	·203683	·285648
·197	·109430	·191777	·252	·155280	·240883	·307	·204605	·286438
·198	·110226	·192696	·253	·156149	·241746	·308	·205527	·287227
·199	·111024	·193614	·254	·157019	·242608	·309	·206451	·288014
·200	·111828	·194531	·255	·157890	·243469	·310	·207376	·288799
·201	·112624	·195447	·256	·158762	·244328	·311	·208301	·289583
·202	·113426	·196362	·257	·159636	·245187	·312	·209227	·290365
·203	·114230	·197277	·258	·160510	·246044	·313	·210154	·291146
·204	·115035	·198190	·259	·161386	·246900	·314	·211082	·291925
·205	·115842	·199108	·260	·162263	·247755	·315	·212011	·292702

Height.	Area of Seg.	Area of Zone.	Height.	Area of Seg.	Area of Zone.	Height.	Area of Seg.	Area of Zone.
·316	·212940	·293478	·371	·265144	·333372	·426	·318970	·366463
·317	·213871	·294252	·372	·266111	·334041	·427	·319959	·366985
·318	·214802	·295025	·373	·267078	·334708	·428	·320948	·367504
·319	·215733	·295796	·374	·268045	·335373	·429	·321938	·368019
·320	·216666	·296565	·375	·269013	·336036	·430	·322928	·368531
·321	·217599	·297333	·376	·269982	·336696	·431	·323918	·369040
·322	·218533	·298098	·377	·270951	·337354	·432	·324909	·369545
·323	·219468	·298863	·378	·271920	·338010	·433	·325900	·370047
·324	·220404	·299625	·379	·272890	·338663	·434	·326892	·370545
·325	·221340	·300386	·380	·273861	·339314	·435	·327882	·371040
·326	·222277	·301145	·381	·274832	·339963	·436	·328874	·371531
·327	·223215	·301902	·382	·275803	·340609	·437	·329866	·372019
·328	·224154	·302658	·383	·276775	·341253	·438	·330858	·372503
·329	·225093	·303412	·384	·277748	·341895	·439	·331850	·372983
·330	·226033	·304164	·385	·278721	·342534	·440	·332843	·373460
·331	·226974	·304914	·386	·279694	·343171	·441	·333836	·373933
·332	·227915	·305663	·387	·280668	·343805	·442	·334829	·374403
·333	·228858	·306410	·388	·281642	·344437	·443	·335822	·374868
·334	·229801	·307155	·389	·282617	·345067	·444	·336816	·375330
·335	·230745	·307898	·390	·283592	·345694	·445	·337810	·375788
·336	·231689	·308640	·391	·284568	·346318	·446	·338804	·376242
·337	·232634	·309379	·392	·285544	·346940	·447	·339798	·376692
·338	·233580	·310117	·393	·286521	·347560	·448	·340793	·377138
·339	·234526	·310853	·394	·287498	·348177	·449	·341787	·377580
·340	·235473	·311588	·395	·288476	·348791	·450	·342782	·378018
·341	·236421	·312319	·396	·289453	·349403	·451	·343777	·378452
·342	·237369	·313050	·397	·290432	·350012	·452	·344772	·378881
·343	·238318	·313778	·398	·291411	·350619	·453	·345768	·379307
·344	·239268	·314505	·399	·292390	·351223	·454	·346764	·379728
·345	·240218	·315230	·400	·293369	·351824	·455	·347759	·380145
·346	·241169	·315952	·401	·294349	·352423	·456	·348755	·380557
·347	·242121	·316673	·402	·295330	·353019	·457	·349752	·380965
·348	·243074	·317393	·403	·296311	·353612	·458	·350748	·381369
·349	·244026	·318110	·404	·297292	·354202	·459	·351745	·381768
·350	·244980	·318825	·405	·298273	·354790	·460	·352742	·382162
·351	·245934	·319538	·406	·299255	·355376	·461	·353739	·382551
·352	·246889	·320249	·407	·300238	·355958	·462	·354736	·382936
·353	·247845	·320958	·408	·301220	·356537	·463	·355732	·383316
·354	·248801	·321666	·409	·302203	·357114	·464	·356730	·383691
·355	·249757	·322371	·410	·303187	·357688	·465	·357727	·384061
·356	·250715	·323075	·411	·304171	·358258	·466	·358725	·384426
·357	·251673	·323775	·412	·305155	·358827	·467	·359723	·384786
·358	·252631	·324474	·413	·306140	·359392	·468	·360721	·385144
·359	·253590	·325171	·414	·307125	·359954	·469	·361719	·385490
·360	·254550	·325866	·415	·308110	·360518	·470	·362717	·385834
·361	·255510	·326559	·416	·309095	·361070	·471	·363715	·386172
·362	·256471	·327250	·417	·310081	·361623	·472	·364713	·386505
·363	·257433	·327939	·418	·311068	·362173	·473	·365712	·386832
·364	·258395	·328625	·419	·312054	·362720	·474	·366710	·387153
·365	·259357	·329310	·420	·313041	·363264	·475	·367709	·387469
·366	·260320	·329992	·421	·314029	·363805	·476	·368708	·387778
·367	·261284	·330673	·422	·315016	·364343	·477	·369707	·388081
·368	·262248	·331351	·423	·316004	·364878	·478	·370706	·388377
·369	·263213	·332027	·424	·316992	·365410	·479	·371704	·388669
·370	·264178	·332700	·425	·317981	·365939	·480	·372704	·388951

Height.	Area of Seg	Area of Zone.	Height.	Area of Seg.	Area of Zone.	Height.	Area of Seg.	Area of Zone.
·481	·373703	·389228	·491	·383699	·391564	·496	·388699	·392362
·482	·374702	·389497	·492	·384699	·391748	·497	·389699	·392480
·483	·375702	·389759	·493	·385699	·391920	·498	·390699	·392580
·484	·376702	·390014	·494	·386699	·392081	·499	·391699	·392657
·435	·377701	·390261	·495	·387699	·392229	·500	·392699	·392699
·486	·378701	·390500						
·487	·379700	·390730						
·488	·380700	·390953						
·489	·381699	·391166						
·490	·382699	·391370						

To find the area of a segment of a circle.

Rule.—Divide the height, or versed sine, by the diameter of the circle, and find the quotient in the column of heights. Then take out the corresponding area, in the column of areas, and multiply it by the square of the diameter; this will give the area of the segment.

Required the area of a segment of a circle, whose height is $3\frac{1}{4}$ feet, and the diameter of the circle 50 feet.

$3\frac{1}{4} = 3{\cdot}25$; and $3{\cdot}25 \div 50 = {\cdot}065$.

·065, by the Table, = ·021659; and $\cdot021659 \times 50^2 = 54{\cdot}147500$, the area required.

To find the area of a circular zone.

Rule 1.—When the zone is less than a semi-circle, divide the height by the longest chord, and seek the quotient in the column of heights. Take out the corresponding area, in the next column on the right hand, and multiply it by the square of the longest chord.

Required the area of a zone whose longest chord is 50, and height 15.

$15 \div 50 = {\cdot}300$; and ·300, by the Table, = ·280876.

Hence $\cdot280876 \times 50^2 = 702{\cdot}19$, the area of the zone.

Rule 2.—When the zone is greater than a semi-circle, take the height on each side of the diameter of the circle.

Required the area of a zone, the diameter of the circle being 50, and the height of the zone on each side of the line which passes through the diameter of the circle 20 and 15 respectively.

$20 \div 50 = {\cdot}400$; ·400, by the Table, = ·351824; and $\cdot351824 \times 50^2 = 879{\cdot}56$.

$15 \div 50 = {\cdot}300$; ·300, by the Table, = ·280876; and $\cdot280876 \times 50^2 = 702{\cdot}19$. Hence $879{\cdot}56 + 702{\cdot}19 = 1581{\cdot}75$.

Approximating rule to find the area of a segment of a circle.

Rule.—Multiply the chord of the segment by the versed sine, divide the product by 3, and multiply the remainder by 2.

Cube the height, or versed sine, find how often twice the length of the chord is contained in it, and add the quotient to the former product; this will give the area of the segment very nearly.

Required the area of the segment of a circle, the chord being 12, and the versed sine 2.

$$12 \times 2 = 24;\ \frac{24}{3} = 8;\ \text{and } 8 \times 2 = 16.$$

$$2^3 \div 24 = {\cdot}3333.$$

Hence $16 + {\cdot}3333 = 16{\cdot}3333$, the area of the segment very nearly.

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.
·100	1·02645	·181	1·08519	·261	1·17275	·341	1·28583	·421	1·42041
·101	1·02698	·182	1·08611	·262	1·17401	·342	1·28739	·422	1·42222
·102	1·02752	·183	1·08704	·263	1·17527	·343	1·28895	·423	1·42402
·103	1·02806	·184	1·08797	·264	1·17655	·344	1·29052	·424	1·42583
·104	1·02860	·185	1·08890	·265	1·17784	·345	1·29209	·425	1·42764
·105	1·02914	·186	1·08984	·266	1·17912	·346	1·29366	·426	1·42945
·106	1·02970	·187	1·09079	·267	1·18040	·347	1·29523	·427	1·43127
·107	1·03026	·188	1·09174	·268	1·18162	·348	1·29681	·428	1·43309
·108	1·03082	·189	1·09269	·269	1·18294	·349	1·29839	·429	1·43491
·109	1·03139	·190	1·09365	·270	1·18428	·350	1·29997	·430	1·43673
·110	1·03196	·191	1·09461	·271	1·18557	·351	1·30156	·431	1·43856
·111	1·03254	·192	1·09557	·272	1·18688	·352	1·30315	·432	1·44039
·112	1·03312	·193	1·09654	·273	1·18819	·353	1·30474	·433	1·44222
·113	1·03371	·194	1·09752	·274	1·18969	·354	1·30634	·434	1·44405
·114	1·03430	·195	1·09850	·275	1·19082	·355	1·30794	·435	1·44589
·115	1·03490	·196	1·09949	·276	1·19214	·356	1·30954	·436	1·44773
·116	1·03551	·197	1·10048	·277	1·19345	·357	1·31115	·437	1·44957
·117	1·03611	·198	1·10147	·278	1·19477	·358	1·31276	·438	1·45142
·118	1·03672	·199	1·10247	·279	1·19610	·359	1·31437	·439	1·45327
·119	1·03734	·200	1·10348	·280	1·19743	·360	1·31599	·440	1·45512
·120	1·03797	·201	1·10447	·281	1·19887	·361	1·31761	·441	1·45697
·121	1·03860	·202	1·10548	·282	1·20011	·362	1·31923	·442	1·45883
·122	1·03923	·203	1·10650	·283	1·20140	·363	1·32086	·443	1·46069
·123	1·03987	·204	1·10752	·284	1·20282	·364	1·32249	·444	1·46255
·124	1·04051	·205	1·10855	·285	1·20419	·365	1·32413	·445	1·46441
·125	1·04116	·206	1·10958	·286	1·20558	·366	1·32577	·446	1·46628
·126	1·04181	·207	1·11062	·287	1·20696	·367	1·32741	·447	1·46815
·127	1·04247	·208	1·11165	·288	1·20828	·368	1·32905	·448	1·47002
·128	1·04313	·209	1·11269	·289	1·20967	·369	1·33069	·449	1·47189
·129	1·04380	·210	1·11374	·290	1·21202	·370	1·33234	·450	1·47377
·130	1·04447	·211	1·11479	·291	1·21239	·371	1·33399	·451	1·47565
·131	1·04515	·212	1·11584	·292	1·21381	·372	1·33564	·452	1·47753
·132	1·04584	·213	1·11692	·293	1·21520	·373	1·33730	·453	1·47942
·133	1·04652	·214	1·11796	·294	1·21658	·374	1·33896	·454	1·48131
·134	1·04722	·215	1·11904	·295	1·21794	·375	1·34063	·455	1·48320
·135	1·04792	·216	1·12011	·296	1·21926	·376	1·34229	·456	1·48509
·136	1·04862	·217	1·12118	·297	1·22061	·377	1·34396	·457	1·48699
·137	1·04932	·218	1·12225	·298	1·22203	·378	1·34563	·458	1·48889
·138	1·05008	·219	1·12334	·299	1·22347	·379	1·34731	·459	1·49079
·139	1·05075	·220	1·12445	·300	1·22495	·380	1·34899	·460	1·49269
·140	1·05147	·221	1·12556	·301	1·22635	·381	1·35068	·461	1·49460
·141	1·05220	·222	1·12663	·302	1·22776	·382	1·35237	·462	1·49651
·142	1·05293	·223	1·12774	·303	1·22918	·383	1·35406	·463	1·49842
·143	1·05367	·224	1·12885	·304	1·23061	·384	1·35575	·464	1·50033
·144	1·05441	·225	1·12997	·305	1·23205	·385	1·35744	·465	1·50224
·145	1·05516	·226	1·13108	·306	1·23349	·386	1·35914	·466	1·50416
·146	1·05591	·227	1·13219	·307	1·23494	·387	1·36084	·467	1·50608
·147	1·05667	·228	1·13331	·308	1·23636	·388	1·36254	·468	1·50800
·148	1·05743	·229	1·13444	·309	1·23780	·389	1·36425	·469	1·50992
·149	1·05819	·230	1·13557	·310	1·23925	·390	1·36596	·470	1·51185
·150	1·05896	·231	1·13671	·311	1·24070	·391	1·36767	·471	1·51378
·151	1·05973	·232	1·13786	·312	1·24216	·392	1·36939	·472	1·51571
·152	1·06051	·233	1·13903	·313	1·24360	·393	1·37111	·473	1·51764
·153	1·06130	·234	1·14020	·314	1·24506	·394	1·37283	·474	1·51958
·154	1·06209	·235	1·14136	·315	1·24654	·395	1·37455	·475	1·52152
·155	1·06288	·236	1·14247	·316	1·24801	·396	1·37628	·476	1·52346
·156	1·06368	·237	1·14363	·317	1·24946	·397	1·37801	·477	1·52541
·157	1·06449	·238	1·14480	·318	1·25095	·398	1·37974	·478	1·52736
·158	1·06530	·239	1·14597	·319	1·25243	·399	1·38148	·479	1·52931
·159	1·06611	·240	1·14714	·320	1·25391	·400	1·38322	·480	1·53126
·160	1·06693	·241	1·14831	·321	1·25539	·401	1·38496	·481	1·53322
·161	1·06775	·242	1·14949	·322	1·25686	·402	1·38671	·482	1·53518
·162	1·06858	·243	1·15067	·323	1·25836	·403	1·38846	·483	1·53714
·163	1·06941	·244	1·15186	·324	1·25987	·404	1·39021	·484	1·53910
·164	1·07025	·245	1·15308	·325	1·26137	·405	1·39196	·485	1·54106
·165	1·07109	·246	1·15429	·326	1·26286	·406	1·39372	·486	1·54302
·166	1·07194	·247	1·15549	·327	1·26437	·407	1·39548	·487	1·54499
·167	1·07279	·248	1·15670	·328	1·26588	·408	1·39724	·488	1·54696
·168	1·07365	·249	1·15791	·329	1·26740	·409	1·39900	·489	1·54893
·169	1·07451	·250	1·15912	·330	1·26892	·410	1·40077	·490	1·55090
·170	1·07537	·251	1·16033	·331	1·27044	·411	1·40254	·491	1·55288
·171	1·07624	·252	1·16157	·332	1·27196	·412	1·40432	·492	1·55486
·172	1·07711	·253	1·16279	·333	1·27349	·413	1·40610	·493	1·55685
·173	1·07799	·254	1·16402	·334	1·27502	·414	1·40788	·494	1·55884
·174	1·07888	·255	1·16526	·335	1·27656	·415	1·40966	·495	1·56083
·175	1·07977	·256	1·16649	·336	1·27810	·416	1·41145	·496	1·56282
·176	1·08066	·257	1·16774	·337	1·27864	·417	1·41324	·497	1·56481
·177	1·08156	·258	1·16899	·338	1·28118	·418	1·41503	·498	1·56680
·178	1·08246	·259	1·17024	·339	1·28273	·419	1·41682	·499	1·56879
·179	1·08337	·260	1·17150	·340	1·28428	·420	1·41861	·500	1·57079
·180	1·08428								

PROPORTIONS OF THE LENGTHS OF SEMI-ELLIPTIC ARCS.

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Hyght of Arc.	Length of Arc.
·100	1·04162	·157	1·10113	·214	1·66678	·271	1·23835	·328	1.31472
·101	1·04262	·158	1·10224	·215	1·16799	·272	1·23966	·329	1·31610
·102	1·04362	·159	1·10335	·216	1·16920	·273	1·24097	·330	1·31748
·103	1·04462	·160	1·10447	·217	1·17041	·274	1·24228	·331	1·31886
·104	1·04562	·161	1·10560	·218	1·17163	·275	1·24359	·332	1·32024
·105	1.04662	·162	1·10672	·219	1·17285	·276	1·24480	·333	1·32162
·106	1·04762	·163	1·10784	·220	1·17407	·277	1·24612	·334	1·32300
·107	1·04862	·164	1·10896	·221	1·17529	·278	1·24744	·335	1·32438
·108	1·04962	·165	1·11008	·222	1·17651	·279	1·24876	·336	1·32576
·109	1·05063	·166	1·11120	·223	1·17774	·280	1·25010	·337	1·32715
·110	1·05164	·167	1·11232	·224	1·17897	·281	1·25142	·338	1·32854
·111	1·05265	·168	1·11344	·225	1·18020	·282	1·25274	·339	1·32993
·112	1·05366	·169	1·11456	·226	1·18143	·283	1·25406	·340	1·33132
·113	1·05467	·170	1·11569	·227	1·18266	·284	1·25538	·341	1·33272
·114	1·05568	·171	1·11682	·228	1·18390	·285	1·25670	·342	1·33412
·115	1·05669	·172	1·11795	·229	1·18514	·286	1·25803	·343	1·33552
·116	1·05770	·173	1·11908	·230	1·18638	·287	1·25936	·344	1·33692
·117	1·05872	·174	1·12021	·231	1·18762	·288	1·26069	·345	1·33833
·118	1·05974	·175	1·12134	·232	1·18886	·289	1·26202	·346	1·33974
·119	1·06076	·176	1·12247	·233	1·19010	·290	1·26335	·347	1·34115
·120	1·06178	·177	1·12360	·234	1·19134	·291	1·26468	·348	1·34256
·121	1·06280	·178	1·12473	·235	1·19258	·292	1·26601	·349	1·34397
·122	1·06382	·179	1·12586	·236	1·19382	·293	1·26734	·350	1·34539
·123	1·06484	·180	1·12699	·237	1·19506	·294	1·26867	·851	1·34681
·124	1·06586	·181	1·12813	·238	1·19630	·295	1·27000	·352	1·34823
·125	1·06689	·182	1·12927	·239	1·19755	·296	1·27133	·353	1·34965
·126	1·06792	·183	1·13041	·240	1·19880	·297	1·27267	·354	1·35108
·127	1·06895	·184	1·13155	·241	1·20005	·298	1·27401	·355	1·35251
·128	1·06998	·185	1·13269	·242	1·20130	·299	1·27535	·356	1·35394
·129	1·07001	·186	1·13383	·243	1·20255	·300	1·27669	·357	1·35537
·130	1·07204	·187	1·13497	·244	1·20380	·301	1·27803	·358	1·35680
·131	1·07308	·188	1·13611	·245	1·20506	·302	1·27937	·359	1·35823
·132	1·07412	·189	1·13726	·246	1·20632	·303	1·28071	·360	1·35967
·133	1·07516	·190	1·13841	·247	1·20758	·304	1·28205	·361	1·36111
·134	1·07621	·191	1·13956	·248	1·20884	·305	1·28339	·362	1·36255
·135	1·07726	·192	1·14071	·249	1·21010	·306	1·28474	·363	1·36399
·136	1·07831	·193	1·14186	·250	1·21136	·307	1·28609	·364	1·36543
·137	1·07937	·194	1·14301	·251	1·21263	·308	1·28744	·365	1·36688
·138	1·08043	·195	1·14416	·252	1·21390	·309	1·28879	·366	1·36833
·139	1·08149	·196	1·14531	·253	1·21517	·310	1·29014	·367	1·36978
·140	1·08255	·197	1·14646	·254	1.21644	·311	1·29149	·368	1·37123
·141	1·08362	·198	1·14762	·255	1·21772	·312	1·29285	·369	1·37268
·142	1·08469	·199	1·14888	·256	1·21900	·313	1·29421	·370	1·37414
·143	1·08576	·200	1·15014	·257	1·22028	·314	2·29557	·371	1·37662
·144	1·08684	·201	1·15131	·258	1·22156	·315	1·29603	·372	1·37708
·145	1·08792	·202	1·15248	·259	1·22284	·316	1·29829	·373	1·37854
·146	1·08901	·203	1·15366	·260	1·22412	·317	1·29965	·374	1·38000
·147	1·09010	·204	1·15484	·261	1·22541	·318	1·30102	·375	1·38146
·148	1·09119	·205	1·15602	·262	1·22670	·319	1·30239	·376	1·38292
·149	1·09228	·206	1·15720	·263	1·22799	·320	1·30376	·377	1·38439
·150	1·09330	·207	1·15838	·264	1·22928	·321	1·30513	·378	1·38585
·151	1·09448	·208	1·15957	·265	1·23057	·322	1·30650	·379	1·38732
·152	1·09558	·209	1·16076	·266	1·23186	·323	1·30787	·380	1·38879
·153	1·09669	·210	1·16196	·267	1·23315	·324	1·30924	·381	1·39024
·154	1·09780	·211	1·16316	·268	1·23445	·325	1·31061	·382	1·39169
·155	1·09891	·212	1·16436	·269	1·23575	·326	1·31198	·383	1·39314
·156	1·10002	·213	1·16557	·270	1·23705	·327	1·31335	·384	1·39459

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.
·385	1·39605	·447	1·48850	·509	1·58474	·571	1·68195	·633	1·78172
·386	1·39751	·448	1·49003	·510	1·58629	·572	1·68354	·634	1·78335
·387	1·39897	·449	1·49157	·511	1·58784	·573	1·68513	·635	1·78498
·388	1·40043	·450	1·49311	·512	1·58940	·574	1·68672	·636	1·78660
·389	1·40189	·451	1·49465	·513	1·59096	·575	1·68831	·637	1·78823
·390	1·40335	·452	1·49618	·514	1·59252	·576	1·68990	·638	1·78986
·391	1·40481	·453	1·49771	·515	1·59408	·577	1·69149	·639	1·79149
·392	1·40627	·454	1·49924	·516	1·59564	·578	1·69308	·640	1·79312
·393	1·40773	·455	1·50077	·517	1·59720	·579	1·69467	·641	1·79475
·394	1·40919	·456	1·50230	·518	1·59876	·580	1·69626	·642	1·79638
·395	1·41065	·457	1·50383	·519	1·60032	·581	1·69785	·643	1·79801
·396	1·41211	·458	1·50536	·520	1·60188	·582	1·69945	·644	1·79964
·397	1·41357	·459	1·50689	·521	1·60344	·583	1·70105	·645	1·80127
·398	1·41504	·460	1·50842	·522	1·60500	·584	1·70264	·646	1·80290
·399	1·41651	·461	1·50996	·523	1·60656	·585	1·70424	·647	1·80454
·400	1·41798	·462	1·51150	·524	1·60812	·586	1·70584	·648	1·80617
·401	1·41945	·463	1·51304	·525	1·60968	·587	1·70745	·649	1·80780
·402	1·42092	·464	1·51458	·526	1·61124	·588	1·70905	·650	1·80943
·403	1·42239	·465	1·51612	·527	1·61280	·589	1·71065	·651	1·81107
·404	1·42386	·466	1·51766	·528	1·61436	·590	1·71225	·652	1·81271
·405	1·42533	·467	1·51920	·529	1·61592	·591	1·71286	·653	1·81435
·406	1·42681	·468	1·52074	·530	1·61748	·592	1·71546	·654	1·81599
·407	1·42829	·469	1·52229	·531	1·61904	·593	1·71707	·655	1·81763
·408	1·42977	·470	1·52384	·532	1·62060	·594	1·71868	·656	1·81928
·409	1·43125	·471	1·52539	·533	1·62216	·595	1·72029	·657	1·82091
·410	1·43273	·472	1·52691	·534	1·62372	·596	1·72190	·658	1·82255
·411	1·43421	·473	1·52849	·535	1·62528	·597	1·72350	·659	1·82419
·412	1·43569	·474	1·53004	·536	1·62684	·598	1·72511	·660	1·82583
·413	1·43718	·475	1·53159	·537	1·62840	·599	1·72672	·661	1·82747
·414	1·43867	·476	1·53314	·538	1·62996	·600	1·72833	·662	1·82911
·415	1·44016	·477	1·53469	·539	1·63152	·601	1·72994	·663	1·83075
·416	1·44165	·478	1·53625	·540	1·63309	·602	1·73155	·664	1·83240
·417	1·44314	·479	1·53781	·541	1·63465	·603	1·73316	·665	1·83404
·418	1·44463	·480	1·53937	·542	1·63623	·604	1·73477	·666	1·83568
·419	1·44613	·481	1·54093	·543	1·63780	·605	1·73638	·667	1·83733
·420	1·44763	·482	1·54249	·544	1·63937	·606	1·73799	·668	1·83897
·421	1·44913	·483	1·54405	·545	1·64094	·607	1·73960	·669	1·84061
·422	1·45064	·484	1·54561	·546	1·64251	·608	1·74121	·670	1·84226
·423	1·45214	·485	1·54718	·547	1·64408	·609	1·74283	·671	1·84391
·424	1·45364	·486	1·54875	·548	1·64565	·610	1·74444	·672	1·84556
·425	1·45515	·487	1·55032	·549	1·64722	·611	1·74605	·673	1·84720
·426	1·45665	·488	1·55189	·550	1·64879	·612	1·74767	·674	1·84885
·427	1·45815	·489	1·55346	·551	1·65036	·613	1·74929	·675	1·85050
·428	1·45966	·490	1·55503	·552	1·65193	·614	1·75091	·676	1·85215
·429	1·46167	·491	1·55660	·553	1·65350	·615	1·75252	·677	1·85379
·430	1·46268	·492	1·55817	·554	1·65507	·616	1·75414	·678	1·85544
·431	1·46419	·493	1·55974	·555	1·65665	·617	1·75576	·679	1·85709
·432	1·46570	·494	1·56131	·556	1·65823	·618	1·75738	·680	1·85874
·433	1·46721	·495	1·56289	·557	1·65981	·619	1·75900	·681	1·86039
·434	1·46872	·496	1·56447	·558	1·66139	·620	1·76062	·682	1·86205
·435	1·47023	·497	1·56605	·559	1·66297	·621	1·76224	·683	1·86370
·436	1·47174	·498	1·56763	·560	1·66455	·622	1·76386	·684	1·86535
·437	1·47326	·499	1·56921	·561	1·66613	·623	1·76548	·685	1·86700
·438	1·47478	·500	1·57089	·562	1·66771	·624	1·76710	·686	1·86866
·439	1·47630	·501	1·57234	·563	1·66929	·625	1·76872	·687	1·87031
·440	1·47782	·502	1·57389	·564	1·67087	·626	1·77034	·688	1·87196
·441	1·47934	·503	1·57544	·565	1·67245	·627	1·77197	·689	1·87362
·442	1·48086	·504	1·57699	·566	1·67403	·628	1·77359	·690	1·87527
·443	1·48238	·505	1·57854	·567	1·67561	·629	1·77521	·691	1·87693
·444	1·48391	·506	1·58009	·568	1·67719	·630	1·77684	·692	1·87859
·445	1·48544	·507	1·58164	·569	1·67877	·631	1·77847	·693	1·88024
·446	1·48697	508	1·58319	·570	1·68036	·632	1·78009	·694	1·88190

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.
·695	1·88356	·757	1·98794	·818	2·09360	·879	2·20292	·940	2·31479
·696	1·88522	·758	1·98964	·819	2·09536	·880	2·20474	·941	2·31666
·697	1·88688	·759	1·99134	·820	2·09712	·881	2·20656	·942	2·31852
·698	1·88854	·760	1·99305	·821	2·09888	·882	2·20839	·943	2·32038
·699	1·89020	·761	1·99476	·822	2·10065	·883	2·21022	·944	2·32224
·700	1·89186	·762	1·99647	·823	2·10242	·884	2·21205	·945	2·32411
·701	1·89352	·763	1·99818	·824	2·10419	·885	2·21388	·946	2·32598
·702	1·89519	·764	1·99989	·825	2·10596	·886	2·21571	·947	2·32785
·703	1·89685	·765	2·00160	·826	2·10773	·887	2·21754	·948	2·32972
·704	1·89851	·766	2·00331	·827	2·10950	·888	2·21937	·949	2·33160
·705	1·90017	·767	2·00502	·828	2·11127	·889	2·22120	·950	2·33348
·706	1·90184	·768	2·00673	·829	2·11304	·890	2·22303	·951	2·33537
·707	1·90350	·769	2·00844	·830	2·11481	·891	2·22486	·952	2·33726
·708	1·90517	·770	2·01016	·831	2·11659	·892	2·22670	·953	2·33915
·709	1·90684	·771	2·01187	·832	2·11837	·893	2·22854	·954	2·34104
·710	1·90852	·772	2·01359	·833	2·12015	·894	2·23038	·955	2·34293
·711	1·91019	·773	2·01531	·834	2·12193	·895	2·23222	·956	2·34483
·712	1·91187	·774	2·01702	·835	2·12371	·896	2·23406	·957	2·34673
·713	1·91355	·775	2·01874	·836	2·12549	·897	2·23590	·958	2·34862
·714	1·91523	·776	2·02045	·837	2·12727	·898	2·23774	·959	2·35051
·715	1·91691	·777	2·02217	·838	2·12905	·899	2·23958	·960	2·35241
·716	1·91859	·778	2·02389	·839	2·13083	·900	2·24142	·961	2·35431
·717	1·92027	·779	2·02561	·840	2·13261	·901	2·24325	·962	2·35621
·718	1·92195	·780	2·02733	·841	2·13439	·902	2·24508	·963	2·35810
·719	1·92363	·781	2·02907	·842	2·13618	·903	2·24691	·964	2·36000
·720	1·92531	·782	2·03080	·843	2·13797	·904	2·24874	·965	2·36191
·721	1·92700	·783	2·03252	·844	2·13976	·905	2·25057	·966	2·36381
·722	1·92868	·784	2·03425	·845	2·14155	·906	2·25240	·967	2·36571
·723	1·93036	·785	2·03598	·846	2·14334	·907	2·25423	·968	2·36762
·724	1·93204	·786	2·03771	·847	2·14513	·908	2·25606	·969	2·36952
·725	1·93373	·787	2·03944	·848	2·14692	·909	2·25789	·970	2·37143
·726	1·93541	·788	2·04117	·849	2·14871	·910	2·25972	·971	2·37334
·727	1·93710	·789	2·04290	·850	2·15050	·911	2·26155	·972	2·37525
·728	1·93878	·790	2·04462	·851	2·15229	·912	2·26338	·973	2·37716
·729	1·94046	·791	2·04635	·852	2·15409	·913	2·26521	·974	2·37908
·730	1·94215	·792	2·04809	·853	2·15589	·914	2·26704	·975	2·38100
·731	1·94383	·793	2·04983	·854	2·15770	·915	2·26888	·976	2·38291
·732	1·94552	·794	2·05157	·855	2·15950	·916	2·27071	·977	2·38482
·733	1·94721	·795	2·05331	·856	2·16130	·917	2·27254	·978	2·38673
·734	1·94890	·796	2·05505	·857	2·16309	·918	2·27437	·979	2·38864
·735	1·95059	·797	2·05679	·858	2·16489	·919	2·27620	·980	2·39055
·736	1·95228	·798	2·05853	·859	2·16668	·920	2·27803	·981	2·39247
·737	1·95397	·799	2·06027	·860	2·16848	·921	2·27987	·982	2·39439
·738	1·95566	·800	2·06202	·861	2·17028	·922	2·28170	·983	2·39631
·739	1·95735	·801	2·06377	·862	2·17209	·923	2·28354	·984	2·39823
·740	1·95994	·802	2·06552	·863	2·17389	·924	2·28587	·985	2·40016
·741	1·96074	·803	2·06727	·864	2·17570	·925	2·28720	·986	2·40208
·742	1·96244	·804	2·06901	·865	2·17751	·926	2·28903	·987	2·40400
·743	1·96414	·805	2·07076	·866	2·17932	·927	2·29086	·988	2·40592
·744	1·96583	·806	2·07251	·867	2·18113	·928	2·29270	·989	2·40784
·745	1·96753	·807	2·07427	·868	2·18294	·929	2·29453	·990	2·40976
·746	1·96923	·808	2·07602	·869	2·18475	·930	2·29636	·991	2·41169
·747	1·97093	·809	2·07777	·870	2·18656	·931	2·29820	·992	2·41362
·748	1·97262	·810	2·07953	·871	2·18837	·932	2·30004	·993	2·41556
·749	1·97432	·811	2·08128	·872	2·19018	·933	2·30188	·994	2·41749
·750	1·97602	·812	2·08304	·873	2·19200	·934	2·30373	·995	2·41943
·751	1·97772	·813	2·08480	·874	2·19382	·935	2·30557	·996	2·42136
·752	1·97943	·814	2·08656	·875	2·19564	·936	2·30741	·997	2·42329
·753	1·98113	·815	2·08832	·876	2·19746	·937	2·30926	·998	2·42522
·754	1·98283	·816	2·09008	·877	2·19928	·938	2·31111	·999	2·42715
·755	1·98453	·817	2·09198	·878	2·20110	·939	2·31295	·1000	2·42908
·756	1·98623								

To find the length of an arc of a circle, or the curve of a right semi-ellipse.

RULE.—Divide the height by the base, and the quotient will be the height of an arc of which the base is unity. Seek, in the Table of Circular or of Semi-elliptical arcs, as the case may be, for a number corresponding to this quotient, and take the length of the arc from the next right-hand column. Multiply the number thus taken out by the base of the arc, and the product will be the length of the arc or curve required.

In a Bridge, suppose the profiles of the arches are the arcs of circles; the span of the middle arch is 240 feet and the height 24 feet; required the length of the arc.

24 ÷ 240 = ·100; and ·100, by the Table, is 1·02645.

Hence 1·02645 × 24 = 246·34800 feet, the length required.

The profiles of the arches of a Bridge are all equal and similar semi-ellipses; the span of each is 120 feet, and the rise 18 feet; required the length of the curve.

28 ÷ 120 = ·233; and ·233 by the Table, is 1·19010.

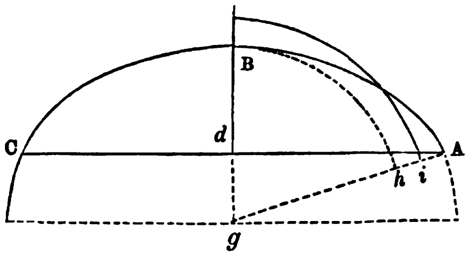
Hence 1·19010 × 120 = 142·81200 feet, the length required.

In this example there is, in the division of 28 by 120, a remainder of 40, or one-third part of the divisor; consequently, the answer, 142·81200, is rather less than the truth. But this difference, in even so large an arch, is little more than half an inch; therefore, except where extreme accuracy is required, it is not worth computing.

These Tables are equally useful in estimating works which may be carried into practice, and the quantity of work to be executed from drawings to a scale.

As the Tables do not afford the means of finding the lengths of the curves of elliptical arcs which are less than half of the entire figure, the following geometrical method is given to supply the defect.

Let the curve, of which the length is required to be found, be ABC.



Produce the height line B*d* to meet the centre of the curve in *g*. Draw the right line A*g*, and from the centre *g*, with the distance *g*B describe an arc B*h*, meeting A*g* in *h*. Bisect A*h* in *i*, and from the centre *g* with the radius *gi* describe the arc *ik*, meeting *d*B produced to *k*; then *ik* is half the arc ABC.

A TABLE *of the Reciprocals of Numbers; or the* DECIMAL FRACTIONS *corresponding to* VULGAR FRACTIONS *of which the Numerator is unity or* 1.

[In the following Tables, the Decimal fractions are Reciprocals of the Denominators of those opposite to them; and their product is = unity.

To find the Decimal corresponding to a fraction having a higher Numerator than 1, multiply the Decimal opposite to the given Denominator, by the given Numerator. Thus, the Decimal corresponding to $\frac{1}{64}$ being ·015625, the Decimal to $\frac{15}{64}$ will be ·015625 × 15 = ·234375.]

Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.
1/2	·5	1/47	·0212766	1/92	·010869565
1/3	·333333333	1/48	·020833333	1/93	·010752688
1/4	·25	1/49	·020408163	1/94	·010638298
1/5	·2	1/50	·02	1/95	·010526316
1/6	·166666667	1/51	·019607843	1/96	·010416667
1/7	·142857143	1/52	·019230769	1/97	·010309278
1/8	·125	1/53	·018867925	1/98	·010204082
1/9	·111111111	1/54	·018518519	1/99	·01010101
1/10	·1	1/55	·018181818	1/100	·01
1/11	·090909091	1/56	·017857143	1/101	·00990099
1/12	·083333333	1/57	·01754386	1/102	·009803922
1/13	·076923077	1/58	·017241379	1/103	·009708738
1/14	·071428571	1/59	·016949153	1/104	·009615385
1/15	·066666667	1/60	·016666667	1/105	·00952381
1/16	·0625	1/61	·016393443	1/106	·009433962
1/17	·058823529	1/62	·016129032	1/107	·009345794
1/18	·055555556	1/63	·015873016	1/108	·009259259
1/19	·052631579	1/64	·015625	1/109	·009174312
1/20	·05	1/65	·015384615	1/110	·009090909
1/21	·047619048	1/66	·015151515	1/111	·009009009
1/22	·045454545	1/67	·014925373	1/112	·008928571
1/23	·043478261	1/68	·014705882	1/113	·008849558
1/24	·041666667	1/69	·014492754	1/114	·00877193
1/25	·04	1/70	·014285714	1/115	·008695652
1/26	·038461538	1/71	·014084517	1/116	·00802069
1/27	·037037037	1/72	·013888889	1/117	·008547009
1/28	·035714286	1/73	·01369863	1/118	·008474576
1/29	·034482759	1/74	·013513514	1/119	·008403361
1/30	·033333333	1/75	·013333333	1/120	·008333333
1/31	·032258065	1/76	·013157895	1/121	·008264463
1/32	·03125	1/77	·012987013	1/122	·008196721
1/33	·030303030	1/78	·012820513	1/123	·008130081
1/34	·029411765	1/79	·012658228	1/124	·008064516
1/35	·028571429	1/80	·0125	1/125	·008
1/36	·027777778	1/81	·012345679	1/126	·007936508
1/37	·027027027	1/82	·012195122	1/127	·007874016
1/38	·026315789	1/83	·012048193	1/128	·0078125
1/39	·025641026	1/84	·011904762	1/129	·007751938
1/40	·025	1/85	·011764706	1/130	·007692308
1/41	·024390244	1/86	·011627907	1/131	·007633588
1/42	·023809524	1/87	·011494253	1/132	·007575758
1/43	·023255814	1/88	·011363636	1/133	·007518797
1/44	·022727273	1/89	·011235955	1/134	·007462687
1/45	·022222222	1/90	·011111111	1/135	·007407407
1/46	·02173913	1/91	·010989011	1/136	·007352941

Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.
1/137	·00729927	1/198	·005050505	1/259	·003861004
1/138	·007246377	1/199	·005025126	1/260	·003846154
1/139	·007194245	1/200	·005	1/261	·003831418
1/140	·007142857	1/201	·004975124	1/262	·003816794
1/141	·007092199	1/202	·004950495	1/263	·003802281
1/142	·007042254	1/203	·004926108	1/264	·003787879
1/143	·006993007	1/204	·004901961	1/265	·003773585
1/144	·006944444	1/205	·004878049	1/266	·003759398
1/145	·006896552	1/206	·004854369	1/267	·003745318
1/146	·006849315	1/207	·004830918	1/268	·003731343
1/147	·006802721	1/208	·004807692	1/269	·003717472
1/148	·006756757	1/209	·004784689	1/270	·003703704
1/149	·006711409	1/210	·004761905	1/271	·003690037
1/150	·006666667	1/211	·004739336	1/272	·003676471
1/151	·006622517	1/212	·004716981	1/273	·003663004
1/152	·006578947	1/213	·004694836	1/274	·003649635
1/153	·006535948	1/214	·004672897	1/275	·003636364
1/154	·006493506	1/215	·004651163	1/276	·003623188
1/155	·006451613	1/216	·00462963	1/277	·003610108
1/156	·006410256	1/217	·004608295	1/278	·003597122
1/157	·006369427	1/218	·004587156	1/279	·003584229
1/158	·006329114	1/219	·00456621	1/280	·003571429
1/159	·006289308	1/220	·004545455	1/281	·003558719
1/160	·00625	1/221	·004524887	1/282	·003546099
1/161	·00621118	1/222	·004504505	1/283	·003533569
1/162	·00617284	1/223	·004484305	1/284	·003522127
1/163	·006134969	1/224	·004464286	1/285	·003508772
1/164	·006097561	1/225	·004444444	1/286	·003496503
1/165	·006060606	1/226	·004424779	1/287	·003484321
1/166	·006024096	1/227	·004405286	1/288	·003472222
1/167	·005988024	1/228	·004385965	1/289	·003460208
1/168	·005952381	1/229	·004366812	1/290	·003448276
1/169	·00591716	1/230	·004347826	1/291	·003436426
1/170	·005882353	1/231	·004329004	1/292	·003424658
1/171	·005847953	1/232	·004310345	1/293	·003412969
1/172	·005813953	1/233	·004291845	1/294	·003401361
1/173	·005780347	1/234	·004273504	1/295	·003389831
1/174	·005747126	1/235	·004255319	1/296	·003378378
1/175	·005714286	1/236	·004237288	1/297	·003367003
1/176	·005681818	1/237	·004219409	1/298	·003355705
1/177	·005649718	1/238	·004201681	1/299	·003344482
1/178	·005617978	1/239	·0041841	1/300	·003333333
1/179	·005586592	1/240	·004166667	1/301	·003322259
1/180	·005555556	1/241	·004149378	1/302	·003311258
1/181	·005524862	1/242	·004132231	1/303	·00330133
1/182	·005494505	1/243	·004115226	1/304	·003289474
1/183	·005464481	1/244	·004098361	1/305	·003278689
1/184	·005434783	1/245	·004081633	1/306	·003267974
1/185	·005405405	1/246	·004065041	1/307	·003257329
1/186	·005376344	1/247	·004048583	1/308	·003246753
1/187	·005347594	1/248	·004032258	1/309	·003236246
1/188	·005319149	1/249	·004016064	1/310	·003225806
1/189	·005291005	1/250	·004	1/311	·003215434
1/190	·005263158	1/251	·003984064	1/312	·003205128
1/191	·005235602	1/252	·003968254	1/313	·003194888
1/192	·005208333	1/253	·003952569	1/314	·003184713
1/193	·005181347	1/254	·003937008	1/315	·003174603
1/194	·005154639	1/255	·003921569	1/316	·003164557
1/195	·005128205	1/256	·00390625	1/317	·003154574
1/196	·005102041	1/257	·003891051	1/318	·003144654
1/197	·005076142	1/258	·003875969	1/319	·003134796

Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.
1/320	·003125	1/381	·002624672	1/442	·002262443
1/321	·003115265	1/382	·002617801	1/443	·002257336
1/322	·00310559	1/383	·002610966	1/444	·002252252
1/323	·003095975	1/384	·002604167	1/445	·002247191
1/324	·00308642	1/385	·002597403	1/446	·002242152
1/325	·003076923	1/386	·002590674	1/447	·002237136
1/326	·003067485	1/387	·002583979	1/448	·002232143
1/327	·003058104	1/388	·00257732	1/449	·002227171
1/328	·00304878	1/389	·002570694	1/450	·002222222
1/329	·003039514	1/390	·002564103	1/451	·002217295
1/330	·003030303	1/391	·002557545	1/452	·002212389
1/331	·003021148	1/392	·00255102	1/453	·002207506
1/332	·003012048	1/393	·002544529	1/454	·002202643
1/333	·003003003	1/394	·002538071	1/455	·002197802
1/334	·002994012	1/395	·002531646	1/456	·002192982
1/335	·002985075	1/396	·002525253	1/457	·002188184
1/336	·00297619	1/397	·002518892	1/458	·002183406
1/337	·002967359	1/398	·002512563	1/459	·002178649
1/338	·00295858	1/399	·002506266	1/460	·002173913
1/339	·002949853	1/400	·0025	1/461	·002169197
1/340	·002941176	1/401	·002493766	1/462	·002164502
1/341	·002932551	1/402	·002487562	1/463	·002159827
1/342	·002923977	1/403	·00248139	1/464	·002155172
1/343	·002915452	1/404	·002475248	1/465	·002150538
1/344	·002906977	1/405	·002469136	1/466	·002145923
1/345	·002898551	1/406	·002463054	1/467	·002141328
1/346	·002890173	1/407	·002457002	1/468	·002136752
1/347	·002881844	1/408	·00245098	1/469	·002132196
1/348	·002873563	1/409	·002444988	1/470	·00212766
1/349	·00286533	1/410	·002439024	1/471	·002123142
1/350	·002857143	1/411	·00243309	1/472	·002118644
1/351	·002849003	1/412	·002427184	1/473	·002114165
1/352	·002840909	1/413	·002421308	1/474	·002109705
1/353	·002832861	1/414	·002415459	1/475	·002105263
1/354	·002824859	1/415	·002409639	1/476	·00210084
1/355	·002816901	1/416	·002406846	1/477	·002096486
1/356	·002808989	1/417	·002398082	1/478	·00209205
1/357	·00280112	1/418	·002392344	1/479	·002087683
1/358	·002793296	1/419	·002386635	1/480	·002083333
1/359	·002785515	1/420	·002380952	1/481	·002079002
1/360	·002777778	1/421	·002375297	1/482	·002074689
1/361	·002770083	1/422	·002369668	1/483	·002070393
1/362	·002762431	1/423	·002364066	1/484	·002066116
1/363	·002754821	1/424	·002358491	1/485	·002061856
1/364	·002747235	1/425	·002352941	1/486	·002057613
1/365	·002739726	1/426	·002347418	1/487	·002053388
1/366	·00273224	1/427	·00234192	1/488	·00204918
1/367	·002724796	1/428	·002336449	1/489	·00204499
1/368	·002717391	1/429	·002331002	1/490	·002040816
1/369	·002710027	1/430	·002325581	1/491	·00203666
1/370	·002702703	1/431	·002320186	1/492	·00203252
1/371	·002695418	1/432	·002314815	1/493	·002028398
1/372	·002688172	1/433	·002309469	1/494	·002024291
1/373	·002680965	1/434	·002304147	1/495	·002020202
1/374	·002673797	1/435	·002298851	1/496	·002016129
1/375	·002666667	1/436	·002293578	1/497	·002012072
1/376	·002659574	1/437	·00228833	1/498	·002008032
1/377	·00265252	1/438	·002283105	1/499	·002004008
1/378	·002645503	1/439	·002277904	1/500	·002
1/379	·002638521	1/440	·002272727	1/501	·001996008
1/380	·002631579	1/441	·002267574	1/502	·001992032

Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.
1/503	·001988072	1/564	·00177305	1/625	·0016
1/504	·001984127	1/565	·001769912	1/626	·001597444
1/505	·001980198	1/566	·001766784	1/627	·001594896
1/506	·001976285	1/567	·001763668	1/628	·001592357
1/507	·001972387	1/568	·001760563	1/629	·001589825
1/508	·001968504	1/569	·001757469	1/630	·001587302
1/509	·001964637	1/570	·001754386	1/631	·001584786
1/510	·001960784	1/571	·001751313	1/632	·001582278
1/511	·001956947	1/572	·001748252	1/633	·001579779
1/512	·001953125	1/573	·001745201	1/634	·001577287
1/513	·001949318	1/574	·00174216	1/635	·001574803
1/514	·001945525	1/575	·00173913	1/636	·001572327
1/515	·001941748	1/576	·001736111	1/637	·001569859
1/516	·001937984	1/577	·001733102	1/638	·001567398
1/517	·001934236	1/578	·001730104	1/639	·001564945
1/518	·001930502	1/579	·001727116	1/640	·0015625
1/519	·001926782	1/580	·001724138	1/641	·001560062
1/520	·001923077	1/581	·00172117	1/642	·001557632
1/521	·001919386	1/582	·001718213	1/643	·00155521
1/522	·001915709	1/583	·001715266	1/644	·001552795
1/523	·001912046	1/584	·001712329	1/645	·001550388
1/524	·001908397	1/585	·001709402	1/646	·001547988
1/525	·001904762	1/586	·001706485	1/647	·001545595
1/526	·001901141	1/587	·001703578	1/648	·00154321
1/527	·001897533	1/588	·00170068	1/649	·001540832
1/528	·001893939	1/589	·001697793	1/650	·001538462
1/529	·001890359	1/590	·001694915	1/651	·001536098
1/530	·001886792	1/591	·001692047	1/652	·001533742
1/531	·001883239	1/592	·001689189	1/653	·001531394
1/532	·001879699	1/593	·001686341	1/654	·001529052
1/533	·001876173	1/594	·001683502	1/655	·001526718
1/534	·001872659	1/595	·001680672	1/656	·00152439
1/535	·001869159	1/596	·001677852	1/657	·00152207
1/536	·001865672	1/597	·001675042	1/658	·001519751
1/537	·001862197	1/598	·001672241	1/659	·001517451
1/538	·001858736	1/599	·001669449	1/660	·001515152
1/539	·001855288	1/600	·001666667	1/661	·001512859
1/540	·001851852	1/601	·001663894	1/662	·001510574
1/541	·001848429	1/602	·00166113	1/663	·001508296
1/542	·001845018	1/603	·001658375	1/664	·001506024
1/543	·001841621	1/604	·001655629	1/665	·001503759
1/544	·001838235	1/605	·001652893	1/666	·001501502
1/545	·001834862	1/606	·001650165	1/667	·00149925
1/546	·001831502	1/607	·001647446	1/668	·001497006
1/547	·001828154	1/608	·001644737	1/669	·001494768
1/548	·001824818	1/609	·001642036	1/670	·001492537
1/549	·001821494	1/610	·001639344	1/671	·001490313
1/550	·001818182	1/611	·001636661	1/672	·001488095
1/551	·001814882	1/612	·001633987	1/673	·001485884
1/552	·001811594	1/613	·001631321	1/674	·00148368
1/553	·001808318	1/614	·001628664	1/675	·001481481
1/554	·001805054	1/615	·001626016	1/676	·00147929
1/555	·001801802	1/616	·001623377	1/677	·001477105
1/556	·001798561	1/617	·001620746	1/678	·001474926
1/557	·001795332	1/618	·001618123	1/679	·001472754
1/558	·001792115	1/619	·001615509	1/680	·001470588
1/559	·001788909	1/620	·001612903	1/681	·001468429
1/560	·001785714	1/621	·001610306	1/682	·001466276
1/561	·001782531	1/622	·001607717	1/683	·001464129
1/562	·001779359	1/623	·001605136	1/684	·001461988
1/563	·001776199	1/624	·001602564	1/685	·001459854

Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.
1/686	·001457726	1/747	·001338688	1/808	·001237624
1/687	·001455604	1/748	·001336898	1/809	·001236094
1/688	·001453488	1/749	·001335113	1/810	·001234568
1/689	·001451879	1/750	·001333333	1/811	·001233046
1/690	·001449275	1/751	·001331558	1/812	·001231527
1/691	·001447178	1/752	·001329787	1/813	·001230012
1/692	·001445087	1/753	·001328021	1/814	·001228501
1/693	·001443001	1/754	·00132626	1/815	·001226994
1/694	·001440922	1/755	·001324503	1/816	·001225499
1/695	·001438849	1/756	·001322751	1/817	·00122399
1/696	·001436782	1/757	·001321004	1/818	·001222494
1/697	·00143472	1/758	·001319261	1/819	·001221001
1/698	·001432665	1/759	·001317523	1/820	·001219512
1/699	·001430615	1/760	·001315789	1/821	·001218027
1/700	·001428571	1/761	·00131406	1/822	·001216545
1/701	·001426534	1/762	·001312336	1/823	·001215067
1/702	·001424501	1/763	·001310616	1/824	·001213592
1/703	·001422475	1/764	·001308901	1/825	·001212121
1/704	·001420455	1/765	·00130719	1/826	·001210654
1/705	·00141844	1/766	·001305483	1/827	·00120919
1/706	·001416431	1/767	·001303781	1/828	·001207729
1/707	·001414427	1/768	·001302083	1/829	·001206273
1/708	·001412429	1/769	·00130039	1/830	·001204819
1/709	·001410437	1/770	·001298701	1/831	·001203369
1/710	·001408451	1/771	·001297017	1/832	·001201923
1/711	·00140647	1/772	·001295337	1/833	·00120048
1/712	·001404494	1/773	·001293661	1/834	·001199041
1/713	·001402525	1/774	·00129199	1/835	·001197605
1/714	·00140056	1/775	·001290323	1/836	·001196172
1/715	·001398601	1/776	·00128866	1/837	·001194743
1/716	·001396648	1/777	·001287001	1/838	·001193317
1/717	·0013947	1/778	·001285347	1/839	·001191895
1/718	·001392758	1/779	·001283697	1/840	·001190476
1/719	·001390821	1/780	·001282051	1/841	·001189061
1/720	·001388889	1/781	·00128041	1/842	·001187648
1/721	·001386963	1/782	·001278772	1/843	·00118624
1/722	·001385042	1/783	·001277139	1/844	·001184834
1/723	·001383126	1/784	·00127551	1/845	·001183432
1/724	·001381215	1/785	·001273885	1/846	·001182033
1/725	·00137931	1/786	·001272265	1/847	·001180638
1/726	·00137741	1/787	·001270648	1/848	·001179245
1/727	·001375516	1/788	·001269036	1/849	·001177856
1/728	·001373626	1/789	·001267427	1/850	·001176471
1/729	·001371742	1/790	·001265823	1/851	·001175088
1/730	·001369863	1/791	·001264223	1/952	·001173709
1/731	·001367989	1/792	·001262626	1/853	·001172333
1/732	·00136612	1/793	·001261034	1/854	·00117096
1/733	·001364256	1/794	·001259446	1/855	·001169591
1/734	·001362398	1/795	·001257862	1/856	·001168224
1/735	·001360544	1/796	·001256281	1/857	·001166861
1/736	·001358696	1/797	·001254705	1/858	·001165501
1/737	·001356852	1/798	·001253133	1/859	·001164144
1/738	·001355014	1/799	·001251364	1/860	·001162791
1/739	·00135318	1/800	·00125	1/861	·00116144
1/740	·001351351	1/801	·001248439	1/862	·001160093
1/741	·001349528	1/802	·001246883	1/863	·001158749
1/742	·001347709	1/803	·00124533	1/864	·001157407
1/743	·001345895	1/804	·001243781	1/865	·001156069
1/744	·001344086	1/805	·001242236	1/866	·001154734
1/745	·001342282	1/806	·001240695	1/867	·001153403
1/746	·001340483	1/807	·001239157	1/868	·001152074

Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.	Fraction or Numb.	Decimal or Reciprocal.
1/869	·001150748	1/913	·00109529	1/957	·001044932
1/870	·001149425	1/914	·001094092	1/958	·001043841
1/871	·001148106	1/915	·001092896	1/959	·001042753
1/872	·001146789	1/916	·001091703	1/960	·001041667
1/873	·001145475	1/917	·001090513	1/961	·001040583
1/874	·001144165	1/918	·001089325	1/962	·001039501
1/875	·001142857	1/919	·001088139	1/963	·001038422
1/876	·001141553	1/920	·001086957	1/964	·001037344
1/877	·001140251	1/921	·001085776	1/965	·001036269
1/878	·001138952	1/922	·001084599	1/966	·001035197
1/879	·001137656	1/923	·001083423	1/967	·001034126
1/880	·001136364	1/924	·001082251	1/968	·001033058
1/881	·001135074	1/925	·001081081	1/969	·001031992
1/882	·001133787	1/926	·001079914	1/970	·001030928
1/883	·001132503	1/927	·001078749	1/971	·001029866
1/884	·001131222	1/928	·001077586	1/972	·001028807
1/885	·001129944	1/929	·001076426	1/973	·001027749
1/886	·001128668	1/930	·001075269	1/974	·001026694
1/887	·001127396	1/931	·001074114	1/975	·001025641
1/888	·001126126	1/932	·001072961	1/976	·00102459
1/889	·001124859	1/933	·001071811	1/977	·001023541
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1/891	·001122334	1/935	·001069519	1/979	·00102145
1/892	·001121076	1/936	·001068376	1/980	·001020408
1/893	·001119821	1/937	·001067236	1/981	·001019168
1/894	·001118568	1/938	·001066098	1/982	·00101833
1/895	·001117818	1/939	·001064963	1/983	·001017294
1/896	·001116071	1/940	·00106383	1/984	·00101626
1/897	·001114827	1/941	·001062699	1/985	·001015228
1/898	·001113586	1/942	·001061571	1/986	·001014199
1/899	·001112347	1/943	·001060445	1/987	·001013171
1/900	·001111111	1/944	·001059322	1/988	·001012146
1/901	·001109878	1/945	·001058201	1/989	·001011122
1/902	·001108647	1/946	·001057082	1/990	·001010101
1/903	·00110742	1/947	·001055966	1/991	·001009082
1/904	·001106195	1/948	·001054852	1/992	·001008065
1/905	·001104972	1/949	·001053741	1/993	·001007049
1/906	·001103753	1/950	·001052632	1/994	·001006036
1/907	·001102536	1/951	·001051525	1/995	·001005025
1/908	·001101322	1/952	·00105042	1/996	·001004016
1/909	·00110011	1/953	·001049318	1/997	·001003009
1/910	·001098901	1/954	·001048218	1/998	·001002004
1/911	·001091695	1/955	·00104712	1/999	·001001001
1/912	·001096491	1/956	·001046025	1/1000	·001

Divide 80000 by 971.

By the above Table we find that 1 divided by 971 gives ·001029866, and ·001029866 × 80000 = 82·38928.

What is the sum of $\frac{5}{883}$ and $\frac{2}{953}$?

$$5 \times \frac{1}{883} = \cdot 001132503 \times 5 = \cdot 005662515$$

$$2 \times \frac{1}{953} = \cdot 001049318 \times 2 = \underline{\cdot 002098636}$$

$$\therefore \frac{5}{883} + \frac{2}{953} \qquad = \underline{\cdot 007761141}$$

WEIGHTS AND VALUES IN DECIMAL PARTS.

TROY WEIGHT.

Dec. parts of a lb.

Ozs.	Decimals.
11	·916666
10	·833333
9	·75
8	·666666
7	·583333
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Dwts.	Decimals.
19	·079166
18	·075
17	·070833
16	·066666
15	·0625
14	·058333
13	·054166
12	·05
11	·045833
10	·041666
9	·0375
8	·083333
7	·029166
6	·025
5	·020833
4	·016666
8	·0125
2	·008333
1	·004166

Grs.	Decimals.
15	·002604
14	·002430
13	·002257
12	·002083
11	·001910
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

AVOIRDUPOIS WEIGHT.

Dec. parts of a cwt.

Qrs.	Decimals.
8	·75
2	·5
1	·25

lbs.	Decimals.
27	·241071
26	·232142
25	·223214
24	·214286
23	·205357
22	·196428
21	·187500
20	·178572
19	·169643
18	·160714
17	·151785
16	·142856
15	·133928
14	·125
13	·116071
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Ozs.	Decimals.
15	·008370
14	·007812
13	·007254
12	·006696
11	·006138
10	·005580
9	·005022
8	·004464
7	·003906
6	·003348
5	·002790
4	·002232
3	·001674
2	·001116
1	·000558

AVOIRDUPOIS WEIGHT.

Dec. parts of a lb.

Ozs.	Decimals.
15	·9375
14	·875
13	·8125
12	·75
11	·6875
10	·625
9	·5625
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

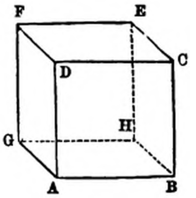
Drs.	Decimals.
15	·058593
14	·054686
18	·050780
12	·046874
11	·042968
10	·039062
9	·035156
8	·03125
7	·027343
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

LONG MEASURE.

Dec. parts of a foot.

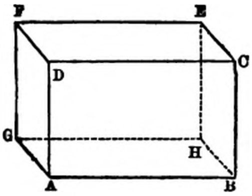
Ins.	Decimals.
11	·916666
10	·833333
9	·75
8	·666666
7	·583333
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

To find the solidity of a cube, the height of one of its sides being given.—Multiply the side of the cube by itself, and that product again by the side, and it will give the solidity required.



The side AB, or BC, of the cube ABCDFGHE, is 25·5: what is the solidity?

Here $AB^3 = (22{\cdot}5)|^3 = 25{\cdot}5 \times 25{\cdot}5 \times 25{\cdot}5 = 25{\cdot}5 \times 650{\cdot}25 = 16581{\cdot}375$, *content of the cube.*

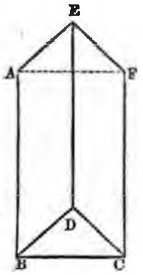


To find the solidity of a parallelopipedon.—Multiply the length by the breadth, and that product again by the depth or altitude, and it will give the solidity required.

Required the solidity of a parallelopipedon ABCDFEHG, whose length AB is 8 feet, its breadth FD $4\frac{1}{2}$ feet, and the depth or altitude AD $6\frac{3}{4}$ feet?

Here $AB \times AD \times FD = 8 \times 6{\cdot}75 \times 4.5 = 54 \times 4{\cdot}5 = 243$ *solid feet, the contents of the parallelopipedon.*

To find the solidity of a prism.—Multiply the area of the base into the perpendicular height of the prism, and the product will be the solidity.



What is the solidity of the triangular prism ABCFED, whose length AB is 10 feet, and either of the equal sides, BC, CD, or DB, of one of its equilateral ends BCD, $2\frac{1}{2}$ feet?

Here $\frac{1}{4} \times 2{\cdot}5^2 \times \sqrt{3} = \frac{1}{4} \times 6{\cdot}25 \times \sqrt{3} = 1{\cdot}5625 \times \sqrt{3} = 1{\cdot}5625 \times 1{\cdot}732 = 2{\cdot}70625 =$ *area of the base* BCD.

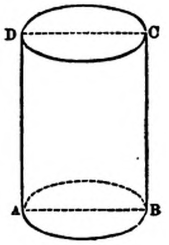
Or, $\frac{2{\cdot}5 + 2{\cdot}5 + 2{\cdot}5}{2} = \frac{7{\cdot}5}{2} = 3{\cdot}75 = \frac{1}{2}$ *sum of the sides,* BC, CD, DB, *of the triangle* CDB.

And $3{\cdot}75 - 2{\cdot}5 = 1{\cdot}25, \therefore 1{\cdot}25, 1{\cdot}25$ *and* $1{\cdot}25 = 3$ *differences.*

Whence $\sqrt{3{\cdot}75 \times 1{\cdot}25 \times 1{\cdot}25 \times 1{\cdot}25} = \sqrt{3{\cdot}75 \times 1{\cdot}25^3} = \sqrt{7{\cdot}32421875} = 2{\cdot}7063 =$ *area of the base as before,*

And $2{\cdot}7063 \times 10 = 27{\cdot}063$ *solid feet, the content of the prism required.*

To find the convex surface of a cylinder.—Multiply the periphery or circumference of the base, by the height of the cylinder, and the product will be the convex surface.



What is the convex surface of the right cylinder ABCD, whose length BC is 20 feet, and the diameter of its base AB 2 feet?

Here $3{\cdot}1416 \times 2 = 6{\cdot}2832 =$ *periphery of the base* AB.

And $6{\cdot}2832 \times 20 = 125{\cdot}6640$ *square feet, the convexity required.*

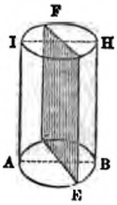
To find the solidity of a cylinder.—Multiply the area of the base by the perpendicular height of the cylinder, and the product will be the solidity.

What is the solidity of the cylinder ABCD, the diameter of whose base AB is 30 inches, and the height BC 50 inches.

Here $\cdot 7854 \times 30^2 = \cdot 7854 \times 900 = 706{\cdot}86 =$ *area of the base* AB.

And $706{\cdot}86 \times 50 = 35343$ *cubic inches; or* $\frac{35343}{1728} = 20{\cdot}4531$ *solid feet.*

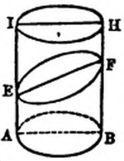
The four following cases contain all the rules for finding the superficies and solidities of *cylindrical ungulas.*



When the section is parallel to the axis of the cylinder.

RULE.—Multiply the length of the arc line of the base by the height of the cylinder, and the product will be the *curve surface.*

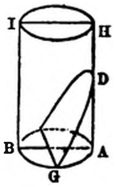
Multiply the area of the base by the height of the cylinder, and the product will be the *solidity.*



When the section passes obliquely through the opposite sides of the cylinder.

RULE.—Multiply the circumference of the base of the cylinder by half the sum of the greatest and least lengths of the ungula, and the product will be the *curve surface.*

Multiply the area of the base of the cylinder by half the sum of the greatest and least lengths of the ungula, and the product will be the *solidity.*



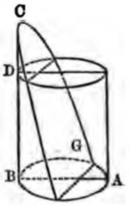
When the section passes through the base of the cylinder, and one of its sides.

RULE.—Multiply the sine of half the arc of the base by the diameter of the cylinder, and from this product subtract the product of the arc and cosine.

Multiply the difference thus found, by the quotient of the height divided by the versed sine, and the product will be the *curve surface.*

From $\frac{2}{3}$ of the cube of the right sine of half the arc of the base, subtract the product of the area of the base and the cosine of the said half arc.

Multiply the difference, thus found, by the quotient arising from the height divided by the versed sine, and the product will be the *solidity.*



When the section passes obliquely through both ends of the cylinder.

RULE.—Conceive the section to be continued, till it meets the side of the cylinder produced; then say, as the difference of the versed sines of half the arcs of the two ends of the ungula is to the versed sine of half the arc of the less end, so is the height of the cylinder to the part of the side produced.

Find the surface of each of the ungulas, thus formed, and their difference will be the *surface.*

In like manner find the solidities of each of the ungulas, and their difference will be the *solidity.*

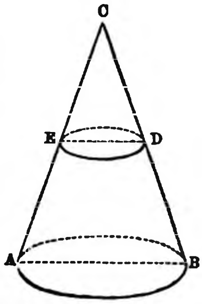
To find the convex surface of a right cone.—Multiply the circumference of the base by the slant height, or the length of the side of the cone, and half the product will be the surface required.

The diameter of the base AB is 3 feet, and the slant height AC or BC 15 feet; required the convex surface of the cone ACB.

Here $3\cdot1416 \times 3 = 9\cdot4248$ = *circumference of the base* AB.

And $\frac{9\cdot4248 \times 15}{2} = \frac{141\cdot3720}{2} = 70\cdot686$ *square feet, the convex surface required.*

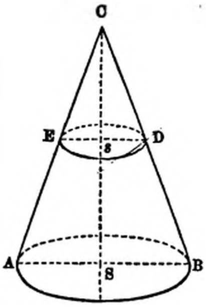
To find the convex surface of the frustum of a right cone.—Multiply the sum of the perimeters of the two ends, by the slant height of the frustum, and half the product will be the surface required.



In the frustum ABDE, the circumferences of the two ends AB and DE are 22·5 and 15·75 respectively, and the slant height BD is 26; what is the convex surface?

Here $\frac{(22\cdot5 + 15\cdot75) \times 26}{2} = \overline{22\cdot5 + 15\cdot75} \times 13 = 38\cdot25 \times 13 = 497\cdot25$ = *convex surface.*

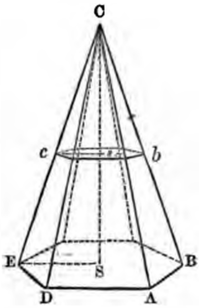
To find the solidity of a cone or pyramid.—Multiply the area of the base by one-third of the perpendicular height of the cone or pyramid, and the product will be the solidity.



Required the solidity of the cone ACB, whose diameter AB is 20, and its perpendicular height CS 24.

Here $\cdot7854 \times 20^2 = \cdot7854 \times 400 = 314\cdot16$ = *area of the base* AB.

And $314\cdot16 \times \frac{24}{3} = 314\cdot16 \times 8 = 2513\cdot28$ = *solidity required.*



Required the solidity of the hexagonal pyramid ECBD, each of the equal sides of its base being 40, and the perpendicular height CS 60.

Here $2\cdot598076$ (*multiplier when the side is* 1) $\times 40^2 = 2\cdot598076 \times 1600 = 4156\cdot9216$ = *area of the base.*

And $4156\cdot9216 \times \frac{60}{3} = 4156\cdot9216 \times 20 = 83138\cdot432$ *solidity.*

To find the solidity of a frustum of a cone or pyramid.—For the frustum of a cone, the diameters or circumferences of the two ends, and the height being given.

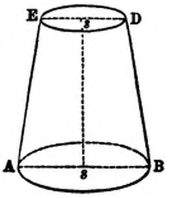
Add together the square of the diameter of the greater end, the square of the diameter of the less end, and the product of the two diameters; multiply the sum by ·7854, and the product by the height; $\frac{1}{3}$ of the last product will be the solidity. Or,

Add together the square of the circumference of the greater end, the square of the circumference of the less end, and the product of the two circumferences; multiply the sum by ·07958, and the product by the height; $\frac{1}{3}$ of the last product will be the solidity.

For the frustum of a pyramid whose sides are regular polygons.—Add together the square of a side of the greater end, the square of a side of the less end, and the product of these two sides; multiply the sum by the proper number in the Table of Superficies, and the product by the height; $\frac{1}{3}$ of the last product will be the solidity.

When the ends of the pyramids are not regular polygons.—Add together the areas of the two ends and the square root of their product; multiply the sum by the height, and $\frac{1}{3}$ of the product will be the solidity.

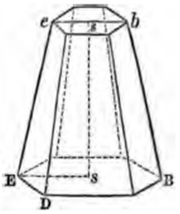
What is the solidity of the frustum of the cone EABD, the diameter of whose greater end AB is 5 feet, that of the less end ED, 3 feet, and the perpendicular height Ss, 9 feet?



$$\frac{(5^2 + 3^2 + \overline{5 \times 3}) \times \cdot7854 \times 9}{3} = \frac{346\cdot3614}{3} =$$

115·4538 *solid feet, the content of the frustum.*

What is the solidity of the frustum *e*EDB*b* of a hexagonal pyramid, the side ED of whose greater end is 4 feet, that *eb* of the less end 3 feet, and the height Ss, 9 feet?



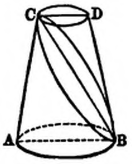
$$\frac{(4^2 + 3^2 + \overline{4 \times 3}) \times 2\cdot598076 \times 9}{3} = \frac{865\cdot159308}{3}$$

$= 288\cdot386436$ *solid feet, the solidity required.*

The following cases contain all the rules for finding the superficies and solidities of conical ungulas.

When the section passes through the opposite extremities of the ends of the frustum.

Let D = AB the diameter of the greater end; d = CD, the diameter of the less end; h = perpendicular height of the frustum, and $n = \cdot7854$.



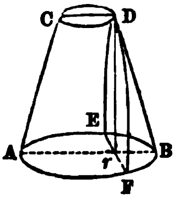
Then $\frac{d^2 - d\sqrt{Dd}}{D - d} \times \frac{nDh}{3}$ = solidity of the greater elliptic ungula ADB.

$\frac{D\sqrt{Dd} - d^2}{D - d} \times \frac{ndh}{3}$ = solidity of the less ungula ACD.

$\frac{(D^{\frac{3}{2}} - d^{\frac{3}{2}})^2}{D - d} \times \frac{nh}{3}$ = difference of these hoofs.

And $\frac{n}{D - d} \sqrt{4h^2 + (D - d^2)} \times (D^2 - \frac{D + d}{2} \sqrt{Dd}$ = curve surface of ADB.

When the section cuts off parts of the base, and makes the angle DrB *less than the angle* CAB.

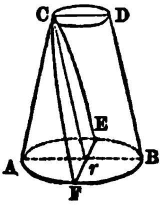


Let S = tabular segment, whose versed sine is $\overline{\mathrm{B}r \div \mathrm{D}}$; s — tab. seg. whose versed sine is $\overline{\mathrm{B}r - (\mathrm{D} - d) \div d}$, and the other letters as above.

The $(\mathrm{S} \times \mathrm{D}^3 - s \times d^3 \times \frac{\mathrm{B}r}{\mathrm{B}r - \overline{\mathrm{D} - d}} \sqrt{\frac{\mathrm{B}r}{\mathrm{B}r - \overline{\mathrm{D} - d}}} \times \frac{\frac{1}{2}h}{\mathrm{D} - d}$ = solidity of the elliptic hoof EFBD.

And $\frac{1}{\mathrm{D} - d} \sqrt{4h^2 + (\mathrm{D} - d)^2} \times (\text{seg. FBE} - \frac{d^2}{\mathrm{D}^2} \times \frac{\frac{1}{2} \times (\mathrm{D} + d) - \mathrm{A}r}{d - \mathrm{A}r} \times \sqrt{\frac{\mathrm{B}r}{d - \mathrm{A}r}} \times$ seg. of the circle AB, whose height is $\mathrm{D} \times \frac{d - \mathrm{A}r}{d})$ = convex surface of EFBD.

When the section is parallel to one of the sides of the frustum.

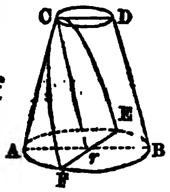


Let A = area of the base FBE, and the other letters as before.

Then $(\frac{\mathrm{A} \times \mathrm{D}}{\mathrm{D} - d} - \frac{4}{3}d \sqrt{(\mathrm{B} - d) \times d}) \times \frac{1}{3}h$ = solidity of the parabolic hoof EFBD.

And $\frac{1}{\mathrm{D} - d} \sqrt{4h^2 \times (\mathrm{D} - d)^2} \times (\text{seg. FBE} - \frac{2}{3}\overline{\mathrm{D} - d} \times \sqrt{d \times \overline{\mathrm{D} - d}})$ = convex surface of EFBD.

When the section cuts off part of the base, and makes the angle DrB *greater than the angle* CAB.



Let the area of the hyperbolic section EDF = A, and the area of the circular seg. EBF = a.

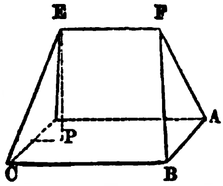
Then $\frac{\frac{1}{3}h}{\mathrm{D} - h} \times (a \times \mathrm{D} - \mathrm{A} \times \frac{d \times \mathrm{E}r}{\mathrm{C}r})$ = solidity of the hyperbolic ungula EFBD.

And $\frac{1}{\mathrm{D} - d} \times \sqrt{4h^2 + (\mathrm{D} - d)^2} \times (\text{cir. seg. EBF} - \frac{d^2}{\mathrm{D}^2} \times \frac{\mathrm{B}r - \frac{1}{2}(\mathrm{D} - d)}{\mathrm{B}r - \overline{\mathrm{D} - d}} \sqrt{\frac{\mathrm{B}r}{\mathrm{B}r - \overline{d - \mathrm{D}}}}$ = curve surface of EFBD.

The transverse diameter of the hyp. seg. $= \frac{d \times \mathrm{C}r}{\mathrm{D} - d - \mathrm{B}r}$ and the conjugate $= d \sqrt{\frac{\mathrm{B}r}{\mathrm{D} - d - \mathrm{B}r}}$, from which its area may be found by the former rules.

To find the solidity of a cuneus or wedge.—Add twice the length of the base to the length of the edge, and reserve the number.

Multiply the height of the wedge by the breadth of the base, and this product by the reserved number; $\frac{1}{6}$ of the last product will be the solidity.



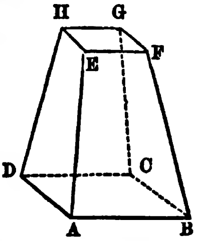
How many solid feet are there in a wedge, whose base is 5 feet 4 inches long, and 9 inches broad, the length of the edge being 3 feet 6 inches, and the perpendicular height 2 feet 4 inches?

Here $\frac{(64 \times 2 + 42) \times 28 \times 9}{6} = \frac{(128 + 42) \times 28 \times 9}{6} = \frac{170 \times 28 \times 9}{6} = \frac{170 \times 28 \times 3}{2} = 170 \times 14 \times 3 = 7140$ *solid inches.*

And $7140 \div 1728 = 4{\cdot}1319$ *solid feet, the content.*

To find the solidity of a prismoid.—To the sum of the areas of the two ends add four times the area of a section parallel to and equally distant from both ends, and this last sum multiplied by $\frac{1}{6}$ of the height will give the solidity.

The length of the middle rectangle is equal to half the sum of the lengths of the rectangles of the two ends, and its breadth equal to half the sum of the breadths of those rectangles.



What is the solidity of a rectangle prismoid, the length and breadth of one end being 14 and 12 inches, and the corresponding sides of the other 6 and 4 inches, and the perpendicular $30\frac{1}{2}$ feet.

Here $14 \times 12 + \overline{6 \times 4} = 168 + 24 = 192 =$ *sum of the area of the two ends.*

Also $\frac{14 + 6}{2} = \frac{20}{2} = 10 =$ *length of the middle rectangle.*

And $\frac{12 + 4}{2} = \frac{16}{2} = 8 =$ *breadth of the middle rectangle.*

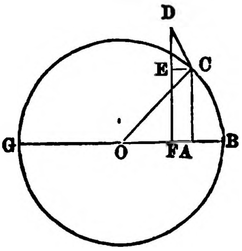
Whence $10 \times 8 \times 4 = 80 \times 4 = 320 = 4$ *times the area of the middle rectangle.*

Or $(320 + 192) \times \frac{366}{6} = 512 \times 61 = 31232$ *solid inches.*

And $31232 \div 1728 = 18{\cdot}074$ *solid feet, the content.*

To find the convex surface of a sphere.—Multiply the diameter of the sphere by its circumference, and the product will be the convex superficies required.

The curve surface of any zone or segment will also be found by multiplying its height by the whole circumference of the sphere.

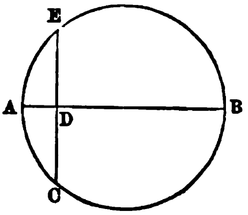


What is the convex superficies of a globe BCG whose diameter BG is 17 inches?

Here $3{\cdot}1416 \times 17 \times 17 = 53{\cdot}4072 \times 17 = 907{\cdot}9224$ *square inches.*

And $907{\cdot}9224 \div 144 = 6{\cdot}305$ *square feet.*

To find the solidity of a sphere or globe.—Multiply the cube of the diameter by ·5236, and the product will be the solidity.



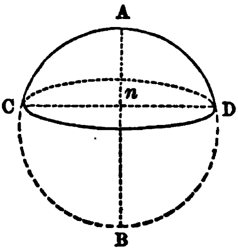
What is the solidity of the sphere AEBC, whose diameter AB is 17 inches?

Here 17^3 × ·5236 = 17 × 17 × 17 × ·5236 = 289 × 17 × 5236 = 4913 × ·5236 = 2572·4468 *solid inches.*

And 2572·4468 ÷ 1728 = 1·48868 *solid feet.*

To find the solidity of the segment of a sphere.—To three times the square of the radius of its base add the square of its height, and this sum multiplied by the height, and the product again by ·5236, will give the solidity. Or,

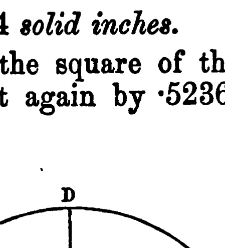
From three times the diameter of the sphere subtract twice the height of the segment, multiply by the square of the height, and that product by ·5236; the last product will be the solidity.



The radius C*n* of the base of the segment CAD is 7 inches, and the height A*n* 4 inches; what is the solidity?

Here $(7^2 \times 3 + 4^2)$ × 4 × ·5236 = $(49\times3+4^2)$ × 4 × ·5236 = $(147 + 4^2)$ × 4 × ·5236 = (147+16) × 4 × ·5236 = 163 × 4 × ·5236 = 652 × ·5236 = 341·3872 *solid inches.*

To find the solidity of a frustum or zone of a sphere.—To the sum of the squares of the radii of the two ends, add one-third of the square of their distance, or of the breadth of the zone, and this sum multiplied by the said breadth, and the product again by 1·5708, will give the solidity.

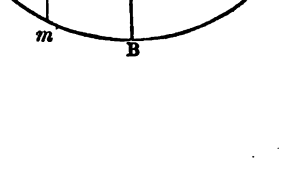


What is the solid content of the zone ABCD, whose greater diameter AB is 20 inches, the less diameter CD 15 inches, and the distance *nm* of the two ends 10 inches?

Here $(10^2 + 7{\cdot}5^2 + \frac{10^2}{3})$ × 10 × 1·5708 = (100 + 56·25 + 33·33) × 10 × 1·5708 = 189·58 × 10 × 1·5708 = 1895·8 × 1·5708 = 2977·92264 *solid inches.*

To find the solidity of a spheroid.—Multiply the square of the revolving axe by the fixed axe, and this product again by ·5236, and it will give the solidity required.

·5236 is = $\frac{1}{6}$ of 3·1416.



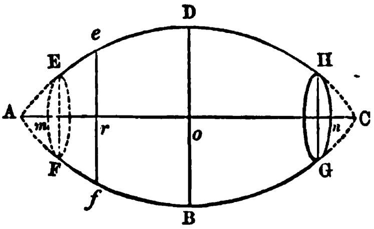
In the prolate spheroid ABCD, the transverse, or fixed axe AC is 90, and the conjugate or revolving axe DB is 70; what is the solidity?

Here DB^2 × AC × ·5236 = 70^2 × 90 × ·5236 = 4900 × 90 × ·5236 = 441000 × ·5236 = 230907·6 = *solidity required.*

To find the content of the middle frustum of a spheroid, its length, the middle diameter, and that of either of the ends, being given, when the ends are circular or parallel to the revolving axis.—To twice the square of the middle diameter add the square of the diameter of either of the ends, and this sum multiplied by the length of the frustum, and the product again by ·2618, will give the solidity.

Where $\cdot 2618 = \frac{1}{12}$ of 3·1416.

In the middle frustum of a spheroid EFGH, the middle diameter DB is 50 inches, and that of either of the ends EF or GH is 40 inches, and its length *nm* 18 inches; what is its solidity?

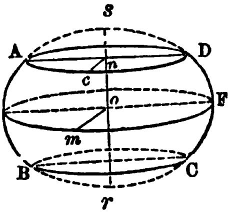


Here $(50^2 \times 2 + 40^2) \times 18 \times \cdot 2618 = (2500 \times 2 + 1600) \times 18 \times \cdot 2618 = (5000 + 1600) \times 18 \times \cdot 2618 = 6600 \times 18 \times \cdot 2618 = 118800 \times \cdot 2613 = 31101 \cdot 84$ *cubic inches.*

When the ends are elliptical or perpendicular to the revolving axis.—Multiply twice the transverse diameter of the middle section by its conjugate diameter, and to this product add the product of the transverse and conjugate diameters of either of the ends.

Multiply the sum thus found by the distance of the ends or the height of the frustum, and the product again by ·2618, and it will give the solidity required.

In the middle frustum ABCD of an oblate spheroid, the diameters of the middle section EF are 50 and 30, those of the end AD 40 and 24, and its height *ne* 18; what is the solidity?

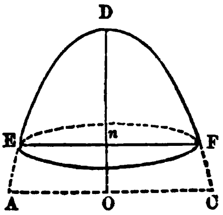


Here $(50 \times 2 \times 30 + \overline{40 \times 24}) \times 18 \times \cdot 2618 = (3000 + 960) \times 18 \times \cdot 2618 = 3960 \times 18 \times \cdot 2618 = 71280 \times \cdot 2618 = 18661 \cdot 104 =$ *the solidity.*

To find the solidity of the segment of a spheroid, when the base is parallel to the revolving axis.—Divide the square of the revolving axis by the square of the fixed axe, and multiply the quotient by the difference between three times the fixed axe and twice the height of the segment.

Multiply the product thus found by the square of the height of the segment, and this product again by ·5236, and it will give the solidity required.

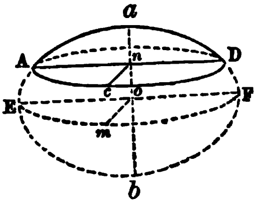
In the prolate spheroid DEFD, the transverse axis 2 DO is 100, the conjugate AC 60, and the height D*n* of the segment EDF 10; what is the solidity?



Here $(\frac{60^2}{100^2} \times \overline{300 - 20}) \times 10^2 \times \cdot 5236 = \cdot 36 \times 280 \times 10^2 \times \cdot 5236 = 100 \cdot 80 \times 100 \times \cdot 5236 = 10080 \times \cdot 5236 = 5277 \cdot 888 =$ *the solidity.*

When the base is perpendicular to the revolving axis.—Divide the fixed axe by the revolving axe, and multiply the quotient by the difference between three times the revolving axe and twice the height of the segment.

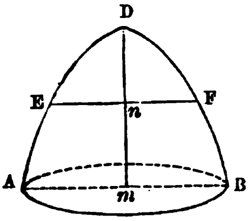
Multiply the product thus found by the square of the height of the segment, and this product again by ·5236, and it will give the solidity required.



In the prolate spheroid *a*E*b*F, the transverse axe EF is 100, the conjugate *ab* 60, and the height *an* of the segment *a*AD 12; what is the solidity?

Here 156 (= *diff. of* 3*ab and* 2*an*) × $1\frac{2}{3}$ (= EF ÷ *ab* × 144 (= *square of an*) × ·5236 $= \frac{156 \times 5}{3}$ × 144 × ·5236 = 52 × 5 × 144 × ·5236 = 260 × 144 × ·5236 = 37440 × ·5236 = 19603·584 = *the solidity.*

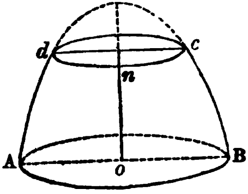
To find the solidity of a parabolic conoid.—Multiply the area of the base by half the altitude, and the product will be the content.



What is the solidity of the paraboloid ADB, whose height D*m* is 84, and the diameter BA of its circular base 48?

Here 48^2 × ·7854 × 42 (= ½ D*m*) = 2304 × ·7854 × 42 = 1809·5616 × 42 = 76001·5872 = *the solidity.*

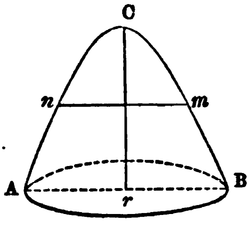
To find the solidity of the frustum of a paraboloid, when its ends are perpendicular to the axe of the solid.—Multiply the sum of the squares of the diameters of the two ends by the height of the frustum, and the product again by ·3927, and it will give the solidity.



Required the solidity of the parabolic frustum ABC*d*, the diameter AB of the greater end being 58, that of the less end *dc* 30, and the height *no* 18.

Here ($58^2 + 30^2$) × 18 × ·3927 = (3364 + 900) × 18 × ·3927 = 4264 × 18 × ·3927 = 76752 × ·3927 = 30140·5104 = *the solidity.*

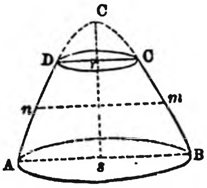
To find the solidity of an hyperboloid.—To the square of the radius of the base add the square of the middle diameter between the base and the vertex, and this sum multiplied by the altitude, and the product again by ·5236 will give the solidity.



In the hyperboloid ACB, the altitude C*r* is 10, the radius A*r* of the base 12, and the middle diameter *nm* 15·8745; what is the solidity?

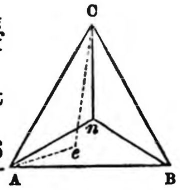
Here $\overline{15{\cdot}8745^2 + 12^2}$ × 10 × ·5236 = $\overline{251{\cdot}99975 + 144}$ × 10 × ·5236 = 395·99975 × 10 × ·5236 = 3959·9975 × ·5236 = 2073·454691 = *the solidity.*

To find the solidity of the frustum of an hyperbolic conoid.—Add together the squares of the greatest and least semi-diameters, and the square of the whole diameter in the middle; then this sum being multiplied by the altitude, and the product again by ·5236, will give the solidity.



In the hyperbolic frustum ADCB, the length *rs* is 20, the diameter AB of the greater end 32, that DC of the less end 24, and the middle diameter *nm* 28·1708; required the solidity.

Here $(16^2 + 12^2 + 28{\cdot}1708^2) \times 20 \times {\cdot}52359 = (256 + 144 + 793{\cdot}5939) \times 20 \times {\cdot}52359 = 1193{\cdot}5939 \times 20 \times {\cdot}52359 = 23871{\cdot}878 \times {\cdot}52359 = 12499{\cdot}07660202 =$ *solidity.*

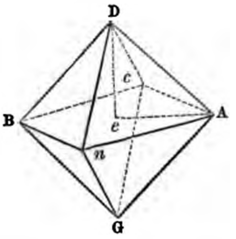


To find the solidity of a tetraedron.—Multiply $\frac{1}{12}$ of the cube of the linear side by the square root of 2, and the product will be the solidity.

The linear side of a tetraedron ABC*n* is 4; what is the solidity?

$$\frac{4^3}{12} \times \sqrt{2} = \frac{4 \times 4 \times 4}{12} \times \sqrt{2} = \frac{4 \times 4}{3} \times \sqrt{2} = \frac{16}{3} \times \sqrt{2} = \frac{16}{3} \times 1{\cdot}414 = \frac{22{\cdot}624}{3} = 7{\cdot}5413 = \textit{solidity.}$$

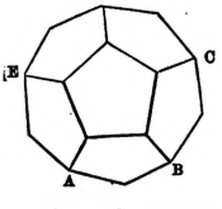
To find the solidity of an octaedron.—Multiply $\frac{1}{3}$ of the cube of the linear side by the square root of 2, and the product will be the solidity.



What is the solidity of the octaedron BGAD, whose linear side is 4?

$$\frac{4^3}{3} \times \sqrt{2} = \frac{64}{3} \times \sqrt{2} = 21{\cdot}333, \times \sqrt{2} = 21{\cdot}333 \times 1{\cdot}414 = 30{\cdot}16486 = \textit{solidity.}$$

To find the solidity of a dodecaedron.—To 21 times the square root of 5 add 47, and divide the sum by 40: then the square root of the quotient being multiplied by five times the cube of the linear side will give the solidity.



The linear side of the dodecaedron ABCDE is 3; what is the solidity?

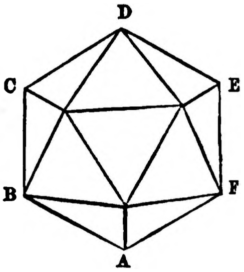
$$\sqrt{\frac{21\sqrt{5}+47}{40}} \times 27 \times 5 = \sqrt{\frac{21 \times 2{\cdot}23606+47}{40}} \times 27 \times 5 = \sqrt{\frac{46{\cdot}95726+47}{40}} \times 135 = 206{\cdot}901$$

solidity.

To find the solidity of an icosaedron.—To three times the square root of 5 add 7, and divide the sum by 2; then the square root of this quotient being multiplied by $\frac{5}{6}$ of the cube of the linear side will give the solidity.

That is $\frac{5}{6}\, S^3 \times \surd\,(\frac{7 + 3 \surd 5}{2}) =$ solidity when S is = to the linear side.

The linear side of the icosaedron ABCDEF is 3; what is the solidity?



$\surd \frac{3 \surd 5 + 7}{2} \times \frac{5 \times 3^2}{6} = \surd \frac{3 \times 2{\cdot}23606 + 7}{2} \times \frac{5 \times 27}{6} = \surd \frac{6{\cdot}70818 + 7}{2} \times \frac{5 \times 9}{2} = \surd \frac{13{\cdot}70818}{2} \times \frac{45}{2} = \surd \overline{6{\cdot}85409} \times 22{\cdot}5 = 2{\cdot}61803 \times 22{\cdot}5 = 58{\cdot}9056 =$ *solidity.*

The superficies and solidity of any of the five regular bodies may be found as follows:

Rule 1. Multiply the tabular area by the square of the linear edge, and the product will be the superficies.

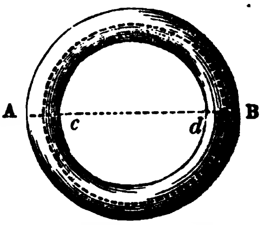
2. Multiply the tabular solidity by the cube of the linear edge, and the product will be the solidity.

Surfaces and Solidities of the Regular Bodies.

No. of Sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64578	7.66312
20	Icosaedron	8.66025	2.18169

To find the convex superficies of a cylindric ring.—To the thickness of the ring add the inner diameter, and this sum being multiplied by the thickness, and the product again by 9.8696, will give the superficies.

The thickness of A*c* of a cylindric ring is 3 inches, and the inner diameter *cd* 12 inches; what is the convex superficies?



$\overline{12 + 3} \times 3 \times 9{\cdot}8696 = 15 \times 3 \times 9{\cdot}8696 = 45 \times 9{\cdot}8696 = 444{\cdot}132 =$ *superficies.*

To find the solidity of a cylindric ring.—To the thickness of the ring add the inner diameter, and this sum being multiplied by the square of half the thickness, and the product again by 9·8696, will give the solidity.

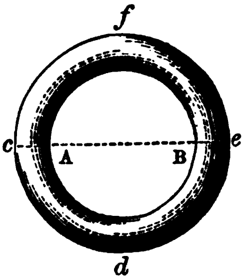
What is the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches?

$\overline{8 + 3} \times \overline{\tfrac{3}{2}}|^{2} \times 9{\cdot}8696 = 11 \times 1{\cdot}5^{2} \times 9{\cdot}8693 = 11 \times 2{\cdot}25 \times 9{\cdot}8696 = 24{\cdot}75 \times 9{\cdot}8696 = 244{\cdot}2726 =$ *solidity.*

The inner diameter AB of the cylindric ring *cdef* equals 18 feet, and the sectional diameter *c*A or B*e* equals 9 inches; required the convex surface and solidity of the ring.

18 feet × 12 = 216 *inches, and* $\overline{216 + 9} \times 9 \times 9{\cdot}8696 = 19985{\cdot}94$ *square inches.*

$\overline{216 + 9} \times 9^{2} \times 2{\cdot}4674 = 44968{\cdot}365$ *cubic inches.*



In the formation of a hoop or ring of wrought iron, it is found in practice that in bending the iron, the side or edge which forms the interior diameter of the hoop is upset or shortened, while at the same time the exterior diameter is drawn or lengthened; therefore, the proper diameter by which to determine the length of the iron in an unbent state, is the distance from centre to centre of the iron of which the hoop is composed: *hence the rule to determine the length of the iron.* If it is the interior diameter of the hoop that is given, add the thickness of the iron; but if the exterior diameter, subtract from the given diameter the thickness of the iron, multiply the sum or remainder by 3·1416, and the product is the length of the iron, in equal terms of unity.

Supposing the interior diameter of a hoop to be 32 inches, and the thickness of the iron 1¼, what must be the proper length of the iron, independent of any allowance for shutting?

$\overline{32 + 1{\cdot}25} = 33{\cdot}25 \times 3{\cdot}1416 = 104{\cdot}458$ *inches.*

But the same is obtained simply by inspection in the Table of Circumferences.

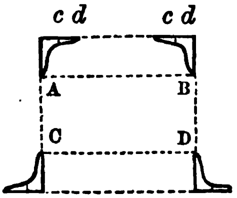
Thus, 33·25 = 2 *feet* 9¼ *in.*, opposite to which is 8 feet 8½ inches.

Again, let it be required to form a hoop of iron ⅞ inch in thickness, and 16½ inches outside diameter.

16·5 − ·875 = 15·625, or 1 foot 3⅝ inches;

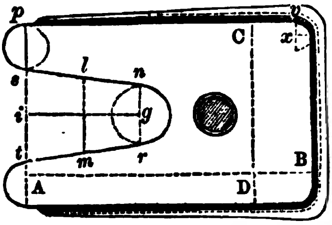
opposite to which, in the Table of Circumferences, is 4 feet 1 inch, independent of any allowance for shutting.

The length for angle iron, of which to form a ring of a given diameter, varies according to the strength of the iron at the root; and the rule is, for a ring with the flange outside, *add* to its required interior diameter, twice the extreme strength of the iron at the root; or, for a ring with the flange inside, *subtract* twice the extreme strength; and the sum or remainder is the diameter by which to determine the length of the angle iron. Thus, suppose two angle iron rings similar to the following be required, the exterior diameter AB, and interior diameter CD, each to be 1 foot 10½ inches, and the extreme strength of the iron at the root *cd*, *cd*, &c, ⅜ of an inch;



twice ⅞ = 1¾, and 1 ft. 10½ in. + 1¾ = 2 ft. ¼ in., opposite to which, in the Table of Circumferences, is 6 ft. 4¼ in., the length of the iron for CD; and 1 ft. 10½ in. − 1¾ = 1 ft. 8¾ in., opposite to which is 5 ft. 5¼ in., the length of the iron for AB.

But observe, as before, that the necessary allowance for shutting must be added to the length of the iron, in addition to the length as expressed by the Table.



Required the capacity in gallons of a locomotive engine tender tank, 2 feet 8 inches in depth, and its superficial dimensions the following, with reference to the annexed plan:

Length, or dist. between	A and B	=	10 ft.	2¾ in. or,	122·75 in.
Breadth "	C and D	=	6	7½	79·5
Length "	*i* and *g*	=	3	10¾	46·75
Mean breadth of coke-space or	*lm*	=	3	1¼	37·25
Diameter of circle	*rn*	=	2	8¼	32·25
" "	*ps*	=	1	6½	18·5
Radius of back corners	*vx*	=		4	4

Then, $122{\cdot}75 \times 79{\cdot}5 = 9758{\cdot}525$ square inches, as a rectangle.

And $18{\cdot}5^2 \times {\cdot}7854 = 268{\cdot}8$ " " area of circle formed by the two ends.

Total $10027{\cdot}325$ " " from which deduct the area of the coke-space, and the difference of area between the semicircle formed by the two back corners, and that of a rectangle of equal length and breadth;

Then $46{\cdot}75 \times 37{\cdot}25 = 1731{\cdot}4375$ area of *r*, *n*, *s*, *t*, in sq. ins.

$$\frac{32{\cdot}25^2 \times {\cdot}7854}{2} = 408{\cdot}4 \quad \text{area of half the circle } rn.$$

Radius of back corners = 4 inches;

consequently $8^2 \times {\cdot}7854 = 25{\cdot}13$, the semicircle's area; and $8 \times 4 = 32 - 25{\cdot}13 = 6{\cdot}87$ inches taken off by rounding the corners.

Hence, $1731{\cdot}4375 + 408{\cdot}4 + 6{\cdot}87 = 2146{\cdot}707$, and

$10027{\cdot}235 - 2146{\cdot}707 = 7880{\cdot}618$ square inches, or whole area in plan,

$7880{\cdot}618 \times 32$ the depth $= 252179{\cdot}776$ cubic inches, and $252179{\cdot}776$ divided by 231 gives $1091{\cdot}6873$ the content in gallons.

TABLES *by which to facilitate the Mensuration of Timber.*

1. Flat or Board Measure.

Breadth in inches.	Area of a lineal foot.	Breadth in inches.	Area of a lineal foot.	Breadth in inches.	Area of a lineal foot.
¼	·0208	4	·3334	8	·6667
½	·0417	4¼	·3542	8¼	·6875
¾	·0625	4½	·375	8½	·7084
1	·0834	4¾	·3958	8¾	·7292
1¼	·1042	5	·4167	9	·75
1½	·125	5¼	·4375	9¼	·7708
1¾	·1459	5½	·4583	9½	·7917
2	·1667	5¾	·4792	9¾	·8125
2¼	·1875	6	·5	10	·8334
2½	·2084	6¼	·5208	10¼	·8542
2¾	·2292	6½	·5416	10½	·875
3	·25	6¾	·5625	10¾	·8959
3¼	·2708	7	·5833	11	·9167
3½	·2916	7¼	·6042	11¼	·9375
3¾	·3125	7½	·625	11½	·9583
		7¾	·6458	11¾	·9792

Application and Use of the Table.

Required the number of square feet in a board or plank 16½ feet in length and 9¾ inches in breadth.

Opposite 9¾ is ·8125 × 16·5 = 13·4 square feet.

A board 1 foot 2¾ inches in breadth, and 21 feet in length; what is its superficial content in square feet?

Opposite 2¾ is ·2292, to which add the 1 foot; then 1·2292 × 21 = 25·8 square feet.

In a board 15½ inches at one end, 9 inches at the other, and 14½ feet in length, how many square feet?

$$\frac{15{\cdot}5 + 9}{2} = 12\tfrac{1}{4}, \text{ or } 1{\cdot}0208; \text{ and } 1{\cdot}0208 \times 14{\cdot}5 = 14{\cdot}8 \text{ sq. ft.}$$

The solidity of round or unsquared timber may be found with much more accuracy by the succeeding Rule:—Multiply the square of one-fifth of the mean girth by twice the length, and the product will be the solidity, very near the truth.

A piece of timber is 30 feet long, and the mean girth is 128 inches, what is the solidity?

$$\frac{128}{5} = 25{\cdot}6.$$

$$\text{Then } \frac{25{\cdot}6^2 \times 60}{144} = 273{\cdot}06 \text{ cubic feet.}$$

This is nearer the truth than if one-fourth the girth be employed.

2. Cubic or Solid Measure.

Mean ¼ girt in inches.	Cubic feet in each lineal foot.	Mean ¼ girt in inches.	Cubic feet in each lineal foot.	Mean ¼ girt in inches.	Cubic feet in each lineal foot.	Mean ¼ girt in inches.	Cubic feet in each lineal foot.
6	·25	12	1	18	2·25	24	4
6¼	·272	12¼	1·042	18¼	2·313	24¼	4·084
6½	·294	12½	1·085	18½	2·376	24½	4·168
6¾	·317	12¾	1·129	18¾	2·442	24¾	4·254
7	·340	13	1·174	19	2·506	25	4·34
7¼	·364	13¼	1·219	19¼	2·574	25¼	4·428
7½	·39	13½	1·265	19½	2·64	25½	4·516
7¾	·417	13¾	1·313	19¾	2·709	25¾	4·605
8	·444	14	1·361	20	2·777	26	4·694
8¼	·472	14¼	1·41	20¼	2·898	26¼	4·785
8½	·501	14½	1·46	20½	2·917	26½	4·876
8¾	·531	14¾	1·511	20¾	2·99	26¾	4·969
9	·562	15	1·562	21	3·062	27	5·062
9¼	·594	15¼	1·615	21¼	3·136	27¼	5·158
9½	·626	15½	1·668	21½	3·209	27½	5·252
9¾	·659	15¾	1·772	21¾	3·285	27¾	5·348
10	·694	16	1·777	22	3·362	28	5·444
10¼	·73	16¼	1·833	22¼	3·438	28¼	5·542
10½	·766	16½	1·89	22½	3·516	28½	5·64
10¾	·803	16¾	1·948	22¾	3·598	28¾	5·74
11	·84	17	2·006	23	3·673	29	5·84
11¼	·878	17¼	2·066	23¼	3·754	29¼	5·941
11½	·918	17½	2·126	23½	3·835	29½	6·044
11¾	·959	17¾	2·187	23¾	3·917	29¾	6·146

In the cubic estimation of timber, custom has established the rule of ¼, the mean girt being the side of the square considered as the cross sectional dimensions; hence, multiply the number of cubic feet by lineal foot as in the Table of Cubic Measure opposite the ¼ girt, and the product is the solidity of the given dimensions in cubic feet.

Suppose the mean ¼ girt of a tree 21¼ inches, and its length 16 feet, what are its contents in cubic feet?

$$3{\cdot}136 \times 16 = 50{\cdot}176 \text{ cubic feet.}$$

Battens, Deals, and Planks are each similar in their various lengths, but differing in their widths and thicknesses, and hence their principal distinction: thus, a batten is 7 inches by 2½, a deal 9 by 3, and a plank 11 by 3, these being what are termed the standard dimensions, by which they are bought and sold, the length of each being taken at 12 feet; therefore, in estimating for the proper value of any quantity, nothing more is required than their lineal dimensions, by which to ascertain the number of times 12 feet, there are in the given whole.

Suppose I wish to purchase the following:

7 of	6 feet	6 × 7 =	42 feet
5	14	14 × 5 =	70
11	19	19 × 11 =	209
and 6	21	21 × 6 =	126

$$12\,)\,\overline{447}\,)\ 37{\cdot}25 \text{ standard deals.}$$

Table *showing the number of Lineal Feet of Scantling of various dimensions, which are equal to a Cubic Foot.*

	Inches.		Ft.	In.
2 inches by	2	require in length	36	0
	2½		28	9
	3		24	0
	3½		20	7
	4		18	0
	4½		16	0
	5		14	5
	5½		13	1
	6		12	0
	6½		11	1
	7		10	3
	7½		9	7
	8		9	0
	8½		8	6
	9		8	0
	9½		7	7
	10		7	3
	10½		6	10
	11		6	6
	11½		6	4
	12		6	0
3 inches by	3		16	0
	3½		13	8
	4		12	0
	4½		10	8
	5		9	7
	5½		9	0
	6		8	0
	6½		7	4
	7		6	10
	7½		6	4
	8		6	0
	8½		5	8
	9		5	4
	9½		5	0
	10		4	10
	10½		4	6
	11		4	4
	11½		4	2
	12		4	0

	Inches.		Ft.	In.
4 inches by	4	require in length	9	0
	4½		8	0
	5		7	2
	5½		6	6
	6		6	0
	6½		5	6
	7		5	1
	7½		4	9
	8		4	6
	8½		4	3
	9		4	0
	9½		3	9
	10		3	7
	10½		3	5
	11		3	3
	11½		3	2
	12		3	0
5 inches by	5		5	9
	5½		5	3
	6		4	10
	6½		4	5
	7		4	1
	7½		3	10
	8		3	7
	8½		3	5
	9		3	2
	9½		3	0
	10		2	10
	10½		2	9
	11		2	8
	11½		2	6
	12		2	4
6 inches by	6		4	0
	6½		3	8
	7		3	5
	7½		3	2
	8		3	0
	8½		2	10
	9		2	8

	Inches.		Ft.	In.
6 inches by	9½	require in length	2	6
	10		2	5
	10½		2	3
	11		2	2
	11½		2	1
	12		2	0
7 inches by	7		2	11
	7½		2	9
	8		2	6
	8½		2	5
	9		2	3
	9½		2	2
	10		2	1
	10½		1	11
	11		1	10
	11½		1	9
	12		1	8
8 inches by	8		2	3
	8½		2	1
	9		2	0
	9½		1	10
	10		1	9
	10½		1	8
	11		1	7
	11½		1	7
	12		1	6
9 inches by	9		1	9
	9½		1	8
	10		1	7
	10½		1	6
	11		1	5
	11½		1	4
	12		1	4
10 in. by	10		1	5
	10½		1	4
	11		1	4
	11½		1	3

Hewn and sawed timber are measured by the cubic foot. The *unit* of board measure is a superficial foot one inch thick.

To measure round timber.—Multiply the length in feet by the square of ¼ of the mean girth in inches, and the product divided by 144 gives the content in cubic feet.

The ¼ girths of a piece of timber, taken at five points, equally distant from each other, are 24, 28, 33, 35, and 40 inches; the length 30 feet, what is the content?

$$\frac{24 + 28 + 33 + 35 + 40}{5} = 32.$$

Then $\frac{32^2 \times 30}{144} = 213\frac{1}{3}$ cubic feet.

TABLE *containing the Superficies and Solid Content of Spheres, from 1 to 12, and advancing by a tenth.*

Diam.	Superficies.	Solidity.	Diam.	Superficies.	Solidity.	Diam.	Superficies.	Solidity.
1·0	3·1416	·5236	4·7	69·3979	54·3617	8·4	221·6712	310·3398
·1	3·8013	·6969	·8	72·3824	57·9059	·5	226·9806	321·5558
·2	4·5239	·9047	·9	75·4298	61·6010	·6	232·3527	333·0389
·3	5·3093	1·1503	5·0	78·5400	65·4500	·7	237·7877	344·7921
·4	6·1575	1·4367	·1	81·7130	69·4560	·8	243·2855	356·8187
·5	7·0686	1·7671	·2	84·9488	73·6223	·9	248·8461	369·1217
·6	8·0424	2·1446	·3	88·2475	77·9519	9·0	254·4696	381·7044
·7	9·0792	2·5724	·4	91·6090	82·4481	·1	260·1558	394·5697
·8	10·1787	3·0536	·5	95·0334	87·1139	·2	265·9130	407·7210
·9	11·3411	3·5913	·6	98·5205	91·9525	·3	271·7169	421·1613
2·0	12·5664	4·1888	·7	102·0705	96·9670	·4	277·5917	434·8937
·1	13·8544	4·8490	·8	105·6834	102·1606	·5	283·5294	448·9215
·2	15·2053	5·5752	·9	109·3590	107·5364	·6	289·5298	463·2477
·3	16·6190	6·3706	6·0	113·0976	113·0976	·7	295·5931	477·7755
·4	18·0956	7·2382	·1	116·8989	118·8472	·8	301·7192	492·8081
·5	19·6350	8·1812	·2	120·7631	124·7885	·9	307·9082	508·0485
·6	21·2372	9·2027	·3	124·6901	130·9246	10·0	314·1600	523·6000
·7	22·9022	10·3060	·4	128·6799	137·2585	·1	320·4746	539·4656
·8	24·6300	11·4940	·5	132·7326	143·7936	·2	326·8520	555·6485
·9	26·4208	12·7700	·6	136·8480	150·5329	·3	333·2923	572·1518
3.0	28·2744	14·1372	·7	141·0264	157·4795	·4	339·7954	588·9784
·1	30·1907	15·5985	·8	145·2675	164·6365	·5	346·3614	606·1324
·2	32·1699	17·1573	·9	149·5715	172·0073	·6	352·9901	623·6159
·3	34·2120	18·8166	7·0	153·9384	179·5948	·7	359·6817	641·48[illegible]5
·4	36·3168	20·5795	·1	158·3680	187·4021	·8	366·4362	659·5852
·5	38·4846	22·4493	·2	162·8605	195·4326	·9	373·2534	678·0771
·6	40·7151	[illegible]4290	·3	167·4158	203·6893	11·0	380·1336	696·9116
·7	43·0085	2[illegible]·5219	[illegible]	[illegible]·0340	212·1752	·1	387·0765	716·0915
·8	45·3647	28·7309	[illegible]	[illegible]·7150	220·[illegible]	[illegible]	394·0823	735·6200
·9	47·7837	31·0594	[illegible]	[illegible]1·4588	229·847[illegible]	[illegible]	401·1509	755·5008
4·0	50·2656	33·5104	·7	186·2654	239·0511	·4	408·2823	775·7364
·1	52·8102	36·0870	·8	191·1349	248·4754	·5	415·4766	796·3301
·2	55·4178	38·7924	·9	196·0672	258·1552	·6	422·7336	817·2851
·3	58·0881	41·6298	8·0	201·0624	268·0832	·7	430·0536	838·6045
·4	60·8213	44·6023	·1	206·1203	278·2625	·8	437·4363	860·2915
·5	63·6174	47·7130	·2	211·2411	288·6962	·9	444·8819	882·3492
·6	66·4782	50·9651	·3	216·4248	299·3876	12·0	452·3904	904·7808

To reduce Solid Inches into Solid Feet.

1728 Solid Inches to one Solid Foot.

Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.
1	=1728	18	=31104	35	=60480	52	=88956	69	=119232	85	=146880
2	3456	19	32832	36	62208	53	91584	70	120960	86	148608
3	5184	20	34560	37	63936	54	93312	71	122688	87	150336
4	6912	21	36288	38	65664	55	95040	72	124416	88	152064
5	8640	22	38016	39	67392	56	96768	73	126144	89	153792
6	10368	23	39744	40	69120	57	98496	74	127872	90	155520
7	12096	24	41472	41	70848	58	100224	75	129600	91	157248
8	13824	25	43200	42	72576	59	101952	76	131328	92	158976
9	15552	26	44928	43	74304	60	103680	77	133056	93	160704
10	17280	27	46656	44	76032	61	105408	78	134784	94	162432
11	19008	28	48384	45	77760	62	107136	79	136512	95	164160
12	20736	29	50112	46	79488	63	108864	80	138240	96	165888
13	22464	30	51840	47	81216	64	110592	81	139968	97	167616
14	24192	31	53568	48	82944	65	112320	82	141696	98	169344
15	25920	32	55296	49	84672	66	114048	83	143424	99	171072
16	27648	33	57024	50	86400	67	115776	84	145152	100	172800
17	29376	34	58752	51	88128	68	117504				

CUTTINGS AND EMBANKMENTS.

The angle of repose upon railways, or that incline on which a carriage would rest in whatever situation it was placed, is said to be at 1 in 280, or nearly 19 feet per mile; at any greater rise than this, the force of gravity overcomes the horizontal traction, and carriages will not rest, or remain quiescent upon the line, but will of themselves run down the line with accelerated velocity. The angle of practical effect is variously stated, ranging from 1 in 75 to 1 in 330.

The width of land required for a railway must vary with the depth of the cuttings and length of embankments, together with the slopes necessary to be given to suit the various materials of which the cuttings are composed: thus, rock will generally stand when the sides are vertical; chalk varies from ⅛ to 1, to 1 to 1; gravel 1½ to 1; coal 1½ to 1; clay 1 to 1, &c.; but where land can be obtained at a reasonable rate, it is always well to be on the safe side.

The following Table is calculated for the purpose of ascertaining the extent of any cutting in cubic yards, for 1 chain, 22 yards, or 66 feet in length, the slopes or angles of the sides being those which are most in general practice, and formation level equal 30 feet.

Slopes **1** *to* **1.**

Depth of cutting in feet.	Half width at top in feet.	Content in cubic yards per chain.	Content of 1 perpendicular ft. in breadth.	Content of 3 perpendicular ft. in breadth.	Content of 6 perpendicular ft. in breadth.	Depth of cutting in feet.	Half width at top in feet.	Content in cubic yards per chain.	Content of 1 perpendicular ft. in breadth.	Content of 3 perpendicular ft. in breadth.	Content of 6 perpendicular ft. in breadth.
1	16	75·78	2·44	7·33	14·67	26	41	3599·11	63·55	190·67	381·33
2	17	156·42	4·89	14·67	29·33	27	42	3762·00	65·99	198·00	396·00
3	18	242·00	7·33	22·00	44·00	28	43	3969·78	68·43	205·33	410·67
4	19	332·44	9·78	29·33	58·67	29	44	4182·44	70·88	212·67	425·33
5	20	427·78	12·22	36·67	73·33	30	45	4400·00	73·32	220·00	440·00
6	21	528·00	14·67	44·00	88·00	31	46	4622·44	75·77	227·33	454·67
7	22	633·11	17·11	51·33	102·67	32	47	4849·78	78·22	234·67	469·33
8	23	743·11	19·56	58·67	117·33	33	48	5082·00	80·67	242·00	484·00
9	24	858·00	22·00	66·00	132·00	34	49	5319·11	83·11	249·33	498·67
10	25	977·78	24·44	73·33	146·67	35	50	5561·11	85·55	256·67	513·33
11	26	1102·44	26·89	80·67	161·33	36	51	5808·00	88·00	264·00	528·00
12	27	1232·00	29·33	88·00	176·00	37	52	6059·78	90·44	271·33	542·67
13	28	1366·44	31·78	95·33	190·67	38	53	6316·44	92·39	278·67	557·33
14	29	1505·78	34·22	102·67	205·33	39	54	6578·00	95·33	286·00	572·00
15	30	1650·00	36·66	110·00	220·00	40	55	6844·44	97·77	293·33	586·67
16	31	1799·11	39·11	117·33	234·67	41	56	7115·78	100·22	300·67	601·33
17	32	1953·11	41·55	124·67	249·33	42	57	7392·00	102·66	308·00	616·00
18	33	2112·00	43·99	132·00	264·00	43	58	7673·11	105·11	315·33	630·67
19	34	2275·78	46·44	139·33	278·67	44	59	7959·11	107·55	322·67	645·33
20	35	2444·44	48·89	146·67	293·33	45	60	8250·00	109·99	330·00	660·00
21	36	2618·00	51·33	154·00	308·00	46	61	8545·78	112·44	337·33	674·67
22	37	2796·44	53·77	161·33	322·67	47	62	8846·44	114·88	344·67	689·33
23	38	2979·78	56·21	168·67	337·33	48	63	9152·00	117·33	352·00	704·00
24	39	3168·00	58·66	176·00	352·00	49	64	9462·44	119·77	359·33	718·67
25	40	3361·11	61·10	183·33	366·67	50	65	9777·78	122·21	366·67	733·33

Slopes 1½ *to* 1.

Depth of cutting in feet.	Half width at top in feet.	Content in cubic yards per chain.	Content of 1 perpendicular ft. in breadth.	Content of 3 perpendicular ft. in breadth.	Content of 6 perpendicular ft. in breadth.	Depth of cutting in feet.	Half width at top in feet.	Content in cubic yards per chain.	Content of 1 perpendicular ft. in breadth.	Content of 3 perpendicular ft. in breadth.	Content of 6 perpendicular ft. in breadth.
1	16½	77·00	2·44	7·33	14·67	26	54	4385·33	63·55	190·67	381·33
2	18	161·33	4·89	14·67	29·33	27	55½	4658·00	65·99	198·00	396·00
3	19½	253·00	7·33	22·00	44·00	28	57	4928·00	68·43	205·33	410·67
4	21	352·00	9·78	29·33	58·67	29	58½	5210·33	70·88	212·67	425·33
5	22½	453·33	12·22	36·67	73·33	30	60	5500·00	73·32	220·00	440·00
6	24	572·00	14·67	44·00	88·00	31	61½	5797·00	75·77	227·33	454·67
7	25½	698·00	17·11	51·33	102·67	32	63	6101·33	78·22	234·67	469·33
8	27	821·33	19·56	58·67	117·33	33	64½	6413·00	80·67	242·00	484·00
9	28½	957·00	22·00	66·00	132·00	34	66	6732·00	83·11	249·33	498·67
10	30	1100·00	24·44	73·33	146·67	35	67½	7058·33	85·55	256·67	513·33
11	31½	1250·33	26·89	80·67	161·33	36	69	7392·00	88·00	264·00	528·00
12	33	1408·00	29·33	88·00	176·00	37	70½	7733·00	90·44	271·33	542·67
13	34½	1573·00	31·78	95·33	190·67	38	72	8081·33	92·39	278·67	557·33
14	36	1745·33	34·22	102·67	205·33	39	73½	8437·00	95·33	286·00	572·00
15	37½	1925·00	36·66	110·00	220·00	40	75	8800·00	97·77	293·33	586·67
16	39	2112·00	39·11	117·33	234·67	41	76½	9170·33	100·22	300·67	601·33
17	40½	2306·33	41·55	124·67	249·33	42	78	9548·00	102·66	308·00	616·00
18	42	2508·00	43·99	132·00	264·00	43	79½	9933·00	105·11	315·33	630·67
19	43½	2717·00	46·44	139·33	278·67	44	81	10325·33	107·55	322·67	645·33
20	45	2933·33	48·89	146·67	293·33	45	82½	10725·00	109·99	330·00	660·00
21	46½	3157·00	51·33	154·00	308·00	46	84	11132·00	112·44	337·33	674·67
22	48	3388·00	53·77	151·33	322·67	47	85½	11546·33	114·88	344·67	689·33
23	49½	3626·33	56·21	168·67	337·33	48	87	11968·00	117·33	352·00	704·00
24	51	3872·00	58·66	176·00	352·00	49	88½	12397·00	119·77	359·33	718·67
25	52½	4125·00	61·10	183·33	366·67	50	90	12833·33	122·21	366·67	733·33

Slopes 2 *to* 1.

Depth of cutting in feet.	Half width at top in feet.	Content in cubic yards per chain.	Content of 1 perpendicular ft. in breadth.	Content of 3 perpendicular ft. in breadth.	Content of 6 perpendicular ft. in breadth.	Depth of cutting in feet.	Half width at top in feet.	Content in cubic yards per chain.	Content of 1 perpendicular ft. in breadth.	Content of 3 perpendicular ft. in breadth.	Content of 6 perpendicular ft. in breadth.
1	17	78·22	2·44	7·33	14·67	26	67	5211·55	63·55	190·67	381·33
2	19	166·22	4·89	14·67	29·33	27	69	5544·00	65·99	198·00	396·00
3	21	264·00	7·33	22·00	44·00	28	71	5886·22	68·43	205·33	410·67
4	23	371·55	9·78	29·33	58·67	29	73	6238·22	70·88	212·67	425·33
5	25	488·89	12·22	36·67	73·33	30	75	6600·00	73·32	220·00	440·00
6	27	616·00	14·67	44·00	88·00	31	77	6971·55	75·77	227·33	454·67
7	29	752·89	17·11	51·33	102·67	32	79	7352·89	78·22	234·67	469·33
8	31	899·55	19·56	58·67	117·33	33	81	7744·00	80·67	242·00	484·00
9	33	1056·00	22·00	66·00	132·00	34	83	8144·89	83·11	249·33	498·67
10	35	1222·22	24·44	73·33	146·67	35	85	8555·55	85·55	256·67	513·33
11	37	1398·22	26·89	80·67	161·33	36	87	8976·00	88·00	264·00	528·00
12	39	1584·00	29·33	88·00	176·00	37	89	9406·22	90·44	271·33	542·67
13	41	1779·55	31·78	95·33	190 67	38	91	9846·22	92·39	278·67	557·33
14	43	1984·89	34·22	102·67	205·33	39	93	10296·00	95·33	286·00	572·00
15	45	2200·00	36·66	110·00	220·00	40	95	10755·55	97·77	293·33	586·67
16	47	2424·89	39·11	117·33	234·67	41	97	11224·89	100·22	300·67	601·33
17	49	2659·55	41·55	124·67	249·33	42	99	11704·00	102·66	308·00	616·00
18	51	2904·00	43·99	132·00	264·00	43	101	12192·89	105·11	315·33	630·67
19	53	3158·22	46·44	139·33	278·67	44	103	12691·55	107·55	322·67	645·33
20	55	34·2222	48·89	146·67	293·33	45	105	13200·00	109·99	330·00	660·00
21	57	3696·00	51·33	154·00	308·00	46	107	13718·22	112·44	337·33	674·67
22	59	3979·55	53·77	161·33	322·67	47	109	14246·22	114·88	344·67	689·33
23	61	4272·89	56·21	168·67	337·33	48	111	14784·00	117·33	352·00	704·00
24	63	4576·00	58·66	176·00	352·00	49	113	15331·55	119·77	359·33	718·67
25	65	4888·89	61·10	183·33	366·67	50	115	15888·89	122·21	366·67	733·33

By the fourth, fifth, and sixth columns in each table, the number of cubic yards is easily ascertained at any other width of formation level above or below 30 feet, having the same slopes as by the tables, thus:—

Suppose an excavation of 40 feet in depth, and 33 feet in width at formation level, whose slopes or sides are at an angle of 2 to 1, required the extent of excavation in cubic yards:

$$10755{\cdot}55 + 293{\cdot}33 = 11048{\cdot}88 \text{ cubic yards.}$$

The number of cubic yards in any other excavation may be ascertained by the following simple rule:

To the width at formation level in feet, add the horizontal length of the side of the triangle formed by the slope, multiply the sum by the depth of the cutting, or excavation, and by the length, also in feet; divide the product by 27, and the quotient is the content in cubic yards.

Suppose a cutting of any length, and of which take 1 chain, its depth being 14½ feet, width at the bottom 28 feet, and whose sides have a slope of 1¼ to 1, required the content in cubic yards:

$$14{\cdot}5 \times 1{\cdot}25 = \overline{18{\cdot}125 + 28} \times 14 = 645{\cdot}75 \times 66 =$$
$$\frac{42619{\cdot}5}{27} = 1578{\cdot}5 \text{ cubic yards.}$$

$$\frac{l}{6}\left\{(b + rh')\,h' + (b + rh)\,h + 4\left[b + r\frac{h + h'}{2}\right]\frac{h + h'}{2}\right\}$$

gives the content of any cutting. In words, this formula will be:— To the area of each end, add four times the middle area; the sum multiplied by the length and divided by 6 gives the content. The breadth at the bottom of cutting $= b$; the perpendicular depth of cutting at the higher end $= h$; the perpendicular depths of cutting at the lower end $= h'$; l, the length of the solid; and rh' the ratio of the perpendicular height of the slope to the horizontal base, multiplied by the height h'. rh, the ratio r, of the perpendicular height of the slope, to the horizontal base, multiplied by the height h.

Let $b = 30$; $h = 50$; $h' = 20$; $l = 84$ feet; and 2 to 5 or $\frac{2}{5}$ the ratio of the perpendicular height of the slope to the horizontal base:

$$\frac{84}{6}\left\{(30 + \tfrac{2}{5} \times 20)\,20 + (30 + \tfrac{2}{5} \times 50)\,50 + 4\left[30 + \tfrac{2}{5}\frac{50 + 20}{2}\right]\right.$$
$$\left.\frac{50 + 20}{2}\right\} = 14\left\{38 \times 20 + 50 \times 50 + 4 \times 44 \times 35\right\} = 131880$$

cubic feet. $\frac{131880}{27} = 4884{\cdot}44$ cubic yards.

This rule is one of the most useful in the mensuration of solids, it will give the content of any irregular solid very nearly, whether it be bounded by right lines or not.

915414

TABLE *of Squares, Cubes, Square and Cube Roots of Numbers.*

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1	1	1	1·0000000	1·0000000	·100000000
2	4	8	1·4142136	1·2599210	·500000000
3	9	27	1·7320508	1·4422496	·333333333
4	16	64	2·0000000	1·5874011	·250000000
5	25	125	2·2360680	1·7099759	·200000000
6	36	216	2·4494897	1·8171206	·166666667
7	49	343	2·6457513	1·9129312	·142857143
8	64	512	2·8284271	2·0000000	·125000000
9	81	729	3·0000000	2·0800837	·111111111
10	100	1000	3·1622777	2·1544347	·100000000
11	121	1331	3·3166248	2·2239801	·090909091
12	144	1728	3·4641016	2·2894286	·083333333
13	169	2197	3·6055513	2·3513347	·076923077
14	196	2744	3·7416574	2·4101422	·071428571
15	225	3375	3·8729833	2·4662121	·066666667
16	256	4096	4·0000000	2·5198421	·062500000
17	289	4913	4·1231056	2·5712816	·058823529
18	324	5832	4·2426407	2·6207414	·055555556
19	361	6859	4·3588989	2·6684016	·052631579
20	400	8000	4·4721360	2·7144177	·050000000
21	441	9261	4·5825757	2·7589243	·047619048
22	484	10648	4·6904158	2·8020393	·045454545
23	529	12167	4·7958315	2·8438670	·043478261
24	576	13824	4·8989795	2·8844991	·041666667
25	625	15625	5·0000000	2·9240177	·040000000
26	676	17576	5·0990195	2·9624960	·038461538
27	729	19683	5·1961524	3·0000000	·037037037
28	784	21952	5·2915026	3·0365889	·035714286
29	841	24389	5·3851648	3·0723168	·034482759
30	900	27000	5·4772256	3·1072325	·033333333
31	961	29791	5·5677644	3·1413806	·032258065
32	1024	32768	5·6568542	3·1748021	·031250000
33	1089	35937	5·7445626	3·2075343	·030303030
34	1156	39304	5·8309519	3·2396118	·029411765
35	1225	42875	5·9160798	3·2710663	·028571429
36	1296	46656	6·0000000	3·3019272	·027777778
37	1369	50653	6·0827625	3·3322218	·027027027
38	1444	54872	6·1644140	3·3619754	·026315789
39	1521	59319	6·2449980	3·3912114	·025641026
40	1600	64000	6·3245553	3·4199519	·025000000
41	1681	68921	6·4031242	3·4482172	·024390244
42	1764	74088	6·4807407	3·4760266	·023809524
43	1849	79507	6·5574385	3·5033981	·023255814
44	1936	85184	6·6332496	3·5303483	·022727273
45	2025	91125	6·7082039	3·5568933	·022222222
46	2116	97336	6·7823300	3·5830479	·021739130
47	2209	103823	6·8556546	3·6088261	·021276600
48	2304	110592	6·9282032	3·6342411	·020833333
49	2401	117649	7·0000000	3·6593057	·020408163
50	2500	125000	7·0710678	3·6840314	·020000000
51	2601	132651	7·1414284	3·7084298	·019607843
52	2704	140608	7·2111026	3·7325111	·019230769
53	2809	148877	7·2801099	3·7562858	·018867925
54	2916	157464	7·3484692	3·7797631	·018518519
55	3025	166375	7·4161985	3·8029525	·018181818
56	3136	175616	7·4833148	3·8258624	·017857143
57	3249	185193	7·5498344	3·8485011	·017543860

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
58	3364	195112	7·6157731	3·8708766	·017241379
59	3481	205379	7·6811457	3·8929965	·016949153
60	3600	216000	7·7459667	3·9148676	·016666667
61	3721	226981	7·8102497	3·9364972	·016393443
62	3844	238328	7·8740079	3·9578915	·016129032
63	3969	250047	7·9372539	3·9790571	·015873016
64	4096	262144	8·0000000	4·0000000	·015625000
65	4225	274625	8·0622577	4·0207256	·015384615
66	4356	287496	8·1240384	4·0412401	·015151515
67	4489	300763	8·1853528	4·0615480	·014925373
68	4624	314432	8·2462113	4·0816551	·014705882
69	4761	328509	8·3066239	4·1015661	·014492754
70	4900	343000	8·3666003	4·1212853	·014285714
71	5041	357911	8·4261498	4·1408178	·014084517
72	5184	373248	8·4852814	4·1601676	·013888889
73	5329	389017	8·5440037	4·1793390	·013698630
74	5476	405224	8·6023253	4·1983364	·013513514
75	5625	421875	8·6602540	4·2171633	·013333333
76	5776	438976	8·7177979	4·2358236	·013157895
77	5929	456533	8·7749644	4·2543210	·012987013
78	6084	474552	8·8317609	4·2726586	·012820513
79	6241	493039	8·8881944	4·2908404	·012658228
80	6400	512000	8·9442719	4·3088695	·012500000
81	6561	531441	9·0000000	4·3267487	·012345679
82	6724	551368	9·0553851	4·3444815	·012195122
83	6889	571787	9·1104336	4·3620707	·012048193
84	7056	592704	9·1651514	4·3795191	·011904762
85	7225	614125	9·2195445	4·3968296	·011764706
86	7396	636056	9·2736185	4·4140049	·011627907
87	7569	658503	9·3273791	4·4310476	·011494253
88	7744	681472	9·3808315	4·4479602	·011363636
89	7921	704969	9·4339811	4·4647451	·011235955
90	8100	729000	9·4868330	4·4814047	·011111111
91	8281	753571	9·5393920	4·4979414	·010989011
92	8464	778688	9·5916630	4·5143574	·010869565
93	8649	804357	9·6436508	4·5306549	·010752688
94	8836	830584	9·6953597	4·5468359	·010638298
95	9025	857375	9·7467943	4·5629026	·010526316
96	9216	884736	9·7979590	4·5788570	·010416667
97	9409	912673	9·8488578	4·5947009	·010309278
98	9604	941192	9·8994949	4·6104363	·010204082
99	9801	970299	9·9498744	4·6260650	·010101010
100	10000	1000000	10·0000000	4·6415888	·010000000
101	10201	1030301	10·0498756	4·6570095	·009900990
102	10404	1061208	10·0995049	4·6723287	·009803922
103	10609	1092727	10·1488916	4·6875482	·009708738
104	10816	1124864	10·1980390	4·7026694	·009615385
105	11025	1157625	10·2469508	4·7176940	·009523810
106	11236	1191016	10·2956301	4·7326235	·009433962
107	11449	1225043	10·3440804	4·7474594	·009345794
108	11664	1259712	10·3923048	4·7622032	·009259259
109	11881	1295029	10·4403065	4·7768562	·009174312
110	12100	1331000	10·4880885	4·7914199	·009090909
111	12321	1367631	10·5356538	4·8058995	·009009009
112	12544	1404928	10·5830052	4·8202845	·008928571
113	12769	1442897	10·6301458	4·8345881	·008849558
114	12996	1481544	10·6770783	4·8488076	·008771930
115	13225	1520875	10·7238053	4·8629442	·008695652
116	13456	1560896	10·7703296	4·8769990	·008620690
117	13689	1601613	10·8166538	4·8909732	·008547009
118	13924	1643032	10·8627805	4·9048681	·008474576
119	14161	1685159	10·9087121	4·9186847	·008403361

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
120	14400	1728000	10·9544512	4·9324242	·008333333
121	14641	1771561	11·0000000	4·9460874	·008264463
122	14884	1815848	11·0453610	4·9596757	·008196721
123	15129	1860867	11·0905365	4·9731898	·008130081
124	15376	1906624	11·1355287	4·9866310	·008064516
125	15625	1953125	11·1803399	5·0000000	·008000000
126	15876	2000376	11·2249722	5·0132979	·007936508
127	16129	2048383	11·2694277	5·0265257	·007874016
128	16384	2097152	11·3137085	5·0396842	·007812500
129	16641	2146689	11·3578167	5·0527743	·007751938
130	16900	2197000	11·4017543	5·0657970	·007692308
131	17161	2248091	11·4455231	5·0787531	·007633588
132	17424	2299968	11·4891253	5·0916434	·007575758
133	17689	2352637	11·5325626	5·1044687	·007518797
134	17956	2406104	11·5758369	5·1172299	·007462687
135	18225	2460375	11·6189500	5·1299278	·007407407
136	18496	2515456	11·6619038	5·1425632	·007352941
137	18769	2571353	11·7046999	5·1551367	·007299270
138	19044	2628072	11·7473444	5·1676493	·007246377
139	19321	2685619	11·7898261	5·1801015	·007194245
140	19600	2744000	11·8321596	5·1924941	·007142857
141	19881	2803221	11·8743421	5·2048279	·007092199
142	20164	2863288	11·9163753	5·2171034	·007042254
143	20449	2924207	11·9582607	5·2293215	·006993007
144	20736	2985984	12·0000000	5·2414828	·006944444
145	21025	3048625	12·0415946	5·2535879	·006896552
146	21316	3112136	12·0830460	5·2656374	·006849315
147	21609	3176523	12·1243557	5·2776321	·006802721
148	21904	3241792	12·1655251	5·2895725	·006756757
149	22201	3307949	12·2065556	5·3014592	·006711409
150	22500	3375000	12·2474487	5·3132928	·006666667
151	22801	3442951	12·2882057	5·3250740	·006622517
152	23104	3511008	12·3288280	5·3368033	·006578947
153	23409	3581577	12·3693169	5·3484812	·006535948
154	23716	3652264	12·4096736	5·3601084	·006493506
155	24025	3723875	12·4498996	5·3716854	·006451613
156	24336	3796416	12·4899960	5·3832126	·006410256
157	24649	3869893	12·5299641	5·3946907	·006369427
158	24964	3944312	12·5698051	5·4061202	·006329114
159	25281	4019679	12·6095202	5·4175015	·006289308
160	25600	4096000	12·6491106	5·4288352	·006250000
161	25921	4173281	12·6885775	5·4401218	·006211180
162	26244	4251528	12·7279221	5·4513618	·006172840
163	26569	4330747	12·7671453	5·4625556	·006134969
164	26896	4410944	12·8062485	5·4737037	·006097561
165	27225	4492125	12·8452326	5·4848066	·006060606
166	27556	4574296	12·8840987	5·4958647	·006024096
167	27889	4657463	12·9228480	5·5068784	·005988024
168	28224	4741632	12·9614814	5·5178484	·005952381
169	28561	4826809	13·0000000	5·5287748	·005917160
170	28900	4913000	13·0384048	5·5396583	·005882353
171	29241	5000211	13·0766968	5·5504991	·005847953
172	29584	5088448	13·1148770	5·5612978	·005813953
173	29929	5177717	13·1529464	5·5720546	·005780347
174	30276	5268024	13·1909060	5·5827702	·005747126
175	30625	5359375	13·2287566	5·5934447	·005714286
176	30976	5451776	13·2664992	5·6040787	·005681818
177	31329	5545233	13·3041347	5·6146724	·005649718
178	31684	5639752	13·3416641	5·6252263	·005617978
179	32041	5735339	13·3790882	5·6357408	·005586592
180	32400	5832000	13·4164079	5·6462162	·005555556
181	32761	5929741	13·4536240	5·6566528	·005524862

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
182	33124	6028568	13·4907376	5·6670511	·005494505
183	33489	6128487	13·5277493	5·6774114	·005464481
184	33856	6229504	13·5646600	5·6877340	·005434783
185	34225	6331625	13·6014705	5·6980192	·005405405
186	34596	6434856	13·6381817	5·7082675	·005376344
187	34969	6539203	13·6747943	5·7184791	·005347594
188	35344	6644672	13·7113092	5·7286543	·005319149
189	35721	6751269	13·7477271	5·7387936	·005291005
190	36100	6859000	13·7840488	5·7488971	·005263158
191	36481	6967871	13·8202750	5·7589652	·005235602
192	36864	7077888	13·8564065	5·7689982	·005208333
193	37249	7189517	13·8924400	5·7789966	·005181347
194	37636	7301384	13·9283883	5·7889604	·005154639
195	38025	7414875	13·9642400	5·7988900	·005128205
196	38416	7529536	14·0000000	5·8087857	·005102041
197	38809	7645373	14·0356688	5·8186479	·005076142
198	39204	7762392	14·0712473	5·8284867	·005050505
199	39601	7880599	14·1067360	5·8382725	·005025126
200	40000	8000000	14·1421356	5·8480355	·005000000
201	40401	8120601	14·1774469	5·8577660	·004975124
202	40804	8242408	14·2126704	5·8674673	·004950495
203	41209	8365427	14·2478068	5·8771307	·004926108
204	41616	8489664	14·2828569	5·8867653	·004901961
205	42025	8615125	14·3178211	5·8963685	·004878049
206	42436	8741816	14·3527001	5·9059406	·004854369
207	42849	8869743	14·3874946	5·9154817	·004830918
208	43264	8998912	14·4222051	5·9249921	·004807692
209	43681	9129329	14·4568323	5·9344721	·004784689
210	44100	9261000	14·4913767	5·9439220	·004761905
211	44521	9393931	14·5258390	5·9533418	·004739336
212	44944	9528128	14·5602198	5·9627320	·004716981
213	45369	9663597	14·5945195	5·9720926	·004694836
214	45796	9800344	14·6287388	5·9814240	·004672897
215	46225	9938375	14·6628783	5·9907264	·004651163
216	46656	10077696	14·6969385	6·0000000	·004629630
217	47089	10218313	14·7309199	6·0092450	·004608295
218	47524	10360232	14·7648231	6·0184617	·004587156
219	47961	10503459	14·7986486	6·0276502	·004566210
220	48400	10648000	14·8323970	6·0368107	·004545455
221	48841	10793861	14·8660687	6·0459435	·004524887
222	49284	10941048	14·8996644	6·0550489	·004504505
223	49729	11089567	14·9331845	6·0641270	·004484305
224	50176	11239424	14·9666295	6·0731779	·004464286
225	50625	11390625	15·0000000	6·0824020	·004444444
226	51076	11543176	15·0332964	6·0991994	·004424779
227	51529	11697083	15·0665192	6·1001702	·004405286
228	51984	11852352	15·0996689	6·1091147	·004385965
229	52441	12008989	15·1327460	6·1180332	·004366812
230	52900	12167000	15·1657509	6·1269257	·004347826
231	53361	12326391	15·1986842	6·1357924	·004329004
232	53824	12487168	15·2315462	6·1446337	·004310345
233	54289	12649337	15·2643375	6·1534495	·004291845
234	54756	12812904	15·2970585	6·1622401	·004273504
235	55225	12977875	15·3297097	6·1710058	·004255319
236	55696	13144256	15·3622915	6·1797466	·004237288
237	56169	13312053	15·3948043	6·1884628	·004219409
238	56644	13481272	15·4272486	6·1971544	·004201681
239	57121	13651919	15·4596248	6·2058218	·004184100
240	57600	13824000	15·4919334	6·2144650	·004166667
241	58081	13997521	15·5241747	6·2230843	·004149378
242	58564	14172488	15·5563492	6·2316797	·004132231
243	59049	14348907	15·5884573	6·2402515	·004115226

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
244	59536	14526784	15·6204994	6·2487998	·004098361
245	60025	14706125	15·6524758	6·2573248	·004081633
246	60516	14886936	15·6843871	6·2658266	·004065041
247	61009	15069223	15·7162336	6·2743054	·004048583
248	61504	15252992	15·7480157	6·2827613	·004032258
249	62001	15438249	15·7797338	6·2911946	·004016064
250	62500	15625000	15·8113883	6·2996053	·004000000
251	63001	15813251	15·8429795	6·3079935	·003984064
252	63504	16003008	15·8745079	6·3163596	·003968254
253	64009	16194277	15·9059737	6·3247035	·003952569
254	64516	16387064	15·9373775	6·3330256	·003937008
255	65025	16581375	15·9687194	6·3413257	·003921569
256	65536	16777216	16·0000000	6·3496042	·003906250
257	66049	16974593	16·0312195	6·3578611	·003891051
258	66564	17173512	16·0623784	6·3660968	·003875969
259	67081	17373979	16·0934769	6·3743111	·003861004
260	67600	17576000	16·1245155	6·3825043	·003846154
261	68121	17779581	16·1554944	6·3906765	·003831418
262	68644	17984728	16·1864141	6·3988279	·003816794
263	69169	18191447	16·2172747	6·4069585	·003802281
264	69696	18399744	16·2480768	6·4150687	·003787879
265	70225	18609625	16·2788206	6·4231583	·003773585
266	70756	18821096	16·3095064	6·4312276	·003759398
267	71289	19034163	16·3401346	6·4392767	·003745318
268	71824	19248832	16·3707055	6·4473057	·003731343
269	72361	19465109	16·4012195	6·4553148	·003717472
270	72900	19683000	16·4316767	6·4633041	·003703704
271	73441	19902511	16·4620776	6·4712736	·003690037
272	73984	20123648	16·4924225	6·4792236	·003676471
273	74529	20346417	16·5227116	6·4871541	·003663004
274	75076	20570824	16·5529454	6·4950653	·003649635
275	75625	20796875	16·5831240	6·5029572	·003636364
276	76176	21024576	16·6132477	6·5108300	·003623188
277	76729	21253933	16·6433170	6·5186839	·003610108
278	77284	21484952	16·6733320	6·5265189	·003597122
279	77841	21717639	16·7032931	6·5343351	·003584229
280	78400	21952000	16·7332005	6·5421326	·003571429
281	78961	22188041	16·7630546	6·5499116	·003558719
282	79524	22425768	16·7928556	6·5576722	·003546099
283	80089	22665187	16·8226038	6·5654144	·003533569
284	80656	22906304	16·8522995	6·5731385	·003522127
285	81225	23149125	16·8819430	6·5808443	·003508772
286	81796	23393656	16·9115345	6·5885323	·003496503
287	82369	23639903	16·9410743	6·5962023	·003484321
288	82944	23887872	16·9705627	6·6038545	·003472222
289	83521	24137569	17·0000000	6·6114890	·003460208
290	84100	24389000	17·0293864	6·6191060	·003448276
291	84681	24642171	17·0587221	6·6267054	·003436426
292	85264	24897088	17·0880075	6·6342874	·003424658
293	85849	25153757	17·1172428	6·6418522	·003412969
294	86436	25412184	17·1464282	6·6493998	·003401361
295	87025	25672375	17·1755640	6·6569302	·003389831
296	87616	25934336	17·2046505	6·6644437	·003378378
297	88209	26198073	17·2336879	6·6719403	·003367003
298	88804	26463592	17·2626765	6·6794200	·003355705
299	89401	26730899	17·2916165	6·6868831	·003344482
300	90000	27000000	17·3205081	6·6943295	·003333333
301	90601	27270901	17·3493516	6·7017593	·003322259
302	91204	27543608	17·3781472	6·7091729	·003311258
303	91809	27818127	17·4068952	6·7165700	·003300330
304	92416	28094464	17·4355958	6·7239508	·003289474
305	93025	28372625	17·4642492	6·7313155	·003278689

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
306	93636	28652616	17·4928557	6·7386641	·003267974
307	94249	28934443	17·5214155	6·7459967	·003257329
308	94864	29218112	17·5499288	6·7533134	·003246753
309	95481	29503609	17·5783958	6·7606143	·003236246
310	96100	29791000	17·6068169	6·7678995	·003225806
311	96721	30080231	17·6351921	6·7751690	·003215434
312	97344	30371328	17·6635217	6·7824229	·003205128
313	97969	30664297	17·6918060	6·7896613	·003194888
314	98596	30959144	17·7200451	6·7968844	·003184713
315	99225	31255875	17·7482393	6·8040921	·003174603
316	99856	31554496	17·7763888	6·8112847	·003164557
317	100489	31855013	17·8044938	6·8184620	·003154574
318	101124	32157432	17·8325545	6·8256242	·003144654
319	101761	32461759	17·8605711	6·8327714	·003134796
320	102400	32768000	17·8885438	6·8399037	·003125000
321	103041	33076161	17·9164729	6·8470213	·003115265
322	103684	33386248	17·9443584	6·8541240	·003105590
323	104329	33698267	17·9722008	6·8612120	·003095975
324	104976	34012224	18·0000000	6·8682855	·003086420
325	105625	34328125	18·0277564	6·8753433	·003076923
326	106276	34645976	18·0554701	6·8823888	·003067485
327	106929	34965783	18·0831413	6·8894188	·003058104
328	107584	35287552	18·1107703	6·8964345	·003048780
329	108241	35611289	18·1383571	6·9034359	·003039514
330	108900	35937000	18·1659021	6·9104232	·003030303
331	109561	36264691	18·1934054	6·9173964	·003021148
332	110224	36594368	18·2208672	6·9243556	·003012048
333	110889	36926037	18·2482876	6·9313088	·003003003
334	111556	37259704	18·2756669	6·9382321	·002994012
335	112225	37595375	18·3030052	6·9451496	·002985075
336	112896	37933056	18·3303028	6·9520533	·002976190
337	113569	38272753	18·3575598	6·9589434	·002967359
338	114244	38614472	18·3847763	6·9658198	·002958580
339	114921	38958219	18·4119526	6·9726826	·002949853
340	115600	39304000	18·4390889	6·9795321	·002941176
341	116281	39651821	18·4661853	6·9863681	·002932551
342	116964	40001688	18·4932420	6·9931906	·002923977
343	117649	40353607	18·5202592	7·0000000	·002915452
344	118336	40707584	18·5472370	7·0067962	·002906977
345	119025	41063625	18·5741756	7·0135791	·002898551
346	119716	41421736	18·6010752	7·0203490	·002890173
347	120409	41781923	18·6279360	7·0271058	·002881844
348	121104	42144192	18·6547581	7·0338497	·002873563
349	121801	42508549	18·6815417	7·0405860	·002865330
350	122500	42875000	18·7082869	7·0472987	·002857143
351	123201	43243551	18·7349940	7·0540041	·002849003
352	123904	43614208	18·7616630	7·0606967	·002840909
353	124609	43986977	18·7882942	7·0673767	·002832861
354	125316	44361864	18·8148877	7·0740440	·002824859
355	126025	44738875	18·8414437	7·0806988	·002816901
356	126736	45118016	18·8679623	7·0873411	·002808989
357	127449	45499293	18·8944436	7·0939709	·002801120
358	128164	45882712	18·9208879	7·1005885	·002793296
359	128881	46268279	18·9472953	7·1071937	·002785515
360	129600	46656000	18·9736660	7·1137866	·002777778
361	130321	47045881	19·0000000	7·1203674	·002770083
362	131044	47437928	19·0262976	7·1269360	·002762431
363	131769	47832147	19·0525589	7·1334925	·002754821
364	132496	48228544	19·0787840	7·1400370	·002747253
365	133225	48627125	19·1049732	7·1465695	·002739726
366	133956	49027896	19·1311265	7·1530901	·002732240
367	134689	49430863	19·1572441	7·1595988	·002724796

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
368	135424	49836032	19·1833261	7·1660957	·002717391
369	136161	50243409	19·2093727	7·1725809	·002710027
370	136900	50653000	19·2353841	7·1790544	·002702703
371	137641	51064811	19·2613603	7·1855162	·002695418
372	138384	51478848	19·2873015	7·1919663	·002688172
373	139129	51895117	19·3132079	7·1984050	·002680965
374	139876	52313624	19·3390796	7·2048322	·002673797
375	140625	52734875	19·3649167	7·2112479	·002666667
376	141376	53157376	19·3907194	7·2176522	·002659574
377	142129	53582633	19·4164878	7·2240450	·002652520
378	142884	54010152	19·4422221	7·2304268	·002645503
379	143641	54439939	19·4679223	7·2367972	·002638521
380	144400	54872000	19·4935887	7·2431565	·002631579
381	145161	55306341	19·5192213	7·2495045	·002624672
382	145924	55742968	19·5448203	7·2558415	·002617801
383	146689	56181887	19·5703858	7·2621675	·002610966
384	147456	56623104	19·5959179	7·2684824	·002604167
385	148225	57066625	19·6214169	7·2747864	·002597403
386	148996	57512456	19·6468827	7·2810794	·002590674
387	149769	57960603	19·6723156	7·2873617	·002583979
388	150544	58411072	19·6977156	7·2936330	·002577320
389	151321	58863869	19·7230829	7·2998936	·002570694
390	152100	59319000	19·7484177	7·3061436	·002564103
391	152881	59776471	19·7737199	7·3123828	·002557545
392	153664	60236288	19·7989899	7·3186114	·002551020
393	154449	60698457	19·8242276	7·3248295	·002544529
394	155236	61162984	19·8494332	7·3310369	·002538071
395	156025	61629875	19·8746069	7·3372339	·002531646
396	156816	62099136	19·8997487	7·3434205	·002525253
397	157609	62570773	19·9248588	7·3495966	·002518892
398	158404	63044792	19·9499373	7·3557624	·002512563
399	159201	63521199	19·9749844	7·3619178	·002506266
400	160000	64000000	20·0000000	7·3680630	·002500000
401	160801	64481201	20·0249844	7·3741979	·002493766
402	161604	64964808	20·0499377	7·3803227	·002487562
403	162409	65450827	20·0748599	7·3864373	·002481390
404	163216	65939264	20·0997512	7·3925418	·002475248
405	164025	66430125	20·1246118	7·3986363	·002469136
406	164836	66923416	20·1494417	7·4047206	·002463054
407	165649	67419143	20·1742410	7·4107950	·002457002
408	166464	67917312	20·1990099	7·4168595	·002450980
409	167281	68417929	20·2237484	7·4229142	·002444988
410	168100	68921000	20·2484567	7·4289589	·002439024
411	168921	69426531	20·2731349	7·4349938	·002433090
412	169744	69934528	20·2977831	7·4410189	·002427184
413	170569	70444997	20·3224014	7·4470343	·002421308
414	171396	70957944	20·3469899	7·4530399	·002415459
415	172225	71473375	20·3715488	7·4590359	·002409639
416	173056	71991296	20·3960781	7·4650223	·002406846
417	173889	72511713	20·4205779	7·4709991	·002398082
418	174724	73034632	20·4450483	7·4769664	·002392344
419	175561	73560059	20·4694895	7·4829242	·002386635
420	176400	74088000	20·4939015	7·4888724	·002380952
421	177241	74618461	20·5182845	7·4948113	·002375297
422	178084	75151448	20·5426386	7·5007406	·002369668
423	178929	75686967	20·5669638	7·5066607	·002364066
424	179776	76225024	20·5912603	7·5125715	·002358491
425	180625	76765625	20·6155281	7·5184730	·002352941
426	181476	77308776	20·6397674	7·5243652	·002347418
427	182329	77854483	20·6639783	7·5302482	·002341920
428	183184	78402752	20·6881609	7·5361221	·002336449
429	184041	78953589	20·7123152	7·5419867	·002331002

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
430	184900	79507000	20·7364414	7·5478423	·002325581
431	185761	80062991	20·7605395	7·5536888	·002320186
432	186624	80621568	20·7846097	7·5595263	·002314815
433	187489	81182737	20·8086520	7·5653548	·002309469
434	188356	81746504	20·8326667	7·5711743	·002304147
435	189225	82312875	20·8566536	7·5769849	·002298851
436	190096	82881856	20·8806130	7·5827865	·002293578
437	190969	83453453	20·9045450	7·5885793	·002288330
438	191844	84027672	20·9284495	7·5943633	·002283105
439	192721	84604519	20·9523268	7·6001385	·002277904
440	193600	85184000	20·9761770	7·6059049	·002272727
441	194481	85766121	21·0000000	7·6116626	·002267574
442	195364	86350888	21·0237960	7·6174116	·002262443
443	196249	86938307	21·0475652	7·6231519	·002257336
444	197136	87528384	21·0713075	7·6288837	·002252252
445	198025	88121125	21·0950231	7·6346067	·002247191
446	198916	88716536	21·1187121	7·6403213	·002242152
447	199809	89314623	21·1423745	7·6460272	·002237136
448	200704	89915392	21·1660105	7·6517247	·002232143
449	201601	90518849	21·1896201	7·6574138	·002227171
450	202500	91125000	21·2132034	7·6630943	·002222222
451	203401	91733851	21·2367606	7·6687665	·002217295
452	204304	92345408	21·2602916	7·6744303	·002212389
453	205209	92959677	21·2837967	7·6800857	·002207506
454	206116	93576664	21·3072758	7·6857328	·002202643
455	207025	94196375	21·3307290	7·6913717	·002197802
456	207936	94818816	21·3541565	7·6970023	·002192982
457	208849	95443993	21·3775583	7·7026246	·002188184
458	209764	96071912	21·4009346	7·7082388	·002183406
459	210681	96702579	21·4242853	7·7188448	·002178649
460	211600	97336000	21·4476106	7·7194426	·002173913
461	212521	97972181	21·4709106	7·7250325	·002169197
462	213444	98611128	21·4941858	7·7306141	·002164502
463	214369	99252847	21·5174348	7·7361877	·002159827
464	215296	99897344	21·5406592	7·7417532	·002155172
465	216225	100544625	21·5638587	7·7473109	·002150538
466	217156	101194696	21·5870331	7·7528606	·002145923
467	218089	101847563	21·6101828	7·7584023	·002141328
468	219024	102503232	21·6333077	7·7639361	·002136752
469	219961	103161709	21·6564078	7·7694620	·002132196
470	220900	103823000	21·6794834	7·7749801	·002127660
471	221841	104487111	21·7025344	7·7804904	·002123142
472	222784	105154048	21·7255610	7·7859928	·002118644
473	223729	105828817	21·7485632	7·7914875	·002114165
474	224676	106496424	21·7715411	7·7969745	·002109705
475	225625	107171875	21·7944947	7·8024538	·002105263
476	226576	107850176	21·8174242	7·8079254	·002100840
477	227529	108531333	21·8403297	7·8133892	·002096486
478	228484	109215352	21·8632111	7·8188456	·002092050
479	229441	109902239	21·8860686	7·8242942	·002087683
480	230400	110592000	21·9089023	7·8297353	·002083333
481	231361	111284641	21·9317122	7·8351688	·002079002
482	232324	111980168	21·9544984	7·8405949	·002074689
483	233289	112678587	21·9772610	7·8460134	·002070393
484	234256	113379904	22·0000000	7·8514244	·002066116
485	235225	114084125	22·0227155	7·8568281	·002061856
486	236196	114791256	22·0454077	7·8622242	·002057613
487	237169	115501303	22·0680765	7·8676130	·002053388
488	238144	116214272	22·0907220	7·8729944	·002049180
489	239121	116930169	22·1133444	7·8783684	·002044990
490	240100	117649000	22·1359436	7·8837352	·002040816
491	241081	118370771	22·1585198	7·8890946	·002036660

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
492	242064	119095488	22·1810730	7·8944468	·002032520
493	243049	119823157	22·2036033	7·8997917	·002028398
494	244036	120553784	22·2261108	7·9051294	·002024291
495	245025	121287375	22·2485955	7·9104599	·002020202
496	246016	122023936	22·2710575	7·9157832	·002016129
497	247009	122763473	22·2934968	7·9210994	·002012072
498	248004	123505992	22·3159136	7·9264085	·002008032
499	249001	124251499	22·3383079	7·9317104	·002004008
500	250000	125000000	22·3606798	7·9370053	·002000000
501	251001	125751501	22·3830293	7·9422931	·001996008
502	252004	126506008	22·4053565	7·9475739	·001992032
503	253009	127263527	22·4276615	7·9528477	·001988072
504	254016	128024064	22·4499443	7·9581144	·001984127
505	255025	128787625	22·4722051	7·9633743	·001980198
506	256036	129554216	22·4944438	7·9686271	·001976285
507	257049	130323843	22·5166605	7·9738731	·001972387
508	258064	131096512	22·5388553	7·9791122	·001968504
509	259081	131872229	22·5610283	7·9843444	·001964637
510	260100	132651000	22·5831796	7·9895697	·001960784
511	261121	133432831	22·6053091	7·9947883	·001956947
512	262144	134217728	22·6274170	8·0000000	·001953125
513	263169	135005697	22·6495033	8·0052049	·001949318
514	264196	135796744	22·6715681	8·0104032	·001945525
515	265225	136590875	22·6936114	8·0155946	·001941748
516	266256	137388096	22·7156334	8·0207794	·001937984
517	267289	138188413	22·7376341	8·0259574	·001934236
518	268324	138991832	22·7596134	8·0311287	·001930502
519	269361	139798359	22·7815715	8·0362935	·001926782
520	270400	140608000	22·8035085	8·0414515	·001923077
521	271441	141420761	22·8254244	8·0466030	·001919386
522	272484	142236648	22·8473193	8·0517479	·001915709
523	273529	143055667	22·8691933	8·0568862	·001912046
524	274576	143877824	22·8910463	8·0620180	·001908397
525	275625	144703125	22·9128785	8·0671432	·001904762
526	276676	145531576	22·9346899	8·0722620	·001901141
527	277729	146363183	22·9564806	8·0773743	·001897533
528	278784	147197952	22·9782506	8·0824800	·001893939
529	279841	148035889	23·0000000	8·0875794	·001890359
530	280900	148877001	23·0217289	8·0926723	·001886792
531	281961	149721291	23·0434372	8·0977589	·001883239
532	283024	150568768	23·0651252	8·1028390	·001879699
533	284089	151419437	23·0867928	8·1079128	·001876173
534	285156	152273304	23·1084400	8·1129803	·001872659
535	286225	153130375	23·1300670	8·1180414	·001869159
536	287296	153990656	23·1516738	8·1230962	·001865672
537	288369	154854153	23·1732605	8·1281447	·001862197
538	289444	155720872	23·1948270	8·1331870	·001858736
539	290521	156590819	23·2163735	8·1382230	·001855288
540	291600	157464000	23·2379001	8·1432529	·001851852
541	292681	158340421	23·2594067	8·1482765	·001848429
542	293764	159220088	23·2808935	8·1532939	·001845018
543	294849	160103007	23·3023604	8·1583051	·001841621
544	295936	160989184	23·3238076	8·1633102	·001838235
545	297025	161878625	23·3452351	8·1683092	·001834862
546	298116	162771336	23·3666429	8·1733020	·001831502
547	299209	163667323	23·3880311	8·1782888	·001828154
548	300304	164566592	23·4093998	8·1832695	·001824818
549	301401	165469149	23·4307490	8·1882441	·001821494
550	302500	166375000	23·4520788	8·1932127	·001818182
551	303601	167284151	23·4733892	8·1981753	·001814882
552	304704	168196608	23·4946802	8·2031319	·001811594
553	305809	169112377	23·5159520	8·2080825	·001808318

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
554	306916	170031464	23·5372046	8·2130271	·001805054
555	308025	170953875	23·5584380	8·2179657	·001801802
556	309136	171879616	23·5796522	8·2228985	·001798561
557	310249	172808693	23·6008474	8·2278254	·001795332
558	311364	173741112	23·6220236	8·2327463	·001792115
559	312481	174676879	23·6431808	8·2376614	·001788909
560	313600	175616000	23·6643191	8·2425706	·001785714
561	314721	176558481	23·6854386	8·2474740	·001782531
562	315844	177504328	23·7065392	8·2523715	·001779359
563	316969	178453547	23·7276210	8·2572635	·001776199
564	318096	179406144	23·7486842	8·2621492	·001773050
565	319225	180362125	23·7697286	8·2670294	·001769912
566	320356	181321496	23·7907545	8·2719039	·001766784
567	321489	182284263	23·8117618	8·2767726	·001763668
568	322624	183250432	23·8327506	8·2816255	·001760563
569	323761	184220009	23·8537209	8·2864928	·001757469
570	324900	185193000	23·8746728	8·2913444	·001754386
571	326041	186169411	23·8956063	8·2961908	·001751313
572	327184	187149248	23·9165215	8·3010304	·001748252
573	328329	188132517	23·9374184	8·3058651	·001745201
574	329476	189119224	23·9582971	8·3106941	·001742160
575	330625	190109375	23·9791576	8·3155175	·001739130
576	331776	191102976	24·0000000	8·3203353	·001736111
577	332927	192100033	24·0208243	8·3251475	·001733102
578	334084	193100552	24·0416306	8·3299542	·001730104
579	335241	194104539	24·0624188	8·3347553	·001727116
580	336400	195112000	24·0831891	8·3395509	·001724138
581	337561	196122941	24·1039416	8·3443410	·001721170
582	338724	197137368	24·1246762	8·3491256	·001718213
583	339889	198155287	24·1453929	8·3539047	·001715266
584	341056	199176704	24·1660919	8·3586784	·001712329
585	342225	200201625	24·1867732	8·3634466	·001709402
586	343396	201230056	24·2074369	8·3682095	·001706485
587	344569	202262003	24·2280829	8·3729668	·001703578
588	345744	203297472	24·2487113	8·3777188	·001700680
589	346921	204336469	24·2693222	8·3824653	·001697793
590	348100	205379000	24·2899156	8·3872065	·001694915
591	349281	206425071	24·3104996	8·3919428	·001692047
592	350464	207474688	24·3310501	8·3966729	·001689189
593	351649	208527857	24·3515913	8·4013981	·001686341
594	352836	209584584	24·3721152	8·4061180	·001683502
595	354025	210644875	24·3926218	8·4108326	·001680672
596	355216	211708736	24·4131112	8·4155419	·001677852
597	356409	212776173	24·4335834	8·4202460	·001675042
598	357604	213847192	24·4540385	8·4249448	·001672241
599	358801	214921799	24·4744765	8·4296383	·001669449
600	360000	216000000	24·4948974	8·4343267	·001666667
601	361201	217081801	24·5153013	8·4390098	·001663894
602	362404	218167208	24·5356883	8·4436877	·001661130
603	363609	219256227	24·5560583	8·4483605	·001658375
604	364816	220348864	24·5764115	8·4530281	·001655629
605	366025	221445125	24·5967478	8·4576906	·001652893
606	367236	222545016	24·6170673	8·4623479	·001650165
607	368449	223648543	24·6373700	8·4670001	·001647446
608	369664	224755712	24·6576560	8·4716471	·001644737
609	370881	225866529	24·6779254	8·4762892	·001642036
610	372100	226981000	24·6981781	8·4809261	·001639344
611	373321	228099131	24·7184142	8·4855579	·001636661
612	374544	229220928	24·7386338	8·4901848	·001633987
613	375769	230346397	24·7588368	8·4948065	·001631321
614	376996	231475544	24·7790234	8·4994233	·001628664
615	378225	232608375	24·7991935	8·5040350	·001626016

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
616	379456	233744896	24·8193473	8·5086417	·001623377
617	380689	234885113	24·8394847	8·5132435	·001620746
618	381924	236029032	24·8596058	8·5178403	·001618123
619	383161	237176659	24·8797106	8·5224331	·001615509
620	384400	238328000	24·8997992	8·5270189	·001612903
621	385641	239483061	24·9198716	8·5316009	·001610306
622	386884	240641848	24·9399278	8·5361780	·001607717
623	388129	241804367	24·9599679	8·5407501	·001605136
624	389376	242970624	24·9799920	8·5453173	·001602564
625	390625	244140625	25·0000000	8·5498797	·001600000
626	391876	245184376	25·0199920	8·5544372	·001597444
627	393129	246491883	25·0399681	8·5589899	·001594896
628	394384	247673152	25·0599282	8·5635377	·001592357
629	395641	248858189	25·0798724	8·5680807	·001589825
630	396900	250047000	25·0998008	8·5726189	·001587302
631	398161	251239591	25·1197134	8·5771523	·001584786
632	399424	252435968	25·1396102	8·5816809	·001582278
633	400689	253636137	25·1594913	8·5862247	·001579779
634	401956	254840104	25·1793566	8·5907238	·001577287
635	403225	256047875	25·1992063	8·5952380	·001574803
636	404496	257259456	25·2190404	8·5997476	·001572327
637	405769	258474853	25·2388589	8·6042525	·001569859
638	407044	259694072	25·2586619	8·6087526	·001567398
639	408321	260917119	25·2784493	8·6132480	·001564945
640	409600	262144000	25·2982213	8·6177388	·001562500
641	410881	263374721	25·3179778	8·6222248	·001560062
642	412164	264609288	25·3377189	8·6267063	·001557632
643	413449	265847707	23·3574447	8·6311830	·001555210
644	414736	267089984	25·3771551	8·6356551	·001552795
645	416125	268336125	25·3968502	8·6401226	·001550388
646	417316	269585136	25·4165302	8·6445855	·001547988
647	418609	270840023	25·4361947	8·6490437	·001545595
648	419904	272097792	25·4558441	8·6534974	·001543210
649	421201	273359449	25·4754784	8·6579465	·001540832
650	422500	274625000	25·4950976	8·6623911	·001538462
651	423801	275894451	25·5147013	8·6668310	·001536098
652	425104	277167808	25·5342907	8·6712665	·001533742
653	426409	278445077	25·5538647	8·6756974	·001531394
654	427716	279726264	25·5734237	8·6801237	·001529052
655	429025	281011375	25·5929678	8·6845456	·001526718
656	430336	282300416	25·6124969	8·6889630	·001524390
657	431639	283593393	25·6320112	8·6933759	·001522070
658	432964	284890312	25·6515107	8·6977843	·001519751
659	434281	286191179	25·6709953	8·7021882	·001517451
660	435600	287496000	25·6904652	8·7065877	·001515152
661	436921	288804781	25·7099208	8·7109827	·001512859
662	438244	290117528	25·7293607	8·7153734	·001510574
663	439569	291434247	25·7487864	8·7197596	·001508296
664	440896	292754944	25·7681975	8·7241414	·001506024
665	442225	294079625	25·7875939	8·7285187	·001503759
666	443556	295408296	25·8069758	8·7328918	·001501502
667	444899	296740963	25·8263431	8·7372604	·001499250
668	446224	298077632	25·8456960	8·7416246	·001497006
669	447561	299418309	25·8650343	8·7459846	·001494768
670	448900	300763000	25·8843582	8·7503401	·001492537
671	450241	302111711	25·9036677	8·7546913	·001490313
672	451584	303464448	25·9229628	8·7590383	·001488095
673	452929	304821217	25·9422435	8·7633809	·001485884
674	454276	306182024	25·9615100	8·7677192	·001483680
675	455625	307546875	25·9807621	8·7720532	·001481481
676	456976	308915776	26·0000000	8·7763830	·001479290
677	458329	310288733	26·0192237	8·7807084	·001477105

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
678	459684	311665752	26·0384331	8·7850296	·001474926
679	461041	313046839	26·0576284	8·7893466	·001472754
680	462400	314432000	26·0768096	8·7936593	·001470588
681	463761	315821241	26·0959767	8·7979679	·001468429
682	465124	317214568	26·1151297	8·8022721	·001466276
683	466489	318611987	26·1342687	8·8065722	·001464129
684	467856	320013504	26·1533937	8·8108681	·001461988
685	469225	321419125	26·1725047	8·8151598	·001459854
686	470596	322828856	26·1916017	8·8194474	·001457726
687	471969	324242703	26·2106848	8·8237307	·001455604
688	473344	325660672	26·2297541	8·8280099	·001453488
689	474721	327082769	26·2488095	8·8322850	·001451379
690	476100	328509000	26·2678511	8·8365559	·001449275
691	477481	329939371	26·2868789	8·8408227	·001447178
692	478864	331373888	26·3058929	8·8450854	·001445087
693	480249	332812557	26·3248932	8·8493440	·001443001
694	481636	334255384	26·3438797	8·8535985	·001440922
695	483025	335702375	26·3628527	8·8578489	·001438849
696	484416	337153536	26·3818119	8·8620952	·001436782
697	485809	338608873	26·4007576	8·8663375	·001434720
698	487204	340068392	26·4196896	8·8705757	·001432665
699	488601	341532099	26·4386081	8·8748099	·001430615
700	490000	343000000	26·4575131	8·8790400	·001428571
701	491401	344472101	26·4764046	8·8832661	·001426534
702	492804	345948408	26·4952826	8·8874882	·001424501
703	494209	347428927	26·5141472	8·8917063	·001422475
704	495616	348913664	26·5329983	8·8959204	·001420455
705	497025	350402625	26·5518361	8·9001304	·001418440
706	498436	351895816	26·5706605	8·9043366	·001416431
707	499849	353393243	26·5894716	8·9085387	·001414427
708	501264	354894912	26·6082694	8·9127369	·001412429
709	502681	356400829	26·6270539	8·9169311	·001410437
710	504100	357911000	26·6458252	8·9211214	·001408451
711	505521	359425431	26·6645833	8·9253078	·001406470
712	506944	360944128	26·6833281	8·9294902	·001404494
713	508369	362467097	26·7020598	8·9336687	·001402525
714	509796	363994344	26·7207784	8·9378433	·001400560
715	511225	365525875	26·7394839	8·9420140	·001398601
716	512656	367061696	26·7581763	8·9461809	·001396648
717	514089	368601813	26·7768557	8·9503438	·001394700
718	515524	370146232	26·7955220	8·9545029	·001392758
719	516961	371694959	26·8141754	8·9586581	·001390821
720	518400	373248000	26·8328157	8·9628095	·001388889
721	519841	374805361	26·8514432	8·9669570	·001386963
722	521284	376367048	26·8700577	8·9711007	·001385042
723	522729	377933067	26·8886593	8·9752406	·001383126
724	524176	379503424	26·9072481	8·9793766	·001381215
725	525625	381078125	26·9258240	8·9835089	·001379310
726	527076	382657176	26·9443872	8·9876373	·001377410
727	528529	384240583	26·9629375	8·9917620	·001375516
728	529984	385828352	26·9814751	8·9958899	·001373626
729	531441	387420489	27·0000000	9·0000000	·001371742
730	532900	389017000	27·0185122	9·0041134	·001369863
731	534361	390617891	27·0370117	9·0082229	·001367989
732	535824	392223168	27·0554985	9·0123288	·001366120
733	537289	393832837	27·0739727	9·0164309	·001364256
734	538756	395446904	27·0924344	9·0205293	·001362398
735	540225	397065375	27·1108834	9·0246289	·001360544
736	541696	398688256	27·1293199	9·0287149	·001358696
737	543169	400315553	27·1477149	9·0328021	·001356852
738	544644	401947272	27·1661554	9·0368857	·001355014
739	546121	403583419	27·1845544	9·0409655	·001353180

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
740	547600	405224000	27·2029140	9·0450419	·001351351
741	549801	406869021	27·2213152	9·0491142	·001349528
742	550564	408518488	27·2396769	9·0531831	·001347709
743	552049	410172407	27·2580263	9·0572482	·001345895
744	553536	411830784	27·2763634	9·0613098	·001344086
745	555025	413493625	27·2946881	9·0653677	·001342282
746	556516	415160936	27·3130006	9·0694220	·001340483
747	558009	416832723	27·3313007	9·0734726	·001338688
748	559504	418508992	27·3495887	9·0775197	·001336898
749	561001	420189749	27·3678644	9·0815631	·001335113
750	562500	421875000	27·3861279	9·0856030	·001333333
751	564001	423564751	27·4043792	9·0896352	·001331558
752	565504	425259008	27·4226184	9·0936719	·001329787
753	567009	426957777	27·4408455	9·0977010	·001328021
754	568516	428661064	27·4590604	9·1017265	·001326260
755	570025	430368875	27·4772633	9·1057485	·001324503
756	571536	432081216	27·4954542	9·1097669	·001322751
757	573049	433798093	27·5136330	9·1137818	·001321004
758	574564	435519512	27·5317998	9·1177931	·001319261
759	576081	437245479	27·5499546	9·1218010	·001317523
760	577600	438976000	27·5680975	9·1258053	·001315789
761	579121	440711081	27·5862284	9·1298061	·001314060
762	580644	442450728	27·6043475	9·1338034	·001312336
763	582169	444194947	27·6224546	9·1377971	·001310616
764	583696	445943744	27·6405499	9·1417874	·001308901
765	585225	447697125	27·6586334	9·1457742	·001307190
766	586756	449455096	27·6767050	9·1497576	·001305483
767	588289	451217663	27·6947648	9·1537375	·001303781
768	589824	452984832	27·7128129	9·1577139	·001302083
769	591361	454756609	27·7308492	9·1616869	·001300390
770	592900	456533000	27·7488739	9·1656565	·001298701
771	594441	458314011	27·7668868	9·1696225	·001297017
772	595984	460099648	27·7848880	9·1735852	·001295337
773	597529	461889917	27·8028775	9·1775445	·001293661
774	599076	463684824	27·8208555	9·1815003	·001291990
775	600625	465484375	27·8388218	9·1854527	·001290323
776	602176	467288576	27·8567766	9·1894018	·001288660
777	603729	469097433	27·8747197	9·1933474	·001287001
778	605284	470910952	27·8926514	9·1972897	·001285347
779	606841	472729139	27·9105715	9·2012286	·001283697
780	608400	474552000	27·9284801	9·2051641	·001282051
781	609961	476379541	27·9463772	9·2090962	·001280410
782	611524	478211768	27·9642629	9·2130250	·001278772
783	613089	480048687	27·9821372	9·2169505	·001277139
784	614656	481890304	28·0000000	9·2208726	·001275510
785	616225	483736625	28·0178515	9·2247914	·001273885
786	617796	485587656	28·0356915	9·2287068	·001272265
787	619369	487443403	28·0535203	9·2326189	·001270648
788	620944	489303872	28·0713377	9·2365277	·001269036
789	622521	491169069	28·0891438	9·2404333	·001267427
790	624100	493039000	28·1069386	9·2443355	·001265823
791	625681	494913671	28·1247222	9·2482344	·001264223
792	627624	496793088	28·1424946	8·2521300	·001262626
793	628849	498677257	28·1602557	9·2560224	·001261034
794	630436	500566184	28·1780056	9·2599114	·001259446
795	632025	502459875	28·1957444	9·2637973	·001257862
796	633616	504358336	28·2134720	9·2676798	·001256281
797	635209	506261573	28·2311884	9·2715592	·001254705
798	636804	508169592	28·2488938	9·2754352	·001253133
799	638401	510082399	28·2665881	9·2793081	·001251364
800	640000	512000000	28·2842712	9·2831777	·001250000
801	641601	513922401	28·3019434	9·2870444	·001248439

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
802	643204	515849608	28·3196045	9·2909072	·001246883
803	644809	517781627	28·3372546	9·2947671	·001245330
804	646416	519718464	28·3548938	9·2986239	·001243781
805	648025	521660125	28·3725219	9·3024775	·001242236
806	649636	523606616	28·3901391	9·3063278	·001240695
807	651249	525557943	28·4077454	9·3101750	·001239157
808	652864	527514112	28·4253408	9·3140190	·001237624
809	654481	529475129	28·4429253	9·3178599	·001236094
810	656100	531441000	28·4604989	9·3216975	·001234568
811	657721	533411731	28·4780617	9·3255320	·001233046
812	659344	535387328	28·4956137	9·3293634	·001231527
813	660969	537367797	28·5131549	9·3331916	·001230012
814	662596	539353144	28·5306852	9·3370167	·001228501
815	664225	541343375	28·5482048	9·3408386	·001226994
816	665856	543338496	28·5657137	9·3446575	·001225499
817	667489	545338513	28·5832119	9·3484731	·001223990
818	669124	547343432	28·6006993	9·3522857	·001222494
819	670761	549353259	28·6181760	9·3560952	·001221001
820	672400	551368000	28·6356421	9·3599016	·001219512
821	674041	553387661	28·6530976	9·3637049	·001218027
822	675684	555412248	28·6705424	9·3675051	·001216545
823	677329	557441767	28·6879716	9·3713022	·001215067
824	678976	559476224	28·7054002	9·3750963	·001213592
825	680625	561515625	28·7228132	9·3788873	·001212121
826	682276	563559976	28·7402157	9·3826752	·001210654
827	683929	565609283	28·7576077	9·3864600	·001209190
828	685584	567663552	28·7749891	9·3902419	·001207729
829	687241	569722789	28·7923601	9·3940206	·001206273
830	688900	571787000	28·8097206	9·3977964	·001204819
831	690561	573856191	28·8270706	9·4015691	·001203369
832	692224	575930368	28·8444102	9·4053387	·001201923
833	693889	578009537	28·8617394	9·4091054	·001200480
834	695556	580093704	28·8790582	9·4128690	·001199041
835	697225	582182875	28·8963666	9·4166297	·001197605
836	698896	584277056	28·9136646	9·4203873	·001196172
837	700569	586376253	28·9309523	9·4241420	·001194743
838	702244	588480472	28·9482297	9·4278936	·001193317
839	703921	590589719	28·9654967	9·4316423	·001191895
840	705600	592704000	28·9827535	9·4353800	·001190476
841	707281	594823321	29·0000000	9·4391307	·001189061
842	708964	596947688	29·0172363	9·4428704	·001187648
843	710649	599077107	29·0344623	9·4466072	·001186240
844	712336	601211584	29·0516781	9·4503410	·001184834
845	714025	603351125	29·0688837	9·4540719	·001183432
846	715716	605495736	29·0860791	9·4577999	·001182033
847	717409	607645423	29·1032644	9·4615249	·001180638
848	719104	609800192	29·1204396	9·4652470	·001179245
849	720801	611960049	29·1376046	9·4689661	·001177856
850	722500	614125000	29·1547595	9·4726824	·001176471
851	724201	616295051	29·1719043	9·4763957	·001175088
852	725904	618470208	29·1890390	9·4801061	·001173709
853	727609	620650477	29·2061637	9·4838136	·001172333
854	729316	622835864	29·2232784	9·4875182	·001170960
855	731025	625026375	29·2403830	9·4912200	·001169591
856	732736	627222016	29·2574777	9·4949188	·001168224
857	734449	629422793	29·2745623	9·4986147	·001166861
858	736164	631628712	29·2916370	9·5023078	·001165501
859	737881	633839779	29·3087018	9·5059980	·001164144
860	739600	636056000	29·3257566	9·5096854	·001162791
861	741321	638277381	29·3428015	9·5133699	·001161440
862	743044	640503928	29·3598365	9·5170515	·001160093
863	744769	642735647	29·3768616	9·5207303	·001158749

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
864	746496	644972544	29·3938769	9·5244063	·001157407
865	748225	647214625	29·4108823	9·5280794	·001156069
866	749956	649461896	29·4278779	9·5317497	·001154734
867	751689	651714363	29·4448637	9·5354172	·001153403
868	753424	653972032	29·4618397	9·5390818	·001152074
869	755161	656234909	29·4788059	9·5427437	·001150748
870	756900	658503000	29·4957624	9·5464027	·001149425
871	758641	660776311	29·5127091	9·5500589	·001148106
872	760384	663054848	29·5296461	9·5537123	·001146789
873	762129	665338617	29·5465734	9·5573630	·001145475
874	763876	667627624	29·5634910	9·5610108	·001144165
875	765625	669921875	29·5803989	9·5646559	·001142857
876	767376	672221376	29·5972972	9·5682782	·001141553
877	769129	674526133	29·6141858	9·5719377	·001140251
878	770884	676836152	29·6310648	9·5755745	·001138952
879	772641	679151439	29·6479342	9·5792085	·001137656
880	774400	681472000	29·6647939	9·5828397	·001136364
881	776161	683797841	29·6816442	9·5864682	·001135074
882	777924	686128968	29·6984848	9·5900937	·001133787
883	779689	688465387	29·7153159	9·5937169	·001132503
884	781456	690807104	29·7321375	9·5973373	·001131222
885	783225	693154125	29·7489496	9·6009548	·001129944
886	784996	695506456	29·7657521	9·6045696	·001128668
887	786769	697864103	29·7825452	9·6081817	·001127396
888	788544	700227072	29·7993289	9·6117911	·001126126
889	790321	702595369	29·8161030	9·6153977	·001124859
890	792100	704969000	29·8328678	9·6190017	·001123596
891	793881	707347971	29·8496231	9·6226030	·001122334
892	795664	707032288	29·8663690	9·6262016	·001121076
893	797449	712121957	29·8831056	9·6297975	·001119821
894	799236	714516984	29·8998328	9·6333907	·001118568
895	801025	716917375	29·9165506	9·6369812	·001117318
896	802816	719323136	29·9332591	9·6405690	·001116071
897	804609	721734273	29·9499583	9·6441542	·001114827
898	806404	724150792	29·9666481	9·6477367	·001113586
899	808201	726572699	29·9833287	9·6513166	·001112347
900	810000	729000000	30·0000000	9·6548938	·001111111
901	811801	731432701	30·0166621	9·6584684	·001109878
902	813604	733870808	30·0333148	9·6620403	·001108647
903	815409	736314327	30·0499584	9·6656096	·001107420
904	817216	738763264	30·0665928	9·6691762	·001106195
905	819025	741217625	30·0832179	9·6727403	·001104972
906	820836	743677416	30·0998339	9·6763017	·001103753
907	822649	746142643	30·1164407	9·6798604	·001102536
908	824464	748613312	30·1330383	9·6834166	·001101322
909	826281	751089429	30·1496269	9·6869701	·001100110
910	828100	753571000	30·1662063	9·6905211	·001098901
911	829921	756058031	30·1827765	9·6940694	·001097695
912	831744	758550825	30·1993377	9·6976151	·001096491
913	833569	761048497	30·2158899	9·7011583	·001095290
914	835396	763551944	30·2324329	9·7046989	·001094092
915	837225	766060875	30·2489669	9·7082369	·001092896
916	839056	768575296	30·2654919	9·7117723	·001091703
917	840889	771095213	30·2820079	9·7153051	·001090513
918	842724	773620632	30·2985148	9·7188354	·001089325
919	844561	776151559	30·3150128	9·7223631	·001088139
920	846400	778688000	30·3315018	9·7258883	·001086957
921	848241	781229961	30·3479818	9·7294109	·001085776
922	850084	783777448	30·3644529	9·7329309	·001084599
923	851929	786330467	30·3809151	9·7364484	·001083423
924	853776	788889024	30·3973683	9·7399634	·001082251
925	855625	791453125	30·4138127	9·7434758	·001081081

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
926	857476	794022776	30·4302481	9·7469857	·001079914
927	859329	796597983	30·4466747	9·7504930	·001078749
928	861184	799178752	30·4630924	9·7539979	·001077586
929	863041	801765089	30·4795013	9·7575002	·001076426
930	864900	804357000	30·4959014	9·7610001	·001075269
931	866761	806954491	30·5122926	9·7644974	·001074114
932	868624	809557568	30·5286750	9·7679922	·001072961
933	870489	812166237	30·5450487	9·7714845	·001071811
934	872356	814780504	30·5614136	9·7749743	·001070664
935	874225	817400375	30·5777697	9·7784616	·001069519
936	876096	820025856	30·5941171	9·7829466	·001068376
937	877969	822656953	30·6104557	9·7854288	·001067236
938	879844	825293672	30·6267857	9·7889087	·001066098
939	881721	827936019	30·6431069	9·7923861	·001064963
940	883600	830584000	30·6594194	9·7958611	·001063830
941	885481	833237621	30·6757233	9·7993336	·001062699
942	887364	835896888	30·6920185	9·8028036	·001061571
943	889249	838561807	30·7083051	9·8062711	·001060445
944	891136	841232384	30·7245830	9·8097362	·001059322
945	893025	843908625	30·7408523	9·8131989	·001058201
946	894916	846590536	30·7571130	9·8166591	·001057082
947	896808	849278123	30·7733651	9·8201169	·001055966
948	898704	851971392	30·7896086	9·8235723	·001054852
949	900601	854670349	30·8058436	9·8270252	·001053741
950	902500	857375000	30·8220700	9·8304757	·001052632
951	904401	860085351	30·8382879	9·8339238	·001051525
952	906304	862801408	30·8544972	9·8373695	·001050420
953	908209	865523177	30·8706981	9·8408127	·001049318
954	910116	868250664	30·8868904	9·8442536	·001048218
955	912025	870983875	30·9030743	9·8476920	·001047120
956	913936	873722816	30·9192477	9·8511280	·001046025
957	915849	876467493	30·9354166	9·8545617	·001044932
958	917764	879217912	30·9515751	9·8579929	·001043841
959	919681	881974079	30·9677251	9·8614218	·001042753
960	921600	884736000	30·9838668	9·8648483	·001041667
961	923521	887503681	31·0000000	9·8682724	·001040583
962	925444	890277128	31·0161248	9·8716941	·001039501
963	927369	893056347	31·0322413	9·8751135	·001038422
964	929296	895841344	31·0483494	9·8785305	·001037344
965	931225	898632125	31·0644491	9·8819451	·001036269
966	933156	901428696	31·0805405	9·8853574	·001035197
967	935089	904231063	31·0966236	9·8887673	·001034126
968	937024	907039232	31·1126984	9·8921749	·001033058
969	938961	909853209	31·1287648	9·8955801	·001031992
970	940900	912673000	31·1448230	9·8989830	·001030928
971	942841	915498611	31·1608729	9·9023835	·001029866
972	944784	918330048	31·1769145	9·9057817	·001028807
973	946729	921167317	31·1929479	9·9091776	·001027749
974	948676	924010424	31·2089731	9·9125712	·001026694
975	950625	926859375	31·2249900	9·9159624	·001025641
976	952576	929714176	31·2409987	9·9193513	·001024590
977	954529	932574833	31·2569992	9·9227379	·001023541
978	956484	935441352	31·2729915	9·9261222	·001022495
979	958441	938313739	31·2889757	9·9295042	·001021450
980	960400	941192000	31·3049517	9·9328839	·001020408
981	962361	944076141	31·3209195	9·9362613	·001019168
982	964324	946966168	31·3368792	9·9396363	·001018330
983	966289	949862087	31·3528308	9·9430092	·001017294
984	968256	952763904	31·3687743	9·9463797	·001016260
985	970225	955671625	31·3847097	9·9497479	·001015228
986	972196	958585256	31·4006369	9·9531138	·001014199
987	974169	961504803	31·4165561	9·9564775	·001013171

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
988	976144	964430272	31·4324673	9·9598389	·001012146
989	978121	967361669	31·4483704	9·9631981	·001011122
990	980100	970299000	31·4642654	9·9665549	·001010101
991	982081	973242271	31·4801525	9·9699055	·001009082
992	984064	976191488	31·4960315	9·9732619	·001008065
993	986049	979146657	31·5119025	9·9766120	·001007049
994	988036	982107784	31·5277655	9·9799599	·001006036
995	990025	985074875	31·5436206	9·9833055	·001005025
996	992016	988047936	31·5594677	9·9866488	·001004016
997	994009	991026973	31·5753068	9·9899900	·001003009
998	996004	994011992	31·5911380	9·9933289	·001002004
999	998001	997002999	31·6069613	9·9966656	·001001001
1000	1000000	1000000000	31·6227766	10·0000000	·001000000
1001	1000201	1003003001	31·6385840	10·0033222	·0009990010
1002	1004004	1006012008	31·6543866	10·0066622	·0009980040
1003	1006009	1009027027	31·6701752	10·0099899	·0009970090
1004	1008016	1012048064	31·6859590	10·0133155	·0009960159
1005	1010025	1015075125	31·7017349	10·0166389	·0009950249
1006	1010036	1018108216	31·7175030	10·0199601	·0009940358
1007	1014049	1021147343	31·7332633	10·0232791	·0009930487
1008	1016064	1024192512	31·7490157	10·0265958	·0009920635
1009	1018081	1027243729	31·7647603	10·0299104	·0009910803
1010	1020100	1030301000	31·7804972	10·0332228	·0009900990
1011	1020121	1033364331	31·7962262	10·0365330	·0009891197
1012	1024144	1036433728	31·8119474	10·0398410	·0009881423
1013	1026169	1039509197	31·8276609	10·0431469	·0009871668
1014	1028196	1042590744	31·8433666	10·0464506	·0009861933
1015	1030225	1045678375	31·8590646	10·0497521	·0009852217
1016	1032256	1048772096	31·8747549	10·0530514	·0009842520
1017	1034289	1051871913	31·8904374	10·0563485	·0009832842
1018	1036324	1054977832	31·9061123	10·0596435	·0009823183
1019	1038361	1058089859	31·9217794	10·0629364	·0009813543
1020	1040400	1061208000	31·9374388	10·0662271	·0009803922
1021	1042441	1064332261	31·9530906	10·0695156	·0009794319
1022	1044484	1067462648	31·9687347	10·0728020	·0009784736
1023	1046529	1070599167	31·9843712	10·0760863	·0009775171
1024	1048576	1073741824	32·0000000	10·0793684	·0009765625
1025	1050625	1076890625	32·0156212	10·0826484	·0009756098
1026	1052676	1080045576	32·0312348	10·0859262	·0009746589
1027	1054729	1083206683	32·0468407	10·0892019	·0009737098
1028	1056784	1086373952	32·0624391	10·0924755	·0009727626
1029	1058841	1089547389	32·0780298	10·0957469	·0009718173
1030	1060900	1092727000	32·0936131	10·0990163	·0009708738
1031	1062961	1095912791	32·1091887	10·1022835	·0009699321
1032	1065024	1099104768	32·1247568	10·1055487	·0009689922
1033	1067089	1102302937	32·1403173	10·1088117	·0009680542
1034	1069156	1105507304	32·1558704	10·1120726	·0009671180
1035	1071225	1108717875	32·1714159	10·1153314	·0009661836
1036	1073296	1111984656	32·1869539	10·1185882	·0009652510
1037	1075369	1115157653	32·2024844	10·1218428	·0009643202
1038	1077444	1118386872	32·2180074	10·1250953	·0009633911
1039	1079521	1121622319	32·2335229	10·1283457	·0009624639
1040	1081600	1124864000	32·2490310	10·1315941	·0009615385
1041	1083681	1128111921	32·2645316	10·1348403	·0009606148
1042	1085764	1131366088	32·2800248	10·1380845	·0009596929
1043	1087849	1134626507	32·2955105	10·1413266	·0009587738
1044	1089936	1137893184	32·3109888	10·1445667	·0009578544
1045	1092025	1141166125	32·3264598	10·1478047	·0009569378
1046	1094116	1144445336	32·3419233	10·1510406	·0009560229
1047	1096209	1147730823	32·3573794	10·1542744	·0009551098
1048	1098304	1151022592	32·3728281	10·1575062	·0009541985
1049	1100401	1154320649	32·3882695	10·1607359	·0009532888

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1050	1102500	1157625000	32·4037035	10·1689636	·0009523810
1051	1104601	1160935651	32·4191301	10·1671893	·0009514748
1052	1106704	1164252608	32·4345495	10·1704129	·0009505703
1053	1108809	1167575877	32·4499615	10·1736844	·0009496676
1054	1110916	1170905464	32·4653662	10·1768539	·0009487666
1055	1113025	1174241375	32·4807635	10·1800714	·0009478673
1056	1115136	1177583616	32·4961536	10·1832868	·0009469697
1057	1117249	1180932193	32·5115364	10·1865002	·0009460738
1058	1119364	1184287112	32·5269119	10·1897116	·0009451796
1059	1121481	1187648379	32·5422802	10·1929209	·0009442871
1060	1123600	1191016000	32·5576412	10·1961283	·0009433962
1061	1125721	1194389981	32·5729949	10·1993336	·0009425071
1062	1127844	1197770328	32·5883415	10·2025369	·0009416196
1063	1129969	1201157047	32·6035807	10·2057382	·0009407338
1064	1132096	1204550144	32·6190129	10·2089375	·0009398496
1065	1134225	1207949625	32·6343377	10·2121347	·0009389671
1066	1136356	1211355496	32·6496554	10·2153300	·0009380863
1067	1138489	1214767763	32·6649659	10·2185233	·0009372071
1068	1140624	1218186432	32·6802693	10·2217146	·0009363296
1069	1142761	1221611509	32·6955654	10·2249039	·0009354537
1070	1144900	1225043000	32·7108544	10·2280912	·0009345794
1071	1147041	1228480911	32·7261363	10·2312766	·0009337068
1072	1149184	1231925248	32·7414111	10·2344599	·0009328358
1073	1151329	1235376017	32·7566787	10·2376413	·0009319664
1074	1153476	1238833224	32·7719392	10·2408207	·0009310987
1075	1155625	1242296875	32·7871926	10·2439981	·0009302326
1076	1157776	1245766976	32·8024398	10·2471735	·0009293680
1077	1159929	1249243533	32·8176782	10·2503470	·0009285051
1078	1162084	1252726552	32·8329103	10·2535186	·0009276438
1079	1164241	1256216039	32·8481354	10·2566881	·0009267841
1080	1166400	1259712000	32·8633535	10·2598557	·0009259259
1081	1168561	1263214441	32·8785644	10·2630213	·0009250694
1082	1170724	1266723368	32·8937684	10·2661850	·0009242144
1083	1172889	1270238787	32·9089653	10·2693467	·0009233610
1084	1175056	1273760704	32·9241553	10·2725065	·0009225092
1085	1177225	1277289125	32·9393382	10·2756644	·0009216590
1086	1179396	1280824056	32·9545141	10·2788203	·0009208103
1087	1181569	1284365503	32·9696830	10·2819743	·0009199632
1088	1183744	1287913472	32·9848450	10·2851264	·0009191176
1089	1185921	1291467969	33·0000000	10·2882765	·0009182736
1090	1188100	1295029000	33·0151480	10·2914247	·0009174312
1091	1190281	1298596571	33·0302891	10·2945709	·0009165903
1092	1192464	1302170688	33·0454233	19·2977153	·0009157509
1093	1194649	1305751357	33·0605505	10·3008577	·0009149131
1094	1196836	1309338584	33·0756708	10·3039982	·0009140768
1095	1199025	1312932375	33·0907842	10·3071368	·0009132420
1096	1201216	1316532736	33·1058907	10·3102735	·0009124008
1097	1203409	1320139673	33·1209903	10·3134083	·0009115770
1098	1205604	1323753192	33·1360830	10·3165411	·0009107468
1099	1207801	1327373299	33·1511689	10·3196721	·0009099181
1100	1210000	1331000000	33·1662479	10·3228012	·0009090909
1101	1212201	1334633301	33·1813200	10·3259284	·0009082652
1102	1214404	1338273208	33·1963853	10·3290537	·0009074410
1103	1216609	1341919727	33·2114438	10·3321770	·0009066183
1104	1218816	1345572864	33·2266955	10·3352985	·0009057971
1105	1221025	1349232625	33·2415403	10·3384181	·0009049774
1106	1223236	1352899016	33·2565783	10·3415358	·0009041591
1107	1225449	1356572043	33·2716095	10·3446517	·0009033424
1108	1227664	1360251712	33·2866339	10·3477657	·0009025271
1109	1229881	1363938029	33·3016516	10·3508778	·0009017133
1110	1232100	1367631000	33·3166625	10·3539880	·0009009009
1111	1234321	1371330631	33·3316666	10·3570964	·0009000900

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1112	1236544	1375036928	33·3466640	10·3602029	·0008992806
1113	1238769	1378749897	33·3616546	10·3633076	·0008984726
1114	1240996	1382469544	33·3766385	10·3664103	·0008976661
1115	1243225	1386195875	33·3916157	10·3695118	·0008968610
1116	1245456	1389928896	33·4065862	10·3726103	·0008960753
1117	1247689	1393668613	33·4215499	10·3757076	·0008952551
1118	1249924	1397415032	33·4365070	10·3788030	·0008944544
1119	1252161	1401168159	33·4514573	10·3818965	·0008936550
1120	1254400	1404928000	33·4664011	10·3849882	·0008928571
1121	1256641	1408694561	33·4813381	10·3880781	·0008960607
1122	1258884	1412467848	33·4962684	10·3911661	·0008912656
1123	1261129	1416247867	33·5111921	10·3942527	·0008904720
1124	1263376	1420034624	33·5261092	10·3973366	·0008896797
1125	1265625	1423828125	33·5410196	10·4004192	·0008888889
1126	1267876	1427628376	33·5559234	10·4034999	·0008880995
1127	1270129	1431435383	33·5708206	10·4065787	·0008873114
1128	1272384	1435249152	33·5857112	10·4096557	·0008865248
1129	1274641	1439069689	33·6005952	10·4127310	·0008857396
1130	1276900	1442897000	33·6154726	10·4158044	·0008849558
1131	1279161	1446731091	33·6303434	10·4188760	·0008841733
1132	1281424	1450571968	33·6452077	10·4219458	·0008833922
1133	1283689	1454419637	33·6600653	10·4250138	·0008826125
1134	1285956	1458274104	33·6749165	10·4280800	·0008818342
1135	1288225	1462135375	33·6897610	10·4311443	·0008810573
1136	1290496	1466003456	33·7045991	10·4342069	·0008802817
1137	1292769	1469878353	33·7174806	10·4372677	·0008795075
1138	1295044	1473760072	33·7340556	10·4403677	·0008787346
1139	1297321	1477648619	33·7490741	10·4433839	·0008779631
1140	1299600	1481544000	33·7638860	10·4464393	·0008771930
1141	1301881	1485446221	33·7786915	10·4494929	·0008764242
1142	1304164	1489355288	33·7934905	10·4525448	·0008756567
1143	1306449	1493271207	33·8082830	10·4555948	·0008748906
1144	1308736	1497193984	33·8230691	10·4586431	·0008741259
1145	1311025	1501123625	33·8378486	10·4616896	·0008733624
1146	1313316	1505060136	33·8526218	10·4647343	·0008726003
1147	1315609	1509003523	33·8673884	10·4677773	·0008718396
1148	1317904	1512953792	33·8821487	10·4708158	·0008710801
1149	1320201	1516910949	33·8969025	10·4738579	·0008703220
1150	1322500	1520875000	33·9116499	10·4768955	·0008695652
1151	1324801	1524845951	33·9263909	10·4799314	·0008688097
1152	1327104	1528823808	33·9411255	10·4829656	·0008680556
1153	1329409	1532808577	33·9558537	10·4859980	·0008673027
1154	1331716	1536800264	33·9705755	10·4890286	·0008665511
1155	1334025	1540798875	33·9852910	10·4920575	·0008658009
1156	1336336	1544804416	34·0000000	10·4950847	·0008650519
1157	1338649	1548816893	34·0147027	10·4981101	·0008643042
1158	1340964	1552836312	34·0293990	10·5011337	·0008635579
1159	1343281	1556862679	34·0440890	10·5041556	·0008628128
1160	1345600	1560896000	34·0587727	10·5071757	·0008620690
1161	1347921	1564936281	34·0734501	10·5101942	·0008613244
1162	1350244	1568983528	34·0881211	10·5132109	·0008605852
1163	1352569	1573037749	34·0127858	10·5162259	·0008598452
1164	1354896	1577098944	34·1174442	10·5192391	·0008591065
1165	1357225	1581167125	34·1320963	10·5222506	·0008583691
1166	1359556	1585242296	34·1467422	10·5252604	·0008576329
1167	1361889	1589324463	34·1613817	10·5282685	·0008568980
1168	1364224	1593413632	34·1760150	10·5312749	·0008561644
1169	1366561	1597509809	34·1906420	10·5342795	·0008554320
1170	1368900	1601613000	34·2052627	10·5372825	·0008547009
1171	1371241	1605723211	34·2198773	10·5402837	·0008539710
1172	1373584	1609840448	34·2344855	10·5432832	·0008532423
1173	1375929	1613964717	34·2490875	10·5462810	·0008525149

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1174	1378276	1618096024	34·2636834	10·5492771	·0008517888
1175	1380625	1622234375	34·2782730	10·5522715	·0008510638
1176	1382976	1626379776	34·2928564	10·5552642	·0008503401
1177	1385329	1630532233	34·3074336	10·5582552	·0008496177
1178	1387684	1634691752	34·3220046	10·5612445	·0008488964
1179	1390041	1638858339	34·3365694	10·5642322	·0008481764
1180	1392400	1643032000	34·3511281	10·5672181	·0008474576
1181	1394761	1647212741	34·3656805	10·5702024	·0008467401
1182	1397124	1651400568	34·3802268	10·5731849	·0008460237
1183	1399489	1655595487	34·3947670	10·5761658	·0008453085
1184	1401856	1659797504	34·4093011	10·5791449	·0008445946
1185	1404225	1664006625	34·4238289	10·5821225	·0008438819
1186	1406596	1668222856	34·4383507	10·5850983	·0008431703
1187	1408969	1672446203	34·4528663	10·5880725	·0008424600
1188	1411344	1676676672	34·4673759	10·5910450	·0008417508
1189	1413721	1680914629	34·4818793	10·5940158	·0008410429
1190	1416100	1685159000	34·4963766	10·5969850	·0008403361
1191	1418481	1689410871	34·5108678	10·5999525	·0008396306
1192	1420864	1693669888	34·5253530	10·6029184	·0008389262
1193	1423249	1697936057	34·5398321	10·6058826	·0008382320
1194	1425636	1702209384	34·5543051	10·6088451	·0008375209
1195	1428025	1706489875	34·5687720	10·6118060	·0008368201
1196	1430416	1710777536	34·5832329	10·6147652	·0008361204
1197	1432809	1715072373	34·5976879	10·6177228	·0008354219
1198	1435204	1719374392	34·6121366	10·6206788	·0008347245
1199	1437601	1723683599	34·6265794	10·6236331	·0008340284
1200	1440000	1728000000	34·6410162	10·6265857	·0008333333
1201	1442401	1732323601	34·6554469	10·6295367	·0008326395
1202	1444804	1736654408	34·6698716	10·6324860	·0008319468
1203	1447209	1740992427	34·6842904	10·6354338	·0008312552
1204	1449616	1745337664	34·6987031	10·6383799	·0008305648
1205	1452025	1749690125	34·7131099	10·6413244	·0008298755
1206	1454436	1754049816	34·7275107	10·6442672	·0008291874
1207	1456849	1758416743	34·7419055	10·6472085	·0008285004
1208	1459264	1762790912	34·7562944	10·6501480	·0008278146
1209	1461681	1767172329	34·7706773	10·6530860	·0008271299
1210	1464100	1771561000	34·7850543	10·6560223	·0008264463
1211	1466521	1775956931	34·7994253	10·6589570	·0008257638
1212	1468944	1780360128	34·8137904	10·6618902	·0008250825
1213	1471369	1784770597	34·8281495	10·6648217	·0008244023
1214	1473796	1789188344	34·8425028	10·6677516	·0008237232
1215	1476225	1793613375	34·8568501	10·6706799	·0008230453
1216	1478656	1798045696	34·8711915	10·6736066	·0008223684
1217	1481089	1802485313	34·8855271	10·6765317	·0008216927
1218	1483524	1806932232	34·8998567	10·6794552	·0008210181
1219	1485961	1811386459	34·9141805	10·6823771	·0008203445
1220	1488400	1815848000	34·9284984	10·6852973	·0008196721
1221	1490841	1820316861	34·9428104	10·6882160	·0008190008
1222	1493284	1824793048	34·9571166	10·6911331	·0008183306
1223	1495729	1829276567	34·9714169	10·6940486	·0008176615
1224	1498176	1833764247	34·9857114	10·6969625	·0008169935
1225	1500625	1838265625	35·0000000	10·6998748	·0008163265
1226	1503276	1842771176	35·0142828	10·7027855	·0008156607
1227	1505529	1847284083	35·0285598	10·7056947	·0008149959
1228	1507984	1851804352	35·0428309	10·7086023	·0008143322
1229	1510441	1856331989	35·0570963	10·7115083	·0008136696
1230	1512900	1860867000	35·0713558	10·7144127	·0008130081
1231	1515361	1865409391	35·0856096	10·7173155	·0008123477
1232	1517824	1869959168	35·0998575	10·7202168	·0008116883
1233	1520289	1874516337	35·1140997	10·7231165	·0008110300
1234	1522756	1879080904	35·1283361	10·7260146	·0008103728
1235	1525225	1883652875	35·1425568	10·7289112	·0008097166

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1236	1527696	1888232256	35·1567917	10·7318062	·0008090615
1237	1530169	1892819053	35·1710108	10·7346997	·0008084074
1238	1532644	1897413272	35·1852242	10·7375916	·0008077544
1239	1535121	1902014919	35·1994318	10·7404819	·0008071025
1240	1537600	1906624000	35·2136337	10·7433707	·0008064516
1241	1540081	1911240521	35·2278299	10·7462579	·0008058018
1242	1542564	1915864488	35·2420204	10·7491436	·0008051530
1243	1545049	1920495907	35·2562051	10·7520277	·0008045052
1244	1547536	1925134784	35·2703842	10·7549103	·0008038585
1245	1550025	1929781125	35·2845575	10·7577913	·0008032129
1246	1552521	1934434936	35·2987252	10·7606708	·0008025682
1247	1555009	1939096223	35·3128872	10·7635488	·0008019246
1248	1557504	1943764992	35·3270435	10·7664252	·0008012821
1249	1560001	1948441249	35·3411941	10·7693001	·0008006405
1250	1562500	1953125000	35·3553391	10·7721735	·0008000000
1251	1565001	1957816251	35·3694784	10·7750453	·0007993605
1252	1567504	1962515008	35·3836120	10·7779156	·0007987220
1253	1570009	1967221277	35·3977400	10·7807843	·0007980846
1254	1572516	1971935064	35·4118624	10·7836516	·0007974482
1255	1575025	1976656875	35·4259792	10·7865173	·0007968127
1256	1577536	1981385216	35·4400903	10·7893815	·0007961783
1257	1580049	1986121593	35·4541958	10·7922441	·0007955449
1258	1582564	1990865512	35·4682957	10·7951053	·0007949126
1259	1585081	1995616979	35·4823900	10·7979649	·0007942812
1260	1587600	2000376000	35·4964787	10·8008230	·0007936508
1261	1590121	2005142581	35·5105618	10·8036797	·0007930214
1262	1592644	2009916728	35·5246393	10·8065848	·0007923930
1263	1595166	2014698447	35·5387113	10·8093884	·0007917656
1264	1597696	2019487744	35·5527777	10·8122404	·0007911392
1265	1600225	2024284625	35·5668385	10·8150909	·0007905138
1266	1602756	2029089096	35·5808937	10·8179400	·0007898894
1267	1605289	2033901163	35·5949434	10·8207876	·0007892660
1268	1607824	2038720832	35·6089876	10·8236336	·0007886435
1269	1610361	2043548109	35·6230262	10·8264782	·0007880221
1270	1612900	2048383000	35·6370593	10·8293213	·0007874016
1271	1615441	2053225511	35·6510869	10·8321629	·0007867821
1272	1617984	2058075648	35·6651090	10·8350030	·0007861635
1273	1620529	2062933417	35·6791255	10·8378416	·0007855460
1274	1623076	2067798824	35·6931366	10·8406788	·0007849294
1275	1625625	2072671875	35·7071421	10·8435144	·0007843137
1276	1628176	2077552576	35·7211422	10·8463485	·0007836991
1277	1630729	2082440933	35·7351367	10·8491812	·0007830854
1278	1633284	2087336952	35·7491258	10·8520125	·0007824726
1279	1635841	2092240639	35·7631095	10·8548422	·0007818608
1280	1638400	2097152000	35·7770876	10·8576704	·0007812500
1281	1640961	2102071841	35·7910603	10·8604972	·0007806401
1282	1643524	2106997768	35·8050276	10·8633225	·0007800312
1283	1646089	2111932187	35·8189894	10·8661454	·0007794232
1284	1648656	2116874304	35·8329457	10·8689687	·0007788162
1285	1651225	2121824125	35·8468966	10·8717897	·0007782101
1286	1653796	2126781656	35·8608421	10·8746091	·0007776050
1287	1656369	2131746903	35·8747822	10·8774271	·0007770008
1288	1658944	2136719872	35·8887169	10·8802436	·0007763975
1289	1661521	2141700569	35·9026461	10·8830587	·0007757952
1290	1664100	2146689000	35·9165699	10·8858723	·0007751938
1291	1666681	2151685171	35·9304884	10·8886845	·0007745933
1292	1669264	2156689088	35·9444015	10·8914952	·0007739938
1293	1671849	2161700757	35·9583092	10·8943044	·0007733952
1294	1674436	2166720184	35·9722115	10·8971123	·0007727975
1295	1677025	2171747875	35·9861084	10·8999186	·0007722008
1296	1679616	2176782336	36·0000000	10·9027235	·0007716049
1297	1682209	2181825073	36·0138862	10·9055269	·0007710100

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1298	1684804	2186875592	36·0277671	10·9083290	·0007704160
1299	1687401	2191933899	36·0416426	10·9111296	·0007698229
1300	1690000	2197000000	36·0555128	10·9139287	·0007692308
1301	1692601	2202073901	36·0693776	10·9167265	·0007686395
1302	1695204	2207155608	36·0832371	10·9195228	·0007680492
1303	1697809	2212245127	36·0970913	10·9223177	·0007674579
1304	1700416	2217342464	36·1109402	10·9251111	·0007668712
1305	1703025	2222447625	36·1247837	10·9279031	·0007662835
1306	1705636	2227560616	36·1386220	10·9306937	·0007656968
1307	1708249	2232681443	36·1524550	10·9334829	·0007651109
1308	1710864	2237810112	36·1662826	10·9362706	·0007645260
1309	1713481	2242946629	36·1801050	10·9390569	·0007639419
1310	1716100	2248091000	36·1939221	10·9418418	·0007633588
1311	1718721	2253243231	36·2077340	10·9446253	·0007627765
1312	1721344	2258403328	36·2215406	10·9475074	·0007621951
1313	1723969	2263571297	36·2353419	10·9501880	·0007616446
1314	1726596	2268747144	36·2491379	10·9529673	·0007610350
1315	1729225	2273930875	36·2626287	10·9557451	·0007604563
1316	1731856	2279122496	36·2767143	10·9585215	·0007598784
1317	1734489	2284322013	36·2904246	10·9612965	·0007593014
1318	1737124	2289529432	36·3042697	10·9640701	·0007587253
1319	1739761	2294744759	36·3180396	10·9668423	·0007581501
1320	1742400	2299968000	36·3318042	10·9696131	·0007575758
1321	1745041	2305199161	36·3455637	10·9723825	·0007570023
1322	1747684	2310438248	36·3593179	10·9751505	·0007564297
1323	1750329	2315685267	36·3730670	10·9779171	·0007558579
1324	1752976	2320940224	36·3868108	10·9806823	·0007552870
1325	1755625	2326203125	36·4005494	10·9834462	·0007547170
1326	1758276	2331473976	36·4142829	10·9862086	·0007541478
1327	1760929	2336752783	36·4280112	10·9889696	·0007535795
1328	1763584	2342039552	36·4417343	10·9917293	·0007530120
1329	1766241	2347334289	36·4554523	10·9944876	·0007524454
1330	1768900	2352637000	36·4691650	10·9972445	·0007518797
1331	1771561	2357947691	36·4828727	11·0000000	·0007513148
1332	1774224	2363266368	36·4965752	11·0027541	·0007507508
1333	1776889	2368593037	36·5102725	11·0055069	·0007501875
1334	1779556	2373927704	36·5239647	11·0082583	·0007496252
1335	1782225	2379270375	36·5376518	11·0110082	·0007490637
1336	1784896	2384621056	36·5513338	11·0137569	·0007485030
1337	1787569	2389979753	36·5650106	11·0165041	·0007479482
1338	1790244	2395346472	36·5786823	11·0192500	·0007473842
1339	1792921	2400721219	36·5923489	11·0219945	·0007468260
1340	1795600	2406104000	36·6060104	11·0247377	·0007462687
1341	1798281	2411494821	36·6196668	11·0274795	·0007457122
1342	1800964	2416893688	36·6333181	11·0302199	·0007451565
1343	1803649	2422300607	36·6469144	11·0329590	·0007446016
1344	1806336	2427715584	36·6606056	11·0356967	·0007440476
1345	1809025	2433138625	36·6742416	11·0384330	·0007434944
1346	1811716	2438569736	36·6878726	11·0411680	·0007429421
1347	1814409	2444008923	36·7014986	11·0439017	·0007423905
1348	1817104	2449456192	36·7151195	11·0466339	·0007418398
1349	1819801	2454911549	36·7287353	11·0493649	·0007412898
1350	1822500	2460375000	36·7423461	11·0520945	·0007407407
1351	1825201	2465846551	36·7559519	11·0548227	·0007401924
1352	1827904	2471326208	36·7695526	11·0575497	·0007396450
1353	1830609	2476813977	36·7831483	11·0602752	·0007390983
1354	1833316	2482309864	36·7967390	11·0629994	·0007385524
1355	1836025	2487813875	36·8103246	11·0657222	·0007380074
1356	1838736	2493326016	36·8239053	11·0684437	·0007374631
1357	1841449	2498846293	36·8374809	11·0711639	·0007369197
1358	1844164	2504374712	36·8510515	11·0738828	·0007363770
1359	1846881	2509911279	36·8646172	11·0766008	·0007358352

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1360	1849600	2515456000	36·8781778	11·0793165	·0007352941
1361	1852321	2521008881	36·8917335	11·0820314	·0007347539
1362	1855044	2526569928	36·9052842	11·0847449	·0007342144
1363	1857769	2532139147	36·9188299	11·0874571	·0007336757
1364	1860496	2537716544	36·9323706	11·0901679	·0007331378
1365	1863225	2543302125	36·9459064	11·0928775	·0007326007
1366	1865956	2548895896	36·9594372	11·0955857	·0007320644
1367	1868689	2554497863	36·9729631	11·0982926	·0007315289
1368	1871424	2560108032	36·9864840	11·1009982	·0007309942
1369	1874161	2565726409	37·0000000	11·1037025	·0007304602
1370	1876900	2571353000	37·0135110	11·1064054	·0007299270
1371	1879641	2576987811	37·0270172	11·1091070	·0007293946
1372	1882384	2582630848	37·0405184	11·1118073	·0007288630
1373	1885129	2588282117	37·0540146	11·1145064	·0007283321
1374	1887876	2593941624	37·0675060	11·1172041	·0007278020
1375	1890625	2599609375	37·0899924	11·1199004	·0007272727
1376	1893376	2605285376	37·0944740	11·1225955	·0007267442
1377	1896129	2610969633	37·1079506	11·1252893	·0007262164
1378	1898884	2616662152	37·1214224	11·1279817	·0007256894
1379	1901641	2622362939	37·1348893	11·1306729	·0007251632
1380	1904400	2628072000	37·1483512	11·1333628	·0007246377
1381	1907161	2633789341	37·1618084	11·1360514	·0007241130
1382	1909924	2639514968	37·1752606	11·1387386	·0007235890
1383	1912689	2645248887	37·1887079	11·1414246	·0007230658
1384	1915456	2650991104	37·2021505	11·1441093	·0007225434
1385	1918225	2656741625	37·2155881	11·1467926	·0007220217
1386	1920996	2662500456	37·2290209	11·1494747	·0007215007
1387	1923769	2668267603	37·2424489	11·1521555	·0007209805
1388	1926544	2674043072	37·2558720	11·1548350	·0007204611
1389	1929321	2679826869	37·2692903	11·1575133	·0007199424
1390	1932100	2685619000	37·2827037	11·1601903	·0007194245
1391	1934881	2691419471	37·2961124	11·1628659	·0007189073
1392	1937664	2697228288	37·3095162	11·1655403	·0007183908
1393	1940449	2703045457	37·3229152	11·1682134	·0007178751
1394	1943236	2708870984	37·3363094	11·1708852	·0007173601
1395	1946025	2714704875	37·3496988	11·1735558	·0007168459
1396	1948816	2720547136	37·3630834	11·1762250	·0007163324
1397	1951609	2726397773	37·3764632	11·1788930	·0007158196
1398	1954404	2732256792	37·3898382	11·1815598	·0007153076
1399	1957201	2738124199	37·4032084	11·1842252	·0007147963
1400	1960000	2744000000	37·4165738	11·1868894	·0007142857
1401	1962801	2749884201	37·4299345	11·1895523	·0007137759
1402	1965604	2755776808	37·4432904	11·1922139	·0007132668
1403	1968409	2761677827	37·4566416	11·1948743	·0007127584
1404	1971216	2767587264	37·4699880	11·1975334	·0007122507
1405	1974025	2773505123	37·4833296	11·2001913	·0007117438
1406	1976836	2779431416	37·4966665	11·2028479	·0007112376
1407	1979649	2785366143	37·5099987	11·2055032	·0007107321
1408	1982464	2791309312	37·5233261	11·2081573	·0007102273
1409	1985281	2797260929	37·5366487	11·2108101	·0007097232
1410	1988100	2803221000	37·5499667	11·2134617	·0007092199
1411	1990921	2809189531	37·5632799	11·2161120	·0007087172
1412	1993744	2815166528	37·5765885	11·2187611	·0007082153
1413	1996569	2821151997	37·5898922	11·2214089	·0007077141
1414	1999396	2827145944	37·6031913	11·2240054	·0007072136
1415	2002225	2833148375	37·6164857	11·2267007	·0007067138
1416	2005056	2839159296	37·6297754	11·2293448	·0007062147
1417	2007889	2845178713	37·6430604	11·2319876	·0007057163
1418	2010724	2851206632	37·6563407	11·2346292	·0007052186
1419	2013561	2857243059	37·6696164	11·2372696	·0007047216
1420	2016400	2863288000	37·6828874	11·2399087	·0007042254
1421	2019241	2869341461	37·6961536	11·2425465	·0007037298

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1422	2022084	2875403448	37·7094153	11·2451831	·0007032349
1423	2024929	2881473967	37·7226722	11·2478185	·0007027407
1424	2027776	2887553024	37·7359245	11·2504527	·0007022472
1425	2030625	2893640625	37·7491722	11·2530856	·0007017544
1426	2033476	2899736776	37·7624152	11·2557173	·0007012623
1427	2036329	2905841483	37·7756535	11·2583478	·0007007708
1428	2039184	2911954752	37·7888873	11·2609770	·0007002801
1429	2042041	2918076589	37·8021163	11·2636050	·0006997901
1430	2044900	2924207000	37·8153408	11·2662318	·0006993007
1431	2047761	2930345991	37·8285606	11·2688573	·0006988120
1432	2050624	2936493568	37·8417759	11·2714816	·0006983240
1433	2053489	2942649737	37·8549864	11·2741047	·0006978367
1434	2056356	2948814504	37·8681924	11·2767266	·0006973501
1435	2059225	2954987875	37·8813938	11·2793472	·0006968641
1436	2062096	2961169856	37·8945906	11·2819666	·0006963788
1437	2064969	2967360453	37·9077828	11·2845849	·0006958942
1438	2067844	2973559672	37·9209704	11·2872019	·0006954103
1439	2070721	2979767519	37·9341538	11·2898177	·0006949270
1440	2073600	2985984000	37·9473319	11·2924323	·0006944444
1441	2076481	2992209121	37·9605058	11·2950457	·0006939625
1442	2079364	3098442888	37·9736751	11·2976579	·0006934813
1443	2082249	3004685307	37·9868398	11·3002688	·0006930007
1444	2085136	3010936384	38·0000000	11·3028786	·0006925208
1445	2088025	3017196125	38·0131556	11·3054871	·0006920415
1446	2080916	3023464536	38·0263067	11·3080945	·0006915629
1447	2093809	3029741623	38·0394532	11·3107006	·0006910850
1448	2096704	3036027392	38·0525952	11·3133056	·0006906078
1449	2099601	3042321849	38·0657326	11·3159094	·0006901312
1450	2102500	3048625000	38·0788655	11·3185119	·0006896552
1451	2105401	3054936851	38·0919939	11·3211132	·0006891799
1452	2108304	3061257408	38·1051178	11·3237134	·0006887052
1453	2111209	3067586777	38·1182371	11·3263124	·0006882312
1454	2114116	3073924664	38·1313519	11·3289102	·0006877579
1455	2117025	3080271375	38·1444622	11·3315067	·0006872852
1456	2119936	3086626816	38·1575681	11·3341022	·0006868132
1457	2122849	3092990993	38·1706693	11·3366964	·0006863412
1458	2125764	3099363912	38·1837662	11·3392894	·0006858711
1459	2128681	3105745579	38·1968585	11·3418813	·0006854010
1460	2131600	3112136000	38·2099463	11·3444719	·0006849315
1461	2134521	3118535181	38·2230297	11·3470614	·0006844627
1462	2137444	3124943128	38·2361085	11·3496497	·0006839945
1463	2140369	3131359847	38·2491829	11·3522368	·0006835270
1464	2143296	3137785344	38·2622529	11·3548227	·0006830601
1465	2146225	3144219625	38·2753184	11·3574075	·0006825939
1466	2149156	3150662696	38·2883794	11·3599911	·0006821282
1467	2152089	3157114563	38·3014360	11·3625735	·0006816633
1468	2155024	3163575232	38·3144881	11·3651547	·0006811989
1469	2157961	3170044709	38·3275358	11·3677347	·0006807352
1470	2160900	3176523000	38·3405790	11·3703136	·0006802721
1471	2163841	3183010111	38·3536178	11·3728914	·0006798097
1472	2166784	3189506048	38·3666522	11·3754679	·0006793478
1473	2169729	3196010817	38·3796821	11·3780433	·0006788866
1474	2172676	3202524424	38·3927076	11·3806175	·0006784261
1475	2175625	3209046875	38·4057287	11·3831906	·0006779661
1476	2178576	3215578176	38·4187454	11·3857625	·0006775068
1477	2181529	3222118333	38·4317577	11·3883332	·0006770481
1478	2184484	3228667352	38·4447656	11·3909028	·0006765900
1479	2187441	3235225239	38·4577691	11·3934712	·0006761325
1480	2190400	3241792000	38·4707681	11·3960384	·0006756757
1481	2193361	3248367641	38·4837627	11·3986045	·0006752194
1482	2196324	3254952168	38·4967530	11·4011695	·0006747638
1483	2199289	3261545587	38·5097390	11·4037332	·0006743088

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1484	2202256	3268147904	38·5227206	11·4062959	·0006738544
1485	2205225	3274759125	38·5356977	11·4088574	·0006734007
1486	2208196	3281379256	38·5486705	11·4114177	·0006729474
1487	2211169	3288008303	38·5616389	11·4139769	·0006724950
1488	2214144	3294646272	38·5746030	11·4165349	·0006720430
1489	2217121	3301293169	38·5875627	11·4190918	·0006715917
1490	2220100	3307949000	38·6005181	11·4206476	·0006711409
1491	2223081	3314613771	38·6134691	11·4242022	·0006706908
1492	2226004	3321287488	38·6264158	11·4267556	·0006702413
1493	2229049	3227970157	38·6393582	11·4293079	·0006697924
1494	2232036	3334661784	38·6522962	11·4318591	·0006693440
1495	2235025	3341362375	38·6652299	11·4344092	·0006688963
1496	2238016	3348071936	38·6781593	11·4369581	·0006684492
1497	2241009	3354790473	38·6910843	11·4395059	·0006680027
1498	2244004	3361517992	38·7040050	11·4420525	·0006675567
1499	2247001	3368254499	38·7169214	11·4445980	·0006671114
1500	2250000	3375000000	38·7298335	11·4471424	·0006666667
1501	2253001	3381754501	38·7427412	11·4496857	·0006662225
1502	2256004	3388518008	38·7556447	11·4522278	·0006657790
1503	2259009	3395290527	38·7685439	11·4547688	·0006553360
1504	2262016	3402072064	38·7814389	11·4573087	·0006648936
1505	2265025	3408862625	38·7943294	11·4598476	·0006644518
1506	2268036	3415662216	38·8072158	11·4623850	·0006640106
1507	2271049	3422470843	38·8200978	11·4649215	·0006635700
1508	2274064	3429288512	38·8329757	11·4674568	·0006631300
1509	2277081	3436115229	38·8458491	11·4699911	·0006626905
1510	2280100	3442951000	38·8587184	11·4725242	·0006622517
1511	2283121	3449795831	38·8715834	11·4750562	·0006618134
1512	2286144	3456649728	38·8844442	11·4775871	·0006613757
1513	2289169	3463512697	38·8973006	11·4801169	·0006609385
1514	2292196	3470384744	38·9101529	11·4826455	·0006605020
1515	2295225	3477265875	38·9230009	11·4851731	·0006600660
1516	2298256	3484156096	38·9358447	11·4876995	·0006596306
1517	2301289	3491055413	38·9486841	11·4902249	·0006591958
1518	2304324	3597963832	38·9615194	11·4927491	·0006587615
1519	2307361	3504881359	38·9743505	11·4952722	·0006583278
1520	2310400	3511808000	38·9871774	11·4977942	·0006578947
1521	2313441	3518743761	39·0000000	11·5003151	·0006574622
1522	2316484	3525688648	39·0128184	11·5028348	·0006570302
1523	2319529	3532642667	39·0256326	11·5053535	·0006565988
1524	2322576	3539605824	39·0384426	11·5078711	·0006561680
1525	2325625	3546578125	39·0512483	11·5103876	·0006557377
1526	2328676	3553559576	39·0640499	11·5129030	·0006553080
1527	2331729	3567549552	39·0768473	11·5154173	·0006548788
1528	2334784	3560558183	39·0896406	11·5179305	·0006544503
1529	2337841	3574558889	39·1024296	11·5204425	·0006540222
1530	2340900	3581577000	39·1152144	11·5229535	·0006535948
1531	2343961	3588604291	39·1279951	11·5254634	·0006531679
1532	2347024	3595640768	39·1407716	11·5279722	·0006527415
1533	2350089	3602686437	39·1535439	11·5304799	·0006523157
1534	2353156	3609741304	39·1663120	11·5329865	·0006518905
1535	2356225	3616805375	39·1790760	11·5354920	·0000514658
1536	2359256	3623878656	39·1918359	11·5379965	·0006510417
1537	2362369	3630961153	39·2045915	11·5404998	·0006506181
1538	2365444	3638052872	39·2173431	11·5430021	·0006501951
1539	2368521	3645153819	39·2300905	11·5455033	·0006497726
1540	2371600	3652264000	39·2428337	11·5480034	·0006493506
1541	2374681	3659383421	39·2555728	11·5505025	·0006489293
1542	2377764	3666512088	39·2683078	11·5530004	·0006485084
1543	2380849	3673650007	39·2810387	11·5554972	·0006480881
1544	2383936	3680797184	39·2937654	11·5579931	·0006476684
1545	2387025	3687953625	39·3064880	11·5604878	·0006472492

Number.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1546	2390116	3695119336	39·3192065	11·5629815	·0006468305
1547	2393209	3702294323	39·3319208	11·5654740	·0006464124
1548	2396304	3709478592	39·3446311	11·5679655	·0006459948
1549	2399401	3716672149	39·3573373	11·5704559	·0006455778
1550	2402500	3723875000	39·3700394	11·5729453	·0006451613
1551	2405601	3731087151	39·3827373	11·5754336	·0006447453
1552	2408704	3738308608	39·3954312	11·5779208	·0006443299
1553	2411809	3745539377	39·4081210	11·5804069	·0006439150
1554	2414916	3752779464	39·4208067	11·5828919	·0006435006
1555	2418025	3760028875	39·4334883	11·5853759	·0006430868
1556	2421136	3767287616	39·4461658	11·5878588	·0006426735
1557	2424249	3774555693	39·4588393	11·5903407	·0006422608
1558	2427364	3781833112	39·4715087	11·5928215	·0006418485
1559	2430481	3789119879	39·4841740	11·5953013	·0006414368
1560	2433600	3796416000	39·4968353	11·5977799	·0006410256
1561	2436721	3803721481	39·5094925	11·6002576	·0006406150
1562	2439844	3811036328	39·5221457	11·6027342	·0006402049
1563	2442969	3818360547	39·5347948	11·6052097	·0006397953
1564	2446096	3825641444	39·5474399	11·6076841	·0006393862
1565	2449225	3833037125	39·5600809	11·6101575	·0006389776
1566	2452356	3840389496	39·5727179	11·6126299	·0006385696
1567	2455489	3847751263	39·5853508	11·6151012	·0006381621
1568	2458624	3855123432	39·5979797	11·6175715	·0006377551
1569	2461761	3862503009	39·6106046	11·6200407	·0006373486
1570	2464900	3869883000	39·6232255	11·6225088	·0006369427
1571	2468041	3877292411	39·6358424	11·6249759	·0006365372
1572	2471184	3884701248	39·6484552	11·6274420	·0006361323
1573	2474329	3892119157	39·6610640	11·6299070	·0006357279
1574	2477476	3899547224	39·6736688	11·6323710	·0006353240
1575	2480625	3906984375	39·6862696	11·6348339	·0006349206
1576	2483776	3914430976	39·6988665	11·6372957	·0006345178
1577	2486929	3921887033	39·7114593	11·6397566	·0006341154
1578	2490084	3929352552	39·7240481	11·6422164	·0006337136
1579	2493241	3936827539	39·7366329	11·6446751	·0006333122
1580	2496400	3944312000	39·7492138	11·6471329	·0006329114
1581	2499561	3951805941	39·7617907	11·6495895	·0006325111
1582	2502724	3959309368	39·7743636	11·6520452	·0006321113
1583	2505889	3966822287	39·7869325	11·6544998	·0006317119
1584	2509056	3974344704	39·7994976	11·6569534	·0006313131
1585	2512225	3981876625	39·8120585	11·6594059	·0006309148
1586	2515396	3989418056	39·8246155	11·6618574	·0006305170
1587	2518569	3996969003	39·8371686	11·6643079	·0006301197
1588	2521744	4004529472	39·8497177	11·6667574	·0006297229
1589	2524921	4012099469	39·8622628	11·6692058	·0006293266
1590	2528100	4014679000	39·8748040	11·6716532	·0006289308
1591	2531281	4027268071	39·8873413	11·6740996	·0006285355
1592	2534464	4034866688	39·8998747	11·6765449	·0006281407
1593	2537649	4042474857	39·9124041	11·6789892	·0006277464
1594	2540836	4050092584	39·9249295	11·6814325	·0006273526
1595	2544025	4057719875	39·9374511	11·6838748	·0006269592
1596	2547216	4065356736	39·9499687	11·6863161	·0006265664
1597	2550409	4073003173	39·9624824	11·6887563	·0006261741
1598	2553604	4080659192	39·9749922	11·6911955	·0006257822
1599	2556801	4088324799	39·9874980	11·6936337	·0006253909
1600	2560000	4096000000	40·0000000	11·6960709	·0006250000

To find the square or cube root of a number consisting of integers and decimals.

RULE.—Multiply the difference between the root of the integer part of the given number, and the root of the next higher integer number, by the decimal part of the given number, and add the product to the root of the given integer number; the sum is the root required.

Required the square root of 20·321.

Square root of 21 = 4·5825
Do. 20 = 4·4721

·1104 × ·321 + 4·4721 = 4·5075384, the square root required.

Required the cube root of 16·42.

Cube root of 17 = 2·5712
Do. 16 = 2·5198

·0514 × ·42 + 2·5198 = 2·541388, the cube root required.

To find the squares of numbers in arithmetical progression; or, to extend the foregoing table of squares.

Rule.—Find, in the usual way, the squares of the first two numbers, and subtract the less from the greater. Set down the square of the larger number, in a separate column, and add to it the difference already found, with the addition of 2, as a constant quantity; the product will be the square of the next following number.

The square of 1500 ....= 2250000.........2250000
The square of 1499..................= 2247001
Difference...... 2999 + 2 = 3001
The square of 1501..2253001
Difference...... 3001 + 2 = 3003
The square of 1502..2256004

To find the square of a greater number than is contained in the table.

Rule 1.—If the number required to be squared exceed by 2, 3, 4, or any other number of times, any number contained in the table, let the square affixed to the number in the table be multiplied by the square of 2, 3, or 4, &c., and the product will be the answer sought.

Required the square of 2595.

2595 is three times greater than 865; and the square of 865, by the table, is 748225.

Then, 748225 × 3^2 = 6734025.

Rule 2.—If the number required to be squared be an odd number, and do not exceed twice the amount of any number contained in the table, find the two numbers nearest to each other, which, added together, make that sum; then the sum of the squares of these two numbers, by the table, multiplied by 2, will exceed the square required by 1.

Required the square of 1865.

The two nearest numbers (932 + 933) = 1865.

Then, by table (932^2 = 868624) + (933^2 = 870489) = 1739113 × 2 = 3478226 − 1 = 3478225.

To find the cube of a greater number than is contained in the table.

RULE.—Proceed, as in squares, to find how many times the number required to be cubed exceeds the number contained in the table. Multiply the cube of that number by the cube of as many times as the number sought exceeds the number in the table, and the product will be the answer required.

Required the cube of 3984.

3984 is 4 times greater than 996; and the cube of 996, by the table, is 988047936.

Then, $988047936 \times 4^3 = 63235067904$.

To find the square or cube root of a higher number than is in the table.

RULE.—Refer to the table, and seek in the column of squares or cubes the number nearest to that number whose root is sought, and the number from which that square or cube is derived will be the answer required, when decimals are not of importance.

Required the square root of 542869.

In the Table of Squares, the nearest number is 543169; and the number from which that square has been obtained is 737.

Therefore, $\sqrt{542869} = 737$ nearly.

To find more nearly the cube root of a higher number than is in the table.

RULE.—Ascertain, by the table, the nearest cube number to the number given, and call it the assumed cube.

Multiply the assumed cube, and the given number, respectively, by 2; to the product of the assumed cube add the given number, and to the product of the given number add the assumed cube.

Then, by proportion, as the sum of the assumed cube is to the sum of the given number, so is the root of the assumed cube to the root of the given number.

Required the cube root of 412568555.

By the table, the nearest number is 411830784, and its cube root is 744.

Therefore, $411830784 \times 2 + 412568555 = 1236230123$.

And, $412568555 \times 2 + 411830784 = 1236967894$.

Hence, as $1236230123 : 1236967894 :: 744 : 744{\cdot}369$, very nearly.

To find the square or cube root of a number containing decimals.

RULE.—Subtract the square root or cube root of the *integer* of the given number from the root of the next higher number, and multiply the difference by the decimal part. The product, added to the root of the integer of the given number will be the answer required.

Required the square root of 321·62.

$\sqrt{321} = 17{\cdot}9164729$, and $\sqrt{322} = 17{\cdot}9443584$; the difference $({\cdot}0278855) \times {\cdot}62 + 17{\cdot}9164729 = 17{\cdot}9337619$.

To obtain the square root or cube root of a number containing decimals, by inspection.

Rule.—The square or cube-root of a number containing decimals may be found at once by inspection of the tables, by taking the figures cut off in the *number*, by the decimal point, in *pairs* if for the square root, and in *triads* if for the cube root. The following example will show the results obtained, by simple inspection of the tables, from the figures 234, and from the numbers formed by the addition of the decimal point or of ciphers.

Number.	Square Root.	Cube Root.
·00234	·0483735465*	·132761439†
·0234	·152970585	·284‡
·2340	·483735465	·61622401
2·34	1·52970585	1·32761439
23·40	4·83735465	2·860
234	15·2970585	6·1622401
2340	48·3735465	13·2761439
23400	152·970585	28·60

To find the cubes of numbers in arithmetical progression, or to extend the preceding table of cubes.

Rule.—Find the cubes of the first two numbers, and subtract the less from the greater. Then, multiply the least of the two numbers cubed by 6, add the product, with the addition of 6 as a constant quantity, to the difference; and thus, adding 6 each time to the sum last added, form a first series of differences.

To form a second series of differences, bring down, in a separate column, the cube of the highest of the above numbers, and add the difference to it. The amount will be the cube of the next general number.

Required the cubes of 1501, 1502, and 1503.

First series of differences.			*Second series of differences.*		
By Tab. 1500 =	3375000000		Then,	3375000000	Cube of 1500
1499 =	3368254499		Diff. for 1500 =	6754501	
	6745501	difference.		3381754501	Cube of 1501
1499 × 6 + 6 =	9000		Diff. for 1501 =	6763507	
	6754501	diff. of 1500		3388518008	Cube of 1502
9000 + 6 =	9006		Diff. for 1502 =	6772519	
	6763507	diff. of 1501		3395290527	Cube of 1503
9006 + 6 =	9012			&c., &c.	
	6772519	diff. of 1502			
	&c., &c.				

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* Derived from ·002340 by means of 2340.

† Derived from ·002340 by means of 2340.

‡ The nearest result by simple inspection is obtained for ·023 by 23. But four places correct can always be obtained by looking in the table of cubes for the nearest triad or triads, in this instance for 23400; the cube beginning with the figures 23393 is that of 2860, whence ·2860 is true to the last place, and is afterwards substituted.

TABLE *of the Fourth and Fifth Powers of Numbers.*

Number.	4th Power.	5th Power.	Number.	4th Power.	5th Power.
1	1	1	76	33362176	2535525376
2	16	32	77	35153041	2706784157
3	81	243	78	37015056	2887174368
4	256	1024	79	38950081	3077056399
5	625	3125	80	40960000	3276800000
6	1296	7776	81	43046721	3486784401
7	2401	16807	82	45212176	3707398432
8	4096	32768	83	47458321	3939040643
9	6561	59049	84	49787136	4182119424
10	10000	100000	85	52200625	4437053125
11	14641	161051	86	54708016	4704270176
12	20736	248832	87	57289761	4984209207
13	28561	371293	88	59969536	5277319168
14	38416	537824	89	62742241	5584059449
15	50625	759375	90	65610000	5904900000
16	65536	1048576	91	68574961	6240321451
17	83521	1419857	92	71639296	6590815232
18	104976	1889568	93	74805201	6596883693
19	130321	2476099	94	78074896	7339040224
20	160000	3200000	95	81450625	7737809375
21	194481	4084101	96	84934656	8153726976
22	234256	5153632	97	88529281	8587340257
23	279841	6436343	98	92236816	9039207968
24	331776	7962624	99	96059601	9509900499
25	390625	9765625	100	100000000	10000000000
26	456976	11881376	101	104060401	10510100501
27	531441	14348907	102	108243216	11040808032
28	614656	17210368	103	112550881	11592740743
29	707281	20511149	104	116985856	12166529024
30	810000	24300000	105	121550625	12762815625
31	923521	28629151	106	126247696	13382255776
32	1048576	33554432	107	131079601	14025517307
33	1185921	39135393	108	136048896	14693280768
34	1336336	45435424	109	141158161	15386239549
35	1500625	52521875	110	146410000	16105100000
36	1679616	60466176	111	151807041	16850581551
37	1874161	69343957	112	157351936	17623416832
38	2085136	79235168	113	163047361	18424351793
39	2313441	90224199	114	168896016	19254145824
40	2560000	102400000	115	174900625	20113571875
41	2825761	115856201	116	181063936	21003416576
42	3111696	130691232	117	187388721	21924480357
43	3418801	147008443	118	193877776	22877577568
44	3748096	164916224	119	200533921	23863536599
45	4100625	184528125	120	207360000	24883200000
46	4477456	205962976	121	214358881	25937424601
47	4879681	229345007	122	221533456	27027081632
48	5308416	254803968	123	228886641	28153056843
49	5764801	282475249	124	236421376	29316250624
50	6250000	312500000	125	244140625	30517578125
51	6765201	345025251	126	252047376	31757969376
52	7311616	380204032	127	260144641	33038369407
53	7890481	418195493	128	268435456	34359738368
54	8503056	459165024	129	276922881	35723051649
55	9150625	503284375	130	285610000	37129300000
56	9834496	550731776	131	294499921	38579489651
57	10556001	601692057	132	303595776	40074642432
58	11316496	656356768	133	312900721	41615795893
59	12117361	714924299	134	322417936	43204003424
60	12960000	777600000	135	332150625	44840334375
61	13845841	844596301	136	342102016	46525874176
62	14776336	916132832	137	352275361	48261724457
63	15752961	992436543	138	362673936	50049003168
64	16777216	1073741824	139	373301041	51888844699
65	17850625	1160290625	140	384160000	53782400000
66	18974736	1252332576	141	395254161	55730836701
67	20151121	1350125107	142	406586896	57735339232
68	21381376	1453933568	143	418161601	59797108943
69	22667121	1564031349	144	429981696	61917364224
70	24010000	1680700000	145	442050625	64097340625
71	25411681	1804229351	146	454371856	66338290976
72	26873856	1934917632	147	466948881	68641485507
73	28398241	2073071593	148	479785216	71008211968
74	29986576	2219006624	149	492884401	73439775749
75	31640625	2373046875	150	506250000	75937500000

TABLE *of Hyperbolic Logarithms.*

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1·01	·0099503	1·58	·4574248	2·15	·7654678	2·72	1·0006318
1·02	·0198026	1·59	·4637340	2·16	·7701082	2·73	1·0043015
1·03	·0295588	1·60	·4700036	2·17	·7747271	2·74	1·0079579
1·04	·0392207	1·61	·4762341	2·18	·7793248	2·75	1·0116008
1·05	·0487902	1·62	·4824261	2·19	·7839015	2·76	1·0152306
1·06	·0582689	1·63	·4885800	2·20	·7884573	2·77	1·0188473
1·07	·0676586	1·64	·4946962	2·21	·7929925	2·78	1·0224509
1·08	·0769610	1·65	·5007752	2·22	·7975071	2·79	1·0260415
1·09	·0861777	1·66	·5068175	2·23	·8020015	2·80	1·0296194
1·10	·0953102	1·67	·5128236	2·24	·8064758	2·81	1·0331844
1·11	·1043600	1·68	·5187937	2·25	·8109302	2·82	1·0367368
1·12	·1133287	1·69	·5247285	2·26	·8153648	2·83	1·0402766
1·13	·1222176	1·70	·5306282	2·27	·8197798	2·84	1·0438040
1·14	·1310283	1·71	·5364933	2·28	·8241754	2·85	1·0473189
1·15	·1397619	1·72	·5423242	2·29	·8285518	2·86	1·0508216
1·16	·1484200	1·73	·5481214	2·30	·8329091	2·87	1·0543120
1·17	·1570037	1·74	·5538851	2·31	·8372475	2·88	1·0577902
1·18	·1655144	1·75	·5596157	2·32	·8415671	2·89	1·0612564
1·19	·1739533	1·76	·5653138	2·33	·8458682	2·90	1·0647107
1·20	·1823215	1·77	·5709795	2·34	·8501509	2·91	1·0681530
1·21	·1906203	1·78	·5766133	2·35	·8544153	2·92	1·0715836
1·22	·1988508	1·79	·5822156	2·36	·8586616	2·93	1·0750024
1·23	·2070141	1·80	·5877866	2·37	·8628899	2·94	1·0784095
1·24	·2151113	1·81	·5933268	2·38	·8671004	2·95	1·0818051
1·25	·2231435	1·82	·5988365	2·39	·8712933	2·96	1·0851892
1·26	·2311117	1·83	·6043159	2·40	·8754687	2·97	1·0885619
1·27	·2390169	1·84	·6097655	2·41	·8796267	2·98	1·0919233
1·28	·2468600	1·85	·6151856	2·42	·8837675	2·99	1·0952733
1·29	·2546422	1·86	·6205764	2·43	·8878912	3·00	1·0986123
1·30	·2623642	1·87	·6259384	2·44	·8919980	3·01	1·1019400
1·31	·2700271	1·88	·6312717	2·45	·8960880	3·02	1·1052568
1·32	·2776317	1·89	·6365768	2·46	·9001613	3·03	1·1085626
1·33	·2851789	1·90	·6418538	2·47	·9042181	3·04	1·1118575
1·34	·2926696	1·91	·6471032	2·48	·9082585	3·05	1·1151415
1·35	·3001045	1·92	·6523251	2·49	·9122826	3·06	1·1184149
1·36	·3074846	1·93	·6575200	2·50	·9162907	3·07	1·1216775
1·37	·3148107	1·94	·6626879	2·51	·9202827	3·08	1·1249295
1·38	·3220834	1·95	·6678293	2·52	·9242589	3·09	1·1281710
1·39	·3293037	1·96	·6729444	2·53	·9282193	3·10	1·1314021
1·40	·3364722	1·97	·6780335	2·54	·9321640	3·11	1·1346227
1·41	·3435897	1·98	·6830968	2·55	·9360933	3·12	1·1378330
1·42	·3506568	1·99	·6881346	2·56	·9400072	3·13	1·1410330
1·43	·3576744	2·00	·6931472	2·57	·9439058	3·14	1·1442227
1·44	·3646431	2·01	·6981347	2·58	·9477893	3·15	1·1474024
1·45	·3715635	2·02	·7030974	2·59	·9516578	3·16	1·1505720
1·46	·3784364	2·03	·7080357	2·60	·9555114	3·17	1·1537315
1·47	·3852624	2·04	·7129497	2·61	·9593502	3·18	1·1568811
1·48	·3920420	2·05	·7178397	2·62	·9631743	3·19	1·1600209
1·49	·3987761	2·06	·7227059	2·63	·9669838	3·20	1·1631508
1·50	·4054651	2·07	·7275485	2·64	·9707789	3·21	1·1662709
1·51	·4121096	2·08	·7323678	2·65	·9745596	3·22	1·1693813
1·52	·4187103	2·09	·7371640	2·66	·9783261	3·23	1·1724821
1·53	·4252677	2·10	·7419373	2·67	·9820784	3·24	1·1755733
1·54	·4317824	2·11	·7466879	2·68	·9858167	3·25	1·1786549
1·55	·4382549	2·12	·7514160	2·69	·9895411	3·26	1·1817271
1·56	·4446858	2·13	·7561219	2·70	·9932517	3·27	1·1847899
1·57	·4510756	2·14	·7608058	2·71	·9969486	3·28	1·1878434

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3·29	1·1908875	3·91	1·3635373	4·53	1·5107219	5·15	1·6389967
3·30	1·1939224	3·92	1·3660916	4·54	1·5129269	5·16	1·6409365
3·31	1·1969481	3·93	1·3686394	4·55	1·5151272	5·17	1·6428726
3·32	1·1999647	3·94	1·3711807	4·56	1·5173226	5·18	1·6448050
3·33	1·2029722	3·95	1·3737156	4·57	1·5195132	5·19	1·6467336
3·34	1·2059707	3·96	1·3762440	4·58	1·5216990	5·20	1·6486586
3·35	1·2089603	3·97	1·3787661	4·59	1·5238800	5·21	1·6505798
3·36	1·2119409	3·98	1·3812818	4·60	1·5260563	5·22	1·6524974
3·37	1·2149127	3·99	1·3837912	4·61	1·5282278	5·23	1·6544112
3·38	1·2178757	4·00	1·3862943	4·62	1·5303947	5·24	1·6563214
3·39	1·2208299	4·01	1·3887912	4·63	1·5325568	5·25	1·6582280
3·40	1·2237754	4·02	1·3912818	4·64	1·5347143	5·26	1·6601310
3·41	1·2267122	4·03	1·3937663	4·65	1·5368672	5·27	1·6620303
3·42	1·2296405	4·04	1·3962446	4·66	1·5390154	5·28	1·6639260
3·43	1·2325605	4·05	1·3987168	4·67	1·5411590	5·29	1·6658182
3·44	1·2354714	4·06	1·4011829	4·68	1·5432981	5·30	1·6677068
3·45	1·2383742	4·07	1·4036429	4·69	1·5454325	5·31	1·6695918
3·46	1·2412685	4·08	1·4060969	4·70	1·5475625	5·32	1·6714733
3·47	1·2441545	4·09	1·4085449	4·71	1·5496879	5·33	1·6733512
3·48	1·2470322	4·10	1·4109869	4·72	1·5518087	5·34	1·6752256
3·49	1·2499017	4·11	1·4134230	4·73	1·5539252	5·35	1·6770965
3·50	1·2527629	4·12	1·4158531	4·74	1·5560371	5·36	1·6789639
3·51	1·2556160	4·13	1·4182774	4·75	1·5581446	5·37	1·6808278
3·52	1·2584609	4·14	1·4206957	4·76	1·5602476	5·38	1·6826882
3·53	1·2612978	4·15	1·4231083	4·77	1·5623462	5·39	1·6845453
3·54	1·2641266	4·16	1·4255150	4·78	1·5644405	5·40	1·6863989
3·55	1·2669475	4·17	1·4279160	4·79	1·5665304	5·41	1·6882491
3·56	1·2697605	4·18	1·4303112	4·80	1·5686159	5·42	1·6900958
3·57	1·2725655	4·19	1·4327007	4·81	1·5706971	5·43	1·6919391
3·58	1·2753627	4·20	1·4350845	4·82	1·5727739	5·44	1·6937790
3·59	1·2781521	4·21	1·4374626	4·83	1·5748464	5·45	1·6956155
3·60	1·2809338	4·22	1·4398351	4·84	1·5769147	5·46	1·6974487
3·61	1·2837077	4·23	1·4422020	4·85	1·5789787	5·47	1·6992786
3·62	1·2864740	4·24	1·4445632	4·86	1·5810384	5·48	1·7011051
3·63	1·2892326	4·25	1·4469189	4·87	1·5830939	5·49	1·7029282
3·64	1·2919836	4·26	1·4492691	4·88	1·5851452	5·50	1·7047481
3·65	1·2947271	4·27	1·4516138	4·89	1·5871923	5·51	1·7065646
3·66	1·2974631	4·28	1·4539530	4·90	1·5892352	5·52	1·7083778
3·67	1·3001916	4·29	1·4562867	4·91	1·5912739	5·53	1·7101878
3·68	1·3029127	4·30	1·4586149	4·92	1·5933085	5·54	1.7119944
3·69	1·3056264	4·31	1·4609379	4·93	1·5953389	5·55	1·7137979
3·70	1·3083328	4·32	1·4632553	4·94	1·5973653	5·56	1·7155981
3·71	1·3110318	4·33	1·4655675	4·95	1·5993875	5·57	1·7173950
3·72	1·3137236	4·34	1·4678743	4·96	1·6014057	5·58	1·7191887
3·73	1·3164082	4·35	1·4701758	4·97	1·6034198	5·59	1·7209792
3·74	1·3190856	4·36	1·4724720	4·98	1·6054298	5·60	1·7227666
3·75	1·3217558	4·37	1·4747630	4·99	1·6074358	5·61	1·7245507
3·76	1·3244189	4·38	1·4770487	5·00	1·6094379	5·62	1·7263316
3·77	1·3270749	4·39	1·4793292	5·01	1·6114359	5·63	1·7281094
3·78	1·3297240	4·40	1·4816045	5·02	1·6134300	5·64	1·7298840
3·79	1·3323660	4·41	1·4838746	5·03	1·6154200	5·65	1·7316555
3·80	1·3350010	4·42	1·4861396	5·04	1·6174060	5·66	1·7334238
3·81	1·3376291	4·43	1·4883995	5·05	1·6193882	5·67	1·7351891
3·82	1·3402504	4·44	1·4906543	5·06	1·6213664	5·68	1·7369512
3·83	1·3428648	4·45	1·4929040	5·07	1·6233408	5·69	1·7387102
3·84	1·3454723	4·46	1·4951487	5·08	1·6253112	5·70	1·7404661
3·85	1·3480731	4·47	1·4973883	5·09	1·6272778	5·71	1·7422189
3·86	1·3506671	4·48	1·4996230	5·10	1·6292405	5·72	1·7439687
3·87	1·3532544	4·49	1·5018527	5·11	1·6311994	5·73	1·7457155
3·88	1·3558351	4·50	1·5040774	5·12	1·6331544	5·74	1·7474591
3·89	1·3584091	4·51	1·5062971	5·13	1·6351056	5·75	1·7491998
3·90	1·3609765	4·52	1·5085119	5·14	1·6370530	5·76	1·7509374

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
5·77	1·7526720	6·39	1·8547342	7·01	1·9473376	7·63	2·0320878
5·78	1·7544036	6·40	1·8562979	7·02	1·9487632	7·64	2·0333976
5·79	1·7561323	6·41	1·8578592	7·03	1·9501866	7·65	2·0347056
5·80	1·7578579	6·42	1·8594181	7·04	1·9516080	7·66	2·0360119
5·81	1·7595805	6·43	1·8609745	7·05	1·9530275	7·67	2·0373166
5·82	1·7613002	6·44	1·8625285	7·06	1·9544449	7·68	2·0386195
5·83	1·7630170	6·45	1·8640801	7·07	1·9558604	7·69	2·0399207
5·84	1·7647308	6·46	1·8656293	7·08	1·9572739	7·70	2·0412203
5·85	1·7664416	6·47	1·8671761	7·09	1·9586853	7·71	2·0425181
5·86	1·7681496	6·48	1·8687205	7·10	1·9600947	7·72	2·0438143
5·87	1·7698546	6·49	1·8702625	7·11	1·9615022	7·73	2·0451088
5·88	1·7715567	6·50	1·8718021	7·12	1·9629077	7·74	2·0464016
5·89	1·7732559	6·51	1·8733394	7·13	1·9643112	7·75	2·0476928
5·90	1·7749523	6·52	1·8748743	7·14	1·9657127	7·76	2·0489823
5·91	1·7766458	6·53	1·8764069	7·15	1·9671123	7·77	2·0502701
5·92	1·7783364	6·54	1·8779371	7·16	1·9685099	7·78	2·0515563
5·93	1·7800242	6·55	1·8794650	7·17	1·9699056	7·79	2·0528408
5·94	1·7817091	6·56	1·8809906	7·18	1·9712993	7·80	2·0541237
5·95	1·7833912	6·57	1·8825138	7·19	1·9726911	7·81	2·0554049
5·96	1·7850704	6·58	1·8840347	7·20	1·9740810	7·82	2·0566845
5·97	1·7867469	6·59	1·8855533	7·21	1·9754689	7·83	2·0579624
5·98	1·7884205	6·60	1·8870696	7·22	1·9768549	7·84	2·0592388
5·99	1·7900914	6·61	1·8885837	7·23	1·9782390	7·85	2·0605135
6·00	1·7917594	6·62	1·8900954	7·24	1·9796212	7·86	2·0617866
6·01	1·7934247	6·63	1·8916048	7·25	1·9810014	7·87	2·0630580
6·02	1·7950872	6·64	1·8931119	7·26	1·9823798	7·88	2·0643278
6·03	1·7967470	6·65	1·8946168	7·27	1·9837562	7·89	2·0655961
6·04	1·7984040	6·66	1·8961194	7·28	1·9851308	7·90	2·0668627
6·05	1·8000582	6·67	1·8976198	7·29	1·9865035	7·91	2·0681277
6·06	1·8017098	6·68	1·8991179	7·30	1·9878743	7·92	2·0693911
6·07	1·8033586	6·69	1·9006138	7·31	1·9892432	7·93	2·0706530
6·08	1·8050047	6·70	1·9021075	7·32	1·9906103	7·94	2·0719132
6·09	1·8066481	6·71	1·9035989	7·33	1·9919754	7·95	2·0731719
6·10	1·8082887	6·72	1·9050881	7·34	1·9933387	7·96	2·0744290
6·11	1·8099267	6·73	1·9065751	7·35	1·9947002	7·97	2·0756845
6·12	1·8115621	6·74	1·9080600	7·36	1·9960599	7·98	2·0769384
6·13	1·8131947	6·75	1·9095425	7·37	1·9974177	7·99	2·0781907
6·14	1·8148247	6·76	1·9110228	7·38	1·9987736	8·00	2·0794415
6·15	1·8164520	6·77	1·9125011	7·39	2·0001278	8·01	2·0806907
6·16	1·8180767	6·78	1·9139771	7·40	2·0014800	8·02	2·0819384
6·17	1·8196988	6·79	1·9154509	7·41	2·0028305	8·03	2·0831845
6·18	1·8213182	6·80	1·9169226	7·42	2·0041790	8·04	2·0844290
6·19	1·8229351	6·81	1·9183921	7·43	2·0055258	8·05	2·0856720
6·20	1·8245493	6·82	1·9198594	7·44	2·0068708	8·06	2·0869135
6·21	1·8261608	6·83	1·9213247	7·45	2·0082140	8·07	2·0881534
6·22	1·8277699	6·84	1·9227877	7·46	2·0095553	8·08	2·0893918
6·23	1·8293763	6·85	1·9242486	7·47	2·0108949	8·09	2·0906287
6·24	1·8309801	6·86	1·9257074	7·48	2·0122327	8·10	2·0918640
6·25	1·8325814	6·87	1·9271641	7·49	2·0135687	8·11	2·0930984
6·26	1·8341801	6·88	1·9286186	7·50	2·0149030	8·12	2·0943306
6·27	1·8357763	6·89	1·9300710	7·51	2·0162354	8·13	2·0955613
6·28	1·8373699	6·90	1·9315214	7·52	2·0175661	8·14	2·0967905
6·29	1·8389610	6·91	1·9329696	7·53	2·0188950	8·15	2·0980182
6·30	1·8405496	6·92	1·9344157	7·54	2·0202221	8·16	2·0992444
6·31	1·8421356	6·93	1·9358598	7·55	2·0215475	8·17	2·1004691
6·32	1·8437191	6·94	1·9373017	7·56	2·0228711	8·18	2·1016923
6·33	1·8453002	6·95	1·9387416	7·57	2·0241929	8·19	2·1029140
6·34	1·8468787	6·96	1·9401794	7·58	2·0255131	8·20	2·1041341
6·35	1·8484547	6·97	1·9416152	7·59	2·0268315	8·21	2·1053529
6·36	1·8500283	6·98	1·9430489	7·60	2·0281482	8·22	2·1065702
6·37	1·8515994	6·99	1·9444805	7·61	2·0294631	8·23	2·1077861
6·38	1·8531680	7·00	1·9459101	7·62	2·0307763	8·24	2·1089998

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
8·25	2·1102128	8·69	2·1621729	9·13	2·2115656	9·57	2·2586332
8·26	2·1114243	8·70	2·1633230	9·14	2·2126603	9·58	2·2596776
8·27	2·1126343	8·71	2·1644718	9·15	2·2137538	9·59	2·2607209
8·28	2·1138428	8·72	2·1656192	9·16	2·2148461	9·60	2·2617631
8·29	2·1150499	8·73	2·1667653	9·17	2·2159372	9·61	2·2628042
8·30	2·1162555	8·74	2·1679101	9·18	2·2170272	9·62	2·2638442
8·31	2·1174596	8·75	2·1690536	9·19	2·2181160	9·63	2·2648832
8·32	2·1186622	8·76	2·1701959	9·20	2·2192034	9·64	2·2659211
8·33	2·1198634	8·77	2·1713367	9·21	2·2202898	9·65	2·2669579
8·34	2·1210632	8·78	2·1724763	9·22	2·2213750	9·66	2·2679936
8·35	2·1222615	8·79	2·1736146	9·23	2·2224590	9·67	2·2690282
8·36	2·1234584	8·80	2·1747517	9·24	2·2235418	9·68	2·2700618
8·37	2·1246539	8·81	2·1758874	9·25	2·2246235	9·69	2·2710944
8·38	2·1258479	8·82	2·1770218	9·26	2·2257040	9·70	2·2721258
8·39	2·1270405	8·83	2·1781550	9·27	2·2267833	9·71	2·2731562
8·40	2·1282317	8·84	2·1792868	9·28	2·2278615	9·72	2·2741856
8·41	2·1294214	8·85	2·1804174	9·29	2·2289385	9·73	2·2752138
8·42	2·1306098	8·86	2·1815467	9·30	2·2300144	9·74	2·2762411
8·43	2·1317967	8·87	2·1826747	9·31	2·2310890	9·75	2·2772673
8·44	2·1329822	8·88	2·1838015	9·32	2·2321626	9·76	2·2782924
8·45	2·1341664	8·89	2·1849270	9·33	2·2332350	9·77	2·2793165
8·46	2·1353491	8·90	2·1860512	9·34	2·2343062	9·78	2·2803395
8·47	2·1365304	8·91	2·1871742	9·35	2·2353763	9·79	2·2813614
8·48	2·1377104	8·92	2·1882959	9·36	2·2364452	9·80	2·2823823
8·49	2·1388889	8·93	2·1894163	9·37	2·2375130	9·81	2·2834022
8·50	2·1400661	8·94	2·1905355	9·38	2·2385797	9·82	2·2844211
8·51	2·1412419	8·95	2·1916535	9·39	2·2396452	9·83	2·2854389
8·52	2·1424163	8·96	2·1927702	9·40	2·2407096	9·84	2·2864556
8·53	2·1435893	8·97	2·1938856	9·41	2·2417729	9·85	2·2874714
8·54	2·1447609	8·98	2·1949998	9·42	2·2428350	9·86	2·2884861
8·55	2·1459312	8·99	2·1961128	9·43	2·2438960	9·87	2·2894998
8·56	2·1471001	9·00	2·1972245	9·44	2·2449559	9·88	2·2905124
8·57	2·1482676	9·01	2·1983350	9·45	2·2460147	9·89	2·2915241
8·58	2·1494339	9·02	2·1994443	9·46	2·2470723	9·90	2·2925347
8·59	2·1505987	9·03	2·2005523	9·47	2·2481288	9·91	2·2635443
8·60	2·1517622	9·04	2·2016591	9·48	2·2491843	9·92	2·2945529
8·61	2·1529243	9·05	2·2027647	9·49	2·2502386	9·93	2·2955604
8·62	2·1540851	9·06	2·2038691	9·50	2·2512917	9·94	2·2965670
8·63	2·1552445	9·07	2·2049722	9·51	2·2523438	9·95	2·2975725
8·64	2·1564026	9·08	2·2060741	9·52	2·2533948	9·96	2·2985770
8·65	2·1575593	9·09	2·2071748	9·53	2·2544446	9·97	2·2995806
8·66	2·1587147	9·10	2·2082744	9·54	2·2554934	9·98	2·3005831
8·67	2·1598687	9·11	2·2093727	9·55	2·2565411	9·99	2·3015846
8·68	2·1610215	9·12	2·2104697	9·56	2·2575877	10·00	2·3025851

Logarithms were invented by Juste Byrge, a Frenchman, and **not by Napier.** See "Biographie Universelle," "The Calculus of Form," article 822, and "The Practical, Short, and Direct Method of Calculating the Logarithm of any given Number and the Number corresponding to any given Logarithm," discovered by Oliver Byrne, the author of the present work. Juste Byrge also invented the proportional compasses, and was a profound astronomer and mathematician. The common Logarithm of a number multiplied by 2·302585052994 gives the hyperbolic Logarithm of that number. The common Logarithm of 2·22 is ·346353 .·. 2·302585 × ·346353 = 7975071 the hyperbolic Logarithm. The application of Logarithms to the calculations of the Engineer will be treated of hereafter.

COMBINATIONS OF ALGEBRAIC QUANTITIES.

THE following practical examples will serve to illustrate the method of combining or representing numbers or quantities algebraically; the chief object of which is, to help the memory with respect to the use of the *signs* and *letters*, or *symbols*.

Let $a = 6$, $b = 4$, $c = 3$, $d = 2$, $e = 1$, and $f = 0$.

Then will, (1) $2a + b = 12 + 4 = 16$.

(2) $ab + 2c - d = 24 + 6 - 2 = 28$.

(3) $a^2 - b^2 + e + f = 36 - 16 + 1 + 0 = 21$.

(4) $b^2 \times (a - b) = 16 \times (6 - 4) = 16 \times 2 = 32$.

(5) $3abc - 7de = 216 - 14 = 202$.

(6) $2\,(a - b)\,(5c - 2d) = (12 - 8) \times (15 - 4) = 44$.

(7) $\dfrac{c^2 - e^2}{d + f} \times (a - c) = \dfrac{9 - 1}{2 + 0} \times (6 - 3) = 4 \times 3 = 12$.

(8) $\sqrt{(a^2 - 2b^2)} + d - f = \sqrt{(36 - 32)} + 2 - 0 = 4$.

(9) $3ab - (a - b - c + d) = 72 - 1 = 71$.

(10) $3ab - (a - b - c - d) = 72 + 3 = 75$.

(11) $\dfrac{\sqrt{2abc}}{\sqrt{(ab - 4d)}} \times (c + d) = \dfrac{\sqrt{144}}{\sqrt{(24 - 8)}} \times (3 + 2) = 15$.

In solving the following questions, the letters a, b, c, &c. are supposed to have the same values as before, namely, 6, 4, 3, &c.; but any other values might have been assigned to them; therefore, do not suppose that a must necessarily be 6, nor that b must be 4, for the letter a may be put for any known *quantity*, *number*, or *magnitude* whatever; thus a may represent 10 *miles*, or 50 *pounds*, or any *number* or *quantity*, or it may represent 1 *globe*, or 2 *cubic feet*, &c.; the same may be said of b, or any other letter.

(1) $a + b - c = 7$.

(2) $3bc - d + e = 35$.

(3) $2a^2 + c^2 - d + f = 79$.

(4) $\dfrac{a^2}{b} \times (b - c + d) = 27$.

(5) $5c^2d - a^2 + 4de = 62$.

(6) $4\,(a^2 - b^2)\,(c - e) = 160$.

(7) $\dfrac{a^2 - b^2}{c + d} \times (d^2 + c^2) = 52$.

(8) $\sqrt{(2a^2 + 2d^2)} + bc - f = 20$.

(9) $4a^2b - (c^2 - d - e) = 570$.

(10) $\dfrac{\sqrt{4a^2}}{\sqrt{(10d^2 - 4cd)}} \times \dfrac{a}{d} - c^2 = 0$.

In the use of algebraic symbols, $3\sqrt[3]{4a - b}$ signifies the same thing as $3\,(4a - b)^{\frac{1}{3}}$.

$4\,(c + d)^{\frac{1}{2}}\,(a + b)^{\frac{1}{3}}$, or $4 \times \overline{c + d}^{\frac{1}{2}} \times \overline{a + b}^{\frac{1}{3}}$, signifies the same thing as $4\sqrt{c + d} \cdot \sqrt[3]{a + b}$.

THE STEAM ENGINE.

THE particular example which we shall select is that of an engine having 8 feet stroke and 64 inch cylinder.

The breadth of the web of the crank at the paddle centre is the breadth which the web would have if it were continued to the paddle centre. Suppose that we wished to know the breadth of the web of crank of an engine whose stroke is 8 feet and diameter of cylinder 64 inches. The proper breadth of the web of crank at paddle centre would in this case be about 18 inches.

To find the breadth of crank at paddle centre.—Multiply the square of the length of the crank in inches by 1·561, and then multiply the square of the diameter of cylinder in inches by ·1235; multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide the product by 45; finally extract the cube root of the quotient. The result is the breadth of the web of crank at paddle centre.

Thus, to apply this rule to the particular example which we have selected, we have

$$\begin{array}{rl}
48 & = \text{length of crank in inches.} \\
\underline{48} & \\
2304 & \\
\underline{1{\cdot}561} & = \text{constant multiplier.} \\
3596{\cdot}5 & \\
\underline{505{\cdot}8} & \text{found below.} \\
4102{\cdot}3 &
\end{array}$$

$$\begin{array}{rl}
64 & = \text{diameter of cylinder.} \\
\underline{64} & \\
4096 & \\
\underline{{\cdot}1235} & = \text{constant multiplier.} \\
505{\cdot}8 &
\end{array}$$

$$\begin{array}{rl}
\text{and } \sqrt{4102{\cdot}3} = & 64{\cdot}05 \text{ nearly.} \\
& \underline{4096} = \text{square of the diameter of the cylinder.} \\
45\overline{)\,262348{\cdot}5} & \\
5829{\cdot}97 & \\
& \text{and } \sqrt[3]{5829{\cdot}97} = 18 \text{ nearly.}
\end{array}$$

Suppose that we wished the proper thickness of the large eye of crank for an engine whose stroke is 8 feet and diameter of cylinder 64 inches. The proper thickness for the large eye of crank is 5·77 inches.

Rule.—*To find the thickness of large eye of crank.*—Multiply the square of the length of the crank in inches by 1·561, and then multiply the square of the diameter of the cylinder in inches by ·1235; multiply the sum of these products by the square of the diameter of the cylinder in inches; afterwards, divide the product by 1828·28; divide this quotient by the length of the crank in inches; finally extract the cube root of the quotient. The result is the proper thickness of the large eye of crank in inches.

Thus, to apply this rule to the particular example which we have selected, we have

48 = length of crank in inches.
48
2304
1·561 constant multiplier.
3596·5
505·8
4102·3
64 = diameter of cylinder in inches.
64
4096
·1235 = constant multiplier.
505·8
4102·3
4096 = square of diameter.
48) 16803020·8
1828·28) 350062·94
191·47
and $\sqrt[3]{191{\cdot}47} = 5{\cdot}77$ nearly.

The proper thickness of the web of crank at paddle shaft centre is the thickness which the web ought to have if continued to centre of the shaft. Suppose that it were required to find the proper thickness of web of crank at shaft centre for an engine whose stroke is 8 feet and diameter of cylinder 64 inches. The proper thickness of the web at shaft centre in this case would be 8·97 inches.

Rule.—*To find the thickness of the web of crank at paddle shaft centre.*—Multiply the square of the length of crank in inches by 1·561, and then multiply the square of the diameter in inches by ·1235; multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide this quotient by 360; finally extract the cube root of the quotient. The result is the thickness of the web of crank at paddle shaft centre in inches.

Thus, to apply the rule to the particular example which we have selected, we have

$$\begin{array}{r l}
48 & = \text{length of crank in inches.} \\
\underline{48} & \\
2304 & \\
\underline{1{\cdot}561} & = \text{constant multiplier.} \\
3596{\cdot}5 & \\
\underline{505{\cdot}8} & \\
4102{\cdot}3 & \\
64 & = \text{diameter of cylinder.} \\
\underline{64} & \\
4096 & \\
\underline{{\cdot}1235} & = \text{constant multiplier.} \\
505{\cdot}8 &
\end{array}$$

$$\begin{array}{r l}
\text{And } \sqrt{4102{\cdot}3} = 64{\cdot}05 & \text{nearly.} \\
\underline{4096} & = \text{square of diameter.} \\
360)\,\underline{262348{\cdot}5} & \\
728{\cdot}75 & \\
\text{And } \sqrt[3]{782{\cdot}75} = 9 & \text{nearly.}
\end{array}$$

Suppose that it were required to find the proper diameter for the paddle shaft journal of an engine whose stroke is 8 feet and diameter of cylinder 64 inches. The proper diameter of the paddle shaft journal in this case is 14·06 inches.

Rule.—*To find the diameter of the paddle shaft journal.*—Multiply the square of the diameter of cylinder in inches by the length of the crank in inches; extract the cube root of the product; finally multiply the result by ·242. The final product is the diameter of the paddle shaft journal in inches.

Thus, to apply this rule to the particular example which we have before selected, we have

$$\begin{array}{r l}
64 & = \text{diameter of cylinder in inches.} \\
\underline{64} & \\
4096 & \\
\underline{48} & = \text{length of crank in inches.} \\
196608 &
\end{array}$$

$$\text{and } \sqrt[3]{196608} = 58{\cdot}148$$

$$\text{but } 58{\cdot}148 \times {\cdot}242 = 14{\cdot}07 \text{ inches.}$$

Suppose it were required to find the proper length of the paddle shaft journal for an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. The proper length of the paddle shaft journal would be, in this case, 17·59 inches.

The following rule serves for engines of all sizes:

Rule.—*To find the length of the paddle shaft journal.*—Multiply the square of the diameter of the cylinder in inches by the length of the crank in inches; extract the cube root of the quotient; multiply the result by ·303. The product is the length of the paddle shaft journal in inches. (The length of the paddle shaft journal is 1¼ times the diameter.)

To apply this rule to the example which we have selected, we have

$$\begin{array}{rl} 64 & = \text{diameter of cylinder in inches.} \\ \underline{64} & \\ 4096 & \\ \underline{48} & = \text{length of crank in inches.} \\ 196608 & \end{array}$$

$$\text{and } \sqrt[3]{196608} = 58{\cdot}148$$

$$\therefore \text{ length of journal} = 58{\cdot}148 \times {\cdot}303 = 17{\cdot}60 \text{ inches.}$$

We shall now calculate the proper dimensions of some of those parts which do not depend upon the length of the stroke. Suppose it were required to find the proper dimensions of the respective parts of a marine engine the diameter of whose cylinder is 64 inches.

Diameter of crank-pin journal = 90·9 inches, or about 9 inches.

Length of crank-pin journal = 10·18 inches, or nearly 10⅕ inches.

Breadth of the eye of cross-head = 2·64 inches, or between 2½ and 2¾ inches.

Depth of the eye of cross-head = 18·37 inches, or very nearly 18½ inches.

Diameter of the journal of cross-head = 5·5 inches, or 5½ inches.

Length of journal of cross-head = 6·19 inches, or very nearly 6⅕ inches.

Thickness of the web of cross-head at middle = 4·6 inches, or somewhat more than 4½ inches.

Breadth of web of cross-head at middle = 17·15 inches, or between 17⅒ and 17⅕ inches.

Thickness of web of cross-head at journal = 3·93 inches, or very nearly 4 inches.

Breadth of web of cross-head at journal = 6·46 inches, or nearly 6½ inches.

Diameter of piston rod = 6·4 inches, or 6⅖ inches.

Length of part of piston rod in piston = 12·8 inches, or 12⅘ inches.

Major diameter of part of piston rod in cross-head = 06·8 inches, or nearly 6⅒ inches.

Minor diameter of part of piston rod in cross-head = 5·76 inches, or 5¾ inches.

Major diameter of part of piston rod in piston = 8·96 inches, or nearly 9 inches.

Minor diameter of part of piston rod in piston = 7·36 inches, or between 7¼ and 7½ inches.

Depth of gibs and cutter through cross-head = 6·72 inches, or very nearly 6¾ inches.

Thickness of gibs and cutter through cross-head = 1·35 inches, or between 1¼ and 1½ inches.

Depth of cutter through piston = 5·45 inches, or nearly 5½ inches.

Thickness of cutter through piston = 2·24 inches, or nearly 2¼ inches.

Diameter of connecting rod at ends = 6·08 inches, or nearly $6\frac{1}{16}$ inches.

Major diameter of part of connecting rod in cross-tail = 6·27 inches, or about 6¼ inches.

Minor diameter of part of connecting rod in cross-tail = 5·76 inches, or nearly 5¾ inches.

Breadth of butt = 9·98 inches, or very nearly 10 inches.

Thickness of butt = 8 inches.

Mean thickness of strap at cutter = 2·75 inches, or 2¾ inches.

Mean thickness of strap above cutter = 2·06 inches, or somewhat more than 2 inches.

Distance of cutter from end of strap = 3·08 inches, or very nearly $3\frac{1}{16}$ inches.

Breadth of gibs and cutter through cross-tail = 6·73 inches, or very nearly 6¾ inches.

Breadth of gibs and cutter through butt = 7·04 inches, or somewhat more than 7 inches.

Thickness of gibs and cutter through butt = 1·84 inches, or between 1¾ and 2 inches.

These results are calculated from the following rules, which give correct results for all sizes of engines.

Rule 1. *To find the diameter of crank-pin journal.*—Multiply the diameter of the cylinder in inches by ·142. The result is the diameter of crank-pin journal in inches.

Rule 2. *To find the length of crank-pin journal.*—Multiply the diameter of the cylinder in inches by ·16. The product is the length of the crank-pin journal in inches.

Rule 3. *To find the breadth of the eye of cross-head.*—Multiply the diameter of the cylinder in inches by ·041. The product is the breadth of the eye in inches.

Rule 4. *To find the depth of the eye of cross-head.*—Multiply the diameter of the cylinder in inches by ·286. The product is the depth of the eye of cross-head in inches.

Rule 5. *To find the diameter of the journal of cross-head.*—Multiply the diameter of the cylinder in inches by ·086. The product is the diameter of the journal in inches.

Rule 6. *To find the length of the journal of cross-head.*—Multiply the diameter of the cylinder in inches by ·097. The product is the length of the journal in inches.

Rule 7. *To find the thickness of the web of cross-head at middle.*—Multiply the diameter of the cylinder in inches by ·072. The product is the thickness of the web of cross-head at middle in inches.

Rule 8. *To find the breadth of web of cross-head at middle.*—Multiply the diameter of the cylinder in inches by ·268. The product is the breadth of the web of cross-head at middle in inches.

RULE 9. *To find the thickness of the web of cross-head at journal.*—Multiply the diameter of the cylinder in inches by ·061. The product is the thickness of the web of cross-head at journal in inches.

RULE 10. *To find the breadth of web of cross-head at journal.*—Multiply the diameter of the cylinder in inches by ·101. The product is the breadth of the web of cross-head at journal in inches.

RULE 11. *To find the diameter of the piston rod.*—Divide the diameter of the cylinder in inches by 10. The quotient is the diameter of the piston rod in inches.

RULE 12. *To find the length of the part of the piston rod in the piston.*—Divide the diameter of the cylinder in inches by 5. The quotient is the length of the part of the piston rod in the piston in inches.

RULE 13. *To find the major diameter of the part of piston rod in cross-head.*—Multiply the diameter of the cylinder in inches by ·095. The product is the major diameter of the part of piston rod in cross-head in inches.

RULE 14. *To find the minor diameter of the part of piston rod in cross-head.*—Multiply the diameter of the cylinder in inches by ·09. The product is the minor diameter of the part of piston rod in cross-head in inches.

RULE 15. *To find the major diameter of the part of piston rod in piston.*—Multiply the diameter of the cylinder in inches by ·14. The product is the major diameter of the part of piston rod in piston in inches.

RULE 16. *To find the minor diameter of the part of piston rod in piston.*—Multiply the diameter of the cylinder in inches by ·115. The product is the minor diameter of the part of piston rod in piston.

RULE 17. *To find the depth of gibs and cutter through cross-head.*—Multiply the diameter of the cylinder in inches by ·105. The product is the depth of the gibs and cutter through cross-head.

RULE 18. *To find the thickness of the gibs and cutter through cross-head.*—Multiply the diameter of the cylinder in inches by ·021. The product is the thickness of the gibs and cutter through cross-head.

RULE 19. *To find the depth of cutter through piston.*—Multiply the diameter of the cylinder in inches by ·085. The product is the depth of the cutter through piston in inches.

RULE 20. *To find the thickness of cutter through piston.*—Multiply the diameter of the cylinder in inches by ·035. The product is the thickness of cutter through piston in inches.

RULE 21. *To find the diameter of connecting rod at ends.*—Multiply the diameter of the cylinder in inches by ·095. The product is the diameter of the connecting rod at ends in inches.

RULE 22. *To find the major diameter of the part of connecting rod in cross-tail.*—Multiply the diameter of the cylinder in inches by ·098. The product is the major diameter of the part of connecting rod in cross-tail.

RULE 23. *To find the minor diameter of the part of connecting rod in cross-tail.*—Multiply the diameter of the cylinder in inches by ·09. The product is the minor diameter of the part of connecting rod in cross-tail in inches.

RULE 24. *To find the breadth of butt.*—Multiply the diameter of the cylinder in inches by ·156. The product is the breadth of the butt in inches.

RULE 25. *To find the thickness of the butt.*—Divide the diameter of the cylinder in inches by 8. The quotient is the thickness of the butt in inches.

RULE 26. *To find the mean thickness of the strap at cutter.*—Multiply the diameter of the cylinder in inches by ·043. The product is the mean thickness of the strap at cutter.

RULE 27. *To find the mean thickness of the strap above cutter.*—Multiply the diameter of the cylinder in inches by ·032. The product is the mean thickness of the strap above cutter.

RULE 28. *To find the distance of cutter from end of strap.*—Multiply the diameter of the cylinder in inches by ·048. The product is the distance of cutter from end of strap in inches.

RULE 29. *To find the breadth of the gibs and cutter through cross-tail.*—Multiply the diameter of the cylinder in inches by ·105. The product is the breadth of the gibs and cutter through cross-tail.

RULE 30. *To find the breadth of the gibs and cutter through butt.*—Multiply the diameter of the cylinder in inches by ·11. The product is the breadth of the gibs and cutter through butt in inches.

RULE 31. *To find the thickness of the gibs and cutter through butt.*—Multiply the diameter of the cylinder in inches by ·029. The product is the thickness of the gibs and cutter through butt in inches.

To find other parts of the engine which do not depend upon the stroke. Suppose it were required to find the thickness of the small eye of crank for an engine the diameter of whose cylinder is 64 inches. According to the rule, the proper thickness of the small eye of crank is 4·04 inches. Again, suppose it were required to find the length of the small eye of crank. Hence, according to the rule, the proper length of the small eye of crank is 11·94 inches. Again, supposing it were required to find the proper thickness of the web of crank at pin centre; that is to say, the thickness which it would have if continued to the pin centre. According to the rule, the proper thickness for the web of crank at pin centre is 7·04 inches. Again, suppose it were required to find the breadth of the web of crank at pin centre; that is to say, the breadth which it would have if it were continued to the pin centre. Hence, according to the rule, the proper breadth of the web of crank at pin centre is 10·24 inches.

These results are calculated from the following rules, which give the proper dimensions for engines of all sizes:

RULE 1. *To find the breadth of the small eye of crank.*—Multiply the diameter of the cylinder in inches by ·063. The product is the proper breadth of the small eye of crank in inches.

RULE 2. *To find the length of the small eye of crank.*—Multiply the diameter of the cylinder in inches by ·187. The product is the proper length of the small eye of crank in inches.

RULE 3. *To find the thickness of the web of crank at pin centre.*—Multiply the diameter of the cylinder in inches by ·11. The product is the proper thickness of the web of crank at pin centre in inches.

RULE 4. *To find the breadth of the web of crank at pin centre.*—Multiply the diameter of the cylinder in inches by ·16. The product is the proper breadth of crank at pin centre in inches.

To illustrate the use of the succeeding rules, let us take the particular example of an engine of 8 feet stroke and 64-inch cylinder, and let us suppose that the length of the connecting rod is 12 feet, and the side rod 10 feet. We find by a previous rule that the diameter of the connecting rod at ends is 6·08, and the ratio between the diameters at middle and ends of a connecting rod, whose length is 12 feet, is 1·504. Hence, the proper diameter at middle of the connecting rod = 6·08 × 1·504 inches = 9·144 inches. And again, we find the diameter of cylinder side rods at ends, for the particular engine which we have selected, is 4·10, and the ratio between the diameters at middle and ends of cylinder side rods, whose lengths are 10 feet, is 1·42. Hence, according to the rules, the proper diameter of the cylinder side rods at middle is equal to 4·1 × 1·42 inches = 5·82 inches.

To find some of those parts of the engine which do not depend upon the stroke. Suppose we take the particular example of an engine the diameter of whose cylinder is 64 inches. We find from the following rules that

Diameter of cylinder side rods at ends = 4·1 inches, or $4\frac{1}{10}$ inches.

Breadth of butt = 4·93 inches, or very nearly 5 inches.

Thickness of butt = 3·9 inches, or $3\frac{9}{10}$ inches.

Mean thickness of strap at cutter = 2·06 inches, or a little more than 2 inches.

Mean thickness of strap below cutter = 1·47 inches, or very nearly $1\frac{1}{2}$ inches.

Depths of gibs and cutter = 5·12 inches, or a little more than $5\frac{1}{10}$ inches.

Thickness of gibs and cutter = 1·03 inches, or a little more than 1 inch.

Diameter of main centre journal = 11·71 inches, or very nearly $11\frac{3}{4}$ inches.

Length of main centre journal = 17·6 inches, or $17\frac{3}{5}$ inches.

Depth of eye round end studs of lever = 4·75 inches, or 4¾ inches.

Thickness of eye round end studs of lever = 3·33 inches, or 3⅓ inches.

Diameter of end studs of lever = 4·48 inches, or very nearly 4½ inches.

Length of end studs of lever = 4·86 inches, or between 4¾ and 5 inches.

Diameter of air-pump studs = 2·91 inches, or nearly 3 inches.

Length of air-pump studs = 3·16 inches, or nearly 3⅙ inches.

These results were obtained from the following rules, which will be found to give the proper dimensions for all sizes of engines.

RULE 1. *To find the diameter of cylinder side rods at ends.*—Multiply the diameter of the cylinder in inches by ·065. The product is the diameter of the cylinder side rods at ends in inches.

RULE 2. *To find the breadth of butt in inches.*—Multiply the diameter of the cylinder in inches by ·077. The product is the breadth of the butt in inches.

RULE 3. *To find the thickness of the butt.*—Multiply the diameter of the cylinder in inches by ·061. The product is the thickness of the butt in inches.

RULE 4. *To find the mean thickness of strap at cutter.*—Multiply the diameter of the cylinder in inches by ·032. The product is the mean thickness of the strap at cutter.

RULE 5. *To find the mean thickness of strap below cutter.*—Multiply the diameter of the cylinder in inches by ·023. The product is the mean thickness of strap below cutter in inches.

RULE 6. *To find the depth of gibs and cutter.*—Multiply the diameter of the cylinder in inches by ·08. The product is the depth of the gibs and cutter in inches.

RULE 7. *To find the thickness of gibs and cutter.*—Multiply the diameter of the cylinder in inches by ·016. The product is the thickness of gibs and cutter in inches.

RULE 8. *To find the diameter of the main centre journal.*—Multiply the diameter of the cylinder in inches by ·183. The product is the diameter of the main centre journal in inches.

RULE 9. *To find the length of the main centre journal.*—Multiply the diameter of the cylinder in inches by ·275. The product is the diameter of the cylinder in inches.

RULE 10. *To find the depth of eye round end studs of lever.*—Multiply the diameter of the cylinder in inches by ·074. The product is the depth of the eye round end studs of lever in inches.

RULE 11. *To find the thickness of eye round end studs of lever.*—Multiply the diameter of the cylinder in inches by ·052. The product is the thickness of eye round end studs of lever in inches.

RULE 12. *To find the diameter of the end studs of lever.*—Multiply the diameter of the cylinder in inches by ·07. The product is the diameter of the end studs of lever in inches.

RULE 13. *To find the length of the end studs of lever.*—Multiply the diameter of the cylinder in inches by ·076. The product is the length of the end studs of lever in inches.

RULE 14. *To find the diameter of the air-pump studs.*—Multiply the diameter of the cylinder in inches by ·045. The product is the diameter of the air-pump studs in inches.

RULE 15. *To find the length of the air-pump studs.*—Multiply the diameter of the cylinder in inches by ·049. The product is the length of the air-pump studs in inches.

The next rule gives the proper depth in inches across the centre of the side lever, when, as is generally the case, the side lever is of cast iron. It will be observed that the depth is made to depend upon the diameter of the cylinder and the length of the lever, and not at all upon the length of the stroke, except indeed in so far as the length of the lever may depend upon the length of the stroke. Suppose it were required to find the proper depth across the centre of a side lever whose length is 20 feet, and the diameter of the cylinder 64 inches. According to the rule, the proper depth across the centre would be 39·26 inches.

The following rule will give the proper dimensions for any size of engine:

RULE.—*To find the depth across the centre of the side lever.*—Multiply the length of the side lever in feet by ·7423; extract the cube root of the product, and reserve the result for a multiplier. Then square the diameter of the cylinder in inches; extract the cube root of the result. The product of the final result and the reserved multiplier is the depth of the side lever in inches across the centre.

Thus, to apply this rule to the particular example which we have selected, we have

20 = length of side lever in feet.
·7423 = constant multiplier.
14·846

and $\sqrt[3]{14{\cdot}846} = 2{\cdot}458$ nearly.

64 = diameter of cylinder in inches.
64
4096

and $\sqrt[3]{4096} = 16$

Hence depth at centre = 16 × 2·458 inches = 39·33 inches, or between 39¼ and 39½ inches.

The next set of rules give the dimensions of several of the parts of the air-pump machinery which depend upon the diameter of the cylinder only. To illustrate the use of these rules, let us take the particular example of an engine the diameter of whose cylinder is 64 inches. We find from the succeeding rules successively,

Diameter of air-pump = 38·4 inches, or 38⅜ inches.

Thickness of the eye of air-pump cross-head = 1·58 inches, or a little more than 1½ inches.

Depth of eye of air-pump cross-head = 11·01, or about 11 inches.

Diameter of end journals of air-pump cross-head = 3·29 inches, or somewhat more than 3¼ inches.

Length of end journals of air-pump cross-head = 3·7 inches, or $3\frac{7}{10}$ inches.

Thickness of the web of air-pump cross-head at middle = 2·76 inches, or a little more than 2¾ inches.

Depth of web of air-pump cross-head at middle = 10·29 inches, or somewhat more than 10¼ inches.

Thickness of web of air-pump cross-head at journal = 2·35 inches, or about 2⅜ inches.

Depth of web of air-pump cross-head at journal = 3·89 inches, or about 3⅞ inches.

Diameter of air-pump piston rod when made of copper = 4·27 inches, or about 4¼ inches.

Depth of gibs and cutter through air-pump cross-head = 4·04 inches, or a little more than 4 inches.

Thickness of gibs and cutter through air-pump cross-head = ·81 inches, or about ⅞ inch.

Depth of cutter through piston = 3·27 inches, or somewhat more than 3¼ inches.

Thickness of cutter through piston = 1·34 inches, or about 1⅜ inches.

These results were obtained from the following rules, and give the proper dimensions for all sizes of engines:

RULE 1. *To find the diameter of the air-pump.*—Multiply the diameter of the cylinder in inches by ·6. The product is the diameter of air-pump in inches.

RULE 2. *To find the thickness of the eye of air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·025. The product is the thickness of the eye of air-pump cross-head in inches.

RULE 3. *To find the depth of eye of air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·171. The product is the depth of the eye of air-pump cross-head in inches.

RULE 4. *To find the diameter of the journals of air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·051. The product is the diameter of the end journals.

RULE 5. *To find the length of the end journals for air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·058. The product is the length of the air-pump cross-head journals in inches.

RULE 6. *To find the thickness of the web of air-pump cross-head at middle.*—Multiply the diameter of the cylinder in inches by ·043. The product is the thickness at middle of the web of air-pump cross-head in inches.

RULE 7. *To find the depth at middle of the web of air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·161.

The product is the depth at middle of air-pump cross-head in inches.

RULE 8. *To find the thickness of the web of air-pump cross-head at journals.*—Multiply the diameter of the cylinder in inches by ·037. The product is the thickness of the web of air-pump cross-head at journals in inches.

RULE 9. *To find the depth of the air-pump cross-head web at journals.*—Multiply the diameter of the cylinder in inches by ·061. The product is the depth at journals of the web of air-pump cross-head.

RULE 10. *To find the diameter of the air-pump piston rod when of copper.*—Multiply the diameter of the cylinder in inches by ·067. The product is the diameter of the air-pump piston rod, when of copper, in inches.

RULE 11. *To find the depth of gibs and cutter through air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·063. The product is the depth of the gibs and cutter through air-pump cross-head in inches.

RULE 12. *To find the thickness of the gibs and cutter through air-pump cross-head.*—Multiply the diameter of the cylinder in inches by ·013. The product is the thickness of the gibs and cutter in inches.

RULE 13. *To find the depth of cutter through piston.*—Multiply the diameter of the cylinder in inches by ·051. The product is the depth of the cutter through piston in inches.

RULE 14. *To find the thickness of cutter through air-pump piston.*—Multiply the diameter of the cylinder in inches by ·021. The product is the thickness of the cutter through air-pump piston.

The next seven rules give the dimensions of the remaining parts of the engine which do not depend upon the stroke. To exemplify their use, suppose it were required to find the corresponding dimensions for an engine the diameter of whose cylinder is 64 inches. According to the rule, the proper diameter of the air-pump side rod would be 2·48 inches. Hence, according to the rule, the proper breadth of butt is 2·95 inches. According to the rule, the proper thickness of butt is 2·35 inches. According to the rule, the mean thickness of strap at cutter ought to be 1·24 inches. Hence, according to the rule, the mean thickness of strap below cutter is ·91 inch. According to the rule, the proper depth for the gibs and cutter is 2·94 inches. According to the rule, the proper thickness of the gibs and cutter is ·63 inches.

The following rules give the correct dimensions for all sizes of engines:

RULE 1. *To find the diameter of air-pump side rod at ends.*—Multiply the diameter of the cylinder in inches by ·039. The product is the diameter of the air-pump side rod at ends in inches.

RULE 2. *To find the breadth of butt for air-pump.*—Multiply the diameter of the cylinder in inches by ·046. The product is the breadth of butt in inches.

RULE 3. *To find the thickness of butt for air-pump.*—Multiply the diameter of the cylinder in inches by ·037. The product is the thickness of butt for air-pump in inches.

RULE 4. *To find the mean thickness of strap at cutter.*—Multiply the diameter of the cylinder in inches by ·019. The product is the mean thickness of strap at cutter for air-pump in inches.

RULE 5. *To find the mean thickness of strap below cutter.*—Multiply the diameter of the cylinder in inches by 0·14. The product is the mean thickness of strap below cutter in inches.

RULE 6. *To find the depth of gibs and cutter for air-pump.*—Multiply the diameter of the cylinder in inches by 0·48. The product is the depth of gibs and cutter for air-pump in inches.

RULE 7. *To find the thickness of gibs and cutter for air-pump.*—Divide the diameter of the cylinder in inches by 100. The quotient is the proper thickness of the gibs and cutter for air-pump in inches.

With regard to other dimensions made to depend upon the nominal horse power of the engine:—Suppose that we take the particular example of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. We find that the nominal horse power of this engine is nearly 175. Hence we have successively,

Diameter of valve shaft at journal in inches = 4·85, or between 4¾ and 5 inches.

Diameter of parallel motion shaft at journal in inches = 3·91, or very nearly 4 inches.

Diameter of valve rod in inches = 2·44, or about 2⅜ inches.

Diameter of radius rod at smallest part in inches = 1·97, or very nearly 2 inches.

Area of eccentric rod, at smallest part, in square inches = 8·37, or about 8⅜ square inches.

Sectional area of eccentric hoop in square inches = 8·75, or 8¾ square inches.

Diameter of eccentric pin in inches = 2·24, or 2¼ inches.

Breadth of valve lever for eccentric pin at eye in inches = 5·7, or very nearly 5¾ inches.

Thickness of valve lever for eccentric pin at eye in inches = 3.

Breadth of parallel motion crank at eye = 4·2 inches, or very nearly 4¼ inches.

Thickness of parallel motion crank at eye = 1·76 inches, or about 1¾ inches.

To find the area in square inches of each steam port. Suppose it were required to find the area of each steam port for an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. According to the rule, the area of each steam port would be 202·26 square inches.

With regard to the rule, we may remark that the area of the steam port ought to depend principally upon the cubical content of the cylinder, which again depends entirely upon the product of the square of the diameter of the cylinder and the length of the stroke of the engine. It is well known, however, that the quantity of steam admitted by a small hole does not bear so great a proportion to the quantity admitted by a larger one, as the area of the one does to the area of the other; and a certain allowance ought to be made for this. In the absence of correct theoretical information on this point, we have attempted to make a proper allowance by supplying a constant; but of course this plan ought only to be regarded as an approximation. Our rule is as follows:

RULE.—*To find the area of each steam port.*—Multiply the square of the diameter of the cylinder in inches by the length of the stroke in feet; multiply this product by 11; divide the last product by 1800; and, finally, to the quotient add 8. The result is the area of each steam port in square inches.

To show the use of this rule, we shall apply it to a particular example. We shall apply it to an engine whose stroke is 6 feet, and diameter of cylinder 30 inches. Then, according to the rule, we have

30 = diameter of the cylinder in inches.
30
900 = square of diameter.
6 = length of stroke in feet.
5400
11
59400 ÷ 1800 = 33
8 = constant to be added.
41 = area of steam port in square inches.

When the length of the opening of steam port is from any circumstance found, the corresponding depth in inches may be found, by dividing the number corresponding to the particular engine, by the given length in inches: conversely, the length may be found, when for some reason or other the depth is fixed, by dividing the number corresponding to the particular engine, by the given depth in inches: the quotient is the length in inches.

The next rule is useful for determining the diameter of the steam pipe branching off to any particular engine. Suppose it were required to find the diameter of the branch steam pipe for an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. According to the rule, the proper diameter of the steam pipe would be 13·16 inches.

The following rule will be found to give the proper diameter of steam pipe for all sizes of engines.

RULE.—*To find the diameter of branch steam pipe.*—Multiply together the square of the diameter of the cylinder in inches, the length of the stroke in feet, and ·00498; to the product add 10·2, and extract the square root of the sum. The result is the diameter of the steam pipe in inches.

To exemplify the use of this rule we shall take an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. In this case we have as follows:—

$$\begin{array}{r l} 64 & = \text{diameter of cylinder in inches.} \\ \underline{64} & \\ 4096 & = \text{square of diameter.} \\ \underline{8} & = \text{length of stroke in feet.} \\ 32768 & \\ \underline{\cdot 00498} & = \text{constant multiplier.} \\ 163\cdot 18 & \\ \underline{10\cdot 2} & = \text{constant to be added.} \\ 173\cdot 38 & \end{array}$$

and $\sqrt{173\cdot 38} = 13\cdot 16$.

To find the diameter of the pipes connected with the engine. They are made to depend upon the nominal horse power of the engine. Suppose it were required to apply this rule to determine the size of the pipes for two marine engines, whose strokes are each 8 feet, and diameters of cylinder each 64 inches. We find the nominal horse power of each of these engines to be 174·3. Hence, according to the rules, we have in succession,

Diameter of waste water pipe = 15·87 inches, or between 15¾ and 16 inches.
Area of foot-valve passage = 323 square inches.
Area of injection pipe = 14·88 square inches.

If the injection pipe be cylindrical, then by referring to the table of areas of circles, we see that its diameter would be about 4⅜ inches.

Diameter of feed pipe = 4·12 inches, or between 4 and 4¼ inches.
Diameter of waste steam pipe = 12·17 inches, or nearly 12¼ inches.
Diameter of safety valve,

When one is used = 14·05 inches.
When two are used = 9·94 inches.
When three are used = 8·12 inches.
When four are used = 7·04 inches.

These results were obtained from the following rules, which will give the correct dimensions for all sizes of engines.

Rule 1. *To find the diameter of waste water pipe.*—Multiply the square root of the nominal horse power of the engine by 1·2. The product is the diameter of the waste water pipe in inches.

Rule 2. *To find the area of foot-valve passage.*—Multiply the nominal horse power of the engine by 9; divide the product by 5; add 8 to the quotient. The sum is the area of foot-valve passage in square inches.

Rule 3. *To find the area of injection pipe.*—Multiply the nominal horse power of the engine by ·069; to the product add 2·81. The sum is the area of the injection pipe in square inches.

Rule 4. *To find the diameter of feed pipe.*—Multiply the nominal horse power of the engine by ·04; to the product add 3; extract the square root of the sum. The result is the diameter of the feed pipe in inches.

Rule 5. *To find the diameter of waste steam pipe.*—Multiply the collective nominal horse power of the engines by ·375; to the product add 16·875; extract the square root of the sum. The final result is the diameter of the waste steam pipe in inches.

Rule 6. *To find the diameter of the safety valve when only one is used.*—To one-half the collective nominal horse power of the engines add 22·5; extract the square root of the sum. The result is the diameter of the safety valve when only one is used.

Rule 7. *To find the diameter of the safety valve when two are used.*—Multiply the collective nominal horse power of the engines by ·25; to the product add 11·25; extract the square root of the sum. The result is the diameter of the safety valve when two are used.

Rule 8. *To find the diameter of the safety valve when three are used.*—To one-sixth of the collective nominal horse power of the engines add 7·5; extract the square root of the sum. The result is the diameter of the safety valve where three are used.

Rule 9. *To find the diameter of the safety valve when four are used.*—Multiply the collective nominal horse power of the engines by ·125; to the product add 5·625; extract the square root of the sum. The result is the diameter of the safety valve when four are used.

Another rule for safety valves, and a preferable one for low pressures, is to allow ·8 of a circular inch of area per nominal horse power.

The next rule is for determining the depth across the web of the main beam of a land engine. Suppose we wished to find the proper depth at the centre of the main beam of a land engine whose main beam is 16 feet long, and diameter of cylinder 64 inches. According to the rule, the proper depth of the web across the centre is 46·17 inches. This rule gives correct dimensions for all sizes of engines.

Rule.—*To find the depth of the web at the centre of the main beam of a land engine.*—Multiply together the square of the diameter of the cylinder in inches, half the length of the main beam in feet, and the number 3; extract the cube root of the product. The result is the proper depth of the web of the main beam across the centre in inches, when the main beam is constructed of cast iron.

To illustrate this rule we shall take the particular example of an engine whose main beam is 20 feet long, and the diameter of the cylinder 64 inches. In this case we have

64 = diameter of cylinder in inches.
64
4096 = square of the diameter.
10 = ½ length of main beam in feet.
40960
3 = constant multiplier.
122880

0	0	122880 (49·714 = $\sqrt[3]{122880}$
4	16	64
4	16	58880
4	32	53649
8	4800	5231
4	1161	5112
120	5961	119
9	1242	74
129	7203	35
9	10	
138	730	
9	10	
147	741	

To find the depth of the main beam across the ends. Suppose it were required to find the depth at ends of a cast-iron main beam whose length is 20 feet, when the diameter of the cylinder is 64 inches. The proper depth will be 19·89 inches.

The following rule gives the proper dimensions for all sizes of engines.

RULE.—*To find the depth of main beam at ends.*—Multiply together the square of the diameter of the cylinder in inches, half the length of the main beam in feet, and the number ·192; extract the cube root of the product. The result is the depth in inches of the main beam at ends, when of cast iron.

To illustrate this rule, let us apply it to the particular example of an engine whose main beam is 20 feet long, and the diameter of the cylinder 64 inches. In this case we have as follows:

64 = diameter of cylinder in inches.
64
4096 = square of diameter of cylinder.
10 = ½ length of main beam in feet.
40960
·192 = constant multiplier.
7864·32

0	0	7864·32 (19·89 = $\sqrt[3]{7864{\cdot}32}$
1	1	1
1	1	6864
1	2	5859
2	300	1005
1	351	898
30	651	107
9	432	
39	1083	
9	4	
48	112	
9	4	
57	116	

so that, according to the rule, the depth at ends is nearly 20 inches.

To find the dimensions of the feed-pump in cubic inches. Suppose we take the particular example of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. The proper content of the feed-pump would be 1093·36 cubic inches. Suppose, now, that the cold-water pump was suspended from the main beam at a fourth of the distance between the centre and the end, so that its stroke would be 2 feet, or 24 inches. In this case the area of the pump would be equal to 1093·36 ÷ 24 = 45·556 square inches; so that we conclude that the diameter is between 7½ and 7¾ inches. Conversely, suppose that it was wished to find the stroke of the pump when the diameter was 5 inches. We find the area of the pump to be 19·635 square inches; so that the stroke of the feed-pump must be equal to 1093·36 ÷ 19·635 = 55·69 inches, or very nearly 55¾ inches.

This rule will be found to give correct dimensions for all sizes of engines:

RULE.—*To find the content of the feed-pump.*—Multiply the square of the diameter of the cylinder in inches by the length of the stroke in feet; divide the product by 30. The quotient is the content of the feed-pump in cubic inches.

Thus, for an engine whose stroke is 6 feet, and diameter of cylinder 50 inches, we have,

$$\begin{array}{r l} 50 & = \text{diameter of cylinder.} \\ 50 & \\ \hline 2500 & = \text{square of the diameter of the cylinder.} \\ 6 & = \text{length of stroke in feet.} \\ \hline 30\,)\,15000 & \\ \hline 500 & = \text{content of feed-pump in cubic inches.} \end{array}$$

To determine the content of the cold-water pump in cubic feet. To illustrate this, suppose we take the particular example of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. Suppose, now, the stroke of the pump to be 5 feet, then the area equal to 7·45 ÷ 5 = 1·49 square feet = 214·56 square inches; we see that the diameter of the pump is about 16½ inches. Again, suppose that the diameter of the cold-water pump was 20 inches, and that it was required to find the length of its stroke. The area of the pump is 314·16 square inches, or 314·16 ÷ 144 = 2·18 square feet; so that the stroke of the pump is equal to 7·45 ÷ 2·18 = 3·42 feet.

The content is calculated from the following rule, which will be found to give correct dimensions for all sizes of engines:

Rule.—*To find the content of the cold-water pump.*—Multiply the square of the diameter of the cylinder in inches by the length of the stroke in feet; divide the product by 4400. The quotient is the content of the cold-water pump in cubic feet.

To explain this rule, we shall take the particular example of an engine whose stroke is 5½ feet, and diameter of cylinder 60 inches. In this case we have in succession,

60 = diameter of cylinder in inches.
60
3600 = square of the diameter of cylinder.
5½ = length of stroke in feet.
4400)19800
4·5 = content of cold water pump in cubic feet.

To determine the proper thickness of the large eye of crank for fly-wheel shaft when the crank is of cast iron. The crank is sometimes cast on the shaft, and of course the thickness of the large eye is not then so great as when the crank is only keyed on the shaft, or rather there is then no large eye at all. To illustrate the use of this rule, we shall apply it to the particular example of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. Hence, according to the rule, the proper thickness of the large eye of crank when of cast iron is 8·07 inches. For a marine engine of 8 feet stroke and 64 inch cylinder, the thickness of the large eye of crank is about 5¾ inches. The difference is thus about 2¼ inches, which is an allowance for the inferiority of cast iron to malleable iron.

The following rule will be found to give correct dimensions for all sizes of engines:

Rule.—*To find the thickness of the large eye of crank for fly-wheel shaft when of cast iron.*—Multiply the square of the length of the crank in inches by 1·561, and then multiply the square of the diameter of the cylinder in inches by ·1235; multiply the sum of these products by the square of the diameter of cylinder in inches; divide this product by 666·283; divide this quotient by the length of the crank in inches; finally extract the cube root of the quotient.

The result is the proper thickness of the large eye of crank for fly-wheel shaft in inches, when of cast iron.

As this rule is rather complicated, we shall show its application to the particular example already selected.

48 = length of crank in inches.
48
2304 = square of length of crank in inches.
1·561 = constant multiplier.
3596·5
64 = diameter of cylinder in inches.
64
4096 = square of the diameter of cylinder.
·1235 = constant multiplier.
505·8
3596·5
4102·3 = sum of products.
4096 = square of the diameter of cylinder.
666·283) 16803020·8
length of crank=48) 25219·045
525·397
and $\sqrt[3]{525·397}$ = 8·07 nearly.

To find the breadth of the web of crank at the centre of the fly-wheel shaft, that is to say, the breadth which it would have if it were continued to the centre of the fly-wheel shaft. Suppose it were required to find the breadth of the crank at the centre of the fly-wheel shaft for an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. According to the rule, the proper breadth is 22·49 inches. According to a former rule, the breadth of the web of a cast iron crank of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches, is about 18 inches. The difference between these two is about 4½ inches; which is not too great an allowance for the inferiority of the cast iron.

The following rule will be found to give correct dimensions for all sizes of engines:

Rule.—*To find the breadth of the web of crank at fly-wheel shaft, when of cast iron.*—Multiply the square of the length of the crank in inches by 1·561, and then multiply the square of the diameter of the cylinder in inches by ·1235; multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide the product by 23·04, and finally extract the cube root of the quotient. The final result is the breadth of the crank at the centre of the fly-wheel shaft, when the crank is of cast iron.

As this rule is rather complicated, we shall illustrate it by showing its application to the particular example of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches.

64 = diameter of cylinder in inches.
64
4096 = square of the diameter of cylinder.
·1235 = constant multiplier.
505·8

48 = length of crank in inches.
48
2304 = square of the length of crank.
1·561 = constant multiplier.
3596·5
505·8
4102·3 = sum of products.
$\sqrt{4102{\cdot}3} = 64{\cdot}05$ nearly.
4096 = square of the diameter of [cylinder.
constant divisor = 23·04)262348·5
11386·66 nearly.
and $\sqrt[3]{11386{\cdot}66} = 22{\cdot}49$.

To determine the thickness of the web of crank at the centre of the fly-wheel shaft; that is to say, the thickness which it would have if it were continued so far. Suppose it were required to find the thickness of web of crank at the centre of fly-wheel shaft of an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. According to the rule, the proper thickness would be 11·26 inches. The proper thickness of web at centre of paddle shaft for a marine engine whose stroke is 8 feet, and diameter of cylinder 64 inches, is nearly 9 inches. The difference between the two thicknesses is about 2¼ inches, which is not too great an allowance for the inferiority of cast iron to malleable iron.

The following rule will be found to give correct dimensions for all sizes of engines:

Rule.—*To find the thickness of the web of crank at centre of fly-wheel shaft, when of cast iron.*—Multiply the square of the length of the crank in inches by 1·561, and then multiply the square of the diameter of the cylinder in inches by ·1235; multiply the square root of the sum of these products by the square of the diameter of the cylinder in inches; divide this product by 184·32; finally extract the cube root of the quotient. The result is the thickness of the web of crank at the centre of the fly-wheel shaft when of cast iron, in inches.

As this rule is rather complicated, we shall illustrate it by applying it to the particular engine which we have already selected.

48 = length of crank in inches.
48
2304 = square of length of crank.
1·561 = constant multiplier.
3596·5

64 = diameter of cylinder in inches.
64
4096 = square of the diameter of cylinder.
·1235 = constant multiplier.
505·8
3596·5
4102·3 = sum of products.

and $\sqrt{4102{\cdot}3} = 64{\cdot}05$ nearly.
4096 = square of diameter.
Constant divisor = 184·32) 262348·5
1423·33

and $\sqrt[3]{1423{\cdot}33} = 11{\cdot}24$

To find the proper diameter of the fly-wheel shaft at its smallest part, when, as is usually the case, it is of cast iron. Suppose it were required to find the diameter of the fly-wheel shaft for an engine whose stroke is 8 feet, and diameter of cylinder 64 inches. According to the rule, the diameter would be 17·59 inches. It is obvious enough that the fly-wheel shaft stands in much the same relation to the land engine, as the paddle shaft does to the marine engine. According to a former rule, the diameter of the paddle shaft journal of a marine engine whose stroke is 8 feet, and diameter of cylinder 64 inches, is about 14 inches. The difference betwixt the diameter of the paddle shaft for the marine engine, and the diameter of the fly-wheel shaft for the corresponding land engine is about 3½ inches. This will be found to be a very proper allowance for the different circumstances connected with the land engine.

The following rule will be found to give correct dimensions for all sizes of engines.

RULE.—*To find the diameter of the fly-wheel shaft at smallest part, when it is of cast iron.*—Multiply the square of the diameter of the cylinder in inches by the length of the crank in inches; extract the cube root of the product; finally multiply the result by ·3025. The result is the diameter of the fly-wheel shaft at smallest part in inches.

We shall illustrate this rule by applying it to the particular engine which we have already selected.

64 = diameter of cylinder in inches.
64

4096 = square of the diameter.
48 = length of crank in inches.

196608

0	0	196608 (58·15 = $\sqrt[3]{196608}$
5	25	125
5	25	71608
5	50	70112
10	7500	1496
5	1264	1011
150	8764	485
8	1328	
158	10092	
8	2	
166	1011	
8	2	
174	1013	

and 58·15 × ·3025 = 1759
which agrees with the number given by a former rule.

To determine the sectional area of the fly-wheel rim when of cast iron. Suppose it were required to find the sectional area of the rim of a fly-wheel for an engine whose stroke is 8 feet, and diameter of cylinder 64 inches, the diameter of the fly-wheel itself being 30 feet. According to the rule, the sectional area of the rim in square inches = 146·4 × ·813 = 119·02. We may remark that this calculation has been made on the supposition that the fly-wheel is so connected with the engine, as to make exactly one revolution for each double stroke of the piston. If the fly-wheel is so connected with the engine as to make more than one revolution for each double stroke, then the rim does not need to be so heavy as we make it. If, on the contrary, the fly-wheel does not make a complete revolution for each double stroke of the engine, then it ought to be heavier than this rule makes it.

RULE.—*To find the sectional area of the rim of the fly-wheel when of cast iron.*—Multiply together the square of the diameter of the cylinder in inches, the square of the length of the stroke in feet, the cube root of the length of the stroke in feet, and 6·125; divide the final product by the cube of the diameter of the fly-wheel in feet. The quotient is the sectional area of the rim of fly-wheel in square inches, provided it is of cast iron.

As this rule is rather complicated, we shall endeavour to illustrate it by showing its application to a particular engine. We shall apply the rule to determine the sectional area of the rim of fly-wheel for an engine whose stroke is 8 feet, diameter of cylinder 50 inches; the diameter of the fly-wheel being 20 feet. For this engine we have as follows:

$$\begin{array}{rl} 2500 & = \text{square of diameter of cylinder.} \\ 64 & = \text{square of the length of stroke.} \\ \hline 160000 & \\ 2 & = \text{cube root of the length of stroke.} \\ \hline 320000 & \\ 6{\cdot}125 & = \text{constant multiplier.} \\ \hline 1960000 & \end{array}$$

therefore sectional area in square inches $= 1960000 \div 20^3 = 1960000 \div 8000 = 1960 \div 8 = 245$.

In the following formulas we denote the diameter of the cylinder in inches by D, the length of the crank in inches by R, the length of the stroke in feet, and the nominal horse power of the engine by H.P.

MARINE ENGINES.—DIMENSIONS OF SEVERAL OF THE PARTS OF THE SIDE LEVER.

Depth of eye round end studs of lever $= {\cdot}074 \times D$.
Thickness of eye round end studs of lever $= {\cdot}052 \times D$.
Diameter of end studs, in inches $= {\cdot}07 \times D$.
Length of end studs, in inches $= {\cdot}076 \times D$.
Diameter of air-pump studs, in inches $= {\cdot}045 \times D$.
Length of air-pump studs, in inches $= {\cdot}049 \times D$.

Depth of cast iron side lever across centre, in inches $= D^{\frac{2}{3}} \times \{{\cdot}7423 \times \text{length of lever in feet}\}^{\frac{1}{3}}$.

MARINE ENGINE.—DIMENSIONS OF SEVERAL PARTS OF AIR-PUMP CROSS-HEAD.

Diameter of air-pump, in inches $= {\cdot}6 \times D$.
Thickness of eye for air-pump rod, in inches $= {\cdot}025 \times D$.
Depth of eye for air-pump rod, in inches $= {\cdot}171 \times D$.
Diameter of end journals, in inches $= {\cdot}051 \times D$.
Length of end journals, in inches $= {\cdot}058 \times D$.
Thickness of web at middle, in inches $= {\cdot}043 \times D$.
Depth of web at middle, in inches $= {\cdot}161 \times D$.
Thickness of web at journal $= {\cdot}037 \times D$.
Depth of web at journal $= {\cdot}061 \times D$.

MARINE ENGINE.—DIMENSIONS OF THE PARTS OF AIR-PUMP PISTON-ROD.

Diameter of air-pump piston-rod, when of copper, in inches $= {\cdot}067 \times D$.

Depth of gibs and cutter through cross-head, in inches $= {\cdot}063 \times D$.

Thickness of gibs and cutter through cross-head, in inches = ·013 × D.

Depth of cutter through piston, in inches = ·051 × D.

Thickness of cutter through piston, in inches = ·021 × D.

MARINE ENGINE.—DIMENSIONS OF THE REMAINING PARTS OF THE AIR-PUMP MACHINERY.

Diameter of air-pump side rods at ends, in inches = ·039 × D.

Breadth of butt, in inches = ·046 × D.

Thickness of butt, in inches = ·037 × D.

Mean thickness of strap at cutter, in inches = ·019 × D.

Mean thickness of strap below cutter, in inches = ·014 × D.

Depth of gibs and cutter, in inches = ·048 × D.

Thickness of gibs and cutter in inches = D ÷ 100.

MARINE AND LAND ENGINES.—AREA OF STEAM PORTS.

Area of each steam port, in square inches = $11 \times l \times D^2 \div 1800 + 8$.

MARINE AND LAND ENGINES.—DIMENSIONS OF BRANCH STEAM PIPES.

Diameter of each branch steam pipe = $\sqrt{\cdot00498 \times l \times D^2 \times 10\cdot2}$.

MARINE ENGINE.—DIMENSIONS OF SEVERAL OF THE PIPES CONNECTED WITH THE ENGINE.

Diameter of waste water pipe, in inches = $1\cdot2 \times \sqrt{\text{H.P.}}$

Area of foot-valve passage, in square inches = 1·8 × H.P. + 8.

Area of injection pipe, in square inches = ·069 × H.P. + 2·81.

Diameter of feed pipe, in inches = $\sqrt{\cdot04 \times \text{H.P.} + 3}$.

Diameter of waste steam pipe in inches = $\sqrt{\cdot375 \times \text{H.P.} + 16\cdot875}$.

MARINE AND LAND ENGINES.—DIMENSIONS OF SAFETY-VALVES.

Diam. of safety-valve, when one only is used = $\sqrt{\cdot5 \times \text{H.P.} + 22\cdot5}$.

Diam. of safety-valve, when two are used = $\sqrt{\cdot25 \times \text{H.P.} + 11\cdot25}$.

Diam. of safety-valve, when three are used = $\sqrt{\cdot167 \times \text{H.P.} + 7\cdot5}$.

Diam. of safety-valve, when four are used = $\sqrt{\cdot125 \times \text{H.P.} + 5\cdot625}$.

LAND ENGINE.—DIMENSIONS OF MAIN BEAM.

Depth of web of main beam across centre =

$$\sqrt[3]{3 \times D^2 \times \text{half length of main beam in feet.}}$$

Depth of main beam at ends =

$$\sqrt[3]{\cdot192 \times D^2 \times \text{half length of main beam, in feet.}}$$

LAND AND MARINE ENGINES.—CONTENT OF FEED-PUMP.

Content of feed-pump, in cubic inches = $D^2 \times l \div 30$.

LAND ENGINES.—CONTENT OF COLD WATER PUMP.

Content of cold water pump, in cubic feet = $D^2 \times l \div 4400$.

LAND ENGINES.—DIMENSIONS OF CRANK.

Thickness of large eye of crank, in inches =

$$\sqrt[3]{D^2 \times (1\cdot561 \times R^2 + \cdot1235\ D^2) \div (R \times 666\cdot283)}.$$

Breadth of web of crank at fly-wheel shaft centre, in inches =

$$\sqrt[3]{D^2 \times \surd\,(1\cdot561 \times R^2 + \cdot1235 \times D^2) \div 23\cdot04}.$$

Thickness of web of crank at fly-wheel shaft centre, in inches =

$$\sqrt[3]{D^2 \times \surd\,(1\cdot561 \times R^2 + \cdot1235 \times D^2) \div 184\cdot32}.$$

LAND ENGINES.—DIMENSIONS OF FLY-WHEEL SHAFT.

Diameter of fly-wheel shaft, when of cast iron $= 3025 \times \sqrt[3]{R \times D^2}$.

DIMENSIONS OF PARTS OF LOCOMOTIVES.

DIAMETER OF CYLINDER.

In locomotive engines, the diameter of the cylinder varies less than either the land or the marine engine. In few of the locomotive engines at present in use is the diameter of the cylinder greater than 16 inches, or less than 12 inches. The length of the stroke of nearly all the locomotive engines at present in use is 18 inches, and there are always two cylinders, which are generally connected to cranks upon the axle, standing at right angles with one another.

AREA OF INDUCTION PORTS.

Rule.—*To find the size of the steam ports for the locomotive engine.*—Multiply the square of the diameter of the cylinder by ·068. The product is the proper size of the steam ports in square inches.

Required the proper size of the steam ports of a locomotive engine whose diameter is 15 inches. Here, according to the rule, size of steam ports = ·068 × 15 × 15 = ·068 × 225 = 15·3 square inches, or between 15¼ and 15½ square inches.

After having determined the area of the ports, we may easily find the depth when the length is given, or, conversely, the length when the depth is given. Thus, suppose we knew that the length was 8 inches, then we find that the depth should be 15·3 ÷ 8 = 1·9125 inches, or nearly 2 inches; or suppose we knew the depth was 2 inches, then we would find that the length was 15·3 ÷ 2 = 7·65 inches, or nearly 7¾ inches.

AREA OF EDUCTION PORTS.

The proper area for the eduction ports may be found from the following rule.

Rule.—*To find the area of the eduction ports.*—Multiply the square of the diameter of the cylinder in inches by ·128. The product is the area of the eduction ports in square inches.

Required the area of the eduction ports of a locomotive engine, when the diameter of the cylinders is 13 inches. In this example we have, according to the rule,

Area of eduction port = $\cdot128 \times 13^2 = \cdot128 \times 169 = 21\cdot632$ inches, or between 21½ and 21¾ square inches.

BREADTH OF BRIDGE BETWEEN PORTS.

The breadth of the bridges between the eduction port and the induction ports is usually between ¾ inch and 1 inch.

DIAMETER OF BOILER.

It is obvious that the diameter of the boiler may vary very considerably; but it is limited chiefly by considerations of strength; and 3 feet are found a convenient diameter. Rules for the strength of boilers will be given hereafter.

Rule.—*To find the inside diameter of the boiler.*—Multiply the diameter of the cylinder in inches by 3·11. The product is the inside diameter of the boiler in inches.

Required the inside diameter of the boiler for a locomotive engine, the diameter of the cylinders being 15 inches.

In this example we have, according to the rule,

Inside diameter of boiler = $15 \times 3\cdot11 = 46\cdot65$ inches,
or about 3 feet 10⅝ inches.

LENGTH OF BOILER.

The length of the boiler is usually in practice between 8 feet and 8½ feet.

DIAMETER OF STEAM DOME, INSIDE.

It is obvious that the diameter of the steam dome may be varied considerably, according to circumstances; but the first indication is to make it large enough. It is usual, however, in practice, to proportion the diameter of the steam dome to the diameter of the cylinder; and there appears to be no great objection to this. The following rule will be found to give the diameter of the dome usually adopted in practice.

Rule.—*To find the diameter of the steam dome.*—Multiply the diameter of the cylinder in inches by 1·43. The product is the diameter of the dome in inches.

Required the diameter of the steam dome for a locomotive engine whose diameter of cylinders is 13 inches. In this example we have, according to the rule,

Diameter of steam dome = $1\cdot43 \times 13 = 18\cdot59$ inches,
or about 18½ inches.

HEIGHT OF STEAM DOME.

The height of the steam dome may vary. Judging from practice, it appears that a uniform height of 2½ feet would answer very well.

DIAMETER OF SAFETY-VALVE.

In practice the diameter of the safety-valve varies considerably. The following rule gives the diameter of the safety-valve usually adopted in practice.

Rule.—*To find the diameter of the safety-valve.*—Divide the diameter of the cylinder in inches by 4. The quotient is the diameter of the safety-valve in inches.

Required the diameter of the safety-valves for the boiler of a locomotive engine, the diameter of the cylinder being 13 inches. Here, according to the rule, diameter of safety-valve $= 13 \div 4 = 3\frac{1}{4}$ inches. A larger size, however, is preferable, as being less likely to stick.

DIAMETER OF VALVE SPINDLE.

The following rule will be found to give the correct diameter of the valve spindle. It is entirely founded on practice.

Rule.—*To find the diameter of the valve spindle.*—Multiply the diameter of the cylinder in inches by ·076. The product is the proper diameter of the valve spindle.

Required the diameter of the valve spindle for a locomotive engine whose cylinders' diameters are 13 inches.

In this example we have, according to the rule, diameter of valve spindle $= 13 \times {\cdot}076 = {\cdot}988$ inches, or very nearly 1 inch.

DIAMETER OF CHIMNEY.

It is usual in practice to make the diameter of the chimney equal to the diameter of the cylinder. Thus a locomotive engine whose cylinders' diameters are 15 inches would have the inside diameter of the chimney also 15 inches, or thereabouts. This rule has, at least, the merit of simplicity.

AREA OF FIRE-GRATE.

The following rule determines the area of the fire-grate usually given in practice. We may remark, that the area of the fire-grate in practice follows a more certain rule than any other part of the engine appears to do; but it is in all cases much too small, and occasions a great loss of power by the urging of the blast it renders necessary, and a rapid deterioration of the furnace plates from excessive heat. There is no good reason why the furnace should not be nearly as long as the boiler: it would then resemble the furnace of a marine boiler, and be as manageable.

Rule.—*To find the area of the fire-grate.*—Multiply the diameter of the cylinder in inches by ·77. The product is the area of the fire-grate in superficial feet.

Required the area of the fire-grate of a locomotive engine, the diameters of the cylinders being 15 inches.

In this example we have, according to the rule,

$$\text{Area of fire-grate} = {\cdot}77 \times 15 = 11{\cdot}55 \text{ square feet,}$$

or about $11\frac{1}{2}$ square feet. Though this rule, however, represents the usual practice, the area of the fire-grate should not be contingent upon the size of the cylinder, but upon the quantity of steam to be raised.

AREA OF HEATING SURFACE.

In the construction of a locomotive engine, one great object is to obtain a boiler which will produce a sufficient quantity of steam with as little bulk and weight as possible. This object is admirably accomplished in the construction of the boiler of the locomotive engine. This little barrel of tubes generates more steam in an hour than was formerly raised from a boiler and fire occupying a considerable house. This favourable result is obtained simply by exposing the water to a greater amount of heating surface.

In the usual construction of the locomotive boiler, it is obvious that we can only consider four of the six faces of the inside fire-box as effective heating surface; viz. the crown of the box, and the three perpendicular sides. The circumferences of the tubes are also effective heating surface; so that the whole effective heating surface of a locomotive boiler may be considered to be the four faces of the inside fire-box, plus the sum of the surfaces of the tubes. Understanding this to be the effective heating surface, the following rule determines the average amount of heating surface usually given in practice.

RULE.—*To find the effective heating surface.*—Multiply the square of the diameter of the cylinder in inches by 5; divide the product by 2. The quotient is the area of the effective heating surface in square feet.

Required the effective heating surface of the boiler of a locomotive engine, the diameters of the cylinders being 15 inches.

In this example we have, according to the rule,

Effective heating surface $= 15^2 \times 5 \div 2 = 225 \times 5 \div 2 = 1125 \div 2 = 562\frac{1}{2}$ square feet.

According to the rule which we have given for the fire-grate, the area of the fire-grate for this boiler would be about $11\frac{1}{2}$ square feet. We may suppose, therefore, the area of the crown of the box to be 12 square feet. The area of the three perpendicular sides of the inside fire-box is usually three times the area of the crown; so that the effective heating surface of the fire-box is 48 square feet. Hence the heating surface of the tubes $= 526·5 - 48 = 478·5$ square feet. The inside diameters of the tubes are generally about $1\frac{3}{4}$ inches; and therefore the circumference of a section of these tubes, according to the table, is 5·4978 inches. Hence, supposing the tube to be $8\frac{1}{2}$ feet long, the surface of one $= 5·4978 \times 8\frac{1}{2} \div 12 = ·45815 \times 8\frac{1}{2} = 3·8943$ square feet; and, therefore, the number of tubes $= 478·5 \div 3·8943 = 123$ nearly. The amount of heating surface, however, like that of grate surface, is properly a function of the quantity of steam to be raised, and the proportions of both, given hereafter, will be found to answer well for boilers of every description.

AREA OF WATER-LEVEL.

This, of course, varies with the different circumstances of the boiler. The average area may be found from the following rule.

RULE.—*To find the area of the water-level.*—Multiply the diameter of the cylinder in inches by 2·08. The product is the area of the water-level in square feet.

Required the area of the water-level for a locomotive engine, whose cylinders' diameters are 14 inches.

In this case we have, according to the rule,

Area of water-level $= 14 \times 2{\cdot}08 = 29{\cdot}12$ square feet.

CUBICAL CONTENT OF WATER IN BOILER.

This, of course, varies not only in different boilers, but also in the same boiler at different times. The following rule is supposed to give the average quantity of water in the boiler.

RULE.—*To find the cubical content of the water in the boiler.*—Multiply the square of the diameter of the cylinder in inches by 9: divide the product by 40. The quotient is the cubical content of the water in the boiler in cubic feet.

Required the average cubical content of the water in the boiler of a locomotive engine, the diameters of the cylinders being 14 inches. In this example we have, according to the rule,

Cubical content of water $= 9 \times 14^2 \div 40 = 44{\cdot}1$ cubic feet.

CONTENT OF FEED-PUMP.

In the locomotive engine, the feed-pump is generally attached to the cross-head, and consequently it has the same stroke as the piston. As we have mentioned before, the stroke of the locomotive engine is generally in practice 18 inches. Hence, assuming the stroke of the feed-pump to be constantly 18 inches, it only remains for us to determine the diameter of the ram. It may be found from the following rule.

RULE.—*To find the diameter of the feed-pump ram.*—Multiply the square of the diameter of the cylinder in inches by ·011. The product is the diameter of the ram in inches.

Required the diameter of the ram for the feed-pump for a locomotive engine whose diameter of cylinder is 14 inches. In this example we have, according to the rule,

Diameter of ram $= {\cdot}011 \times 14^2 = {\cdot}011 \times 196 = 2{\cdot}156$ inches, or between 2 and $2\frac{1}{4}$ inches.

CUBICAL CONTENT OF STEAM ROOM.

The quantity of steam in the boiler varies not only for different boilers, but even for the same boiler in different circumstances. But when the locomotive is in motion, there is usually a certain proportion of the boiler filled with the steam. Including the dome and the steam pipe, the content of the steam room will be found usually to be somewhat less than the cubical content of the water.

But as it is desirable that it should be increased, we give the following rule.

RULE.—*To find the cubical content of the steam room.*—Multiply the square of the diameter of the cylinder in inches by 9; divide the product by 40. The quotient is the cubical content of the steam room in cubic feet.

Required the cubical content of the steam room in a locomotive boiler, the diameters of the cylinders being 12 inches.

In this example we have, according to the rule,

Cubical content of steam room $= 9 \times 12^2 \div 40 = 9 \times 144 \div 40 =$ 32·4 cubic feet.

CUBICAL CONTENT OF INSIDE FIRE-BOX ABOVE FIRE-BARS.

The following rule determines the cubical content of fire-box usually given in practice.

RULE.—*To find the cubical content of inside fire-box above fire-bars.*—Divide the square of the diameter of the cylinder in inches by 4. The quotient is the content of the inside fire-box above fire-bars in cubic feet.

Required the content of inside fire-box above fire-bars in a locomotive engine, when the diameters of the cylinders are each 15 inches.

In this example we have, according to the rule,

Content of inside fire-box above fire-bars $= 15^2 \div 4 = 225 \div 4 =$ $56\frac{1}{4}$ cubic feet.

THICKNESS OF THE PLATES OF BOILER.

In general, the thickness of the plates of the locomotive boiler is $\frac{3}{8}$ inch. In some cases, however, the thickness is only $\frac{5}{16}$ inch.

INSIDE DIAMETER OF STEAM PIPE.

The diameter usually given to the steam pipe of the locomotive engine may be found from the following rule.

RULE.—*To find the diameter of the steam pipe of the locomotive engine.*—Multiply the square of the diameter of the cylinder in inches by ·03. The product is the diameter of the steam pipe in inches.

Required the diameter of the steam pipe of a locomotive engine, the diameter of the cylinder being 13 inches. Here, according to the rule, diameter of steam pipe $= \cdot 03 \times 13^2 = \cdot 03 \times 169 = 5\cdot 07$ inches; or a very little more than 5 inches. The steam pipe is usually made too small in engines intended for high speeds.

DIAMETER OF BRANCH STEAM PIPES.

The following rule gives the usual diameter of the branch steam pipe for locomotive engines.

RULE.—*To find the diameter of the branch steam pipe for the locomotive engine.*—Multiply the square of the diameter of the cylinder in inches by ·021. The product is the diameter of the branch steam pipe for the locomotive engine in inches.

Required the diameter of the branch steam pipes for a locomotive engine, when the cylinder's diameter is 15 inches. Here, according to the rule, diameter of branch pipe $= {\cdot}021 \times 15^2 = {\cdot}021 \times 225 = 4{\cdot}725$ inches, or about 4¾ inches.

DIAMETER OF TOP OF BLAST PIPE.

The diameter of the top of the blast pipe may be found from the following rule.

Rule.—*To find the diameter of the top of the blast pipe.*—Multiply the square of the diameter of the cylinder in inches by 0·17. The product is the diameter of the top of the blast pipe in inches.

The diameter of a locomotive engine is 13 inches; required the diameter of the blast pipe at top. Here, according to the rule, diameter of blast pipe at top $= {\cdot}017 \times 13^2 = {\cdot}017 \times 169 = 2{\cdot}873$ inches, or between 2¾ and 3 inches; but the orifice of the blast pipe should always be made as large as the demands of the blast will permit.

DIAMETER OF FEED PIPES.

There appear to be no theoretical considerations which would lead us to determine exactly the proper size of the feed pipes. Judging from practice, however, the following rule will be found to give the proper dimensions.

Rule.—*To find the diameter of the feed pipes.*—Multiply the diameter of the cylinder in inches by ·141. The product is the proper diameter of the feed pipes.

Required the diameter of the feed pipes for a locomotive engine, the diameter of the cylinder being 15 inches.

In this example we have, according to the rule,

Diameter of feed-pipe $= 15 \times {\cdot}141 = 2{\cdot}115$ inches,

or between 2 and 2¼ inches.

DIAMETER OF PISTON ROD.

The diameter of the piston rod for the locomotive engine is usually about one-seventh the diameter of the cylinder. Making practice our guide, therefore, we have the following rule.

Rule.—*To find the diameter of the piston rod for the locomotive engine.*—Divide the diameter of the cylinder in inches by 7. The quotient is the diameter of the piston rod in inches.

The diameter of the cylinder of a locomotive engine is 15 inches; required the diameter of the piston rod. Here, according to the rule, diameter of piston rod $= 15 \div 7 = 2\frac{1}{7}$ inches.

THICKNESS OF PISTON.

The thickness of the piston in locomotive engines is usually about two-sevenths of the diameter of the cylinder. Making practice our guide, therefore, we have the following rule.

Rule.—*To find the thickness of the piston in the locomotive engine.*—Multiply the diameter of the cylinder in inches by 2; divide the product by 7. The quotient is the thickness of the piston in inches.

The diameter of the cylinder of a locomotive engine is 14 inches; required the thickness of the piston. Here, according to the rule, thickness of piston = 2 × 14 ÷ 7 = 4 inches.

DIAMETER OF CONNECTING RODS AT MIDDLE.

The following rule gives the diameter of the connecting rod at middle. The rule, we may remark, is entirely founded on practice.

Rule.—*To find the diameter of the connecting rod at middle of the locomotive engine.*—Multiply the diameter of the cylinder in inches by ·21. The product is the diameter of the connecting rod at middle in inches.

Required the diameter of the connecting rods at middle for a locomotive engine, the diameter of the cylinders being twelve inches.

For this example we have, according to the rule,

Diameter of connecting rods at middle = 12 × ·21 = 2·52 inches, or $2\frac{1}{2}$ inches.

DIAMETER OF BALL ON CROSS-HEAD SPINDLE.

The diameter of the ball on the cross-head spindle may be found from the following rule.

Rule.—*To find the diameter of the ball on cross-head spindle of a locomotive engine.*—Multiply the diameter of the cylinder in inches by ·23. The product is the diameter of the ball on the cross-head spindle.

Required the diameter of the ball on the cross-head spindle of a locomotive engine, when the diameter of the cylinder is 15 inches. Here, according to the rule,

Diameter of ball = ·23 × 15 = 3·45 inches, or nearly $3\frac{1}{2}$ inches.

DIAMETER OF THE INSIDE BEARINGS OF THE CRANK AXLE.

It is obvious that the inside bearings of the crank axle of the locomotive engine correspond to the paddle-shaft journal of the marine engine, and to the fly-wheel shaft journal of the land-engine. We may conclude, therefore, that the proper diameter of these bearings ought to depend jointly upon the length of the stroke and the diameter of the cylinder. In the locomotive engine the stroke is usually 18 inches, so that we may consider that the diameter of the bearing depends solely upon the diameter of the cylinder. The following rule will give the diameter of the inside bearing.

Rule.—*To find the diameter of the inside bearing for the locomotive engine.*—Extract the cube root of the square of the diameter of the cylinder in inches; multiply the result by ·96. The product is the proper diameter of the inside bearing of the crank axle for the locomotive engine.

Required the diameter of the inside bearing of the crank axle for a locomotive engine whose cylinders are of 13-inch diameters. In this example we have, according to the rule,

13 = diameter of cylinder in inches.
13
169 = square of the diameter of cylinder.

0	0	169(5·5289 = $\sqrt[3]{169}$
5	25	125
5	25	44000
5	50	41375
10	7500	2625
5	775	1820
150	8275	805
5	800	726
155	9075	79
5	3	
160	910	
5	3	
165	913	

and diameter of bearing = 5·5289 × ·96 = 5·31 inches nearly; or between 5¼ and 5½ inches.

DIAMETER OF THE OUTSIDE BEARINGS OF THE CRANK AXLE.

The crank axle, in addition to resting upon the inside bearings, is sometimes also made to rest partly upon outside bearings. These outside bearings are added only for the sake of steadiness, and they do not need to be so strong as the inside bearings. The proper size of the diameter of these bearings may be found from the following rule.

Rule.—*To find the diameter of outside bearings for the locomotive engine.*—Multiply the square of the diameters of the cylinders in inches by ·396; extract the cube root of the product. The result is the diameter of the outside bearings in inches.

Required the proper diameter of the outside bearings for a locomotive engine, the diameter of its cylinders being 15 inches.

In this example we have, according to the rule,

15 = diameter of cylinders in inches.
15
225 = square of diameter of cylinder.
·396 = constant multiplier.
89·1

0	0	89·1(4·466 = $\sqrt[3]{89\cdot1}$
4	16	64
4	16	25100
4	32	21184
8	4800	3916
4	496	3528
120	5296	388
4	512	358
124	5808	
4	8	
128	588	
4	8	
132	596	

Hence diameter of outside bearing = 4·466 inches, or very nearly 4½ inches.

DIAMETER OF PLAIN PART OF CRANK AXLE.

It is usual to make the plain part of crank axle of the same sectional area as the inside bearings. Hence, to determine the sectional area of the plain part when it is cylindrical, we have the following rule.

Rule.—*To determine the diameter of the plain part of crank axle for the locomotive engine.*—Extract the cube root of the square of the diameter of the cylinder in inches; multiply the result by ·96. The product is the proper diameter of the plain part of the crank axle of the locomotive engine in inches.

Required the diameter of the plain part of the crank axle for the locomotive engine, whose cylinders' diameters are 14 inches. In this example we have, according to the rule,

14 = diameter of cylinder in inches.
14
196 = square of the diameter of cylinder.

0	·0	196(5·808 = $\sqrt[3]{196}$
5	25	125
5	25	71·000
5	50	70·112
10	7500	·888
5	1264	
150	8764	
8	1328	
158	10092	
8		
166		
8		
174		

Hence the plain part of crank axle = 5·808 × ·96 = 5·58 nearly, or a little more than 5½ inches.

DIAMETER OF CRANK PIN.

The following rule gives the proper diameter of the crank pin. It is obvious that the crank pin of the locomotive engine is not altogether analogous to the crank pin of the marine or land engine, and, like them, ought to depend upon the diameter of the cylinder, as it is usually formed out of the solid axle.

Rule.—*To find the diameter of the crank pin for the locomotive engine.*—Multiply the diameter of the cylinder in inches by ·404. The product is the diameter of the crank pin in inches.

Required the diameter of the crank pin of a locomotive engine whose cylinders' diameters are 15 inches.

In this example we have, according to the rule,

Diameter of crank pin = 15 × ·404 = 6·06 inches, or about 6 inches.

LENGTH OF CRANK PIN.

The length of the crank pin usually given in practice may be found from the following rule.

Rule.—*To find the length of the crank pin.*—Multiply the diameter of the cylinder in inches by ·233. The product is the length of the crank pins in inches.

Required the length of the crank pins for a locomotive engine with a diameter of cylinder of 13 inches.

In this example we have, according to the rule,

Length of crank pin = 13 × ·233 = 3·029 inches,

or about 3 inches. The part of the crank axle answering to the crank pin is usually rounded very much at the corners, both to give additional strength, and to prevent side play.

These then are the chief dimensions of locomotive engines according to the practice most generally followed. The establishment of express trains and the general exigencies of steam locomotion are daily introducing innovations, the effect of which is to make the engines of greater size and power: but it cannot be said that a plan of locomotive engine has yet been contrived that is free from grave objections. The most material of these defects is the necessity that yet exists of expending a large proportion of the power in the production of a draft; and this evil is traceable to the inadequate area of the fire-grate, which makes an enormous rush of air through the fire necessary to accomplish the combustion of the fuel requisite for the production of the steam. To gain a sufficient area of fire-grate, an entirely new arrangement of engine must be adopted: the furnace must be greatly lengthened, and perhaps it may be found that short upright tubes, or the very ingenious arrangement of Mr. Dimpfell, of Philadelphia, may be introduced with advantage. Upright tubes have been found to be more effectual in raising steam than horizontal tubes; but the tube plate in the case of upright tubes would be more liable to burn.

We here give the preceding rules in formulas, in the belief that those well acquainted with algebraic symbols prefer to have a rule expressed as a formula, as they can thus see at once the different operations to be performed. In the following formulas we denote the diameter of the cylinder in inches by D.

LOCOMOTIVE ENGINE.—PARTS OF THE CYLINDER.

Area of induction ports, in square inches $= {\cdot}068 \times D^2$.
Area of eduction ports, in square inches $= {\cdot}128 \times D^2$.
Breadth of bridge between ports between $\frac{3}{4}$ inch and 1 inch.

LOCOMOTIVE ENGINE.—PARTS OF BOILER.

Diameter of boiler, in inches $= 3{\cdot}11 \times D$.
Length of boiler between 8 feet and 12 feet.
Diameter of steam dome, inside, in inches $= 1{\cdot}43 \times D$.
Height of steam dome $= 2\frac{1}{2}$ feet.
Diameter of safety valve, in inches $= D \div 4$.
Diameter of valve spindle, in inches $= {\cdot}076 \times D$.
Diameter of chimney, in inches $= D$.
Area of fire-grate, in square feet $= {\cdot}77 \times D$.
Area of heating surface, in square feet $= 5 \times D^2 \div 2$.
Area of water level, in square feet $= 2{\cdot}08 \times D$.
Cubical content of water in boiler, in cubic feet $= 9 \times D^2 \div 40$.
Diameter of feed-pump ram, in inches $= {\cdot}011 \times D^2$.
Cubical content of steam room, in cubic feet $= 9 \times D^2 \div 40$.
Cubical content of inside fire-box above fire bars, in cubic feet $= D^2 \div 4$.
Thickness of the plates of boiler $= \frac{3}{8}$ inch.

LOCOMOTIVE ENGINE.—DIMENSIONS OF SEVERAL PIPES.

Inside diameter of steam pipe, in inches $= {\cdot}03 \times D^2$.
Inside diameter of branch steam pipe, in inches $= {\cdot}021 \times D^2$.
Inside diameter of the top of blast pipe $= {\cdot}017 \times D^2$.
Inside diameter of the feed pipes $= {\cdot}141 \times D$.

LOCOMOTIVE ENGINE.—DIMENSIONS OF SEVERAL MOVING PARTS.

Diameter of piston rod, in inches $= D \div 7$.
Thickness of piston, in inches $= 2\,D \div 7$.
Diameter of connecting rods at middle, in inches $= {\cdot}21 \times D$.
Diameter of the ball on cross-head spindle, in inches $= {\cdot}23 \times D$.
Diameter of the inside bearings of the crank axle, in inches $= {\cdot}96 \times \sqrt[3]{D^2}$.
Diameter of the plain part of crank axle, in inches $= {\cdot}96 \times \sqrt[3]{D^2}$.
Diameter of the outside bearings of the crank axle, in inches $= \sqrt[3]{{\cdot}396 \times D^2}$.
Diameter of crank pin, in inches $= {\cdot}404 \times D$.
Length of crank pin, in inches $= {\cdot}233 \times D$.

TABLE *of the Pressure of Steam, in Inches of Mercury, at different Temperatures.*

Temperature, Fahrenheit.	Dalton.	Ure.	Young.	Ivory.	Tredgold.	Southern.	Robison.	Watt.
0°	0·08	...	...	...	...	...	...	...
10	0·12	...	...	...	...	...	...	...
20	0·17	...	0·11	...	...	...	...	...
32	0·26	0·20	0·18	...	0·17	0·16	0·00	...
40	0·34	0·25	0·20	...	0·24	0·22	0·10	...
50	0·49	0·36	0·36	0·36	0·37	0·33	0·20	...
60	0·65	0·52	0·53	...	0·55	0·48	0·35	...
70	0·87	0·73	0·75	0·73	0·78	0·68	0·55	0·77
80	1·16	1·01	1·05	...	1·11	0·95	0·82	...
90	1·59	1·36	1·44	1·36	1·53	1·34	1·18	...
100	2·12	1·86	1·95	...	2·08	1·84	1·60	1·55
110	2·79	2·45	2·62	2·46	2·79	2·56	2·25	...
120	3·63	3·30	3·46	...	3·68	3·46	3·00	...
130	4·71	4·37	4·54	4·41	4·81	4·43	3·95	...
140	6·05	5·78	5·88	...	6·21	5·75	5·15	5·14
150	7·73	7·53	7·55	7·42	7·94	7·46	6·72	...
160	9·79	9·60	9·62	...	10·05	9·52	8·65	8·92
170	12·31	12·05	12·14	12·05	12·60	12·14	11·05	11·37
180	15·38	15·16	15·23	...	15·67	15·20	14·05	12·73
190	18·98	19·00	18·96	18·93	19·00	...	17·85	19·00
200	23·51	23·60	23·44	...	23·71	...	22·65	...
210	28·82	28·88	28·81	28·81	28·86	...	28·62	...
212	30·00	30·00	30·00	30·00	30·00	30·00	30·00	29·40
220	35·18	35·54	35·19	...	34·92	...	35·8	33·65
230	44·60	43·10	42·47	42·68	42·00	...	44·5	40
240	53·45	51·70	51·66	...	50·24	...	54·9	49·0

TABLE *of the Temperature of Steam at different Pressures in Atmospheres.*

Pressure in Atmospheres.	French Academy.	Dr. Ure.	Young.	Ivory.	Tredgold.	Southern.	Robison.	Watt.	Franklin Institute.
1st At.	212·0°	212°	212°	212°	212°	...	212°	212°	212°
2d At.	250·5	250·0	240·3	249	250	250·3	...	252·5	250·0
3d At.	275·2	275·0	271	...	274	...	267	...	275·2
4th At.	293·7	291·5	288	290	294	293·4	...	...	291·5
5th At.	308·8	304·5	302	...	309	...	...	...	304·5
6th At.	320·4	315·5	...	...	322	...	...	...	315·5
7th At.	331·7	325·5	...	...	...	...	...	...	326·5
8th At.	342·0	336·0	...	337	342	343·6	...	...	336·0
9th At.	350·0	345·0	...	...	...	...	...	...	345·0
10th At.	358·9	...	...	...	...	...	...	...	352·5
11th At.	366·8	...	...	...	...	...	...	...	...
12th At.	374·0	...	...	...	372	...	...	...	...
13th At.	380·6	...	...	...	...	...	...	...	...
14th At.	386·9	...	...	...	...	...	...	...	...
15th At.	392·8	...	...	...	...	...	...	...	383·8
16th At.	398·5	...	...	...	...	...	...	...	...
17th At.	403·8	...	...	...	...	...	...	...	...
18th At.	408·9	...	...	...	...	...	...	...	...
19th At.	413·9	...	...	...	...	...	...	...	...
20th At.	418·5	...	...	...	414	...	...	...	405
30th At.	457·2	...	...	...	...	...	...	...	...
40th At.	466·6	...	...	...	...	...	...	...	...
50th At.	510·6	...	...	...	...	...	...	...	...

TABLE *of the Expansion of Air by Heat.*

Fahren.		Fahren.		Fahren.	
32	1000	61	1069	90	1132
33	1002	62	1071	91	1134
34	1004	63	1073	92	1136
35	1007	64	1075	93	1138
36	1009	65	1077	94	1140
37	1012	66	1080	95	1142
38	1015	67	1080	96	1144
39	1018	68	1034	97	1146
40	1021	69	1087	98	1148
41	1023	70	1089	99	1150
42	1025	71	1091	100	1152
43	1027	72	1093	110	1173
44	1030	73	1095	120	1194
45	1032	74	1097	130	1215
46	1034	75	1099	140	1235
47	1036	76	1101	150	1255
48	1038	77	1104	160	1275
49	1040	78	1106	170	1295
50	1043	79	1108	180	1315
51	1045	80	1110	190	1334
52	1047	81	1112	200	1364
53	1050	82	1114	210	1372
54	1052	83	1116	212	1376
55	1055	84	1118	302	1558
56	1057	85	1121	392	1739
57	1059	86	1123	482	1919
58	1062	87	1125	572	2098
59	1064	88	1128	680	2312
60	1066	89	1130		

STRENGTH OF MATERIALS.

The chief materials, of which it is necessary to record the strength in this place, are cast and malleable iron; and many experiments have been made at different times upon each of these substances, though not with any very close correspondence. The following is a summary of them:—

Materials.	C	S	E	M
Iron, cast from	16300	8100	69120000	5530000
Iron, cast to	36000			
—— Malleable	60000	9000	91440000	6770000
—— Wire	80000			

The first column of figures, marked C, contains the mean strength of cohesion on an inch section of the material; the second, marked S, the constant for transverse strains; the third, marked E, the constant for deflections; and the fourth, marked M, the modulus of elasticity. The introduction of the hot blast iron brought with it the impression that it was less strong than that previously in use, and the experiments which had previously been confided in as giving results near enough the truth, for all practical purposes, were no longer considered to be applicable to the new state of things. New experiments were therefore made. The following Table gives, we have no doubt, results as nearly correct as can be required or attained:—

RESULTS OF EXPERIMENTS ON THE STRENGTH AND OTHER PROPERTIES OF CAST IRON.

In the following Table each bar is reduced to exactly one inch square; and the transverse strength, which may be taken as a criterion of the value of each Iron, is obtained from a mean between the experiments upon it;—first on bars 4 ft. 6 in. between the supports; and next on those of half the length, or 2 ft. 3 in. between the supports. All the other results are deduced from the 4 ft. 6 in. bars. In all cases the weights were laid on the middle of the bar.

Name of Iron.	Specific Gravity.	Modulus of elasticity in lbs. per sq. inch, or stiffness.	Breaking weight in lbs. of bars 4 ft. 6 in. between supports.	Breaking weight in lbs. of bars 2 ft. 3 in. reduced to 4 ft. 6 in. between supports.	Mean breaking weight in lbs. (S.)	Ultimate deflection of 4 ft. 6 in. bars, in parts of an inch.	Power of the 4 ft. 6 in. bars to resist impact.	Colour.
Dickerson's, Newark, N. J	7·030	18470000	510	532	600	1·530	991	Gray
Ponkey, No. 3. Cold Blast	7·122	17211000	567	595	581	1·747	992	Whitish gray
Devon, No. 3. Hot Blast*	7·251	22473650	537	—	537	1·09	589	White
Oldberry, No. 3. Hot Blast	7·300	22733400	543	517	530	1·005	549	White
Pattison, N. J. Hot Blast*	7·056	17873100	520	534	527	1·365	710	Whitish gray
Beaufort, No. 3. Hot Blast	7·069	16802000	505	529	517	1·599	807	Dullish gray
Pennsylvanian	7·8	15379500	500	515	502	1·815	889	Dark gray
Bute, No. 1. Cold Blast	7·066	15103000	495	487	491	1·764	872	Bluish gray
Wind Mill End, No. 2. Cold Blast	7·071	16490000	483	495	489	1·581	765	Dark gray
Old Park, No. 2. Cold Blast	7·049	14607000	441	529	485	1·621	718	Gray
Beaufort, No. 2. Hot Blast	7·108	16301000	478	470	474	1·512	729	Dull gray
Low Moor, No. 2. Cold Blast	7·055	14509500	462	483	472	1·852	855	Dark gray
Buffery, No. 1. Cold Blast*	7·079	15381200	463	—	463	1·55	721	Gray
Brimbo, No. 2. Cold Blast	7·017	14911666	466	453	459	1·748	815	Light gray
Apedale, No. 2. Hot Blast	7·017	14852000	457	455	456	1·730	791	Light gray
Oldberry, No. 2. Cold Blast	7·059	14307500	453	457	455	1·811	822	Dark gray
Pentwyn, No. 2	7·038	15193000	438	473	455	1·484	650	Bluish gray
Maesteg, No. 2	7·038	13959500	453	455	454	1·957	886	Dark gray
Muirkirk, No. 1. Cold Blast*	7·113	14003550	443	464	453	1·734	770	Bright gray
Adelphi, No. 2. Cold Blast	7·080	13815500	441	457	449	1·759	777	Light gray
Blania, No. 3. Cold Blast	7·159	14281466	433	464	448	1·726	747	Bright gray
Devon, No. 3. Cold Blast*	7·285	22907700	448	—	448	·790	353	Light gray
Gartsherrie, No. 3. Hot Blast	7·017	13894000	427	467	447	1·557	998	Light gray
Frood, No. 2. Cold Blast	7·031	13112666	460	434	447	1·825	841	Light gray
Lane End, No. 2.	7·028	15787666	444	—	444	1·414	629	Dark gray
Carron, No. 3. Cold Blast*	7·094	16246966	444	443	443	1·336	593	Gray
Dundyvan, No. 3. Cold Blast	7·087	16534000	456	430	443	1·469	674	Dull gray
Maesteg (Marked Red)	7·038	13971500	440	444	442	1·887	830	Bluish gray
Corbyns Hall, No. 2	7·007	13845866	430	454	442	1·687	727	Gray
Pontypool, No. 2	7·080	13136500	439	441	440	1·857	816	Dull blue
Wallbrook, No. 3	6·979	15394766	432	449	440	1·443	625	Light gray
Milton, No. 3. Hot Blast	7·051	15852500	427	449	438	1·368	585	Gray
Buffery, No. 1. Hot Blast*	6·998	13730500	436	—	436	1·64	721	Dull gray
Level, No. 1. Hot Blast	7·080	15452500	461	403	432	1·516	699	Light gray
Pant, No. 2	6·975	15280900	408	455	431	1·251	511	Light gray
Level, No. 2. Hot Blast	7·031	15241000	419	439	429	1·358	570	Dull gray
W. S. S., No. 2	7·041	14953333	413	446	429	1·339	554	Light gray
Eagle Foundry, No. 2. Hot Blast	7·038	14211000	408	446	427	1·512	618	Bluish gray
Elsicar, No. 2. Cold Blast	6·928	12586500	446	408	427	2·224	992	Gray
Varteg, No. 2. Hot Blast	7·007	15012000	422	430	426	1·450	621	Gray
Coltham, No. 1. Hot Blast	7·128	15510066	464	385	424	1·532	716	Whitish gray
Carroll, No. 2. Cold Blast	7·069	17036000	430	408	419	1·231	530	Gray
Muirkirk, No. 1. Hot Blast*	6·953	13294400	417	419	418	1·570	656	Bluish gray
Bierley, No. 2	7·185	16156133	404	432	418	1·222	494	Dark gray
Coed-Talon, No. 2. Hot Blast*	6·969	14322500	409	424	416	1·882	771	Bright gray
Coed-Talon, No. 2. Cold Blast*	6·955	14304000	408	418	413	1·470	600	Gray
Monkland, No. 2. Hot Blast	6·916	12259500	402	404	403	1·762	709	Bluish gray
Ley's Works, No. 1. Hot Blast	6·957	11539333	392	—	392	1·890	742	Bluish gray
Milton, No. 1. Hot Blast	6·976	11974500	353	366	359	1·525	538	Gray
Plaskynaston, No. 2. Hot Blast	6·916	13341633	378	337	357	1·366	517	Light gray

The irons with asterisks are taken from Experiments on Hot and Cold Blast Iron.

Rule.—To find from the above Table the breaking weight in rectangular bars, generally. Calling b and d the breadth and depth in inches, and l the distance between the supports, in feet, and putting 4·5 for 4 ft. 6 in., we have $\frac{4{\cdot}5 \times b\, d^2\, S}{l}$ = breaking weight in lbs.,—the value of S being taken from the above Table.

For example:—What weight would be necessary to break a bar of Low Moor Iron, 2 inches broad, 3 inches deep, and 6 feet between the supports? According to the rule given above, we have $b = 2$ inches, $d = 3$ inches, $l = 6$ feet, $S = 472$ from the Table. Then $\frac{4{\cdot}5 \times b\, d^2\, S}{l} = \frac{4{\cdot}5 \times 2 \times 3^2 \times 472}{6} = 6372$ lbs., the breaking weight.

Table *of the Cohesive Power of Bodies whose Cross Sectional Areas equal one Square Inch.*

Metals.	Cohesive Power in lbs.
Swedish bar iron	65,000
Russian do	59,470
English do	56,000
Cast steel	134,256
Blistered do	133,152
Shear do	127,632
Wrought copper	33,892
Hard gun-metal	36,368
Cast copper	19,072
Yellow brass, cast	17,968
Cast iron	17,628
Tin, cast	4,736
Bismuth, cast	3,250
Lead, cast	1,824
Elastic power or direct tension of wrought iron, medium quality	22,400

Note.—A bar of iron is extended ·000096, or nearly one ten-thousandth part of its length, for every ton of direct strain per square inch of sectional area.

CENTRE OF GRAVITY.

The centre of gravity of a body is that point within it which continually endeavours to gain the lowest possible situation; or it is that point on which the body, being freely suspended, will remain at rest in all positions. The centre of gravity of a body does not always exist within the matter of which the body is composed, there being bodies of such forms as to preclude the possibility of this being the case, but it must either be surrounded by the constituent matter, or so placed that the particles shall be symmetrically situated, with respect to a vertical line in which the position of the centre occurs. Thus, the centre of gravity of a ring is not in the substance of the ring itself, but, if the ring be uniform, it will be in the axis of its circumscribing cylinder; and if the ring varies in form or density, it will be situated nearest to those parts where the weight or density is greatest. Varying the position of a body will not cause any change in the situation of the centre of gravity; for any change of position the body undergoes will only have the effect of altering the directions of the sustaining forces, which will still preserve their parallelism. When a body is suspended by any other point than its centre of gravity, it will not rest unless that centre be in the same vertical line with the point of suspension; for, in every other position, the force which is intended to insure the equilibrium will not directly oppose the resultant of gravity upon the particles of the body, and of course the equilibrium will not obtain; the directions of the forces of gravity upon the constituent particles are all parallel to one another and perpendicular to the horizon. If a heavy body be sustained by two or more forces, their lines of direction must meet either at the centre of gravity, or in the vertical line in which it occurs.

A body cannot descend or fall downwards, unless it be in such a position that by its motion the centre of gravity descends. If a body stands on a plane, and a line be drawn perpendicular to the horizon, and if this perpendicular line fall within the base of the body, it will be supported without falling; but if the perpendicular falls without the base of the body, it will overset. For when the perpendicular falls within the base, the body cannot be moved at all without raising the centre of gravity; but when the perpendicular falls without the base towards any side, if the body be moved towards that side, the centre of gravity will descend, and consequently the body will overset in that direction. If a perpendicular to the horizon from the centre of gravity fall upon the extremity of the base, the body may continue to stand, but the least force that can be applied will cause it to overset in that direction; and the nearer the perpendicular is to any side the easier the body will be made to fall on that side, but the nearer the perpendicular is to the middle of the base the firmer the body will stand. If the centre of gravity of a body be supported, the whole body is supported, and the place of the centre of gravity must be considered as the place of the body, and it is always in a line which is perpendicular to the horizon.

In any two bodies, the common centre of gravity divides the line that joins their individual centres into two parts that are to one another reciprocally as the magnitudes of the bodies. The products of the bodies multiplied by their respective distances from the common centre of gravity are equal. If a weight be laid upon any point of an inflexible lever which is supported at the ends, the pressure on each point of the support will be inversely as the respective distances from the point where the weight is applied. In a system of three bodies, if a line be drawn from the centre of gravity of any one of them to the common centre of the other two, then the common centre of all the three bodies divides the line into two parts that are to each other reciprocally as the magnitude of the body from which the line is drawn to the sum of the magnitudes of the other two; and, consequently, the single body multiplied by its distance from the common centre of gravity is equal to the sum of the other bodies multiplied by the distance of their common centre from the common centre of the system.

If there be taken any point in the straight line or lever joining the centres of gravity of two bodies, the sum of the two products of each body multiplied by its distance from that point is equal to the product of the sum of the bodies multiplied by the distance of their common centre of gravity from the same point. The two bodies have, therefore, the same tendency to turn the lever about the assumed point, as if they were both placed in their common centre of gravity. Or, if the line with the bodies moves about the assumed point, the sum of the momenta is equal to the momentum of the sum of the bodies placed at their common centre of gravity. The same property holds with respect to any number of bodies whatever, and also when the bodies are not placed in the line, but in perpendiculars to it passing through the bodies. If any plane pass through the assumed point, perpendicular to the line in which it subsists, then the distance of the common centre of gravity of all the bodies from that plain is equal to the sum of all the momenta divided by the sum of all the bodies. We may here specify the positions of the centre of gravity in several figures of very frequent occurrence.

In a straight line, or in a straight bar or rod of uniform figure and density, the position of the centre of gravity is at the middle of its length. In the plane of a triangle the centre of gravity is situated in the straight line drawn from any one of the angles to the middle of the opposite side, and at two-thirds of this line distant from the angle where it originates, or one-third distant from the base. In the surface of a trapezium the centre of gravity is in the intersections of the straight lines that join the centres of the opposite triangles made by the two diagonals. The centre of gravity of the surface of a parallelogram is at the intersection of the diagonals, or at the intersection of the two lines which bisect the figure from its opposite sides. In any regular polygon the centre of gravity is at the same point as the centre of magnitude. In a circular arc the position of the centre of gravity is distant from the centre of the circle by the measure of a fourth proportional to the arc, radius, and chord. In a semicircular arc the position of the centre of gravity is distant from the centre by the measure of a third proportional to the arc of the quadrant and the radius. In the sector of a circle the position of the centre of gravity is distant from the centre of the circle by a fourth proportional to three times the arc of the sector, the chord of the arc, and the diameter of the circle. In a circular segment, the position of the centre of gravity is distant from the centre of the circle by a space which is equal to the cube or third power of the chord divided by twelve times the area of the segment. In a semicircle the position of the centre of gravity is distant from the centre of the circle by a space which is equal to four times the radius divided by the constant number $3{\cdot}1416 \times 3 = 9{\cdot}4248$. In a parabola the position of the centre of gravity is distant from the vertex by three-fifths of the axis. In a semi-parabola the position of the centre of gravity is at the intersection of the co-ordinates, one of which is parallel to the base, and distant from it by two-fifths of the axis, and the other parallel to the axis, but distant from it by three-eighths of the semi-base.

The centres of gravity of the surface of a cylinder, a cone, and conic frustum, are respectively at the same distances from the origin as are the centres of gravity of the parallelogram, the triangle, and the trapezoid, which are sections passing along the axes of the respective solids. The centre of gravity of the surface of a spheric segment is at the middle of the versed sine or height. The centre of gravity of the convex surface of a spherical zone is at the middle of that portion of the axis of the sphere intercepted by its two bases. In prisms and cylinders the position of the centre of gravity is at the middle of the straight line that joins the centres of gravity of their opposite ends. In pyramids and cones the centre of gravity is in the straight line that joins the vertex with the centre of gravity of the base, and at three-fourths of its length from the vertex, and one-fourth from the base. In a semisphere, or semispheroid, the position of the centre of gravity is distant from the centre by three-eighths of the radius. In a parabolic conoid the position of the centre of gravity is distant from the base by one-third of the axis, or two-thirds of the axis distant from the vertex. There are several other bodies and figures of which the position of the centre of gravity is known; but as the position in those cases cannot be defined without algebra, we omit them.

CENTRIPETAL AND CENTRIFUGAL FORCES.

Central forces are of two kinds, *centripetal* and *centrifugal*. *Centripetal force* is that force by which a body is attracted or impelled towards a certain fixed point as a centre, and that point towards which the body is urged is called the *centre of attraction* or the *centre of force.* *Centrifugal force* is that force by which a body endeavours to recede from the centre of attraction, and from which it would actually fly off in the direction of a tangent if it were not prevented by the action of the centripetal force. These two forces are therefore antagonistic; the action of the one being directly opposed to that of the other. It is on the joint action of these two forces that all curvilinear motion depends. Circular motion is that affection of curvilinear motion where the body is constrained to move in the circumference of a circle: if it continues to move so as to describe the entire circle, it is denominated *rotatory motion*, and the body is said to revolve in a circular orbit, the centre of which is called the centre of motion. In all circular motions the deflection or deviation from the rectilinear course is constantly the same at every point of the orbit, in which case the centripetal and centrifugal forces are equal to one another. In circular orbits the centripetal forces, by which equal bodies placed at equal distances from the centres of force are attracted or drawn towards those centres, are proportional to the quantities of matter in the central bodies. This is manifest, for since all attraction takes place towards some particular body, every particle in the attracting body must produce its individual effect; consequently, a body containing twice the quantity of matter will exert twice the attractive energy, and a body containing thrice the quantity of matter will operate with thrice the attractive force, and so on according to the quantity of matter in the attracting body.

Any body, whether large or small, when placed at the same distance from the centre of force, is attracted or drawn through equal spaces in the same time by the action of the central body. This is obvious from the consideration that although a body two or three times greater is urged with two or three times greater an attractive force, yet there is two or three times the quantity of matter to be moved; and, as we have shown elsewhere, the velocity generated in a given time is directly proportional to the force by which it is generated, and inversely as the quantity of matter in the moving or attracted body. But the force which in the present instance is the weight of the body is proportional to the quantity of matter which it contains; consequently, the velocity generated is directly and inversely proportional to the quantity of matter in the attracted body, and is, therefore, a given or a constant quantity. Hence, the centripetal force, or force towards the centre of the circular orbit, is not measured by the magnitude of the revolving body, but only by the space which it describes or passes over in a given time. When a body revolves in a circular orbit, and is retained in it by means of a centripetal force directed to the centre, the actual velocity of the revolving body at every point of its revolution is equal to that which it would acquire by falling perpendicularly with the same uniform force through one-fourth of the diameter, or one-half the radius of its orbit; and this velocity is the same as would be acquired by a second body in falling through half the radius, whilst the first body, in revolving in its orbit, describes a portion of the circumference which is equal in length to half the diameter of the circle. Consequently, if a body revolves uniformly in the circumference of a circle by means of a given centripetal force, the portion of the circumference which it describes in any time is a mean proportional between the diameter of the circle and the space which the body would descend perpendicularly in the same time, and with the same given force continued uniformly.

The *periodic time*, in the doctrine of central forces, is the time occupied by a body in performing a complete revolution round the centre, when that body is constrained to move in the circumference by means of a centripetal force directed to that point; and when the body revolves in a circular orbit, the periodic time, or the time of performing a complete revolution, is expressed by the term $\pi t \sqrt{\frac{d}{s}}$, and the velocity or space passed over in the time t will be $\sqrt{ds}$; in which expressions d denotes the diameter of the circular orbit described by the revolving body, s the space descended in any time by a body falling perpendicularly downwards with the same uniform force, t the time of descending through the space, s and π the circumference of a circle whose diameter is unity. If several bodies revolving in circles round the same or different centres be retained in their orbits by the action of centripetal forces directed to those points, the periodic times will be directly as the square roots of the radii or distances of the revolving bodies, and inversely as the square roots of the centripetal forces, or, what is the same thing, the squares of the periodic times are directly as the radii, and inversely as the centripetal forces.

CENTRE OF GYRATION.

The *centre of gyration* is that point in which, if all the constituent particles, or all the matter contained in a revolving body, or system of bodies, were concentrated, the same angular velocity would be generated in the same time by a given force acting at any place as would be generated by the same force acting similarly on the body or system itself according to its formation.

The *angular motion* of a body, or system of bodies, is the motion of a line connecting any point with the centre or axis of motion, and is the same in all parts of the same revolving system.

In different unconnected bodies, each revolving about a centre, the angular velocity is directly proportional to the absolute velocity, and inversely as the distance from the centre of motion; so that, if the absolute velocities of the revolving bodies be proportional to their radii or distances, the angular velocities will be equal. If the axis of motion passes through the centre of gravity, then is this centre called the principal centre of gyration.

The distance of the centre of gyration from the point of suspension, or the axis of motion in any body or system of bodies, is a geometrical mean between the centres of gravity and oscillation from the same point or axis; consequently, having found the distances of these centres in any proposed case, the square root of their product will give the distance of the centre of gyration. If any part of a system be conceived to be collected in the centre of gyration of that particular part, the centre of gyration of the whole system will continue the same as before; for the same force that moved this part of the system before along with the rest will move it now without any change; and consequently, if each part of the system be collected into its own particular centre, the common centre of the whole system will continue the same. If a circle be described about the centre of gravity of any system, and the axis of rotation be made to pass through any point of the circumference, the distance of the centre of gyration from that point will always be the same.

If the periphery of a circle revolve about an axis passing through the centre, and at right angles to its plane, it is the same thing as if all the matter were collected into any one point in the periphery. And moreover, the plane of a circle or a disk containing twice the quantity of matter as the said periphery, and having the same diameter, will in an equal time acquire the same angular velocity. If the matter of a revolving body were actually to be placed in the centre of gyration, it ought either to be arranged in the circumference, or in two points of the circumference diametrically opposite to each other, and equally distant from the centre of motion, for by this means the centre of motion will coincide with the centre of gravity, and the body will revolve without any lateral force on any side. These are the chief properties connected with the centre of gyration, and the following are a few of the cases in which its position has been ascertained.

In a right line, or a cylinder of very small diameter revolving about one of its extremities, the distance of the centre of gyration from the centre of motion is equal to the length of the revolving line or cylinder multiplied by the square root of $\frac{1}{3}$. In the plane of a circle, or a cylinder revolving about the axis, it is equal to the radius multiplied by the square root of $\frac{1}{2}$. In the circumference of a circle revolving about the diameter it is equal to the radius multiplied by the square root of $\frac{1}{2}$. In the plane of a circle revolving about the diameter it is equal to one-half the radius. In a thin circular ring revolving about one of its diameters as an axis it is equal to the radius multiplied by the square root of $\frac{1}{2}$. In a solid globe revolving about the diameter it is equal to the radius multiplied by the square root of $\frac{2}{5}$. In the surface of a sphere revolving about the diameter it is equal to the radius multiplied by the square root of $\frac{2}{3}$. In a right cone revolving about the axis it is equal to the radius of the base multiplied by the square root of $\frac{3}{10}$. In all these cases the distance is estimated from the centre of the axis of motion. We shall have occasion to illustrate these principles when we come to treat of fly-wheels in the construction of the different parts of steam engines.

When bodies revolving in the circumferences of different circles are retained in their orbits by centripetal forces directed to the centres, the periodic times of revolution are directly proportional to the distances or radii of the circles, and inversely as the velocities of motion; and the periodic times, under like circumstances, are directly as the velocities of motion, and inversely as the centripetal forces. If the times of revolution are equal, the velocities and centripetal forces are directly as the distances or radii of the circles. If the centripetal forces are equal, the squares of the times of revolution and the squares of the velocities are as the distances or radii of the circles. If the times of revolution are as the radii of the circles, the velocities will be equal, and the centripetal forces reciprocally as the radii.

If several bodies revolve in circular orbits round the same or different centres, the velocities are directly as the distances or radii, and inversely as the times of revolution. The velocities are directly as the centripetal forces and the times of revolution. The squares of the velocities are proportional to the centripetal forces, and the distances or radii of the circles. When the velocities are equal, the times of revolution are proportional to the radii of the circles in which the bodies revolve, and the radii of the circles are inversely as the centripetal forces. If the velocities be proportional to the distances or radii of the circles, the centripetal forces will be in the same ratio, and the times of revolution will be equal.

If several bodies revolve in circular orbits about the same or different centres, the centripetal forces are proportional to the distances or radii of the circles directly, and inversely as the squares of the times of revolution. The centripetal forces are directly proportional to the velocities, and inversely as the times of revolution. The centripetal forces are directly as the squares of the velocities, and inversely as the distances or radii of the circles. When the centripetal forces are equal, the velocities are proportional to the times of revolution, and the distances as the squares of the times or as the squares of the velocities. When the central forces are proportional to the distances or radii of the circles, the times of revolution are equal. If several bodies revolve in circular orbits about the same or different centres, the radii of the circles are directly proportional to the centripetal forces, and the squares of the periodic times. The distances or radii of the circles are directly as the velocities and periodic times. The distances or radii of the circles are directly as the squares of the velocities, and reciprocally as the centripetal forces. If the distances are equal, the centripetal forces are directly as the squares of the velocities, and reciprocally as the squares of the times of revolution; the velocities also are reciprocally as the times of revolution. The converse of these principles and properties are equally true; and all that has been here stated in regard to centripetal forces is similarly true of centrifugal forces, they being equal and contrary to each other.

The quantities of matter in all attracting bodies, having other bodies revolving about them in circular orbits, are proportional to the cubes of the distances directly, and to the squares of the times of revolution reciprocally. The attractive force of a body is directly proportional to the quantity of matter, and inversely as the square of the distance. If the centripetal force of a body revolving in a circular orbit be proportional to the distance from the centre, a body let fall from the upper extremity of the vertical diameter will reach the centre in the same time that the revolving body describes one-fourth part of the orbit. The velocity of the descending body at any point of the diameter is proportional to the ordinate of the circle at that point; and the time of falling through any portion of the diameter is proportional to the arc of the circumference whose versed sine is the space fallen through. All the times of falling from any altitudes whatever to the centre of the orbit will be equal; for these times are equal to one-fourth of the periodic times, and these times, under the specified conditions, are equal. The velocity of the descending body at the centre of the circular orbit is equal to the velocity of the revolving body.

These are the chief principles that we need consider regarding the motion of bodies in circular orbits; and from them we are led to the consideration of bodies suspended on a centre, and made to revolve in a circle beneath the suspending point, so that when the body describes the circumference of a circle, the string or wire by which it is suspended describes the surface of a cone. A body thus revolving is called a *conical pendulum*, and this species of pendulum, or, as it is usually termed, the *governor*, is of great importance in mechanical arrangements, being employed to regulate the movements of steam engines, water-wheels, and other mechanism. As we shall have occasion to show the construction and use of this instrument when treating of the parts and proportions of engines, we need not do more at present than state the principles on which its action depends. We must, however, previously say a few words on the properties of the simple pendulum, or that which, being suspended from a centre, is made to vibrate from side to side in the same vertical plane.

PENDULUMS.

If a pendulum vibrates in a small circular arc, the time of performing one vibration is to the time occupied by a heavy body in falling perpendicularly through half the length of the pendulum as the circumference of a circle is to its diameter. All vibrations of the same pendulum made in very small circular arcs, are made in very nearly the same time. The space described by a falling body in the time of one vibration is to half the length of the pendulum as the square of the circumference of a circle is to the square of the diameter. The lengths of two pendulums which by vibrating describe similar circular arcs are to each other as the squares of the times of vibration. The times of pendulums vibrating in small circular arcs are as the square roots of the lengths of the pendulums. The velocity of a pendulum at the lowest point of its path is proportional to the chord of the arc through which it descends to acquire that velocity. Pendulums of the same length vibrate in the same time, whatever the weights may be. From which we infer, that all bodies near the earth's surface, whether they be heavy or light, will fall through equal spaces in equal times, the resistance of the air not being considered.

The lengths of pendulums vibrating in the same time in different positions of the earth's surface are as the forces of gravity in those positions. The times wherein pendulums of the same length will vibrate by different forces of gravity are inversely as the square roots of the forces. The lengths of pendulums vibrating in different places are as the forces of gravity at those places and the squares of the times of vibration. The times in which pendulums of any length perform their vibrations are directly as the square roots of their lengths, and inversely as the square roots of the gravitating forces. The forces of gravity at different places on the earth's surface are directly as the lengths of the pendulums, and inversely as the squares of the times of vibration. These are the chief properties of a simple pendulum vibrating in a vertical plane, and the principal problems that arise in connection with it are the following, viz.:

To find the length of a pendulum that shall make any number of vibrations in a given time; and secondly, *having given the length of a pendulum, to find the number of vibrations it will make in any time given.*—These are problems of very easy solution, and the rules for resolving them are simply as follow:—For the first, the rule is, multiply the square of the number of seconds in the given time by the constant number 39·1015, and divide the product by the square of the number of vibrations, for the length of the pendulum in inches. For the second, it is, multiply the square of the number of seconds in the given time by the constant number 39·1393, divide the product by the given length of the pendulum in inches, and extract the square root of the quotient for the number of vibrations sought. The number 39·1015 is the length of a pendulum in inches, that vibrates seconds, or sixty times in a minute, in the latitude of Philadelphia.

Suppose a pendulum is found to make 35 vibrations in a minute; what is the distance from the centre of suspension to the centre of oscillation?

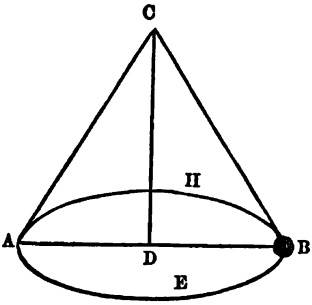
Here, by the rule, the number of seconds in the given time is 60; hence we get $60 \times 60 \times 39{\cdot}1015 = 140765{\cdot}4$, which, being divided by $35 \times 35 = 1225$, gives $140765{\cdot}4 \div 1225 = 114{\cdot}9105$ inches for the length required.

The length of a pendulum between the centre of suspension and the centre of oscillation is 64 inches; what number of vibrations will it make in 60 seconds?

By the rule we have $60 \times 60 \times 39{\cdot}1015 = 140765{\cdot}4$, which, being divided by 64, gives $140765{\cdot}4 \div 64 = 2199{\cdot}46$, and the square root of this is $2199{\cdot}46 = 46{\cdot}9$, number of vibrations sought. When the given time is a minute, or 60 seconds, as in the two examples proposed above, the product of the constant number 39·1015 by the square of the time, or 140765·4, is itself a constant quantity, which, being kept in mind, will in some measure facilitate the process of calculation in all similar cases. We now return to the consideration of the conical pendulum, or that in which the ball revolves about a vertical axis in the circumference of a circular plane which is parallel to the horizon.

CONICAL PENDULUM.

If a pendulum be suspended from the upper extremity of a vertical axis, and be made to revolve about that axis by a conical motion, which constrains the revolving body to move in the circumference of a circle whose plane is parallel to the horizon, then the time in which the pendulum performs a revolution about the axis can easily be found.



Let CD be the pendulum in question, suspended from C, the upper extremity of the vertical axis CD, and let the ball or body B, by revolving about the said axis, describe the circle BE AH, the plane of which is parallel to the horizon; it is proposed to assign the time of description, or the time in which the body B performs a revolution about the axis CD, at the distance BD.

Conceive the axis CD to denote the weight of the revolving body, or its force in the direction of gravity; then, by the Composition and Resolution of Forces, CB will denote the force or tension of the string or wire that retains the revolving body in the direction CB, and BD the force tending to the centre of the plane of revolution at D. But, by the general laws of motion and forces previously laid down, if the time be given, the space described will be directly proportional to the force; but, by the laws of gravity, the space fallen perpendicularly from rest, in one second of time, is $g = 16\frac{1}{12}$ feet; consequently we have CD : BD : : $16\frac{1}{12} : \frac{16\frac{1}{12}.\mathrm{BD}}{\mathrm{CD}}$, the space described towards D by the force in BD in one second. Consequently, by the laws of centripetal forces, the periodic time, or the time of the body revolving in the circle BEAH, is expressed by the term $\pi\sqrt{\frac{2\cdot\mathrm{CD}}{16\frac{1}{12}}}$, where $\pi = 3\cdot1416$, the circumference of a circle whose diameter is unity; or putting t to denote the time, and expressing the height CD in feet, we get $t = 6\cdot2832\sqrt{\frac{\mathrm{CD}}{12 \times 32\frac{1}{6}}}$, or, by reducing the expression to its simplest form, it becomes $t = 0\cdot31986\sqrt{\mathrm{CD}}$, where CD must be estimated in inches, and t in seconds. Here we have obtained an expression of great simplicity, and the practical rule for reducing it may be expressed in words as follows:

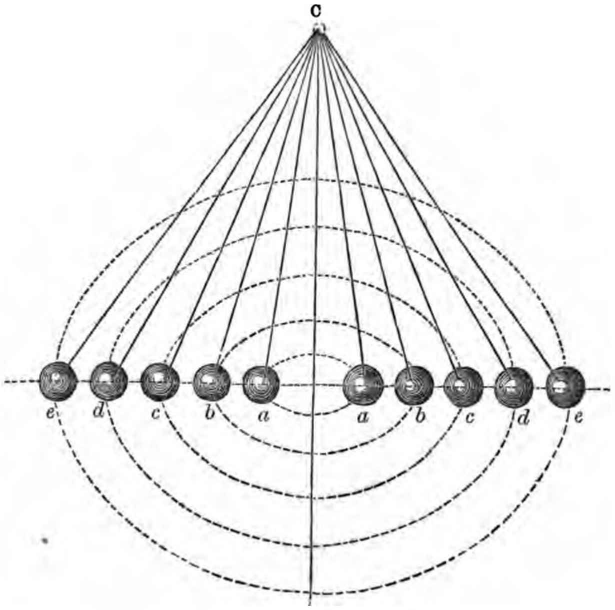
Rule.—Multiply the square root of the height, or the distance between the point of suspension and the centre of the plane of revolution, in inches, by the constant fraction 0·31986, and the product will be the time of revolution in seconds.

In what time will a conical pendulum revolve about its vertical axis, supposing the distance between the point of suspension and the centre of the plane of revolution to be 39·1393 inches, which is the length of a simple pendulum that vibrates seconds in latitude 51° 30′?

The square root of 39·1393 is 6·2561; consequently, by the rule, we have, $6·2561 \times 0·31986 = 2·0011$ seconds for the time of revolution sought. It consequently revolves 30 times in a minute, as it ought to do by the theory of the simple pendulum.

By reversing the process, the height of the cone, or the distance between the point of suspension and the centre of the plane of revolution, corresponding to any given time, can easily be ascertained; for we have only to divide the number of seconds in the given time by the constant decimal 0·31986, and the square of the quotient will be the required height in inches. Thus, suppose it were required to find the height of a conical pendulum that would revolve 30 times in a minute. Here the time of revolution is 2 seconds for $60 \div 30 = 2$; therefore, by division, it is $2 \div 0·31986 = 6·2527$, which, being squared, gives $6·2527 = 39·0961$ inches, or the length of a simple pendulum that vibrates seconds very nearly. In all conical pendulums the times of revolution, or the periodic times, are proportional to the square roots of the heights of the cones. This is manifest, for in the foregoing equation of the periodic time the numbers 6·2832 and 386, or $12 \times 32\frac{1}{6}$, are constant quantities, consequently t varies as $\sqrt{\mathrm{CD}}$.

If the heights of the cones, or the distances between the points of suspension and the centres of the planes of revolution, be the same, the periodic times, or the times of revolution, will be the same, whatever may be the radii of the circles described by the revolving bodies. This will be clearly understood by contemplating the subjoined diagram, where all the pendulums Ca, Cb, Cc, Cd, and Ce, having the common axis CD, will revolve in the same time; and



if they are all in the same vertical plane when first put in motion, they will continue to revolve in that plane, whatever be the velocity, so long as the common axis or height of the cone remains the same. This will become manifest, if we conceive an inflexible bar or rod of iron to pass through the centres of all the balls as well as the common axis, for then the bar and the several balls must all revolve in the same time; but if any one of them should be allowed to rise higher, its velocity would be increased; and if it descends, the velocity will be decreased.

Half the periodic time of a conical pendulum is equal to the time of vibration of a simple pendulum, the length of which is equal to the axis or height of the cone; that is, the simple pendulum makes two oscillations or vibrations from side to side, or it arrives at the same point from which it departed, in the same time that the conical pendulum revolves about its axis. The space descended by a falling body in the time of one revolution of the conical pendulum is equal to $3{\cdot}1416^2$ multiplied by twice the height or axis of the cone. The periodic time, or the time of one revolution is equal to the product of $3{\cdot}1416 \sqrt{2}$ multiplied by the time of falling through the height of the cone. The weight of a conical pendulum, when revolving in the circumference of a circle, bears the same proportion to the centrifugal force, or its tendency to fly off in a straight line, as the axis or height of the cone bears to the radius of the plane of revolution; consequently, when the height of the cone is equal to the radius of its base, the centripetal or centrifugal force is equal to the power of gravity.

These are the principles on which the action of the conical pendulum depends; but as we shall hereafter have occasion to consider it more at large, we need not say more respecting it in this place. Before dismissing the subject, however, it may be proper to put the reader in possession of the rules for calculating the position of the centre of oscillation in vibrating bodies, in a few cases where it has been determined, these being the cases that are of the most frequent occurrence in practice.

The *centre of oscillation* in a vibrating body is that point in the line of suspension, in which, if all the matter of the system were collected, any force applied there would generate the same angular motion in a given time as the same force applied at the centre of gravity. The centres of oscillation for several figures of very frequent use, suspended from their vertices and vibrating flatwise, are as follow :—

In a right line, or parallelogram, or a cylinder of very small diameter, the centre of oscillation is at two-thirds of the length from the point of suspension. In an isosceles triangle the centre of oscillation is at three-fourths of the altitude. In a circle it is five-fourths of the radius. In the common parabola it is five-sevenths of its altitude. In a parabola of any order it is $\left(\frac{2\,n + 1}{3\,n + 1}\right) \times$ altitude, where n denotes the order of the figure.

In bodies vibrating laterally, or in their own plane, the centres of oscillation are situated as follows; namely, in a circle the centre of oscillation is at three-fourths of the diameter; in a rectangle, suspended at one of its angles, it is at two-thirds of the diagonal; in a parabola, suspended by the vertex, it is five-sevenths of the axis, increased by one-third of the parameter; in a parabola, suspended by the middle of its base, it is four-sevenths of the axis, increased by half the parameter; in the sector of a circle it is three times the arc of the sector multiplied by the radius, and divided by four times the chord; in a right cone it is four-fifths of the axis or height, increased by the quotient that arises when the square of the radius of the base is divided by five times the height; in a globe or sphere it is the radius of the sphere, plus the length of the thread by which it is suspended, plus the quotient that arises when twice the square of the radius is divided by five times the sum of the radius and the length of the suspending thread. In all these cases the distance is estimated from the point of suspension, and since the centres of oscillation and percussion are in one and the same point, whatever has been said of the one is equally true of the other.

THE TEMPERATURE AND ELASTIC FORCE OF STEAM.

In estimating the mechanical action of steam, the intensity of its elastic force must be referred to some known standard measure, such as the pressure which it exerts against a square inch of the surface that contains it, usually reckoned by so many pounds avoirdupois upon the square inch. The intensity of the elastic force is also estimated by the inches in height of a vertical column of mercury, whose weight is equal to the pressure exerted by the steam on a surface equal to the base of the mercurial column. It may also be estimated by the height of a vertical column of water measured in feet; or generally, the elastic force of any fluid may be compared with that of atmospheric air when in its usual state of temperature and density; this is equal to a column of mercury 30 inches or 2½ feet in height.

When the temperature of steam is increased, respect being had to its density, the elastic force, or the effort to separate the parts of the containing vessel and occupy a larger space, is also increased; and when the temperature is diminished, a corresponding and proportionate diminution takes place in the intensity of the emancipating effort or elastic power. It consequently follows that there must be some law or principle connecting the temperature of steam with its elastic force; and an intimate acquaintance with this law, in so far as it is known, must be of the greatest importance in all our researches respecting the theory and the mechanical operations of the *steam engine.*

To find a theorem, by means of which it may be ascertained when a general law exists, and to determine what that law is, in cases where it is known to obtain.—Suppose, for example, that it is required to assign the nature of the law that subsists between the temperature of steam and its elastic force, on the supposition that the elasticity is proportional to some power of the temperature, and unaffected by any other constant or co-efficient, except the exponent by which the law is indicated. Let E and e be any two values of the elasticity, and T, t, the corresponding temperatures deducted from observation. It is proposed to ascertain the powers of T and t, to which E and e are respectively proportional. Let n denote the index or exponent of the required power; then by the conditions of the problem admitting that a law exists, we get, $T^n : t^n :: E : e$; but by the principles of proportion, it is $\frac{t^n}{T^n} = \frac{e}{E}$; and if this be expressed logarithmically, it is $n \times \log. \frac{t}{T} = \log. \frac{e}{E}$, and by reducing the equation in respect of n, it finally becomes

$$n = \frac{\log. e - \log. E}{\log. t - \log. T}.$$

The theorem that we have here obtained is in its form sufficiently simple for practical application; it is of frequent occurrence in physical science, but especially so in inquiries respecting the motion of bodies moving in air and other resisting media; and it is even applicable to the determination of the planetary motions themselves. The process indicated by it in the case that we have chosen, is simply, *To divide the difference of the logarithms of the elasticities by the difference of the logarithms of the corresponding temperatures, and the quotient will express that power of the temperature to which the elasticity is proportional.*

Take as an example the following data:—In two experiments it was found that when the temperature of steam was 250·3 and 343·6 degrees of Fahrenheit's scale, the corresponding elastic forces were 59·6 and 238·4 inches of the mercurial column respectively. From these data it is required to determine the law which connects the temperature with the elastic force on the supposition that a law does actually exist under the specified conditions. The process by the rule is as follows:

Greater temperature, 343·6	log. 2·5352941
Lesser temperature, 250·3	log. 2·3984608
Remainder	= 0·1368333
Greater elastic force, 238·4	log. 2·3773063
Lesser elastic force, 59·6	log. 1·7752463
Remainder	= 0·6020600

Let the second of these remainders be divided by the first, as directed in the rule, and we get $n = 6020600 \div 1368333 = 4·3998$, the exponent sought. Consequently, by taking the nearest unit, for the sake of simplicity, we shall have, according to this result, the following analogy, viz.:

$$T^{4·4} : t^{4·4} :: E : e;$$

that is, the elasticities are proportional to the 4·4-power of the temperatures very nearly.

Now this law is rigorously correct, as applied to the particular cases that furnished it; for if the two temperatures and one elasticity be given, the other elasticity will be found as indicated by the above analogy; or if the two elasticities and one temperature be given, the other temperature will be found by a similar process. It by no means follows, however, that the principle is general, nor could we venture to affirm that the exponent here obtained will accurately represent the result of any other experiments than those from which it is deduced, whether the temperature be higher or lower than that of boiling water; but this we learn from it, that the index which represents the law of elasticity is of a very high order, and that the general equation, whatever its form may be, must involve other conditions than those which we have assumed in the foregoing investigation. The theorem, however, is valuable to practical men, not only as being applicable to numerous other branches of mechanical inquiry, but as leading directly to the methods by which some of the best rules have been obtained for calculating the elasticity of steam, when in contact with the liquid from which it is generated.

We now proceed to apply our formula to the determination of a general law, or such as will nearly represent the class of experiments on which it rests; and for this purpose we must first assign the limits, and then inquire under what conditions the limitations take place, for by these limitations we must in a great measure be guided in determining the ultimate form of the equation which represents the law of elasticity.

The limits of elasticity will be readily assigned from the following considerations, viz.: In the first place, it is obvious that steam cannot exist when the cohesive attraction of the particles is of greater intensity than the repulsive energy of the caloric or matter of heat interposed between them; for in this case, the change from an elastic fluid to a solid may take place without passing through the intermediate stage of liquidity: hence we infer that there must be a temperature at which the elastic force is nothing, and this temperature, whatever may be its value, corresponds to the lower limit of elasticity. The higher limit will be discovered by similar considerations, for it must take place when the density of steam is the same as that of water, which therefore depends on the *modulus of elasticity* of water. The modulus of elasticity of any substance is the measure of its elastic force; that of water at 60° of temperature is 22,100 atmospheres. Thus, for instance, suppose a given quantity of water to be confined in a close vessel which it exactly fills, and let it be exposed to a high degree of temperature, then it is obvious that in this state no steam would be produced, and the force which is exerted to separate the parts of the vessel is simply the expansive force of compressed water; we therefore have the following proportion. As the expanded volume of water is to the quantity of expansion, so is the modulus of elasticity of water to the elastic force of steam of the same density as water.

Having therefore assigned the limits beyond which the elastic force of steam cannot reach, we shall now proceed to apply the principle of our formula to the determination of the general law which connects the temperature with the elastic force; and for this purpose, in addition to the notation which we have already laid down, let c denote some constant quantity that affects the elasticity, and d the temperature at which the elasticity vanishes; then since this temperature must be applied subtractively, we have from the foregoing principle, $c\,E = (T - \delta)^n$, and $c\,e = (t - \delta)^n$. From either of these equations, therefore, the constant quantity c can be determined in terms of the rest when they are known; thus we have $c = \frac{(T - \delta)^n}{E}$, and $c = \frac{(t - \delta)^n}{e}$, and by comparing these two independent values of c, the value of n becomes known; for $\frac{(T - \delta)^n}{E} = \frac{(t - \delta)^n}{e}$, and consequently

$$n = \frac{\log. e - \log. E}{\log. (t - \delta) - \log. (T - \delta).} \quad . \; . \; . \; . \; (A).$$

In this equation the value of the symbol δ is unknown; in order therefore to determine it, we must have another independent expression for the value of n; and in order to this, let the elasticities E and e become E$'$ and e' respectively; while the corresponding temperatures T and t assume the values T$'$ and t'; then by a similar process to the above, we get $\frac{(T' - \delta)^n}{E'} = \frac{(t' - \delta)}{e'}$, and

$$n = \frac{\log. e' - \log. E'}{\log. (t' - \delta) - \log. (T' - \delta)} \quad . \; . \; . \; (B).$$

Let the equations (A) and (B) be compared with each other, and we shall then have an expression involving only the unknown quantity δ, for it must be understood that the several temperatures with their corresponding elasticities are to be deduced from experiment; and in consequence, the law that we derive from them must be strictly empirical; thus we have

$$\frac{\log. e - \log. E.}{\log. (t - \delta) - \log. (T - \delta)} = \frac{\log. e' - \log. E}{\log. (t' - \delta) - \log. (T' - \delta)} \quad . \; . \; (C).$$

We have no direct method of reducing expressions of this sort, and the usual process is therefore by approximation, or by the rule of trial and error, and it is in this way that the value of the quantity δ must be found; and for the purpose of performing the reduction, we shall select experiments performed with great care, and may consequently be considered as representing the law of elasticity with very great nicety.

$T = 212{\cdot}0$ Fahrenheit $E = 29{\cdot}8$ inches of mercury.
$t = 250{\cdot}3$ $e = 59{\cdot}6$
$T' = 293{\cdot}4$ $E' = 119{\cdot}2$
$t' = 343{\cdot}6$ $e' = 238{\cdot}4$

Therefore, by substituting these numbers in equation (C), and making a few trials, we find that $\delta = -50°$, and substituting this in either of the equations (A) or (B), we get $n = 5{\cdot}08$; and finally, by substituting these values of δ and n in either of the expressions for the constant quantity c, we get $c = 64674730000$, the 5·08 root of which is 134·27 very nearly; hence we have

$$F = \left\{ \frac{t + 50}{134{\cdot}27} \right\}^{5{\cdot}08} \quad . \quad . \quad . \quad . \text{(D)}.$$

Where the symbol F denotes generally the elastic force of the steam in inches of mercury, and t the corresponding temperature in degrees of Fahrenheit's thermometer, the logarithm of the denominator of the fraction is 2·1279717, which may be used as a constant in calculating the elastic force corresponding to any given temperature. We have thus discovered a rule of a very simple form; it errs in defect; but this might have been remedied by assuming two points near one extremity of the range of experiment, and two points near the other extremity; and by substituting the observed numbers in equation (C), different constants and a more correct exponent would accordingly have been obtained. Mr. Southern has, by pursuing a method somewhat analogous to that which is here described, found his experiments to be very nearly represented by

$$F = \left\{ \frac{t + 51{\cdot}3}{135{\cdot}767} \right\}^{5{\cdot}13}$$

But even here the formula errs in defect, for he has found it necessary to correct it by adding the arbitrary decimal 0·1; and thus modified, it becomes

$$F = \left\{ \frac{t + 51.3}{135{\cdot}767} \right\}^{5{\cdot}13} + 0{\cdot}1. \quad . \quad . \quad . \quad . \text{(E)}.$$

Our own formula may also be corrected by the application of some arbitrary constant of greater magnitude; but as our motive for tracing the steps of investigation in the foregoing case was to exemplify the method of determining the law of elasticity, our end is answered; for we consider it a very unsatisfactcry thing merely to be put in possession of a formula purporting to be applicable to some particular purpose, without at the same time being put in possession of the method by which that formula was obtained, and the principles on which it rests. Having thus exhibited the principles and the method of reduction, the reader will have greater confidence as regards the consistency of the processes that he may be called upon to perform. The operation implied by equation (E) may be expressed in words as follows:—

Rule.—To the given temperature in degrees of Fahrenheit's thermometer add 51·3 degrees and divide the sum by 135·767; to the 5·13 power of the quotient add the constant fraction $\frac{1}{10}$, and the sum will be the elastic force in inches of mercury.

The process here described is that which is performed by the rules of common arithmetic; but since the index is affected by a fraction, it is difficult to perform in that way: we must therefore have recourse to logarithms as the only means of avoiding the difficulty. The rule adapted to these numbers is as follows:—

RULE FOR LOGARITHMS.—To the given temperature in degrees of Fahrenheit's thermometer add 51·3 degrees; then, from the logarithm of the sum subtract 2·1327940 or the logarithm of 135·767, the denominator of the fraction; multiply the remainder by the index 5·13, and to the natural number answering to the sum add the constant fraction $\frac{1}{10}$; the sum will be the elastic force in inches of mercury.

If the temperature of steam be 250·3 degrees as indicated by Fahrenheit's thermometer, what is the corresponding elastic force in inches of mercury?

$$\begin{array}{lrl}
\text{By the rule it is } 250{\cdot}3 + 51{\cdot}3 = 301{\cdot}6 & \text{log. } 2{\cdot}4794313 & \\
\text{constant den.} = 135{\cdot}767 & \text{log. } \underline{2{\cdot}1327940} & \text{subtract} \\
\text{remainder} = & 0{\cdot}3466373 & \\
& \underline{31{\cdot}5} & \text{inverted} \\
& 17331865 & \\
& 346637 & \\
& \underline{103991} & \\
\text{natural number } 60{\cdot}013 & \text{log. } 1{\cdot}7782493 &
\end{array}$$

If this be increased by $\frac{1}{10}$, we get 60·113 inches of mercury for the elastic force of steam at 250·3 degrees of Fahrenheit.

By simply reversing the process or transposing equation (E), the temperature corresponding to any given elastic force can easily be found; the transformed expression is as follows, viz.:

$$t = 135{\cdot}767\,(F - 0{\cdot}1)^{\frac{1}{5{\cdot}13}} - 51{\cdot}3 \quad . \quad . \quad . \quad . \quad (F).$$

Since, in consequence of the complicated index, the process of calculation cannot easily be performed by common arithmetic, it is needless to give a rule for reducing the equation in that way; we shall therefore at once give the rule for performing the process by logarithms.

RULE.—From the given elastic force in inches of mercury, subtract the constant fraction 0·1; divide the logarithm of the remainder by 5·13, and to the quotient add the logarithm 2·1327940; find the natural number answering to the sum of the logarithms, and from the number thus found subtract the constant 51·3, and the remainder will be the temperature sought.

Supposing the elastic force of steam or the vapour of water to be equivalent to the weight of a vertical column of mercury, the height of which is 238·4 inches; what is the corresponding temperature in degrees of Fahrenheit's thermometer?

Here, by proceeding as directed in the rule, we have 238·4 − 0·1 =

238·3, and dividing the logarithm of this remainder by the constant exponent 5·13, we get

log. 238·3 ÷ 5·13 = 2·3771240 ÷ 5·13 = 0·4633770
constant co-efficient = 135·767 - - log. 2·1327940 add
natural number = 394·61 - - - log. 2·5961710 sum
constant temperature = 51·3 subtract
required temperature = 343·31 degrees of Fahrenheit's thermometer.

The temperature by observation is 343·6 degrees, giving a difference of only 0·29 of a degree in defect. For low temperature or low pressure steam, that is, steam not exceeding the simple pressure of the atmosphere, M. Pambour gives

$$p = 0{\cdot}04948 + \left(\frac{t + 51{\cdot}3}{155{\cdot}7256}\right)^{5{\cdot}13} \quad . \ . \ . \ . \ (G).$$

In which equation the symbol p denotes the pressure in pounds avoirdupois per square inch, and t the temperature in degrees of Fahrenheit's thermometer. When this expression is reduced in reference to temperature, it is

$$t = 155{\cdot}7256\,(p - 0{\cdot}04948)^{\frac{1}{5{\cdot}13}} - 51{\cdot}3 \quad . \ . \ . \ . \ (H).$$

The formula of Tredgold is well known. The equation, in its original form, is

$$177 f^{\frac{1}{6}} = t + 100 \quad . \ . \ . \ . \ (I):$$

where f denotes the elastic force of steam in inches of mercury, and t the temperature in degrees of Fahrenheit's thermometer. The same formula, as modified and corrected by M. Millet, becomes

$$179{\cdot}0773 f^{\frac{1}{6}} = t + 103 \quad . \ . \ . \ . \ (K).$$

Dr. Young of Dublin constructed a formula which was adapted to the experiments of his countryman Dr. Dalton: it assumed a form sufficiently simple and elegant; it is thus expressed—

$$f = (1 + 0.0029\,t)^7 \quad . \ . \ . \ . \ (L):$$

where the symbol f denotes the elastic force of steam expressed in atmospheres of 30 inches of mercury, and t the temperature in degrees estimated above 212 of Fahrenheit. This formula is not applicable in practice, especially in high temperatures, as it deviates very widely and rapidly from the results of observation: it is chiefly remarkable as being made the basis of a numerous class of theorems somewhat varied, but of a more correct and satisfactory character. The Commission of the French Academy represented their experiments by means of a formula constructed on the same principles: it is thus expressed—

$$f = (1 + 0{\cdot}7153\,t)^5 \quad . \ . \ . \ . \ (M):$$

where f denotes the elastic force of the steam expressed in atmospheres of 0·76 metres or 29·922 inches of mercury, and t the temperature estimated above 100 degrees of the centigrade thermometer; but when the same formula is so transformed as to be expressed in the usual terms adopted in practice, it is

$$p = (0{\cdot}2679 + 0{\cdot}0067585\,t)^5 \quad . \quad . \quad . \quad . \quad (\text{N}):$$

where p is the pressure in pounds per square inch, and t the temperature in degrees of Fahrenheit's scale, estimated above 212 or simple atmospheric pressure.

The committee of the Franklin Institute adopted the exponent 6, and found it necessary to change the constant 0·0029 into 0·00333; thus modified, they represented their experiments by the equation

$$p = (0{\cdot}460467 + 0{\cdot}00521478\,t)^6 \quad . \quad . \quad . \quad . \quad (\text{O}).$$

By combining Dr. Dalton's experiments with the mean between those of the French Academy and the Franklin Institute, we obtain the following equations, the one being applicable for temperatures below 212 degrees, and the other for temperatures above that point as far as 50 atmospheres. Thus, for low pressure steam, that is, for steam of less temperature than 212, it is

$$f = \left(\frac{t + 175}{387}\right)^{7{\cdot}71307} \quad . \quad . \quad . \quad . \quad (\text{P}):$$

and for steam above the temperature of 212, it is

$$f = \left(\frac{t + 121}{333}\right)^{6{\cdot}42} \quad . \quad . \quad . \quad . \quad (\text{Q}).$$

In consequence therefore of the high and imposing authority from which these formulas are deduced, we shall adopt them in all our subsequent calculations relative to the steam engine; and in order to render their application easy and familiar, we shall translate them into rules in words at length, and illustrate them by the resolution of appropriate numerical examples; and for the sake of a systematic arrangement, we think proper to branch the subject into a series of problems, as follows:

The temperature of steam being given in degrees of Fahrenheit's thermometer, to find the corresponding elastic force in inches of mercury.—The problem, as here propounded, is resolved by one or other of the last two equations, and the process indicated by the arrangement is thus expressed:—

RULE.—To the given temperature expressed in degrees of Fahrenheit's thermometer, add the constant temperature 175; find the logarithm answering to the sum, from which subtract the constant 2·587711; multiply the remainder by the index 7·71307, and the product will be the logarithm of the elastic force in atmospheres of 30 inches of mercury when the given temperature is less than 212 degrees. But when the temperature is greater than 212, increase it by 121; then, from the logarithm of the temperature thus increased, subtract the constant logarithm 2·522444, multiply the remainder by the exponent 6·42, and the product will be the logarithm of the elastic force in atmospheres of 30 inches of mercury; which being multiplied by 30 will give the force in inches, or if multiplied by 14·76 the result will be expressed in pounds avoirdupois per square inch.

When steam is generated under a temperature of 187 degrees of Fahrenheit's thermometer, what is its corresponding elastic force in atmospheres of 30 inches of mercury?

In this example, the given temperature is less than 212 degrees: it will therefore be resolved by the first clause of the preceding rule, in which the additive constant is 175; hence we get

$$\begin{array}{rl} 187 + 175 = 362\ldots\text{log.} & 2{\cdot}558709 \\ \text{Constant divisor} = 387\ldots\text{log.} & \underline{2{\cdot}587711}\ \text{subtract} \\ & 9{\cdot}970998 \times 7{\cdot}71307 = 9{\cdot}773393 \end{array}$$

And the corresponding natural number is 0·5934 atmospheres, or 17·802 inches of mercury, the elastic force required, or if expressed in pounds per square inch, it is 0·5934 × 14·76 = 8·76 lbs. very nearly. If the temperature be 250 degrees of Fahrenheit, the process is as follows:

$$\begin{array}{rl} 250 + 121 = 371\ldots\text{log.} & 2{\cdot}569374 \\ \text{Constant divisor} = 333\ldots\text{log.} & \underline{2{\cdot}522444}\ \text{subtract} \\ & 0{\cdot}046930 \times 6{\cdot}42 = 0{\cdot}301291 \end{array}$$

And the corresponding natural number is 2·0012 atmospheres, or 60·036 inches of mercury, and in pounds per square inch it is 2·0012 × 14·76 = 29·54 lbs. very nearly.

It is sometimes convenient to express the results in inches of mercury, without a previous determination in atmospheres, and for this purpose the rule is simply as follows:

Rule.—Multiply the given temperature in degrees of Fahrenheit's thermometer by the constant coefficient 1·5542, and to the product add the constant number 271·985; then from the logarithm of the sum subtract the constant logarithm 2·587711, and multiply the remainder by the exponent 7·71307; the natural number answering to the product, considered as a logarithm, will give the elastic force in inches of mercury. This answers to the case when the temperature is less than 212 degrees; but when it is above that point proceed as follows:

Multiply the given temperature in degrees of Fahrenheit's thermometer by the constant coefficient 1·69856, and to the product add the constant number 205·526; then from the logarithm of the sum subtract the constant logarithm 2·522444, and multiply the remainder by the exponent 6·42; the natural number answering to the product considered as a logarithm, will give the elastic force in inches of mercury. Take, for example, the temperatures as assumed above, and the process, according to the rule, is as follows:

187 × 1·5542 = 290·6354
Constant = 271·985 add
Sum = 562·6204...log. 2·750216
Constant = 387..........log. 2·587711 subtract
0·162505 × 7·71307 = 1·253408

And the natural number answering to this logarithm is 17·923 inches of mercury. By the preceding calculation the result is 17·802; the slight difference arises from the introduction of the decimal constants, which in consequence of not terminating at the proper place are taken to the nearest unit in the last figure, but the process is equally true notwithstanding. For the higher temperature, we get

250 × 1·69856 = 424·640
Constant = 205·526 add
Sum = 630.166......log. 2·799456
Constant = 333............log. 2·522444 subtract
0·277011 × 6·42 = 1·778410

And the natural number answering to this logarithm is 60·036 inches of mercury, agreeing exactly with the result obtained as above.

It is moreover sometimes convenient to express the force of the steam in pounds per square inch, without a previous determination in atmospheres or inches of mercury; and when the equations are modified for that purpose, they supply us with the following process, viz.:

Multiply the given temperature by the constant coefficient 1·41666, and to the product add the constant number 247·9155; then, from the logarithm of the sum subtract the constant logarithm 2·587711, and multiply the remainder by the index 7·71307; the natural number answering to the product will give the pressure in pounds per square inch, when the temperature is less than 212 degrees; but for all greater temperatures the process is as follows:

Multiply the given temperature by the constant coefficient 1·5209, and to the product add the constant number 184·0289; then, from the logarithm of the sum subtract the constant logarithm 2·522444, and multiply the remainder by the exponent 6·42; the natural or common number answering to the product, will express the force of the steam in pounds per square inch. If any of these results be multiplied by the decimal 0·7854, the product will be the corresponding pressure in pounds per circular inch. Taking, therefore, the temperatures previously employed, the operation is as follows:

187 × 1·41666 = 264·9155
Constant = 247·9155 add
Sum = 512.8310.log. 2·709974
Constant = 387........log. 2·587711 subtract
0·122263 × 7·71307 = 0·942656

And the number answering to this logarithm is 8·763 lbs. per square inch, and 8·763 × 0·7854 = 6·8824 lbs. per circular inch, the proportion in the two cases being as 1 to 0·7554. Again, for the higher temperature, it is

$$\begin{array}{rl} 250 \times 1{\cdot}5209 = & 380{\cdot}2250 \\ \text{Constant} = & \underline{184{\cdot}0289} \text{ add} \\ \text{Sum} = & 564{\cdot}2539 \ldots\ldots \text{log. } 2{\cdot}751475 \\ \text{Constant} = & 333 \ldots\ldots\ldots\ldots \text{log. } \underline{2{\cdot}522444} \text{ subtract} \\ & \qquad\qquad\qquad 0{\cdot}229031 \times 6{\cdot}42 = 1{\cdot}470279 \end{array}$$

And the number answering to this logarithm is 29·568 lbs. per square inch, or 29568 × 0·7854 = 23·2226 lbs. per circular inch.

We have now to reverse the process, and determine the temperature corresponding to any given power of the steam, and for this purpose we must so transpose the formulas (P) and (Q), as to express the temperature in terms of the elastic force, combined with given constant numbers; but as it is probable that many of our readers would prefer to see the theorems from which the rules are deduced, we here subjoin them.

For the lower temperature, or that which does not exceed the temperature of boiling water, we get

$$t = 249 f^{\frac{1}{7.71307}} - 175 \quad . \; . \; . \; . \quad \text{(R)}.$$

Where t denotes the temperature in degrees of Fahrenheit's thermometer, and f the elastic force in inches of mercury, less than 30 inches, or one atmosphere; but when the elastic force is greater than one atmosphere, the formula for the corresponding temperature is as follows:

$$t = 196 f^{\frac{1}{6{\cdot}42}} - 121 \quad . \; . \; . \; . \quad \text{(S)}.$$

In the construction of these formulas, we have, for the sake of simplicity, omitted the fractions that obtain in the coefficient of f; for since they are very small, the omission will not produce an error of any consequence; indeed, no error will arise on this account, as we retain the correct logarithms, a circumstance that enables the computer to ascertain the true value of the coefficients whenever it is necessary so to do; but in all cases of actual practice, the results derived from the integral coefficients will be quite sufficient. The rule supplied by the equations (R) and (S) is thus expressed:

When the elastic force is less than the pressure of the atmosphere, that is, less than 30 inches of the mercurial column,—

RULE.—Divide the logarithm of the given elastic force in inches of mercury, by the constant index 7·71307, and to the quotient add the constant logarithm 2·396204; then from the common or natural number answering to the sum, subtract the constant temperature 175 degrees, and the remainder will be the temperature sought in degrees of Fahrenheit's thermometer. But when the elastic force exceeds 30 inches, or one atmosphere, the following rule applies:

Divide the logarithm of the given elastic force in inches of mercury by the constant index 6·42, and to the quotient add the constant logarithm 2·292363: then, from the natural number answering to the sum subtract the constant temperature 121 degrees, and the remainder will be the temperature sought. Similar rules might be constructed for determining the temperature, when the pressure in pounds per square inch is given; but since this is a less useful case of the problem, we have thought proper to omit it. We therefore proceed to exemplify the above rules, and for this purpose we shall suppose the pressure in the two cases to be equivalent to the weight of 19 and 60 inches of mercury respectively. The operations will therefore be as follows:

Log. 19 ÷ 7·71307 = 1·278754 ÷ 7·71307 = 0·165791
Constant coefficient = 249....................log. 2·396204 add

Natural number = 364·75.................log. 2·561994
Constant temperature = 175 subtract

Required temperature = 189·75 degrees of Fahrenheit's scale.

For the higher elastic force the operation is as follows:

Log. 60 ÷ 6·42 = 1·778151 ÷ 6·42 = 0·276969
Constant coefficient = 196 ....log. 2·292363 add

Natural number = 370·97............log. 2·569332
Constant temperature = 121 subtract

Required temperature = 249·97 degrees of Fahrenheit's scale.

All the preceding results, as computed by our rules, agree as nearly with observation as can be desired: but they have all been obtained on the supposition that the steam is in contact with the liquid from which it is generated; and in this case it is evident that the steam must always attain an elastic force corresponding to the temperature; and in accordance to any increase of pressure, supposing the temperature to remain the same, a quantity of it corresponding to the degree of compression must simply be condensed into water, and in consequence will leave the diminished space occupied by steam of the original degree of tension; or otherwise to express it, if the temperature and pressure invariably correspond with each other, it is impossible to increase the density and elasticity of the steam except by increasing the temperature at the same time; and, contrariwise, the temperature cannot be increased without at the same time increasing the elasticity and density. This being admitted, it is obvious that under these circumstances the steam must always maintain its maximum of pressure and density: but if it be separated from the liquid that produces it, and if its temperature in this case be increased, it will be found not to possess a higher degree of elasticity than a volume of atmospheric air similarly confined, and heated to the same temperature. Under this new condition, the state of maximum density and elasticity ceases; for it is obvious that since no water is present, there cannot be any more steam generated by an increase of temperature; and consequently the force of the steam is only that which confines it to its original bulk, and is measured by the effort which it exerts to expand itself. Our next object, therefore, is to inquire what is the law of elasticity of steam under the conditions that we have here specified.

The specific gravity of steam, its density, and the volume which it occupies at different temperatures, have been determined by experiment with very great precision; and it has also been ascertained that the expansion of vapour by means of heat is regulated by the same laws as the expansion of the other gases, viz. that all gases expand from unity to 1·375 in bulk by 180 degrees of temperature; and again, that steam obeys the law discovered by Boyle and Mariotte, contracting in volume in proportion to the degree of pressure which it sustains. We have therefore to inquire what space a given quantity of water converted into steam will occupy at a given pressure; and from thence we can ascertain the specific gravity, density, and volume at all other pressures.

When a gas or vapour is submitted to a constant pressure, the quantity which it expands by a given rise of temperature is calculated by the following theorem,

$$v' = v\left(\frac{t' + 459}{t + 459}\right)\text{............(T)}$$

where t and t' are the temperatures, and v, v' the corresponding volumes before and after expansion; hence this rule.

RULE.—To each of the temperatures before and after expansion, add the constant experimental number 459; divide the greater sum by the lesser, and multiply the quotient by the volume at the lower temperature, and the product will give the expanded volume.

If the volume of steam at the temperature of 212 degrees of Fahrenheit be 1711 times the bulk of the water that produces it, what will be its volume at the temperature of 250·3 degrees, supposing the pressure to be the same in both cases?

Here, by the rule, we have $212 + 459 = 671$, and $250{\cdot}3 + 459 = 709{\cdot}3$; consequently, by dividing the greater by the lesser, and multiplying by the given volume, we get $\frac{709{\cdot}3}{671} \times 1711 = 1808{\cdot}66$ for the volume at the temperature of 250·3 degrees.

Again, if the elastic force at the lower temperature and the corresponding volume be given, the elastic force at the higher temperature can readily be found; for it is simply as the volume the vapour occupies at the lower temperature is to the volume at the higher temperature, or what it would become by expansion, so is the elastic force given to that required.

If the volume which steam occupies under any given pressure and temperature be given, the volume which it will occupy under any proposed pressure can readily be found by reversing the preceding process, or by referring to chemical tables containing the specific gravity of the gases compared with air as unity at the same pressure and temperature. Now, air at the mean state of the atmosphere has a specific gravity of $1\frac{2}{9}$ as compared with water at 1000; and the bulks are inversely as the specific gravities, according to the general laws of the properties of matter previously announced; hence it follows that air is 818 times the bulk of an equal weight of water, for $1000 \div 1\frac{2}{9} = 818{\cdot}18$. But, by the experiments of Dr. Dalton, it has been found that steam of the same pressure and temperature has a specific gravity of ·625 compared with air as unity; consequently, we have only to divide the number 818·18 by ·625, and the quotient will give the proportion of volume of the vapour to one of the liquid from which it is generated; thus we get $818{\cdot}18 \div {\cdot}625 = 1309$; that is, the volume of steam at 60 degrees of Fahrenheit, its force being 30 inches of mercury, is 1309 times the volume of an equal weight of water; hence it follows, from equation (T), that when the temperature increases to t', the volume becomes

$$v' = 1309 \times \left(\frac{459 + t'}{459 + 60}\right) = 2{\cdot}524(459 + t');$$

and from this expression, the volume corresponding to any specified elastic force f, and temperature t', may easily be found; for it is inversely as the compressing force: that is,

$$f : 30 :: 2{\cdot}525(459 + t') : v';$$

consequently, by working out the analogy, we get

$$v = \frac{75{\cdot}67(459 + t')}{f} \quad \ldots \ldots \quad (\mathrm{U}).$$

By this theorem is found the volume of steam as compared with that of the water producing it, when under a pressure corresponding to the temperature. The rule in words is as follows:

RULE.—Calculate the elastic force in inches of mercury by the rule already given for that purpose, and reserve it for a divisor. To the given temperature add the constant number 459, and multiply the sum by 75·67; then divide the product by the reserved divisor, and the quotient will give the volume sought.

When the temperature of steam is 250·3 degrees of Fahrenheit's thermometer, what is the volume, compared with that of water?

The temperature being greater than 212 degrees, the force is calculated by the rule to equation (Q), and the process is as follows:

$$\begin{array}{lll}
250{\cdot}3 + 121 = 371{\cdot}3 & \text{log. } 2{\cdot}5697249 & \\
\text{Constant divisor} = 333 & \text{log. } \underline{2{\cdot}5224442} & \text{subtract} \\
 & 0{\cdot}0472807 \times 6{\cdot}42 = 0{\cdot}3035421 & \\
\text{Atmosphere} = 30 \text{ inches of mercury} & \text{log. } \underline{1{\cdot}4771213} & \text{add}
\end{array}$$

$$\left.\begin{array}{ll}
\text{Elastic force} = 60{\cdot}348 & \text{log. } 1{\cdot}7806634 \\
\text{Again it is,} & \\
\left.\begin{array}{l} 459 + 250{\cdot}3 = 709{\cdot}3 \text{ log. } 2{\cdot}8508300 \\ \text{Constant coefficient} = 75{\cdot}67 \text{ log. } 1{\cdot}8789237 \end{array}\right\}\text{add} & \underline{4{\cdot}7297537}
\end{array}\right\}\text{sub.}$$

Volume = 889·39 times that of water, log. 2·9490903 remainder.

Thus we have given the method of calculating the elastic force of steam when the temperature is given either in atmospheres or inches of mercury, and also in pounds or the square or circular inch: we have also reversed the process, and determined the temperature corresponding to any given elastic force. We have, moreover, shown how to find the volume corresponding to different temperatures, when the pressure is constant; and, finally, we have calculated the volume, when under a pressure due to the elastic force. These are the chief subjects of calculation as regards the properties of steam; and we earnestly advise our readers to render themselves familiar with the several operations. The calculations as regards the motion of steam in the parts of an engine to produce power, will be considered in another part of the present treatise.

The equation (U), we may add, can be exhibited in a different form involving only the temperature and known quantities; for since the expressions (P) and (Q) represent the elastic force in terms of the temperature, according as it is under or above 212 degrees of Fahrenheit, we have only to substitute those values of the elastic force when reduced to inches of mercury, instead of the symbol *f* in equation (U), and we obtain, when the temperature is less than 212 degrees,

$$\text{Vol.} = 75{\cdot}67(\text{tem.} + 459) \div ({\cdot}004016 \times \text{tem.} + {\cdot}702807)^{7{\cdot}71307} \text{. (V).}$$

and when the temperature exceeds 212 degrees, the expression becomes

$$\text{Vol.} = 75{\cdot}67(\text{tem.} + 459) \div {\cdot}005101 \times \text{tem.} + {\cdot}617195)^{6{\cdot}42} \text{ . (W.)}$$

These expressions are simple in their form, and easily reduced; but, in pursuance of the plan we have adopted, it becomes necessary to express the manner of their reduction in words at length, as follows:

RULE.—When the given temperature is under 212 degrees, multiply the temperature in degrees of Fahrenheit's thermometer by the constant fraction ·004016, and to the product add the constant increment ·702807; multiply the logarithm of the sum by the index 7·71307, and find the natural or common number answering to the product, which reserve for a divisor. To the temperature add the constant number 459, and multiply the sum by the coefficient 75·67 for a dividend; divide the latter result by the former, and the quotient will express the volume of steam when that of water is unity.

Again, when the given temperature is greater than 212 degrees, multiply it by the fraction ·005101, and to the product add the constant increment ·617195; multiply the logarithm of the sum by the index 6·42, and reserve the natural number answering to the product for a divisor; find the dividend as directed above, which, being divided by the divisor, will give the volume of steam when that of the water is unity.

How many cubic feet of steam will be supplied by one cubic foot of water, under the respective temperatures of 187 and 293·4 degrees of Fahrenheit's thermometer?

Here, by the rule, we have

$$
\begin{array}{lr}
 & 187\times0{\cdot}004016=0{\cdot}750992 \\
\text{Constant increment} & =\underline{0{\cdot}702807} \\
 & \text{Sum }=1{\cdot}453799\ \log.{\cdot}1625043\times7{\cdot}71307=1{\cdot}2534069
\end{array}
$$

and the number answering to this logarithm is 17·92284, the divisor. But 187 + 459 = 646, and 646 × 75·67 = 48882·82, the dividend; hence, by division, we get 48882·82 ÷ 17·92284 = 2727·4 cubic feet of steam from one cubic foot of water.

Again, for the higher temperature, it is

$$
\begin{array}{lr}
 & 293{\cdot}4\times0{\cdot}005101=1{\cdot}496633 \\
\text{Constant increment} & =\underline{0{\cdot}617195} \\
 & \text{Sum }=2{\cdot}113828\ \log.\ 0{\cdot}3250696\times642=2{\cdot}0869468;
\end{array}
$$

and the number answering to this logarithm is 122·165, the divisor. But 293·4 + 459 = 752·4, and 752·4 × 75·67 = 56934·108, the dividend; therefore, by division, we get 56934·108 ÷ 122·165 = 466·04 cubic feet of steam from one cubic foot of water.

The preceding is a very simple process for calculating the volume which the steam of a cubic foot of water will occupy when under a pressure due to a given temperature and elastic force; and since a knowledge of this particular is of the utmost importance in calculations connected with the steam engine, it is presumed that our readers will find it to their advantage to render themselves familiar with the method of obtaining it. The above example includes both cases of the problem, a circumstance which gives to the operation, considered as a whole, a somewhat formidable appearance: but it would be difficult to conceive a case in actual practice where the application of both the formulas will be required at one and the same time; the entire process must therefore be considered as embracing only one of the cases above exemplified; and consequently it can be performed with the greatest facility by every person who is acquainted with the use of logarithms; and those unacquainted with the application of logarithms ought to make themselves masters of that very simple mode of computation.

Another thing which it is necessary sometimes to discover in reasoning on the properties of steam as referred to its action in a steam engine, is the weight of a cubic foot, or any other quantity of it, expressed in grains, corresponding to a given temperature and pressure. Now, it has been ascertained by experiment, that when the temperature of steam is 60 degrees of Fahrenheit, and the pressure equal to 30 inches of mercury, the weight of a cubic foot in grains is 329·4; but the weight is directly proportional to the elastic force, for the elastic force is proportional to the density: consequently, if f denote any other elastic force, and w the weight in grains corresponding thereto, then we have

$$30 : f :: 329{\cdot}4 : w = 10{\cdot}98\,f,$$

the weight of a cubic foot of vapour at the force f, and temperature 60 degrees of Fahrenheit. Let t denote the temperature at the force f; then by equation (T), we have $v = \frac{459 + t}{459 + 60} = \frac{459 + t}{519}$, the volume at the temperature t, supposing the volume at 60 degrees to be unity; that is, one cubic foot. Now, since the densities are inversely proportional to the spaces which the vapour occupies, we have $\frac{(459 + t)}{519} : 1 :: w : w' = \frac{519w}{459 + t}$; but by the preceding analogy, the value of w is $10{\cdot}98f$; therefore, by substitution, we get

$$w' = \frac{5698{\cdot}62f}{459 + t} \quad . \quad . \quad . \quad . \quad (X).$$

This equation expresses the weight in grains of a cubic foot of steam at the temperature t and force f; and if we substitute the value of f, from equations (P) and (Q), reduced to inches of mercury, and modified for the two cases of temperature below and above 212 degrees of Fahrenheit, we shall obtain, in the first case,

$$w' = (0{\cdot}012324 \times \text{temp.} + 2{\cdot}155611)^{7{\cdot}71307} \div (\text{temp.} + 459) \ldots \text{(Y)}$$

and for the second case, where the temperature exceeds 212, it is

$$w' = (0{\cdot}01962 \times \text{temp.} + 2{\cdot}37374)^{6{\cdot}42} \div (\text{temp.} + 459) \ldots \text{(Z)}$$

These two equations, like those marked (V) and (W) are sufficiently simple in their form, and offer but little difficulty in their application. The rule for their reduction when expressed in words at length, is as follows:

RULE.—When the temperature is less than 212 degrees, multiply the given temperature, in degrees of Fahrenheit's thermometer, by the fraction 0·012324, and to the product add the constant increment 2·155611; then multiply the logarithm of the sum by the index 7·71307, and from the product subtract the logarithm of the temperature, increased by 459; the natural number answering to the remainder will be the weight of a cubic foot in grains.

Again, when the temperature exceeds 212, multiply it by the fraction 0·01962, and to the product add the constant increment 2·37374; then multiply the logarithm of the sum by the index 6·42, and from the product subtract the logarithm of the temperature increased by 459; the natural number answering to the remainder will be the weight of a cubic foot in grains.

Supposing the temperatures to be as in the preceding example, what will be the weight of a cubic foot in grains for the two cases?

Here, by the rule, we have

187 × 0·012324 = 2·304588
Constant increment = 2·155611
Sum = 4·460199 log. 0·6493542 × 7·71307 = 5·0085143
187 + 459 = 646 . . . . . . log. 2·8102325, subtract
Natural number = 157·863 grains per cubic foot log. 2·1982818

For the higher temperature, it is

293·4 × 0·01962 = 5·756508

Constant increment = 2·373740

Sum = 8·130248 log. 0·9101038 × 6·42 = 5·8428664

293·4 + 459 = 752·4 . . . . log. 2·8764488, subtract

Natural number = 925·59 grains per cubic foot . log. 2·9664176

Here again the operation resolves both cases of the problem; but in practice only one of them can be required.

THE MOTION OF ELASTIC FLUIDS.

The next subject that claims our attention is the velocity with which elastic fluids or vapours move in pipes or confined passages. It is a well-known fact in the doctrine of pneumatics, that the motion of free elastic fluids depends upon the temperature and pressure of the atmosphere; and, consequently, when an elastic fluid is confined in a close vessel, it must be similarly circumstanced with regard to temperature and pressure as it would be in an atmosphere competent to exert the same pressure upon it. The simplest and most convenient way of estimating the motion of an elastic fluid is to assign the height of a column of uniform density, capable of producing the same pressure as that which the fluid sustains in its state of confinement; for under the pressure of such a column, the velocity into a perfect vacuum will be the same as that acquired by a heavy body in falling through the height of the homogeneous column, a proper allowance being made for the contraction at the aperture or orifice through which the fluid flows.

When a passage is opened between two vessels containing fluids of different densities, the fluid of greatest density rushes out of the vessel that contains it, into the one containing the rarer fluid, and the velocity of influx at the first instant of the motion is equal to that which a heavy body acquires in falling through a certain height, and that height is equal to the difference of two uniform columns of the fluid of greatest density, competent to produce the pressures under which the fluids are originally confined; and the velocity of motion at any other instant is proportional to the square root of the difference between the heights of the uniform columns producing the pressures at that instant. Hence we infer that the velocity of motion continually decreases,—the density of the fluids in the two vessels approaching nearer and nearer to an equality, and after a certain time an equilibrium obtains, and the velocity of motion ceases.

It is abundantly confirmed by observation and experiment, that oblique action produces very nearly the same effect in the motion of elastic fluids through apertures as it does in the case of water; and it has moreover been ascertained that eddies take place under similar circumstances, and these eddies must of course have a tendency to retard the motion: it therefore becomes necessary, in all the calculations of practice, to make some allowance for the retardation that takes place in passing the orifice; and this end is most conveniently answered by modifying the constant coefficient according to the nature of the aperture through which the motion is made. Numerous experiments have been made to ascertain the effect of contraction in orifices of different forms and under different conditions, and amongst those which have proved the most successful in this respect, we may mention the experiments of Du Buat and Eytelwein, the latter of whom has supplied us with a series of coefficients, which, although not exclusively applicable to the case of the steam engine, yet, on account of their extensive utility, we take the liberty to transcribe. They are as follow:—

1. For the velocity of motion that would result from the direct unretarded action of the column of the fluid that produces it, we have .... $8\,V = \sqrt{579h}$
2. For an orifice or tube in the form of the contracted vein .... $10\,V = \sqrt{6084h}$
3. For wide openings having the sill on a level with the bottom of the reservoir ...
4. For sluices with walls in a line with the orifice .... $10\,V = \sqrt{5929h}$
5. For bridges with pointed piers ....
6. For narrow openings having the sill on a level with the bottom of the reservoir ...
7. For small openings in a sluice with side walls .... $10\,V = \sqrt{4761h}$
8. For abrupt projections ....
9. For bridges with square piers ....
10. For openings in sluices without side walls $10\,V = \sqrt{2601h}$
11. For openings or orifices in a thin plate ..... $V = \sqrt{25h}$
12. For a straight tube from 2 to 3 diameters in length projecting outwards .... $10\,V = \sqrt{4225}$
13. For a tube from 2 to 3 diameters in length projecting inwards .... $10\,V = \sqrt{2976{\cdot}25h}$

It is necessary to observe, that in all these equations V is the velocity of motion in feet per second, and h the height of the column producing it, estimated also in feet. Nos. 1, 2, 11, 12, and 13 are those which more particularly apply to the usual passages for the steam in a steam engine; but since all the others meet their application in the every-day practice of the civil engineer, we have thought it useful to supply them.

MOTION OF STEAM IN AN ENGINE.

We have already stated that the best method of estimating the motion of an elastic fluid, such as steam or the vapour of water, is to assign the height of a uniform column of that fluid capable of producing the pressure: the determination of this column is therefore the leading step of the inquiry; and since the elastic force of steam is usually reckoned in inches of mercury, 30 inches being equal to the pressure of the atmosphere, the subject presents but little difficulty; for we have already seen that the height of a column of water of the temperature of 60 degrees, balancing a column of 30 inches of mercury, is 34·023 feet; the corresponding column of steam must therefore be as its relative bulk and elastic force; hence we have $30 : 34{\cdot}023 :: f v : h = 1{\cdot}1341 f v$, where f is the elastic force of the steam in inches of mercury, v the corresponding volume or bulk when that of water is unity, and h the height of a uniform column of the fluid capable of producing the pressure due to the elastic force; consequently, in the case of a direct unretarded action, the velocity into a perfect vacuum, according to No. 1 of the preceding class of formulas, is $V = 8{\cdot}542 \sqrt{f v}$; but for the best form of pipes, or a conical tube in form of the contracted vein, the velocity into a vacuum, according to No. 2, becomes $V = 8{\cdot}307 \sqrt{f v}$; and for pipes of the usual construction, No. 12 gives $V = 6{\cdot}922 \sqrt{f v}$; No. 13 gives $V = 5{\cdot}804 \sqrt{f v}$; and in the case of a simple orifice in a thin plate, we get from No. 11 $V = 5{\cdot}322 \sqrt{f v}$. The consideration of all these equations may occasionally be required, but our researches will at present be limited to that arising from No. 12, as being the best adapted for general practice; and for the purpose of shortening the investigation, we shall take no further notice of the case in which the temperature of the steam is below 212 degrees of Fahrenheit; for the expression which indicates the velocity into a vacuum being independent of the elastic force, a separate consideration for the two cases is here unnecessary.

It has been shown in the equation marked (U), that the volume of steam which is generated from an unit of water, is $v = \frac{75{\cdot}67\,(\text{temp.} + 459)}{f}$; let this value of v be substituted for it in the equation $V = 6{\cdot}922 \sqrt{f v}$, and we obtain for the velocity into a vacuum for the usual form of steam passages, as follows, viz.:

$$V = 60{\cdot}2143 \sqrt{(\text{temp.} + 459)}.$$

This is a very neat and simple expression, and the object determined by it is a very important one: it therefore merits the reader's utmost attention, especially if he is desirous of becoming familiar with the calculations in reference to the motion of steam. The rule which the equation supplies, when expressed in words at length, is as follows:—

Rule.—To the temperature of the steam, in degrees of Fahrenheit's thermometer, add the constant number or increment 459, and multiply the square root of the sum by 60·2143; the product will be the velocity with which the steam rushes into a vacuum in feet per second.

With what velocity will steam of 293·4 degrees of Fahrenheit's thermometer rush into a vacuum when under a pressure due to the elastic force corresponding to the given temperature.

By the rule it is 293·4 + 459 = 752·4..................½ log. 1·4382244
Constant coefficient = 60·2143....................log. 1·7797018 add
Velocity into a vacuum in feet per second = 1651·68.............log. 3·2179262

This is the velocity into a perfect vacuum, when the motion is made through a straight pipe of uniform diameter; but when the pipe is alternately enlarged and contracted, the velocity must necessarily be reduced in proportion to the nature of the contraction; and it is further manifest, that every bend and angle in a pipe will be attended with a correspondent diminution in the velocity of motion: it therefore behoves us, in the actual construction of steam passages, to avoid these causes of loss as much as possible; and where they cannot be avoided altogether, such forms should be adopted as will produce the smallest possible retarding effect. In cases where the forms are limited by the situation and conditions of construction, such corrections should be applied as the circumstances of the case demand; and the amount of these corrections must be estimated according to the nature of the obstructions themselves. For each right-angled bend, the diminution of velocity is usually set down as being about one-tenth of its unobstructed value; but whether this conclusion be correct or not, it is at least certain that the obstruction in the case of a right-angled bend is much greater than in that of a gradually curved one. It is a very common thing, especially in steam vessels, for the main steam pipe to send off branches at right angles to each cylinder, and it is easy to see that a great diminution in the velocity of the steam must take place here. In the expansion valve chest a further obstruction must be met with, probably to the extent of reducing the velocity of the steam two-tenths of its whole amount.

These proportional corrections are not to be taken as the results of experiments that have been performed for the purpose of determining the effect of the above causes of retardation: we have no experiments of this sort on which reliance can be placed; and, in consequence, such elements can only be inferred from a comparison of the principles that regulate the motion of other fluids under similar circumstances: they will, however, greatly assist the engineer in arriving at an approximate estimate of the diminution that takes place in the velocity in passing any number of obstructions, when the precise nature of those obstructions can be ascertained. In the generality of practical cases, if the constant coefficient 60·2143 be reduced in the ratio of 650 to 450, the resulting constant 41·6868 may be employed without introducing an error of any consequence.

OF THE ASCENT OF SMOKE AND HEATED AIR IN CHIMNEYS.

The subject of chimney flues, with the ascent of smoke and heated air, is another case of the motion of elastic fluids, in which, by a change of temperature, an atmospheric column assumes a different density from another, where no such alteration of temperature occurs. The proper construction of chimneys is a matter of very great importance to the practical engineer, for in a close fireplace, designed for the generation of steam, there must be a considerable draught to accomplish the intended purpose, and this depends upon the three following particulars, viz.:

1. The height of the chimney from the throat to the top.
2. The area of the transverse section.
3. The temperature at which the smoke and heated air are allowed to enter it.

The formula for determining the power of the chimney may be investigated in the following manner:

Put h = the height in feet from the place where the flue enters to the top of the chimney,
b = the number of cubic feet of air of atmospheric density that the chimney must discharge per hour,
a = the area of the aperture in square inches through which b cubic feet of air must pass when expanded by a change of temperature,
v = the velocity of ascent in feet per second,
t' = the temperature of the external air, and
t = the temperature of the air to be discharged by the chimney.

Now the force producing the motion in this case is manifestly the difference between the weight of a column of the atmospheric air and another of the air discharged by the chimney: and when the temperature of the atmospheric air is at 52 degrees of Fahrenheit's thermometer, this difference will be indicated by the term $h\left(\frac{t'-t}{t'+459}\right)$; the velocity of ascent will therefore be

$v = \sqrt{64\tfrac{2}{3}\, h\left\{\frac{t'-t}{t'+459}\right\}}$ feet per second, and the quantity of air

discharged per second will therefore be, $a\sqrt{64\tfrac{2}{3}\left\{\frac{t'-t}{t'+459}\right\}}$, supposing that there is no contraction in the stream of air; but it is found by experiment, that in all cases the contraction that takes place diminishes the quantity discharged, by about three-eighths of the whole; consequently, the quantity discharged per hour in cubic feet becomes

$$b = 125{\cdot}69\, a\sqrt{\frac{h\,(t'-t)}{t'+459}}.$$

This would be the quantity discharged, provided there were no increase of volume in consequence of the change of temperature; but air expands from b to $\frac{b\,(t'+459)}{t+459}$ for $t'-t$ degrees of temperature, as has been shown elsewhere; consequently, by comparison, we have

$$\frac{b\,(t'+459)}{t+459} = 125{\cdot}69\, a\sqrt{\frac{h\,(t'-t)}{t+459}}.$$

From this equation, therefore, any one of the quantities which it involves can be found, when the others are given: it however supposes that there is no other cause of diminution but the contraction at the aperture; but this can seldom if ever be the case; for eddies, loss of heat, obstructions, and change of direction in the chimney, will diminish the velocity, and consequently a larger area will be required to suffer the heated air to pass. A sufficient allowance for these causes of retardation will be made, if we change the coefficient 125·69 to 100; and in this case the equation for the area of section becomes

$$a = b \sqrt{(t' + 459)^3} \div 100\,(t + 459) \sqrt{h\,(t' - t)}.$$

And if we take the mean temperature of the air of the atmosphere at 52 degrees of Fahrenheit, and make an allowance of 16 degrees for the difference of density between atmospheric air and coal smoke, our equation will ultimately assume the form

$$a = b \sqrt{(t' + 459)^3} \div 51100 \sqrt{h\,(t' - t - 16)}.$$

It has been found by experiment that 200 cubic feet of air of atmospheric density are required for the complete combustion of one pound of coal, and the consumption of ten pounds of coal per hour is usually reckoned equivalent to one horse power: it therefore appears that 2000 cubic feet of air per hour must pass through the fire for each horse power of the engine. This is a large allowance, but it is the safest plan to calculate in excess in the first instance; for the chimney may afterwards be convenient, even if considerably larger than is necessary. The rule for reducing the equation is as follows:—

RULE.—Multiply the number of horse power of the engine by the $\frac{3}{2}$ power of the temperature at which the air enters the chimney, increased by 459; then divide the product by 25·55 times the square root of the height of the chimney in feet, multiplied by the difference of temperature, less 16 degrees, and the quotient will be the area of the chimney in square inches.

Suppose the height of the chimney for a 40-horse engine to be 70 feet, what should be its area when the difference between the temperature at which the air enters the flue, and that of the atmosphere is 250 degrees?

Here, by the rule, we have,

250 + 52 = 302, the temperature at which the air enters [the flue.

Constant increment = 459

Sum = 761..............................log. 2·8813847

3

2)8·6441541

4·3220770

Number of horse power = 40........................log. 1·6020600

5·9241370

$$\left.\begin{array}{llrr}
 & & & 5\cdot9241370 \\
250 - 16 = 234 \;.\;.\;.\;. & \log. & 2\cdot3692159 & \\
\quad\text{height} = 70 \text{ feet} \;.\;. & \log. & \underline{1\cdot8450980} & \\
 & & 2)\underline{4\cdot2143139} & \\
 & & \left.\begin{array}{r} 2\cdot1071569 \\ 1.4073909 \end{array}\right\} & \\
\text{Constant} = 25\cdot55 \;.\;. & \log. & & .\;.\;.\; 3\cdot5145478
\end{array}\right\}$$

Hence the area of the chimney in square inches is 256·79, log. 2·4095892; and in this way may the area be calculated for any other case; but particular care must be taken to have the data accurately determined before the calculation is begun. In the above example the particulars are merely assumed; but even that is sufficient to show the process of calculation, which is more immediately the object of the present inquiry. It is right, however, to add, that recent experiments have greatly shaken the doctrine that it is beneficial to make chimneys small at the top, though such is the way in which they are, nevertheless, still constructed, and our rules must have reference to the present practice. It appears, however, that it would be the best way to make chimneys expand as they ascend, after the manner of a trumpet, with its mouth turned downwards: but these experiments require further confirmation.

The method of calculation adopted above is founded on the principle of correcting the temperature for the difference between the specific gravity of atmospheric air and that of coal-smoke, the one being unity and the other 1·05; there is, however, another method, somewhat more elegant and legitimate, by employing the specific gravity of coal-smoke itself: the investigation is rather tedious and prolix, but the resulting formula is by no means difficult; and since both methods give the same result when properly calculated, we make no further apology for presenting our readers with another rule for obtaining the same object. The formula is as follows:

$$a = \frac{b\,(t' + 459)}{2757\cdot5}\sqrt{\frac{1}{h\,(t' - 77\cdot55)}}$$

where a is the area of the transverse section of the chimney in square inches, b the quantity of atmospheric air required for combustion of the coal in cubic feet per hour, h the height of the chimney in feet, and t' the temperature at which the air enters the flue after passing through the fire. The rule for performing this process is thus expressed:

RULE.—From the temperature at which the air enters the chimney, subtract the constant decrement 77·55; multiply the remainder by the height of the chimney in feet, divide unity by the product, and extract the square root of the quotient. To the temperature of the heated air, add the constant number 459; multiply the sum by the number of cubic feet required for combustion per hour, and divide the product by the number 2757·5; then multiply the quotient by the square root found as above, and the product will be the number of square inches in the transverse section of the chimney.

Suppose a mass of fuel in a state of combustion to require 5000 cubic feet of air per hour, what must be the size of the chimney when its height is 100 feet, the temperature at which the heated air enters the chimney being 200 degrees of Fahrenheit's thermometer?

$$\begin{array}{lr}
\text{By the rule we have } 200-77{\cdot}55=122{\cdot}45 \;.\;.\; \text{log.} & 2{\cdot}0879588 \\
\text{Height of the chimney}=100 \;.\;.\;.\;.\; \text{log.} & \underline{2{\cdot}0000000} \\
 & \underline{4{\cdot}0879588} \\
 & 2)\underline{5{\cdot}9120412} \\
 & 7{\cdot}9560206
\end{array}$$

$$\left.\begin{array}{rl}
200+459=659 \;.\;.\;.\; & \text{log. } 2{\cdot}8188854 \\
5000 \;.\;.\;.\; & \text{log. } 3{\cdot}6989700 \\
2757{\cdot}5 \text{ ar. co.} & \text{log. } 6{\cdot}5594845
\end{array}\right\}\text{add } \begin{array}{l} 3{\cdot}0773399 \\ \hline 1{\cdot}0333605 \quad 10{\cdot}798 \text{ in.}\end{array}$$

This appears to be a very small flue for the quantity of air that passes through it per hour; but it must be observed that we have assumed a great height for the shaft, which has the effect of creating a very powerful draught, thereby drawing off the heated air with great rapidity.

The advantage of a high flue is so very great, that the reader may be desirous of knowing to what height a chimney of a given base may be carried with safety, in cases where it is inconvenient to secure it with lateral stays; and, as an approximate rule for this purpose is not difficult of investigation, we think proper to supply it here.

When the chimney is equally wide throughout its whole height, the formula is

$$s = h\sqrt{\frac{156}{12000 - \frac{1}{8}\,h w}};$$

but when the side of the base is double the size of the top, the equation becomes

$$s = h\sqrt{\frac{104}{12000 - 0{\cdot}42\,h w}};$$

where s is the side of the base in feet, h the height, and m the weight of one cubic foot of the material. When the chimney stalk is not square, but longer on the one side than the other, s must be the least dimension. The proportion of solid wall to a given base, as sanctioned by experience, is about two-thirds of its area, consequently w ought to be two-thirds of the weight of a cubic foot of brickwork. Now, a cubic foot of dried brickwork is, on an average, 114 lbs.; consequently $w = 76$ lbs.; and if this be substituted in the foregoing equations, we get for a chimney of equal size throughout,

$$s = h\sqrt{\frac{156}{1200 - 25\,h}};$$

and when the chimney tapers to one-half the size at top, it is

$$s = h\sqrt{\frac{104}{12000 - 32\,h}};$$

where it may be remarked that 12000 lbs. is the cohesive force of one square foot of mortar; and in the investigation of the formulas we have assumed the greatest force of the wind on a square foot of surface at 52 lbs. These equations are too simple in their form to require elucidation from us; we therefore leave the reduction as an exercise to the reader, who it is presumed will find no difficulty in resolving the several cases that may arise in the course of his practice.

$$v = \sqrt{\frac{2\,g\,\mathrm{H}\,a\,t\,\mathrm{D}}{\mathrm{D} + 2\,g\,\mathrm{K}\,(\mathrm{L} + \mathrm{H}}},$$

is the expression given by M. Péclet for the velocity of smoke in a chimney. v, the velocity; t, the temperature, whose maximum value is about 300° centigrade; $g = 32\frac{1}{6}$ feet; D, the diameter of the chimney; H, the height; L, the length of horizontal flues, supposing them formed into a cylinder of the same diameter as that of the chimney. $\mathrm{K} = \cdot 0127$ for brick, $= \cdot 005$ for sheet-iron, and $= \cdot 0025$ for cast-iron chimneys. $a = \cdot 00365$.

Let $\mathrm{L}=60$; $\mathrm{H}=150$; $\mathrm{D}=5$; $\mathrm{K}=\cdot 005$; $2g=64\frac{1}{3}$; $t=300°$; $a=\cdot 00365$. Then $v=\sqrt{\dfrac{2\,g\,\mathrm{H}\,a\,t\,\mathrm{D}}{\mathrm{D}+2\,g\,\mathrm{K}(\mathrm{H}+\mathrm{L})}}=26\cdot 986$ feet.

A cubic foot of water raised into steam is reckoned equivalent to a horse power, and to generate the steam with sufficient rapidity, an allowance of one square foot of fire-bars, and one square yard of effective heating surface, are very commonly made in practice, at least in land engines. These proportions, however, greatly vary in different cases; and in some of the best marine engine boilers, where the area of fire-grate is restricted by the breadth of the vessel, and the impossibility of firing long furnaces effectually at sea, half a square foot of fire-grate per horse power is a very common proportion. Ten cubic feet of water in the boiler per horse power, and ten cubic feet of steam room per horse power, have been assigned as the average proportion of these elements; but the fact is, no general rule can be formed upon the subject, for the proportions which would be suitable for a wagon boiler would be inapplicable to a tubular boiler, whether marine or locomotive; and good examples will in such cases be found a safer guide than rules which must often give a false result. A capacity of three cubic feet per horse power is a common enough proportion of furnace-room, and it is a good plan to make the furnaces of a considerable width, as they can then be fired more effectually, and do not produce so much smoke as if they are made narrow. As regards the question of draft, there is a great difference of opinion among engineers upon the subject, some preferring a very slow draft and others a rapid one. It is obvious that the question of draft is virtually that of the area of fire-grate, or of the quantity of fuel consumed upon a given area of grate surface, and the weight of fuel burned on a foot of fire-grate per hour varies in different cases in practice from 3½ to 80 lbs. Upon the quickness of the draft again hinges the question of the proper thickness of the stratum of incandescent fuel upon the grate; for if the draft be very strong, and the fire at the same time be thin, a great deal of uncombined oxygen will escape up through the fire, and a needless refrigeration of the contents of the flues will be thereby occasioned; whereas, if the fire be thick, and the draft be sluggish, much of the useful effect of the coal will be lost by the formation of carbonic oxide. The length of the circuit made by the smoke varies in almost every boiler, and the same may be said of the area of the flue in its cross section, through which the smoke has to pass. As an average, about one-fifth of the area of fire-grate for the area of the flue behind the bridge, diminished to half that amount for the area of the chimney, has been given as a good proportion, but the examples which we have given, and the average flue area of the boilers which we shall describe, may be taken as a safer guide than any such loose statements. When the flue is too long, or its sectional area is insufficient, the draft becomes insufficient to furnish the requisite quantity of steam; whereas if the flue be too short or too large in its area, a large quantity of the heat escapes up the chimney, and a deposition of soot in the flues also takes place. This last fault is one of material consequence in the case of tubular boilers consuming bituminous coal, though indeed the evil might be remedied by blocking some of the tubes up. The area of water-level is about 5 feet per horse power in land boilers. In many cases, however, it is much less; but it is always desirable to make the area of the water-level as large as possible, as, when it is contracted, not only is the water-level subject to sudden and dangerous fluctuations, but water is almost sure to be carried into the cylinder with the steam, in consequence of the violent agitation of the water, caused by the ascent of a large volume of steam through a small superficies. It would be an improvement in boilers, we think, to place over each furnace an inverted vessel immerged in the water, which might catch the steam in its ascent, and deliver it quietly by a pipe rising above the water-level. The water-level would thus be preserved from any inconvenient agitation, and the weight of water within the boiler would be diminished at the same time that the original depth of water over the furnaces was preserved. It would also be an improvement to make the sides of the furnaces of marine boilers sloping, instead of vertical, as is the common practice, for the steam could then ascend freely at the instant of its formation, instead of being entangled among the rivets and landings of the plates, and superinducing an overheating of the plates by preventing a free access of the water to the metal.

We have, in the following table, collected a few of the principal results of experiments made on steam boilers.

TABLE I.

NATURE OF THE BOILERS USED.	Mean of Huel Towan, and United Mines boilers, in Cornwall.	Boiler, at Warwick.	Mean of 8 experiments at the Albion Mills, Clithero, Preston, and New River Water Company.	Atmospheric Engine, at Long Benton, 1772.	Mean of 11 of M. de Pambour's experiments.	Cornish boiler at the East London Water Works, 1839.	Another boiler at the East London Water Works, 1839.
	Cylindrical with internal flue.	Wagon.	Wagon.	Circular or Hay-stack.	Locomotive.	Cylindrical with internal flue.	Wagon with internal flue.
Total area of heated surface in square feet······	962	152	342·8	459	334·6	798	588
Length of circuit made by the heat in feet ········	155	50 66	72·5	52·8	7·0	83·1	78
Area of fire grates in square feet ····················	23·60	23·33	26·09	35·10	7·08	14·25	37·20
Weight of fuel burned on each square foot of grate, per hour, in lbs.········	3·46	4·00	10·75	20·34	79·33	46·82	13·31
Cub. ft. of water evaporated from initial temperature by 112 lbs. of fuel ······	18·87	16·44	13·91	14·11	11·14		
Cubic feet of water evaporated per hour from initial temperature·····	13·81	13·79	34·40	00·7	55·18		
Square feet of heated surface for each cubic foot of water evaporated per hour ··················	69·58	11·00	9·96	5·06	0·06	17·17	
Square feet of heated surface for each square foot of grate················	40·65	6·51	13·13	13·08	47·59	56·0	15·78
Pressure of steam above the atmosphere in lbs.··	42·2	2·5	3·68	1·5	50	15·45	

The economical effects of expansion will be found to be very clearly exhibited in the next table. The duties are recorded in the fifth line from the top, and the degree of expansion in the bottom line. It will be observed, that the order in which the different engines stand in respect of superiority of duty is the same as in respect of amount of expansion. The Holmbush engine has a duty of 140,484,848 lbs. raised 1 foot by 1 cwt. of coals, and the steam acts expansively over ·83 of the whole stroke; while the water-works' Cornish engine has only a duty of 105,664,118 lbs., and expands the steam over only ·687 of the whole stroke. Again, comparing the second and last engines together, the Albion Mills engine has a duty of 25,756,752 lbs., and no expansive action. The water-works' engine, again, acts expansively over one-half of its stroke, and has an increased duty of 46,602,333 lbs. Other causes, of course, may influence these comparisons, especially the last, where one engine is a double-acting rotative engine, and the other a single-acting pumping one; but there can be no doubt that the expansive action in the latter is the *principal* cause of its more economical performance.

The heating surface per horse power allowed by some engineers is about 9 square feet in wagon boilers, reckoning the total surface as effective surface, if the boilers be of a considerable size; but in the case of small boilers, the proportion is larger. The total

TABLE II.

	Atmospheric Engine, Long Benton, Northumberland, date 1772.	Non-expansive rotative condensing Engine, Albion Mills, London, date 1786.	Holmbush, Cornish, condensing Engine, single acting for pumping water. Steam acts expansively after the first sixth of the stroke. 1836.	Noncondensing double-acting Engine, nonexpansive, Congleton, Cheshire. 1823.	Cornish Engine, East London Water Works.	Pumping Engine at East London Water Works.
Diameter of cylinder in inches	52	84	50	18	79¾	59⅝
Length of stroke in feet	7	8	9·1	4	10	7·91
Number of strokes per minute	12	16	4·68	27·5	7	11·5
Pressure on the piston, above or below the atmosphere in lbs., per square inch	. . . .	Estimated at —2·5	+80	+20	+5·17	+2·15
Weight in lbs. raised one foot by 112 lbs. of coals	12,600,000	25,756,752	140,484,848	12,418,560	105,664,118	46,602,333
Do. do. by one pound of water, as steam	14,280	28,489	119,097	15,840	110,716	53,369
Effective power of the engine at time of experiment in horse power	40·5	50·0	26·48	12·0	. . . .	
Efficiency of the steam, its efficiency in the Albion Mills being unity	·501	1·000	4·180	·556	8·89	1·87
Efficiency of the fuel, its efficiency in the Albion Mills being unity	·480	1·000	5·454	·482	4·1	1·81
Distance of the piston from the end of its stroke when the steam is cut off in parts of the length of stroke.	0	0	·833	0	·687	·5

heating surface of a two horse power wagon boiler is, according to Fitzgerald's proportions, 30 square feet, or 15 ft. per horse power; whereas, in the case of a 45 horse power boiler the total heating surface is 438 square feet, or 9·6 ft. per horse power. The capacity of steam room is 8¾ cubic feet per horse power, in the two horse power boiler, and 5¾ cubic feet in the 20 horse power boiler; and in the larger class of boilers, such as those suitable for 30 and 45 horse power engines, the capacity of the steam room does not fall below this amount, and indeed is nearer 6 than 5¾ cubic feet per horse power. The content of water is 18½ cubic feet per horse power in the two horse power boiler, and 15 cubic feet per horse power in the 20 horse power boiler. In marine boilers about the same proportions obtain in most particulars. The original boilers of one or two large steamers were proportioned with about half a square foot of fire grate per horse power, and 10 square feet of flue and furnace surface, reckoning the total amount as effective; but in the boilers of other vessels a somewhat smaller proportion of heating surface was adopted. In some cases we have found that, in their marine flue boilers, 9 square feet of flue and furnace surface are requisite to boil off a cubic foot of water per hour, which is the proportion that obtains in some land boilers; but inasmuch as in modern engines the nominal considerably exceeds the actual power, they allow 11 square feet of heating surface per nominal horse power in their marine boilers, and they reckon, as effective heating surface, the tops of the flues, and the whole of the sides of the flues, but not the bottoms. They have been in the habit of allowing for the capacity of the steam space in marine boilers 16 times the content of the cylinder; but as there are two cylinders, this is equivalent to 8 times the content of both cylinders, which is the proportion commonly followed in land engines, and which agrees very nearly with the proportion of between 5 and 6 cubic feet of steam room per horse power. Taking, for example, an engine with 23 inches diameter of cylinder and 4 feet stroke, which will be 18·4 horse power—the area of the cylinder will be 415·476 square inches, which, multiplied by 48, the number of inches in the stroke, will give 19942·848 for the capacity of the cylinder in cubic inches; 8 times this is 159542·784 cubic inches, or 92·3 cubic feet; 92·3 divided by 18·4 is rather more than 5 cubic feet per horse power. There is less necessity, however, that the steam space should be large when the flow of steam from the boiler is very uniform, as it will be where there are two engines attached to the boiler at right angles with one another, or where the engines work at a great speed, as in the case of locomotive engines. A high steam chest too, by rendering boiling over into the steam pipes, or priming as it is called, more difficult, obviates the necessity for so large a steam space; and the use of steam of a high pressure, worked expansively, has the same operation; so that in modern marine boilers, of the tubular construction, where the whole of these modifying circumstances exist, there is no necessity for so large a proportion of steam room as 5 or 6 cubic feet per horse power, and about half that amount more nearly represents the general practice. Many allow 0·64 of a square foot per nominal horse power of grate bars in their marine boilers, and a good effect arises from this proportion; but sometimes so large an area of fire grate cannot be conveniently got, and the proportion of half a square foot per horse power seems to answer very well in engines working with some expansion, and is now very widely adopted. With this allowance, there will be about 22 square feet of heating surface per square foot of fire grate; and if the consumption of fuel be taken at 6 lbs. per nominal horse power per hour, there will be 12 lbs. of coal consumed per hour on each square foot of grate. The flues of all flue boilers diminish in their calorimeter as they approach the chimney; some very satisfactory boilers have been made by allowing a proportion of 0·6 of a square foot of fire grate per nominal horse power, and making the sectional area of the flue at the largest part $\frac{1}{4}$th of the area of fire grate, and the smallest part, where it enters the chimney, $\frac{1}{11}$th of the area of the fire grate; but in some of the boilers proportioned on this plan the maximum sectional area is only $\frac{1}{7\cdot5}$ or $\frac{1}{8\cdot5}$, according to the purposes of the boiler. These proportions are retained whether the boiler is flue or tubular, and from 14 to 16 square feet of tube surface is allowed per nominal horse power; but such boilers, although they may give abundance of steam, are generally, perhaps needlessly, bulky.

We shall therefore conclude our remarks upon the subject by introducing a table of the comparative evaporative power of different kinds of coal, which will prove useful, by affording data for the comparison of experiments upon different boilers when different kinds of coal are used.

TABLE *of the Comparative Evaporative Power of different kinds of Coal.*

No.	Description of Coals.	Water evaporated per lb. of Coals.
		Lbs.
1	The best Welsh..............................	9·493
2	Anthracite American..........................	9·14
3	The best small Pittsburgh..................	8·526
4	Average small Newcastle.....................	8·074
5	Pennsylvanian.................................	10·45
6	Coke from Gas-works........................	7·908
7	Coke and Newcastle, small, ½ and ½....	7·897
8	Welsh and Newcastle, mixed ½ and ½...	7·865
9	Derbyshire and small Newcastle, ½ and ½	7·710
10	Average large Newcastle.....................	7·658
11	Derbyshire.....................................	6·772
12	Blythe Main, Northumberland..........	6·600

Strength of boilers.—The extension of the expansive method of employing steam to boilers of every denomination, and the gradual introduction in connection therewith of a higher pressure than formerly, makes the question of the strength of boilers one of great and increasing importance. This topic was very successfully elucidated, a few years ago, by a committee of the Franklin Institute, Philadelphia, and we shall here recapitulate a few of the more important of the conclusions at which they arrived. Iron boiler plate was found to increase in tenacity as its temperature was raised, until it reached a temperature of 550° above the freezing point, at which point its tenacity began to diminish. The following table exhibits the cohesive strength at different temperatures.

At 32° to 80° the tenacity was = 56,000 lbs., or 1-7th below its maximum.
At 570° —— = 66,500 lbs., the maximum.
At 720° —— = 55,000 lbs., the same nearly as at 32°.
At 1050° —— = 32,000 lbs., nearly ½ of the maximum.
At 1240° —— = 22,000 lbs., nearly ⅓ of the maximum.
At 1317° —— = 9,000 lbs., nearly 1-7th of the maximum.
At 3000° iron becomes fluid.

The difference in strength between strips of iron cut in the direction of the fibre, and strips cut across the grain, was found to be about 6 per cent. in favour of the former. Repeated piling and welding was found to increase the tenacity and closeness of the iron, but welding together different kinds of iron was found to give an unfavourable result; riveting plates was found to occasion a diminution in their strength, to the extent of about one-third. The accidental overheating of a boiler was found to reduce its strength from 65,000 lbs. to 45,000 lbs. per square inch. Taking into account all these contingencies, it appears expedient to limit the tensile force upon boilers in actual use to about 3000 lbs. per square inch of iron.

Copper follows a different law, and appears to diminish in strength by every addition of heat, reckoning from the freezing point. The square of the diminution of strength seems to keep pace with the cube of the temperature, as appears by the following table:—

TABLE *showing the Diminution of Strength of* COPPER *Boiler Plates by additions to the Temperature, the Cohesion at* 32° *being* 32,800 *lbs. per Square Inch.*

No.	Temperature above 32°.	Diminution of Strength.	No.	Temperature above 32°.	Diminution of Strength.
1	90°	0·0175	9	660°	0·3425
2	180	0·0540	10	769	0·4398
3	270	0·0926	11	812	0·4944
4	360	0·1513	12	880	0·5581
5	450	0·2046	13	984	0·6691
6	460	0·2133	14	1000	0·6741
7	513	0·2446	15	1200	0·8861
8	529	0·2558	16	1300	1·0000

In the case of iron, the following are the results when tabulated after a similar fashion.

TABLE *of Experiments on* IRON *Boiler Plate at High Temperature; the Mean Maximum Tenacity being at* 550° = 65,000 *lbs. per Square Inch.*

Temperature observed.	Diminution of Tenacity observed.	Temperature observed.	Diminution of Tenacity observed.
550°	0·0000	824°	0·2010
570	0·0869	932	0·3324
596	0·0899	947	0·3598
600	0·0964	1030	0·4478
630	0·1047	1111	0·5514
562	0·1155	1155	0·6000
722	0·1436	1159	0·6011
782	0·1491	1187	0·6352
784	0·1535	1237	0·6622
766	0·1589	1245	0·6715
770	0·1627	1817	0·7001

The application of stays to marine boilers, especially in those parts of the water spaces which lie in the wake of the furnace bars, has given engineers much trouble; the ⅜ plate, of which ordinary boilers are composed, is hardly thick enough to retain a stay with security by merely tapping the plate, whereas, if the stay be riveted, the head of the rivet will in all probability be soon burnt away. The best practice appears to be to run the stays used for the water spaces in this situation, in a line somewhat beneath the level of the bars, so that they may be shielded as much as possible from the fire, while those which are required above the level of the bars should be kept as nearly as possible towards the crown of the furnace, so as to be removed from the immediate contact of the fire. Screw bolts with a fine thread tapped into the plate, and with a thin head upon the one side, and a thin nut made of a piece of boiler plate on the other, appear to be the best description of stay that has yet been contrived. The stays between the sides of the boiler shell, or the bottom of the boiler and the top, present little difficulty in their application, and the chief thing that is to be attended to is to take care that there be plenty of them; but we may here remark that we think it an indispensable thing, when there is any high pressure of steam to be employed, that the furnace crown be stayed to the top of the boiler. This, it will be observed, is done in the boilers of the Tagus and Infernal; and we know of no better specimen of staying than is afforded by those boilers.

AREA OF STEAM PASSAGES.

RULE.—To the temperature of steam in the boiler add the constant increment 459; multiply the sum by 11025; and extract the square root of the product. Multiply the length of stroke by the number of strokes per minute; divide the product by the square root just found; and multiply the square root of the quotient by the diameter of the cylinder; the product will be the diameter of the steam passages.

Let it be required to determine the diameter of the steam passages in an engine of which the diameter of the cylinder is 48 inches, the length of stroke 4½ feet, and the number of strokes per minute 26, supposing the temperature under which the steam is generated to be 250 degrees of Fahrenheit's thermometer.

Here by the rule we get $\sqrt{11025(250 + 459)} = 2795{\cdot}84$; the number of strokes is 26, and the length of stroke 4½ feet; hence it is $\delta = d\sqrt{\dfrac{117}{2795{\cdot}84}} = 0{\cdot}20456d = 0{\cdot}20456 \times 48 = 9{\cdot}819$ inches; so that the diameter of the steam passages is a little more than one-fifth of the diameter of the cylinder. The same rule will answer for high and low pressure engines, and also for the passages into the condenser.

LOSS OF FORCE BY THE DECREASE OF TEMPERATURE IN THE STEAM PIPES.

Rule.—From the temperature of the surface of the steam pipes subtract the temperature of the external air; multiply the remainder by the length of the pipes in feet, and again by the constant number or coefficient 1·68; then divide the product by the diameter of the pipe in inches drawn into the velocity of the steam in feet per second, and the quotient will express the diminution of temperature in degrees of Fahrenheit's thermometer.

Let the length of the steam pipe be 16 feet and its diameter 5 inches, and suppose the velocity of the steam to be about 95 feet per second, what will be the diminution of temperature, on the supposition that the steam is at 250° and the external air at 60° of Fahrenheit?

Here, by the note to the above rule, the temperature of the surface of the steam pipe is $250 - 250 \times 0{\cdot}05 = 237{\cdot}5$; hence we get

$$t'' = \frac{1{\cdot}68 \times 16(237{\cdot}5 - 60)}{5 \times 95} = 10{\cdot}044 \text{ degrees.}$$

If we examine the manner of the composition of the above equation, it will be perceived that, since the diameter of the pipe and the velocity of motion enter as divisors, the loss of heat will be less as these factors are greater; but, on the other hand, the loss of heat will be greater in proportion to the length of pipe and the temperature of the steam. Since the steam is reduced from a higher to a lower temperature during its passage through the steam pipes, it must be attended with a corresponding diminution in the elastic force; it therefore becomes necessary to ascertain to what extent the force is reduced, in consequence of the loss of heat that takes place in passing along the pipes. This is an inquiry of some importance to the manufacturers of steam engines, as it serves to guard them against a very common mistake into which they are liable to fall, especially in reference to steamboat engines, where it is usual to cause the pipe to pass round the cylinder, instead of carrying it in the shortest direction from the boiler, in order to decrease the quantity of surface exposed to the cooling effect of the atmosphere.

RULE.—From the temperature of the surface of the steam pipe subtract the temperature of the external air; multiply the remainder by the length of the pipe in feet, and again by the constant fractional coefficient 0·00168; divide the product by the diameter of the pipe in inches drawn into the velocity of steam in feet per second, and subtract the quotient from unity; then multiply the difference thus obtained by the elastic force corresponding to the temperature of steam in the boiler, and the product will be the elastic force of the steam as reduced by cooling in passing through the pipes.

Let the dimensions of the pipe, the temperature of the steam, and its velocity through the passages, be the same as in the preceding example, what will be the quantity of reduction in the elastic force occasioned by the effect of cooling in traversing the steam pipe?

Since the elastic force of the steam in the boiler enters the equation from which the above rule is deduced, it becomes necessary in the first place to calculate its value; and this is to be done by a rule already given, which answers to the case in which the temperature is greater than 212°; thus we have

$$\begin{array}{rl} 250 \times 1{\cdot}69856 = & 424{\cdot}640 \\ \text{Constant number} = & \underline{205{\cdot}526} \text{ add} \\ \text{Sum} = & 630{\cdot}166 \ldots\ldots \text{log. } 2{\cdot}79945 \\ \text{Constant divisor} = & 333 \ldots\ldots\ldots\ldots \text{log. } \underline{2{\cdot}522444} \text{ subtract} \\ & \qquad 0{\cdot}277011 \times 6{\cdot}42 = 1{\cdot}778410, \end{array}$$

which is the logarithm of 60·036 inches of mercury.

Again, we have $250 - 0{\cdot}05 \times 250 = 237{\cdot}5$; consequently, by multiplying as directed in the rule, we get $237{\cdot}5 \times 0{\cdot}00168 \times 16 = 6{\cdot}384$, which being divided by $95 \times 5 = 475$, gives 0·01344; and by taking this from unity and multiplying the remainder by the elastic force as calculated above, the value of the reduced elastic force becomes

$$f' = 60{\cdot}036\ (1 - 0{\cdot}01344) = 59{\cdot}229 \text{ inches of mercury.}$$

The loss of force is therefore $60{\cdot}036 - 59{\cdot}229 = 0{\cdot}807$ inches of mercury, which amounts to $\frac{1}{75}$th part of the entire elastic force of the steam in the boiler as generated under the given temperature, being a quantity of sufficient importance to claim the attention of our engineers.

FEED WATER.

The quantity of water required to supply the waste occasioned by evaporation from a boiler, or, as it is technically termed, the "feed water" required by a boiler working with any given pressure, is easily determinable. For, since the relative volumes of water and steam at any given pressure are known, it becomes necessary merely to restore the quantity of water by the feed pump equivalent to that abstracted in the form of steam, which the known relation of the density to the pressure of the steam renders of easy accomplishment. In practice, however, it is necessary that the feed pump should be able to supply a much larger quantity of water than what theory prescribes, as a great waste of water sometimes occurs from leakage or priming, and it is necessary to provide against such contingencies. The feed pump is usually made of such dimensions as to be capable of supplying 3½ times the water that the boiler will evaporate, and in low pressure engines, where the cylinder is double acting and the feed pump single acting, this proportion will be maintained by making the pump a 240th of the capacity of the cylinder. In low pressure engines the pressure in the boiler may be taken at 5 lbs. above the pressure of the atmosphere, or 20 lbs. in all; and as high pressure steam is merely low pressure steam compressed into a smaller compass, the size of the feed pump relatively to the size of the cylinder must obviously vary in the direct proportion of the pressure. If, then, the feed pump be 1-240th of the capacity of the cylinder when the total pressure of the steam is 20 lbs., it must be 1-120th of the capacity of the cylinder when the total pressure of the steam is 40 lbs., or 25 lbs. above the atmosphere. This law of variation is expressed by the following rule, which gives the capacity of feed pump proper for all pressures:—Multiply the capacity of the cylinder in cubic inches by the *total* pressure of the steam in lbs. per square inch, or the pressure in lbs. per square inch on the safety valve, plus 15, and divide the product by 4800; the quotient is the capacity of the feed pump in cubic inches, when the feed pump is single acting and the engine double acting. If the feed pump be double acting, or the engine single acting, the capacity of the pump must be just one-half what is given by this rule.

CONDENSING WATER.

It was found that the most beneficial temperature of the hot well was 100 degrees. If, therefore, the temperature of the steam be 212°, and the latent heat 1000°, then 1212° may be taken to represent the heat contained in the steam, or 1112° if we deduct the temperature of the hot well. If the temperature of the injection water be 50°, then 50 degrees of cold are available for the abstraction of heat, and as the total quantity of heat to be abstracted is that requisite to raise the quantity of water in the steam 1112 degrees, or 1112 times that quantity, one degree, it would raise one-fiftieth of this, or 22·24 times the quantity of water in the steam, 50 degrees. A cubic inch of water, therefore, raised into steam, will require 22·24 cubic inches of water at 50 degrees for its condensation, and will form therewith 23·24 cubic inches of hot water at 100 degrees. It has been a practice to allow about a wine pint (28·9 cubic inches) of injection water for every cubic inch of water evaporated from the boiler. The usual capacity for the cold water pump is $\frac{1}{48}$th of the capacity of the cylinder, which allows some water to run to waste. As a maximum effect is obtained when the temperature of the hot well is about 100°, it will not be advisable to reduce it below that temperature in practice. With the superior vacuum due to a temperature of 70° or 80° the admission of so much cold water into the condenser becomes necessary,—and which has afterwards to be pumped out in opposition to the pressure of the atmosphere,—so that the gain in the vacuum does not equal the loss of power occasioned by the additional load upon the pump, and there is, therefore, a clear loss by the reduction of the temperature below 100°, if such reduction be caused by the admission of an additional quantity of water. If the reduction of temperature, however, be caused by the use of colder water, there is a gain produced by it, though the gain will within certain limits be greater, if advantage be taken of the lowness of the temperature to diminish the quantity of injection.

SAFETY VALVES.

Rule.—Add 459 to the temperature of the steam in degrees of Fahrenheit; divide the sum by the product of the elastic force of the steam in inches of mercury, into its excess above the weight of the atmosphere in inches of mercury; multiply the square root of the quotient by ·0653; multiply this product by the number of cubic feet per hour of water evaporated, and this last product is the theoretical area of the orifice of the safety valve in square inches.

To apply this to an example—which, however, it must be remembered, will give a result much too small for practice.

Required the least area of a safety valve of a boiler suited for a 250 horse power engine, working with steam 6 lbs. more than the atmosphere on the square inch.

In this case the total pressure is equal to 21 lbs. per square inch; and as in round numbers one pound of pressure is equal to about two inches of mercury, it follows that $f = 42$ inches of mercury.

It will be necessary to calculate t from formula (S) already given. The operation is as follows:—

$$\begin{array}{rr}
\log. 42 \div 6{\cdot}42 = 1{\cdot}623249 \div 6{\cdot}42 = & 0{\cdot}252842 \\
\text{constant co-efficient} = 196 \qquad & 2{\cdot}292363 \\
\hline
& 2{\cdot}545205
\end{array}$$

$$\begin{array}{rl}
\text{natural number} = & 350{\cdot}92 \\
\text{constant temperature} = & 121 \\
\hline
t = & 229{\cdot}92
\end{array}$$

$$\text{therefore } \sqrt{\frac{459 + t}{f\,(f - 30)}} = \sqrt{\frac{459 + 229{\cdot}92}{42 \times 12}}$$

$$= \sqrt{\frac{688{\cdot}92}{50{\cdot}4}} = \sqrt{1{\cdot}3669} = 1{\cdot}168;$$

$$\text{therefore } x = {\cdot}0653 \times 1{\cdot}168 \times \text{N} = {\cdot}0757\ \text{N}.$$

We have stated in a former part of this work that a cubic foot of water evaporated per hour is equivalent to one horse power; therefore in this case $N = 250$ and $x = 18{\cdot}925$ sq. in.

As another example. Required the proper area of the safety valve of a boiler suited to an engine of 500 horse power, when it is wished that the steam should never acquire an elastic force greater than 60 lbs. on the square inch above the atmosphere.

In this case the whole elastic force of the steam is 75 lbs.; and as 1 pound corresponds in round numbers to 2 inches of mercury, it follows that $f = 150$. It will be necessary to calculate the temperature corresponding to this force. The operation is as follows:—

$$\begin{array}{rll}
\text{Log. } 150 \div 6{\cdot}42 = & 2{\cdot}176091 \div 6{\cdot}42 = & \cdot 338955 \\
\text{constant co-efficient} = & 196 & \text{log. } 2{\cdot}292363 \text{ add} \\
\text{natural number} = & 427{\cdot}876 & \overline{2{\cdot}631318} \\
\text{constant temperature} = & 121 & \\
\text{required temperature} & 306{\cdot}876 \text{ degrees of Fahrenheit's scale} &
\end{array}$$

$$\text{therefore } \frac{459 + t}{f(f - 30)} = \frac{459 + 306{\cdot}876}{150\,(150 - 30)} = \frac{765{\cdot}876}{150 \times 120} = \frac{765{\cdot}896}{18000}$$

$$= \cdot 043549\text{; therefore } \sqrt{\frac{459 + t}{f(f - 30)}} = \sqrt{\cdot 042549} = \cdot 20628.$$

Hence the required area $= \cdot 0653 \times \cdot 20628 \times 500 = \cdot 01347 \times 500 = 6{\cdot}735$ square inches.

If the area of the safety valve of a boiler suited for an engine of 500 horse power be required, when it is wished the steam should never acquire a greater temperature than 300°, it will be necessary to calculate the elastic force corresponding to this temperature; and by formula for this purpose, the required area $= \cdot 0653 \times \cdot 231 \times 500 = \cdot 0151 \times 500 = 7{\cdot}55$ square inches. It will be perceived from these examples that the greater the elasticity and the higher the corresponding temperature the less is the area of the safety valve. This is just as might have been expected, for then the steam can escape with increased velocity. We may repeat that the results we have arrived at are much less than those used in practice. For the sake of safety, the orifices of the safety valve are intentionally made much larger than what theory requires; usually $\frac{8}{10}$ of a square inch per horse power is the ordinary proportion allowed in the case of low pressure engines.

THE SLIDE VALVE.

The four following practical rules are applicable alike to short slide and long D valves.

RULE I.—*To find how much cover must be given on the steam side in order to cut the steam off at any given part of the stroke.*—From the length of the stroke of the piston, subtract the length of that part of the stroke that is to be made before the steam is cut off. Divide the remainder by the length of the stroke of the piston, and extract the square root of the quotient. Multiply the square root thus found by half the length of the stroke of the valve, and from the product take half the lead, and the remainder will be the cover required.

RULE II.—*To find at what part of the stroke any given amount of cover on the steam side will cut off the steam.*—Add the cover on the steam side to the lead; divide the sum by half the length of stroke of the valve. In a table of natural sines find the arc whose sine is equal to the quotient thus obtained. To this arc add 90°, and from the sum of these two arcs subtract the arc whose cosine is equal to the cover on the steam side divided by half the stroke of the valve. Find the cosine of the remaining arc, add 1 to it, and multiply the sum by half the stroke of the piston, and the product is the length of that part of the stroke that will be made by the piston before the steam is cut off.

RULE III.—*To find how much before the end of the stroke, the exhaustion of the steam in front of the piston will be cut off.*—To the cover on the steam side add the lead, and divide the sum by half the length of the stroke of the valve. Find the arc whose sine is equal to the quotient, and add 90° to it. Divide the cover on the exhausting side by half the stroke of the valve, and find the arc whose cosine is equal to the quotient. Subtract this arc from the one last obtained, and find the cosine of the remainder. Subtract this cosine from 2, and multiply the remainder by half the stroke of the piston. The product is the distance of the piston from the end of its stroke when the exhaustion is cut off.

RULE IV.—*To find how far the piston is from the end of its stroke, when the steam that is propelling it by expansion is allowed to escape to the condenser.*—To the cover on the steam side add the lead, divide the sum by half the stroke of the valve, and find the arc whose sine is equal to the quotient. Find the arc whose cosine is equal to the cover on the exhausting side, divided by half the stroke of the valve. Add these two arcs together, and subtract 90°. Find the cosine of the residue, subtract it from 1, and multiply the remainder by half the stroke of the piston. The product is the distance of the piston from the end of its stroke, when the steam that is propelling it is allowed to escape to the condenser. In using these rules, all the dimensions are to be taken in inches, and the answers will be found in inches also.

From an examination of the formulas we have given on this subject, it will be perceived (supposing that there is no lead) that the part of the stroke where the steam is cut off, is determined by the proportion which the cover on the steam side bears to the length of the stroke of the valve: so that in all cases where the cover bears the same proportion to the length of the stroke of the valve, the steam will be cut off at the same part of the stroke of the piston.

In the first line, accordingly, of Table I., will be found eight different parts of the stroke of the piston designated; and directly below each, in the second line, is given the quantity of cover requisite to cause the steam to be cut off at that particular part of the stroke. The different sizes of the cover are given in the second line, in decimal parts of the length of the stroke of the valve; so that, to get the quantity of cover corresponding to any of the given degrees of expansion, it is only necessary to take the decimal in the second line, which stands under the fraction in the first, that marks the degree of expansion, and multiply that decimal by the length you intend to make the stroke of the valve. Thus, suppose you have an engine in which you wish to have the steam cut off when the piston is a quarter of the length of its stroke from the end of it, look in the table, and you will find in the third column from the left, ¼. Directly under that, in the second line, you have the decimal ·250. Suppose that you think 18 inches will be a convenient length for the stroke of the valve, multiply the decimal ·250 by 18, which gives 4½. Hence we learn that with an 18 inch stroke for the valve, 4½ inches of cover on the steam side will cause the steam to be cut off when the piston has still a quarter of its stroke to perform.

Half the stroke of the valve must always be *at least* equal to the cover on the steam side added to the breadth of the port. By the "breadth" of the port, we mean its dimension in the direction of the valve's motion; in short, its perpendicular depth when the cylinder is upright. The words "cover" and "lap" are synonymous. Consequently, as the cover, in this case, must be 4½ inches, and as half the stroke of the valve is 9 inches, the breadth of the port cannot be more than (9 — 4½ = 4½) 4½ inches. If this breadth of port is not enough, we must increase the stroke of the valve; by which means we shall get both the cover and the breadth of the port proportionally increased. Thus, if we make the length of valve stroke 20 inches, we shall have for the cover ·250 × 20 = 5 inches, and for the breadth of the port 10 — 5 = 5 inches.

TABLE I.

Distance of the piston from the termination of its stroke, when the steam is cut off, in parts of the length of its stroke.	$\frac{8}{24}$ or $\frac{1}{3}$	$\frac{7}{24}$	$\frac{6}{24}$ or $\frac{1}{4}$	$\frac{5}{24}$	$\frac{4}{24}$ or $\frac{1}{6}$	$\frac{8}{24}$ or $\frac{1}{8}$	$\frac{2}{24}$ or $\frac{1}{12}$	$\frac{1}{24}$
Cover on the steam side of the valve, in decimal parts of the length of its stroke.	·289	·270	·250	·228	·204	·177	·144	·102

This table, as we have already intimated, is computed on the supposition that the valve is to have no lead; but, if it is to have lead, all that is necessary is to subtract half the proposed lead from the cover found from the table, and the remainder will be the proper quantity of cover to give to the valve. Suppose that, in the last example, the valve was to have ¼ inch of lead, we would subtract ¼ inch from the 5 inches found for the cover by the table: that would leave 4¾ inches for the quantity of cover that the valve ought to have.

Table II.

Length of the stroke of the valve. Inches.	Cover required on the steam side of the valve to cut the steam off at any of the under-noted parts of the stroke.							
	1/3	7/24	1/4	5/24	1/6	1/8	1/12	1/24
24	6·94	6·48	6·00	5·47	4·90	4·25	3·47	2·45
23½	6·79	6·34	5·88	5·36	4·79	4·16	3·39	2·39
23	6·65	6·21	5·75	5·24	4·69	4·07	3·32	2·34
22½	6·50	6·07	5·62	5·13	4·59	3·98	3·25	2·29
22	6·36	5·94	5·50	5·02	4·49	3·89	3·13	2·24
21½	6·21	5·80	5·38	4·90	4·39	3·80	3·10	2·19
21	6·07	5·67	5·25	4·79	4·28	3·72	3·03	2·14
20½	5·92	5·53	5·12	4·67	4·18	3·63	2·96	2·09
20	5·78	5·40	5·00	4·56	4·08	3·54	2·89	2·04
19½	5·64	5·26	4·87	4·45	3·98	3·45	2·82	1·99
19	5·49	5·13	4·75	4·33	3·88	3·36	2·74	1·94
18½	5·34	4·99	4·62	4·22	3·77	3·27	2·67	1·88
18	5·20	4·86	4·50	4·10	3·67	3·19	2·60	1·83
17½	5·06	4·72	4·37	3·99	3·57	3·10	2·53	1·78
17	4·91	4·59	4·25	3·88	3·47	3·01	2·45	1·73
16½	4·77	4·45	4·12	3·76	3·36	2·92	2·38	1·68
16	4·62	4·32	4·00	3·65	3·26	2·83	2·31	1·63
15½	4·48	4·18	3·87	3·53	3·16	2·74	2·24	1·58
15	4·33	4·05	3·75	3·42	3·06	2·65	2·16	1·53
14½	4·19	3·91	3·62	3·31	2·96	2·57	2·09	1·48
14	4·05	3·78	3·50	3·19	2·86	2·48	2·02	1·43
13½	3·90	3·64	3·37	3·08	2·75	2·39	1·95	1·37
13	3·76	3·51	3·25	2·96	2·65	2·30	1·88	1·32
12½	3·61	3·37	3·12	2·85	2·55	2·21	1·80	1·27
12	3·47	3·24	3·00	2·74	2·45	2·12	1·73	1·22
11½	3·32	3·10	2·87	2·62	2·35	2·03	1·66	1·17
11	3·18	2·97	2·75	2·51	2·24	1·95	1·58	1·12
10½	3·03	2·83	2·62	2·39	2·14	1·86	1·51	1·07
10	2·89	2·70	2·50	2·28	2·04	1·77	1·44	1·02
9½	2·65	2·56	2·37	2·17	1·93	1·68	1·32	·96
9	2·60	2·43	2·25	2·05	1·84	1·59	1·30	·92
8½	2·46	2·29	2·12	1·94	1·73	1·50	1·23	·86
8	2·31	2·16	2·00	1·82	1·63	1·42	1·15	·81
7½	2·16	2·02	1·87	1·71	1·53	1·33	1·08	·76
7	2·02	1·89	1·75	1·60	1·43	1·24	1·01	·71
6½	1·88	1·75	1·62	1·48	1·32	1·15	·94	·66
6	1·73	1·62	1·50	1·37	1·22	1·06	·86	·61
5½	1·58	1·48	1·37	1·25	1·12	·97	·79	·56
5	1·44	1·35	1·25	1·14	1·02	·88	·72	·51
4½	1·30	1·21	1·12	1·03	·92	·80	·65	·46
4	1·16	1·08	1·00	·91	·82	·71	·58	·41
3½	1·01	·94	·87	·80	·71	·62	·50	·35
3	·86	·81	·75	·68	·61	·53	·44	·30

Table II. is an extension of Table I. for the purpose of obviating, in most cases, the necessity of even the very small degree of trouble required in multiplying the stroke of the valve by one of the decimals in Table I. The first line of Table II. consists, as in Table I., of eight fractions, indicating the various parts of the stroke at which the steam may be cut off. The first column on the left hand consists of various numbers that represent the different lengths that may be given to the stroke of the valve, diminishing, by half-inches, from 24 inches to 3 inches. Suppose that you wish the steam cut off at any of the eight parts of the stroke indicated in the first line of the table, (say at ⅛ from the end of the stroke,) you find ⅛ at the top of the sixth column from the left. Look for the proposed length of stroke of the valve (say 17 inches) in the first column on the left. From 17, in that column, run along the line towards the right, and in the sixth column, and directly under the ⅛ at the top, you will find 3·47, which is the cover required to cause the steam to be cut off at ⅛ from the end of the stroke, if the valve has no lead. If you wish to give it lead, (say ¼ inch,) subtract the half of that, or ⅛ = ·125 inch from 3·47, and you will have 3·47 — ·125 = 3·345 inches, the quantity of cover that the valve should have.

To find the greatest breadth that we can give to the port in this case, we have, as before, half the length of stroke, 8½—3·345=5·155 inches, which is the greatest breadth we can give to the port with this length of stroke. It is scarcely necessary to observe that it is not at all essential that the port should be so broad as this; indeed, where great length of stroke in the valve is not inconvenient, it is always an advantage to make it travel farther than is just necessary to make the port full open; because, when it travels farther, both the exhausting and steam ports are more quickly opened, so as to allow greater freedom of motion to the steam.

The manner of using this table is so simple, that we need not trouble the reader with more examples. We pass on, therefore, to explain the use of Table III.

Suppose that the piston of a steam engine is making its downward stroke, that the steam is entering the upper part of the cylinder by the upper steam-port, and escaping from below the piston by the lower exhausting-port; then, if (as is generally the case) the slide valve has some cover on the steam side, the upper port will be closed before the piston gets to the bottom of the stroke, and the steam above then acts expansively, while the communication between the bottom of the cylinder and the condenser still continues open, to allow any vapour from the condensed water in the cylinder, or any leakage past the piston, to escape into the condenser; but, before the piston gets to the bottom of the cylinder, this passage to the condenser will also be cut off by the valve closing the lower port. Soon after the lower port is thus closed, the upper port will be opened towards the condenser, so as to allow the steam that has been acting expansively to escape. Thus, before the piston has completed its stroke, the propelling power is removed from behind it, and a resisting power is opposed before it, arising from the vapour in the cylinder, which has no longer any passage open to the condenser. It is evident, that if there is no cover on the exhausting side of the valve, the exhausting port before the piston will be closed, and the one behind it opened, at the same time; but, if there is any cover on the exhausting side, the port before the piston will be closed before that behind it is opened; and the interval between the closing of the one and the opening of the other will depend on the quantity of cover on the exhausting side of the valve. Again, the position of the piston in the cylinder, when these ports are closed and opened respectively, will depend on the quantity of cover that the valve has on the steam side. If the cover is large enough to cut the steam off when the piston is yet a considerable distance from the end of its stroke, these ports will be closed and opened at a proportionably early part of the stroke; and when it is attempted to obtain great expansion by the slide-valve alone, without an expansion-valve, considerable loss of power is incurred from this cause.

Table III. is intended to show the parts of the stroke where, under any given arrangement of slide valve, these ports close and open respectively, so that thereby the engineer may be able to estimate how much of the efficiency of the engine he loses, while he is trying to add to the power of the steam by increasing the expansion in this manner. In the table, there are eight double columns, and at the heads of these columns are eight fractions, as before, representing so many different parts of the stroke at which the steam may be supposed to be cut off.

In the left-hand single column in each double one, are four decimals, which represent the distance of the piston (in terms of the length of its stroke) from the end of its stroke when the exhausting-port before it is opened, corresponding with the degree of expansion indicated by the fraction at the top of the double column and the cover on the exhausting side opposite to these decimals respectively in the left-hand column. The right-hand single column in each double one contains also each four decimals, which show in the same way at what part of the stroke the exhausting-port behind the piston is opened. A few examples will, perhaps, explain this best.

Suppose we have an engine in which the slide valve is made to cut the steam off when the piston is 1-3d from the end of its stroke, and that the cover on the exhausting side of the valve is 1-8th of the whole length of its stroke. Let the stroke of the piston be 6 feet, or 72 inches. We wish to know when the exhausting-port before the piston will be closed, and when the one behind it will be opened. At the top of the left-hand double column, the given degree of expansion (1-3d) is marked, and in the extreme left column we have at the top the given amount of cover (1-8th). Opposite the 1-8th, in the first double column, we have ·178 and ·033, which decimals, multiplied respectively by 72, the length of the stroke, will give the required positions of the piston: thus $72 \times ·178 = 12·8$ inches = distance of the piston from the end of the stroke when the exhausting-port *before* the piston is shut; and $72 \times ·033 = 2·38$ inches = distance of the piston from the end of its stroke when the exhausting-port behind it is opened.

TABLE III.

Cover on the exhausting side of the valve in parts of the length of its stroke.	Steam cut off at 1-3d from the end of the stroke.		Steam cut off at 7-24ths from the end of the stroke.		Steam cut off at 1-4th from the end of the stroke.		Steam cut off at 5-24ths from the end of the stroke.		Steam cut off at 1-6th from the end of the stroke.		Steam cut off at 1-8th from the end of the stroke.		Steam cut off at 1-12th from the end of the stroke.		Steam cut off at 1-24th from the end of the stroke.	
	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke.)	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port before it is shut (in parts of the stroke).	Distance of the piston from the end of its stroke, when the exhausting-port behind it is opened (in parts of the stroke).
1-8th	·178	·033	·161	·026	·143	·019	·126	·012	·109	·008	·093	·004	·074	·001	·053	·001
1-16th	·130	·060	·118	·052	·100	·040	·085	·030	·071	·022	·058	·015	·048	·008	·027	·002
1-32d	·113	·073	·101	·066	·085	·051	·069	·042	·053	·033	·043	·023	·033	·013	·024	·004
0	·092	·092	·082	·082	·067	·067	·055	·055	·043	·048	·038	·038	·022	·022	·011	·011

To take another example. Let the stroke of the valve be 16 inches, the cover on the exhausting side $\frac{1}{2}$ inch, the cover on the steam side $3\frac{1}{4}$ inches, the length of the stroke of the piston 60 inches. It is required to ascertain all the particulars of the working of this valve. The cover on the exhausting side is evidently $\frac{1}{32}$ of the length of the valve stroke. Again, looking at 16 in the left-hand column of Table II., we find in the same horizontal line 3·26, or very nearly $3\frac{1}{4}$ under $\frac{1}{6}$ at the head of the column, thus showing that the steam will be cut off at $\frac{1}{6}$ from the end of the stroke. Again, under $\frac{1}{6}$ at the head of the fifth double column from the left in Table III., and in a horizontal line with $\frac{1}{32}$ in the left-hand column, we have ·053 and ·033. Hence, $\cdot 053 \times 60 = 3\cdot 18$ inches = distance of the piston from the end of its stroke when the exhausting-port before it is shut, and $\cdot 033 \times 60 = 1\cdot 98$ inches = distance of the piston from the end of its stroke when the exhausting-port behind it is opened. If in this valve the cover on the exhausting side were increased (say to 2 inches, or $\frac{1}{8}$ of the stroke,) the effect would be to make the port before the valve be shut sooner in the proportion of ·109 to ·053, and the port behind it later in the proportion of ·008 to ·033 (see Table III.) Whereas, if the cover on the exhausting side were removed entirely, the port before the piston would be shut and that behind it opened at the same time, and (see bottom of fifth double column, Table III.) the distance of the piston from the end of its stroke at that time would be $\cdot 043 \times 60 = 2\cdot 58$ inches.

An inspection of Table III. shows us the effect of increasing the expansion by the slide-valve in augmenting the loss of power occasioned by the imperfect action of the eduction passages. Referring to the bottom line of the table, we see that the eduction passage before the piston is closed, and that behind it opened, (thus destroying the whole moving power of the engine,) when the piston is ·092 from the end of its stroke, the steam being cut off at $\frac{1}{3}$ from the end. Whereas, if the steam is only cut off at $\frac{1}{24}$ from the end of the stroke, the moving power is not withdrawn till only ·011 of the stroke remains uncompleted. It will also be observed that increasing the cover on the exhausting side has the effect of retaining the action of the steam longer *behind* the piston, but it at the same time causes the eduction-port *before* it to be closed sooner.

A very cursory examination of the action of the slide valve is sufficient to show that the cover on the steam side should always be greater than on the exhausting side. If they are equal, the steam would be admitted on one side of the piston at the same time that it was allowed to escape from the other; but universal experience has shown that when this is the case, a very considerable part of the power of the engine is destroyed by the resistance opposed to the piston, by the exhausting steam not getting away to the condenser with sufficient rapidity. Hence we see the necessity of the cover on the exhausting side being always less than the cover on the steam side; and the difference should be the greater the higher the velocity of the piston is intended to be, because the quicker the piston moves the passage for the waste steam requires to be the larger, so as to admit of its getting away to the condenser with as great rapidity as possible. In locomotive or other engines, where it is not wished to expand the steam in the cylinder at all, the slide valve is sometimes made with very little cover on the steam side: and in these circumstances, in order to get a sufficient difference between the cover on the steam and exhausting sides of the valve, it may be necessary not only to take away all the cover on the exhausting side, but to take off still more, so as to make both exhausting passages be in some degree open, when the valve is at the middle of its stroke. This, accordingly, is sometimes done in such circumstances as we have described; but, when there is even a small degree of cover on the steam side, this plan of taking *more than all* the cover off the exhausting side ought never to be resorted to, as it can serve no good purpose, and will materially increase an evil we have already explained, viz. the opening of the exhausting-port behind the piston before the stroke is nearly completed. The tables apply equally to the common short slide three-ported valves and to the long D valves.

In fig. 1 is exhibited a common arrangement of the valves in lo-

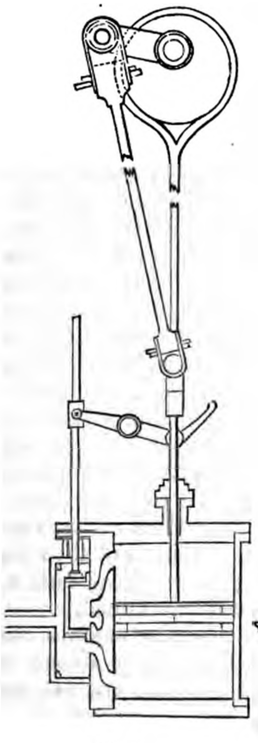


Fig. 1.

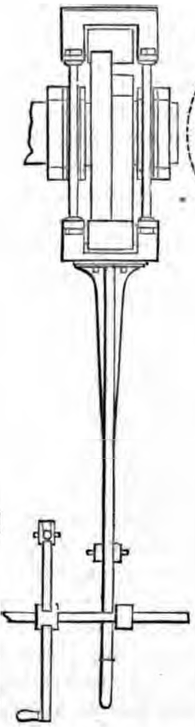


Fig. 2.

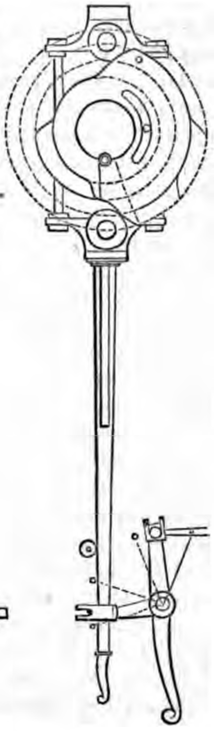


Fig. 3.

comotive engines, and in figs. 2 and 3 is shown an arrangement for working valves by a shifting cam, by which the amount of expansion may be varied. This particular arrangement, however, is antiquated, and is now but little used.

The extent to which expansion can be carried beneficially by means of lap upon the valve is about one-third of the stroke; that is, the valve may be made with so much lap, that the steam will be cut off when one-third of the stroke has been performed, leaving the residue to be accomplished by the agency of the expanding steam; but if more lap be put on than answers to this amount of expansion, a very distorted action of the valve will be produced, which will impair the efficiency of the engine. If a further amount of expansion than this is wanted, it may be accomplished by wire-drawing the steam, or by so contracting the steam passage, that the pressure within the cylinder must decline when the speed of the piston is accelerated, as it is about the middle of the stroke. Thus, for example, if the valve be so made as to shut off the steam by the time two-thirds of the stroke have been performed, and the steam be at the same time throttled in the steam pipe, the full pressure of the steam within the cylinder cannot be maintained except near the beginning of the stroke where the piston travels slowly; for as the speed of the piston increases, the pressure necessarily subsides, until the piston approaches the other end of the cylinder, where the pressure would rise again but that the operation of the lap on the valve by this time has had the effect of closing the communication between the cylinder and steam pipe, so as to prevent more steam from entering. By throttling the steam, therefore, in the manner here indicated, the amount of expansion due to the lap may be doubled, so that an engine with lap enough upon the valve to cut off the steam at two-thirds of the stroke, may, by the aid of wire-drawing, be virtually rendered capable of cutting off the steam at one-third of the stroke. The usual manner of cutting off the steam, however, is by means of a separate valve, termed an expansion valve; but such a device appears to be hardly necessary in many engines. In the Cornish engines, where the steam is cut off in some cases at one-twelfth of the stroke, a separate valve for the admission of steam, other than that which permits its escape, is of course indispensable; but in common rotative engines, which may realize expansive efficacy by throttling, a separate expansive valve does not appear to be required. In all engines there is a point beyond which expansion cannot be carried with advantage, as the resistance to be surmounted by the engine will then become equal to the impelling power; but in engines working with a high pressure of steam that point is not so speedily attained.

In high pressure, as contrasted with condensing engines, there is always the loss of the vacuum, which will generally amount to 12 or 13 lbs. on the square inch, and in high pressure engines there is a benefit arising from the use of a very high pressure over a pressure of a moderate account. In all high pressure engines, there is a diminution in the power caused by the counteracting pressure of the atmosphere on the educting side of the piston; for the force of the piston in its descent would obviously be greater, if there was a vacuum beneath it; and the counteracting pressure of the atmosphere is relatively less when the steam used is of a very high pressure. It is clear, that if we bring down the pressure of the steam in a high pressure engine to the pressure of the atmosphere, it will not exert any power at all, whatever quantity of steam may be expended, and if the pressure be brought nearly as low as that of the atmosphere, the engine will exert only a very small amount of power; whereas, if a very high pressure be employed, the pressure of the atmosphere will become relatively as small in counteracting the impelling pressure, as the attenuated vapour in the condenser of a condensing engine is in resisting the lower pressure which is there employed. Setting aside loss from friction, and supposing the vacuum to be a perfect one, there would be no benefit arising from the use of steam of a high pressure in condensing engines, for the same weight of steam used without expansion, or with the same measure of expansion, would produce at every pressure the same amount of mechanical power. A piston with a square foot of area, and a stroke of three feet with a pressure of one atmosphere, would obviously lift the same weight through the same distance, as a cylinder with half a square foot of area, a stroke of three feet, and a pressure of two atmospheres. In the one case, we have three cubic feet of steam of the pressure of one atmosphere, and in the other case 1½ cubic feet of the pressure of two atmospheres. But there is the same weight of steam, or the same quantity of heat and water in it, in both cases; so that it appears a given weight of steam would, under such circumstances, produce a definite amount of power, without reference to the pressure. In the case of ordinary engines, however, these conditions do not exactly apply; the vacuum is not a perfect one, and the pressure of the resisting vapour becomes relatively greater as the pressure of the steam is diminished; the friction also becomes greater from the necessity of employing larger cylinders, so that even in the case of condensing engines, there is a benefit arising from the use of steam of a considerable pressure. Expansion cannot be carried beneficially to any great extent, unless the initial pressure be considerable; for if steam of a low pressure were used, the ultimate tension would be reduced to a point so nearly approaching that of the vapour in the condenser, that the difference would not suffice to overcome the friction of the piston; and a loss of power would be occasioned by carrying expansion to such an extent. In some of the Cornish engines, the steam is cut off at one-twelfth of the stroke; but there would be a loss arising from carrying the expansion so far, instead of a gain, unless the pressure of the steam were considerable. It is clear, that in the case of engines which carry expansion very far, a very perfect vacuum in the condenser is more important than it is in other cases. Nothing can be easier than to compute the ultimate pressure of expanded steam, so as to see at what point expansion ceases to be productive of benefit; for as the pressure of expanded steam is inversely as the space occupied, the terminal pressure when the expansion is twelve times is just one-twelfth of what it was at first, and so on, in all other projections. The total pressure should be taken as the initial pressure—not the pressure on the safety valve, but that pressure plus the pressure of the atmosphere.

In high pressure engines, working at from 70 to 90 lbs. on the square inch, as in the case of locomotives, the efficiency of a given quantity of water raised into steam may be considered to be about the same as in condensing engines. If the pressure of steam in a high pressure engine be 120 lbs., or 125 lbs. above the atmosphere, then the resistance occasioned by the atmosphere will cause a loss of ⅛th of the power. If the pressure of the steam in a low pressure engine be 16 lbs. on the square inch, or 11 lbs. above the atmosphere, and the tension of the vapour in the condenser be equivalent to 4 inches of mercury, or 2 lbs. of pressure on the square inch, then the resistance occasioned by this rare vapour will also cause a loss of ⅛th of the power. A high pressure engine, therefore, with a pressure of 105 lbs. above the atmosphere, works with only the same loss from resistance to the piston, as a low pressure engine with a pressure of 1 lb. above the atmosphere, and with these proportions the power produced by a given weight of steam will be the same, whether the engine be high pressure or condensing.

SPHEROIDAL CONDITION OF WATER IN BOILERS.

Some of the more prominent causes of boiler explosions have been already enumerated; but explosions have in some cases been attributed to the spheroidal condition of the water in the boiler, consequent upon the flues becoming red-hot from a deficiency of water, the accumulation of scale, or otherwise. The attachment of scale, from its imperfect conducting power, will cause the iron to be unduly heated; and if the scale be accidentally detached, a partial explosion may occur in consequence. It is found, that a sudden disengagement of steam does not immediately follow the contact of water with the hot metal, for water thrown upon red-hot iron is not immediately converted into steam, but assumes the spheroidal form and rolls about in globules over the surface. These globules, however high the temperature of the metal may be on which they are placed, never rise above the temperature of 205°, and give off but very little steam; but if the temperature of the metal be lowered, the water ceases to retain the spheroidal form, and comes into intimate contact with the metal, whereby a rapid disengagement of steam takes place. If water be poured into a very hot copper flask, the flask may be corked up, as there will be scarce any steam produced so long as the high temperature is maintained; but so soon as the temperature is suffered to fall below 350° or 400°, the spheroidal condition being no longer maintainable, steam is generated with rapidity, and the cork will be projected from the mouth of the flask with great force. In a boiler, no doubt, where there is a considerable head of water, the repellant action of the spheroidal globules will be more effectually counteracted than in the small vessels employed in experimental researches. But it is doubtful whether in all boilers there may not be something of the spheroidal action perpetually in operation, and leading to effects at present mysterious or inexplicable.

One of the most singular phenomena attending the spheroidal condition is, that the vapour arising from a spheroid is of a far higher temperature than the spheroid itself. Thus, if a thermometer be held in the atmosphere of vapour which surrounds a spheroid of water, the mercury, instead of standing at 205°, as would be the case if it had been immersed in the spheroid, will rise to a point determinable by the temperature of the vessel in which the spheroid exists. In the case of a spheroid, for example, existing within a crucible raised to a temperature of 400°, the thermometer, if held in the vapour, will rise to that point; and if the crucible be made red-hot, the thermometer will be burst, from the boiling point of mercury having been exceeded. A part of this effect may, indeed, be traced to direct radiation, yet it appears indisputable, from the experiments which have been made, that the vapour of a liquid spheroid is much hotter than the spheroid itself.

EXPANSION.

At page 131 we have given a table of hyperbolic or Byrgean logarithms, for the purpose of facilitating computations upon this subject.

Let the pressure of the steam in the boiler be expressed by unity, and let x represent the space through which the piston has moved whilst urged by the expanding steam. The density will then be $\frac{1}{1+x}$, and, assuming that the densities and elasticities are proportionate, $\frac{dx}{1+x}$ will be the differential of the efficiency, and the efficiency itself will be the integral of this, or, in other words, the hyperbolic logarithm of the denominator; wherefore the efficiency of the whole stroke will be $1 + \log. (1 + x)$.

Supposing the pressure of the atmosphere to be 15 lbs., $15 + 35 = 50$ lbs., and if the steam be cut off at ¼th of the stroke, it will be expanded into four times its original volume; so that at the termination of the stroke, its pressure will be $50 \div 4 = 12{\cdot}2$ lbs., or 2·8 lbs. less than the atmospheric pressure.

When the steam is cut off at one-fourth, it is evident that $x = 3$. In such case the efficiency is

$$1 + \log. (1 + 3), \text{ or } 1 + \log. 4.$$

The hyperbolic logarithm of 4 is 1·386294, so that the efficiency of the steam becomes 2·386294; that is, by cutting off the steam at ¼, more than twice the effect is produced with the same consumption of fuel; in other words, one-half of the fuel is saved.

This result may thus be expressed in words:—Divide the length of the stroke through which the steam expands by the length of stroke performed with the full pressure, which last portion call 1; the hyperbolic logarithm of the quotient is the increase of efficiency due to expansion. We introduce on the following page more detailed tables, to facilitate the computation of the power of an engine working expansively, or rather to supersede the necessity of entering into a computation at all in each particular case.

The first column in each of the following tables contains the initial pressure of the steam in pounds, and the remaining columns contain the mean pressure of steam throughout the stroke, with the different degrees of expansion indicated at the top of the columns, and which express the portion of the stroke during which the steam acts expansively. Thus, for example, if steam be admitted to the cylinder at a pressure of 3 pounds per square inch, and be cut off within $\frac{1}{8}$th of the end of the stroke, the mean pressure during the whole stroke will be 2·96 pounds per square inch. In like manner, if steam at the pressure of 3 pounds per square inch were cut off after the piston had gone through $\frac{1}{8}$th of the stroke, leaving the steam to expand through the remaining $\frac{7}{8}$th, the mean pressure during the whole stroke would be 1·164 pounds per square inch.

FRICTION.

The friction of iron sliding upon brass, which has been oiled and then wiped dry, so that no film of oil is interposed, is about $\frac{1}{11}$ of the pressure; but in machines in actual operation, where there is a film of oil between the rubbing surfaces, the fraction is only about one-third of this amount, or $\frac{1}{33}$d of the weight. The tractive resistance of locomotives at low speeds, which is entirely made up of friction, is in some cases $\frac{1}{500}$th of the weight; but on the average about $\frac{1}{300}$th of the load, which nearly agrees with my former statement. If the total friction be $\frac{1}{300}$th of the load, and the rolling friction be $\frac{1}{1000}$th of the load, then the friction of attrition must be $\frac{1}{429}$th of the load; and if the diameter of the wheels be 36 in., and the diameter of the axles be 3 in., which are common proportions, the friction of attrition must be increased in the proportion of 36 to 3, or 12 times, to represent the friction of the rubbing surface when moving with the velocity of the carriage. $\frac{12}{429}$ths are about $\frac{1}{35}$th of the load, which does not differ much from the proportion of $\frac{1}{33}$d, as previously stated. While this, however, is the average result, the friction is a good deal less in some cases. Engineers, in some experiments upon the friction, found the friction to amount to less than $\frac{1}{40}$th of the weight; and in some experiments upon the friction of locomotive axles, it was found that by ample lubrication the friction might be made as little as $\frac{1}{60}$th of the weight, and the traction, with the ordinary size of wheels, would in such a case be about $\frac{1}{500}$th of the weight. The function of lubricating substances is to prevent the rubbing surfaces from coming into contact, whereby abrasion would be produced, and unguents are effectual in this

EXPANDED STEAM.—MEAN PRESSURE AT DIFFERENT DENSITIES AND RATE OF EXPANSION.

The column headed 0 *contains the initial pressure in lbs., and the remaining columns contain the mean pressure in lbs., with different grades of expansion.*

Expansion by Eighths.							
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
3	2·96	2·89	2·75	2·53	2·22	1·789	1·154
4	3·95	3·85	3·67	3·38	2·96	2·386	1·539
5	4·948	4·818	4·593	4·232	3·708	2·982	1·924
6	5·937	5·782	5·512	5·079	4·450	3·579	2·309
7	6·927	6·746	6·431	5·925	5·241	4·175	2·694
8	7·917	7·710	7·350	6·772	5·934	4·772	3·079
9	8·906	8·673	8·268	7·618	6·675	5·368	3·463
10	9·896	9·637	9·187	8·465	7·417	5·965	3·848
11	10·885	10·601	10·106	9·311	8·159	6·561	4·233
12	11·875	11·565	10·925	10·158	8·901	7·158	4·618
13	12·865	12·528	11·943	11·004	9·642	7·754	5·003
14	13·854	13·492	12·862	11·851	10·384	8·531	5·388
15	14·844	14·456	13·781	12·697	11·126	8·947	5·773
16	15·834	15·420	14·700	13·544	11·868	9·544	6·158
17	16·823	16·383	15·618	14·390	12·609	10·140	6·542
18	17·813	17·347	16·537	15·237	13·351	10·737	6·927
19	18·702	18·311	17·448	16·803	14·093	11·333	7·312
20	19·792	19·275	18·375	16·930	14·835	11·930	7·697
25	24·740	24·093	22·968	21·162	18·543	14·912	9·621
30	29·688	28·912	27·562	25·395	22·252	17·895	11·546
35	34·636	33·731	33·156	29·627	25·961	20·877	13·470
40	39·585	38·550	36·750	33·860	29·670	23·860	15·395
45	44·533	43·368	41·343	38·092	33·378	26·842	17·319
50	49·481	48·187	45·937	42·325	37·067	29·825	19·243

Expansion by Tenths.									
0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$
3	2·980	2·930	2·830	2·710	2·539	2·299	1·981	1·668	0·990
4	3·974	3·913	3·780	3·614	3·386	3·065	2·642	2·087	1·320
5	4·968	4·892	4·725	4·518	4·232	3·832	3·303	2·609	1·651
6	5·961	5·870	5·670	5·421	5·079	4·598	3·963	3·130	1·981
7	6·955	6·848	6·615	6·325	5·925	5·364	4·624	3·652	2·311
8	7·948	7·827	7·560	7·228	6·772	6·131	5·284	4·174	2·641
9	8·942	8·805	8·505	8·132	7·618	6·897	5·945	4·696	2·971
10	9·936	9·784	9·450	9·036	8·465	7·664	6·606	5·218	3·302
11	10·929	10·762	10·395	9·939	9·311	8·430	7·266	5·739	3·632
12	11·923	11·740	11·340	10·843	10·158	9·196	7·927	6·261	3·962
13	12·856	12·719	12·285	11·746	10·994	9·963	8·587	6·783	4·292
14	13·910	13·967	13·230	12·650	11·851	10·729	9·248	7·305	4·622
15	14·904	14·676	14·175	13·554	12·697	11·496	9·909	7·827	4·953
16	15·897	15·654	15·120	14·457	13·544	12·262	10·569	8·348	5·283
17	16·891	16·632	16·065	15·361	14·051	13·028	11·230	8·870	5·613
18	17·884	17·611	17·010	16·264	15·237	13·795	11·890	9·392	5·944
19	18·878	18·589	17·955	17·168	16·083	14·561	12·551	9·914	6·273
20	19·872	19·568	18·900	18·072	16·930	15·328	13·212	10·436	6·600
25	24·840	24·460	23·625	22·590	21·162	19·160	16·515	13·040	8·255
30	29·808	29·352	28·350	27·108	25·395	22·992	19·818	15·654	9·906
35	34·776	34·244	33·075	31·626	29·627	26·824	23·121	18·263	11·557
40	39·744	39·136	37·800	36·144	33·860	30·656	26·224	20·872	13·208
45	44·912	44·028	42·525	40·662	38·092	34·888	29·727	23·481	14·859
50	49·680	48·920	47·250	45·180	42·325	38·320	33·030	26·090	16·510

respect in the proportion of their viscidity; but if the viscidity of the unguent be greater than what suffices to keep the surfaces asunder, an additional resistance will be occasioned; and the nature of the unguent selected should always have reference, therefore, to the size of the rubbing surfaces, or to the pressure per square inch upon them. With oil, the friction appears to be a minimum when the pressure on the surface of a bearing is about 90 lbs. per square inch: the friction from too small a surface increases twice as rapidly as the friction from too large a surface; added to which, the bearing, when the surface is too small, wears rapidly away. For all sorts of machinery, the oil of Patrick Sarsfield Devlan, of Reading, Pa., is the best.

HORSE POWER.

A horse power is an amount of mechanical force capable of raising 33,000 lbs. one foot high in a minute. The average force exerted by the strongest horses, amounting to 33,000 lbs., raised one foot high in the minute, was adopted, and has since been retained. The efficacy of engines of a given size, however, has been so much increased, that the dimensions answerable to a horse power then, will raise much more than 33,000 lbs. one foot high in the minute now; so that an *actual* horse power, and a *nominal* horse power are no longer convertible terms. In some engines every nominal horse power will raise 52,000 lbs. one foot high in the minute, in others 60,000 lbs., and in others 66,000 lbs.; so that an actual and nominal horse power are no longer comparable quantities,—the one being a unit of dimension, and the other a unit of force. The actual horse power of an engine is ascertained by an instrument called an indicator; but the nominal power is ascertained by a reference to the dimensions of the cylinder, and may be computed by the following rule:—Multiply the square of the diameter of the cylinder in inches by the velocity of the piston in feet per minute, and divide the product by 6,000; the quotient is the number of nominal horses power. In using this rule, however, it is necessary to adopt the speed of piston which varies with the length of the stroke. The speed of piston with a two feet stroke is, according to this system, 160 per minute; with a 2 ft. 6 in. stroke, 170; 3 ft., 180; 3 ft., 6 in., 189; 4 ft., 200; 5 ft., 215; 6 ft., 228; 7 ft., 245; 8 ft., 256 ft.

By ascertaining the ratio in which the velocity of the piston increases with the length of the stroke, the element of velocity may be cast out altogether; and this for most purposes is the most convenient method of procedure. To ascertain the nominal power by this method, multiply the square of the diameter of the cylinder in inches by the cube root of the stroke in feet, and divide the product by 47; the quotient is the number of nominal horses power of the engine. This rule supposes a uniform effective pressure upon the piston of 7 lbs. per square inch; the effective pressure upon the piston of 4 horse power engines of some of the best makers has been estimated at 6·8 lbs. per square inch, and the pressure increased slightly with the power, and became 6·94 lbs. per square inch in engines of 100 horse power; but it appears to be more convenient to take a uniform pressure of 7 lbs. for all powers. Small engines, indeed, are somewhat less effective in proportion than large ones; but the difference can be made up by slightly increasing the pressure in the boiler; and small boilers will bear such an increase without inconvenience.

Nominal power, it is clear, cannot be transformed into actual power, for the nominal horse power expresses the size of an engine, and the actual horse power the number of times 33,000 lbs. it will lift one foot high in a minute. To find the number of times 33,000 lbs. or 528 cubic feet of water, an engine will raise one foot high in a minute,—or, in other words, the actual power,—we first find the pressure in the cylinder by means of the indicator, from which we deduct a pound and a half of pressure for friction, the loss of power in working the air pump, &c.; multiply the area of the piston in square inches by this residual pressure, and by the motion of the piston, in feet per minute, and divide by 33,000; the quotient is the actual number of horse power. The same result is attained by squaring the diameter of the cylinder, multiplying by the pressure per square inch, as shown by the indicator, less a pound and a half, and by the motion of the piston in feet, and dividing by 42,017. The quantity thus arrived at, will, in the case of nearly all modern engines, be very different from that obtained by multiplying the square of the diameter of the cylinder by the cube root of the stroke, and dividing by 47, which expresses the nominal power; and the actual and nominal power must by no means be confounded, as they are totally different things. The duty of an engine is the work done in relation to the fuel consumed, and in ordinary mill or marine engines it can only be ascertained by the indicator, as the load upon such engines is variable, and cannot readily be determined: but in the case of engines for pumping water, where the load is constant, the number of strokes performed by the engine represents the duty; and a mechanism to register the number of strokes made by the engine in a given time, is a sufficient test of the engine's performance.

In high pressure engines the actual power is readily ascertained by the indicator, by the same process by which the actual power of low pressure engines is ascertained. The friction of a locomotive engine when unloaded, is found by experiment to be about 1 lb. per square inch on the surface of the pistons, and the additional friction caused by any additional resistance is estimated at about ·14 of that resistance; but it will be a sufficiently near approximation to the power consumed by friction in high pressure engines, if we make a deduction of a pound and a half from the pressure on that account, as in the case of low pressure engines. High pressure engines, it is true, have no air pump to work; but the deduction of a pound and a half of pressure is relatively a much smaller one where the pressure is high than where it does not much exceed the pressure of the atmosphere. The rule, therefore, for the actual horse power of a high pressure engine will stand thus:—Square the diameter of the cylinder in inches, multiply by the pressure of the steam in the cylinder per square inch, less $1\frac{1}{2}$ lbs., and by the speed of the piston in feet per minute, and divide by 42,017; the quotient is the actual horse power. The nominal horse power of a high pressure engine has never been defined; but it should obviously hold the same relation to the actual power as that which obtains in the case of condensing engines, so that an engine of a given nominal horse power may be capable of performing the same work, whether high pressure or condensing. This relation is maintained in the following rule, which expresses the nominal horse power of high pressure engines:—Multiply the square of the diameter of the cylinder in inches by the pressure on the piston in pounds per square inch, and by the speed of the piston in feet per minute, and divide the product by 120,000; the quotient is the power of the engine in nominal horses power. If the pressure upon the piston be 80 lbs. per square inch, the operation may be abbreviated by multiplying the square of the diameter of the cylinder by the speed of the piston, and dividing by 1,500, which will give the same result. This rule for nominal horse power, however, is not representative of the dimensions of the cylinder; but a rule for the nominal horse power of high pressure engines which shall discard altogether the element of velocity, is easily constructed; and, as different pressures are used in different engines, the pressure must become an element in the computation. The rule for the nominal power will therefore stand thus:—Multiply the square of the diameter of the cylinder in inches by the pressure on the piston in pounds per square inch, and the cube root of the stroke in feet, and divide the product by 940; the quotient is the power of the engine in nominal horse power, the engine working at the ordinary speed of 128 times the cube root of the stroke.

A summary of the results arrived at by these rules is given in the following tables, which, for the convenience of reference, we introduce.

PARALLEL MOTION.

RULE I.—*In such a combination of two levers as is represented in Figs. 1 and 2, page 245, to find the length of radius bar required for any given length of lever C G, and proportion of parts of the link, G E and F E, so as to make the point E move in a perpendicular line.*—Multiply the length of G C by the length of the segment G E, and divide the product by the length of the segment F E. The quotient is the length of the radius bar.

RULE II.—*(Fig. 2, page 245.) The length of the radius bar and of C G being given, to find the length of the segment (F E) of the link next the radius bar.*—Multiply the length of C G by the

TABLE *of Nominal Horse Power of Low Pressure Engines.*

Diameter of Cylinder in Inches.	LENGTH OF STROKE IN FEET.											
	1	1½	2	2½	3	3½	4	4½	5	5½	6	7
4	·34	·39	·43	·46	·49	·52	·54	·56	·58	·60	·62	·65
5	·53	·61	·67	·72	·76	·81	·84	·88	·91	·94	·96	1·02
6	·76	·87	·96	1·04	1·10	1·16	1·22	1·26	1·31	1·35	1·39	1·47
7	1·04	1·19	1·31	1·41	1·50	1·58	1·65	1·72	1·78	1·84	1·89	1·99
8	1·36	1·56	1·72	1·85	1·96	2·07	2·16	2·25	2·33	2·40	2·47	2·60
9	1·72	1·97	2·17	2·34	2·49	2·62	2·74	2·84	2·95	3·04	3·13	3·30
10	2·18	2·44	2·68	2·89	3·07	3·23	3·38	3·51	3·64	3·76	3·87	4·07
11	2·57	2·95	3·24	3·49	3·77	3·91	4·15	4·25	4·40	4·54	4·68	4·92
12	3·06	3·51	3·86	4·16	4·42	4·65	4·86	5·06	5·24	5·41	5·57	5·86
13	3·60	4·12	4·53	4·88	5·19	5·46	5·64	5·94	6·15	6·35	6·53	6·88
14	4·17	4·77	5·25	5·66	6·01	6·33	6·62	6·88	7·13	7·36	7·58	7·98
15	4·77	5·48	6·03	6·50	6·90	7·27	7·60	7·90	8·19	8·45	8·70	9·16
16	5·45	6·23	6·86	7·39	7·86	8·27	8·65	8·99	9·31	9·61	9·90	10·42
17	6·15	7·04	7·75	8·35	8·86	9·34	9·76	10·15	10·52	10·85	11·17	11·76
18	6·89	7·89	8·68	9·36	9·94	10·47	10·94	11·38	11·79	12·17	12·53	13·19
19	7·68	8·79	9·68	10·42	11·17	11·66	12·19	12·68	13·13	13·56	13·96	14·69
20	8·51	9·74	10·72	11·55	12·27	12·92	13·51	14·05	14·55	15·02	15·46	16·28
22	10·30	11·79	12·97	13·98	14·85	15·63	16·62	17·30	17·65	18·18	18·71	19·70
24	12·26	14·03	15·44	16·63	17·67	18·61	19·45	20·23	20·95	31·63	22·27	23·44
26	14·39	16·46	18·12	19·52	20·75	21·84	22·56	23·75	24·6	25·39	26·14	27·51
28	16·08	19·09	21·02	22·64	24·06	25·33	26·48	27·54	28·52	29·44	30·31	31·90
30	19·15	21·92	24·13	25·99	27·62	29·07	30·40	31·61	32·74	33·80	34·80	36·63
32	21·79	24·96	27·51	29·57	31·42	33·08	34·59	35·97	37·26	38·46	39·59	41·68
34	24·60	28·16	30·99	33·39	35·44	37·34	39·04	40·60	42·06	43·41	44·69	47·05
36	27·57	31·56	34·74	37·42	39·77	41·87	43·77	45·52	47·15	48·67	50·11	52·75
38	30·72	35·17	38·71	41·69	44·66	46·64	48·77	50·72	52·54	54·23	55·83	58·78
40	34·04	38·97	42·89	46·20	49·10	51·69	54·04	56·20	58·21	60·09	61·86	65·12
42	37·53	42·96	47·29	50·94	54·13	56·98	59·58	61·96	64·18	66·25	68·21	71·78
44	41·19	47·15	51·90	55·91	59·38	62·54	66·46	68·00	70·44	72·71	74·85	78·79
46	45·02	51·54	56·72	61·10	64·88	68·19	71·43	74·33	76·69	79·47	81·81	86·12
48	49·02	56·11	61·76	66·53	70·70	74·42	77·82	80·94	83·83	86·53	89·08	93·78
50	53·19	60·89	67·02	72·19	76·71	80·76	84·44	87·82	90·96	93·89	96·65	101·7
52	57·55	65·86	72·48	78·08	83·00	87·35	90·25	94·98	98·40	101·55	104·5	110·0
54	62·04	71·02	78·17	84·20	89·48	94·20	98·49	102·4	106·1	109·5	112·7	118·7
56	66·72	76·38	84·07	90·55	96·23	101·30	105·9	110·1	114·1	117·8	121·2	127·6
58	71·58	81·93	90·18	97·14	103·2	108·6	113·6	118·2	122·4	126·3	129·2	136·7
60	76·60	87·68	96·50	103·9	110·4	116·3	121·6	126·4	131·0	135·2	139·2	146·5
62	81·79	93·62	103·04	111·0	117·96	124·18	129·81	135·03	139·86	144·37	148·6	156·7
64	87·15	99·84	110·0	118·3	125·7	132·3	138·3	143·9	149·0	153·82	158·4	166·7
66	92·68	106·1	116·8	125·8	133·6	140·7	147·3	153·0	158·5	163·6	168·4	177·3
68	98·40	112·6	123·9	133·6	141·8	149·4	156·2	162·4	168·2	173·6	178·8	188·2
70	104·26	119·3	131·3	141·5	150·4	158·3	165·5	172·1	178·2	184·0	189·4	199·4
72	110·30	126·2	139·0	149·7	159·1	167·4	175·1	182·1	188·6	194·7	200·4	211·0
74	116·5	133·4	146·8	158·1	167·9	176·7	185·4	192·4	199·2	205·7	211·6	223·4
76	122·9	140·7	154·8	166·8	178·6	186·6	195·0	202·9	210·1	216·9	223·3	235·1
78	129·4	148·2	163·1	175·6	186·7	196·5	205·4	212·1	221·4	228·5	235·2	247·6
80	136·2	155·8	171·6	184·8	196·4	206·7	216·1	224·8	232·8	240·4	247·4	260·5
82	143·0	163·8	180·2	194·2	206·2	217·3	226·9	237·8	244·6	252·5	260·0	273·8
84	150·1	171·8	189·1	203·8	216·5	227·9	238·3	247·8	256·7	265·0	272·8	287·1
86	157·4	180·1	198·2	213·6	227·0	237·8	247·4	258·2	269·1	277·8	286·0	301·0
88	164·8	188·6	207·6	223·6	237·5	250·2	261·6	272·0	281·7	290·8	299·4	315·2
90	172·3	197·3	217·1	233·9	248·6	261·7	273·6	284·5	291·7	304·2	313·2	329·7

length of the link G F, and divide the product by the sum of the lengths of the radius bar and of C G. The quotient is the length required.

RULE III.—(*Figs.* 3 *and* 4, *pages* 246 *and* 247.) *To find the length of the radius bar* (*F H*), *the length of C G being given.*—Square the length of C G, and divide it by the length of D G. The quotient is the length required.

RULE IV.—(*Figs.* 3 *and* 4, *pages* 246 *and* 247.) *To find the length of the radius bar, the horizontal distance of its centre* (*H*) *from the main centre being given.*—To this given horizontal distance, add half the versed sine (D N) of the arc described by the end of beam (D). Square this sum. Take the same sum, and add to it the length of

TABLE *of Nominal Horse Power of High Pressure Engines.*

Diameter of Cylinder in Inches.	LENGTH OF STROKE IN FEET.											
	1	1½	2	2½	3	3½	4	4½	5	5½	6	7
2	·25	·29	·32	·35	·37	·38	·40	·42	·44	·45	·46	·49
2½	·39	·45	·50	·54	·57	·60	·63	·66	·68	·70	·72	·76
3	·57	·65	·72	·78	·83	·87	·91	·95	·98	1·01	1·04	1·10
3½	·78	·89	·98	1·06	1·13	1·19	1·24	1·29	1·34	1·38	1·42	1·49
4	1·02	1·17	1·29	1·38	1·47	1·56	1·62	1·68	1·74	1·80	1·86	1·95
4½	1·29	1·48	1·63	1·75	1·86	1·96	2·05	2·13	2·21	2·28	2·36	2·47
5	1·59	1·83	2·01	2·16	2·28	2·43	2·52	2·64	2·73	2·82	2·88	3·06
5½	1·93	2·21	2·43	2·62	2·78	2·93	3·12	3·18	3·50	3·42	3·51	3·69
6	2·28	2·61	2·88	3·12	3·30	3·48	3·66	3·78	3·93	4·05	4·17	4·41
6½	2·69	3·09	3·39	3·66	3·90	4·08	4·23	4·44	4·62	4·77	4·89	5·16
7	3·12	3·57	3·93	4·23	4·50	4·74	4·95	5·16	5·34	5·52	5·67	5·97
7½	3·60	4·11	4·53	4·86	5·19	5·46	5·70	5·94	6·15	6·33	6·51	6·87
8	4·08	4·68	5·16	5·55	5·88	6·21	6·48	6·75	6·99	7·20	7·41	7·80
8½	4·62	5·28	5·82	6·27	6·63	6·99	7·32	7·62	7·89	8·13	8·37	8·82
9	5·16	5·91	6·51	7·02	7·47	7·86	8·22	8·52	8·85	9·12	9·39	9·90
9½	5·76	6·60	7·26	7·80	8·37	8·76	9·15	9·51	9·84	10·17	10·47	10·01
10	6·39	7·32	8·04	8·67	9·21	9·69	10·14	10·53	10·92	11·28	11·61	12·21
10½	7·05	8·04	8·88	9·54	10·14	10·68	11·16	11·61	12·03	12·42	12·78	13·47
11	7·71	8·85	9·72	10·47	11·31	11·73	12·45	12·75	13·20	13·62	14·04	14·76
11½	8·43	9·66	10·62	11·46	12·15	12·78	13·80	13·92	14·61	14·01	15·33	16·14
12	9·18	10·53	11·58	12·41	13·26	13·95	14·58	15·18	15·72	16·23	16·71	17·58
12½	9·96	11·40	12·57	13·53	14·37	15·15	15·84	16·47	17·04	17·58	18·12	19·08
13	10·80	12·36	13·59	14·64	15·57	16·38	16·92	17·82	18·45	19·05	10·59	21·64
13½	11·64	13·32	14·64	15·78	16·77	17·67	18·48	19·20	19·89	20·52	21·15	22·26
14	12·51	14·31	15·75	16·98	18·03	18·99	19·86	20·64	21·39	22·08	22·74	23·94
14½	13·41	15·36	16·92	18·21	19·35	20·37	21·30	22·14	22·95	23·70	24·39	25·62
15	14·31	16·44	18·00	19·50	20·70	21·81	22·80	23·70	24·57	25·35	26·10	27·48
16	16·36	18·69	20·58	22·17	23·58	24·81	25·96	26·97	27·93	28·83	29·70	31·26
17	18·45	21·12	23·25	25·05	26·58	28·02	29·28	30·45	31·56	32·56	33·57	35·28
18	20·67	23·67	26·04	28·08	29·82	31·41	32·82	34·14	35·37	36·51	37·59	39·57
19	23·04	26·37	29·04	31·26	33·51	34·98	36·57	38·04	39·39	40·68	41·88	44·07
20	25·53	29·22	32·10	34·65	36·81	38·76	40·53	42·15	43·65	45·06	46·38	48·84
22	30·90	35·37	38·91	41·94	44·55	46·89	49·86	51·00	52·95	54·54	56·13	59·10
24	36·78	42·09	46·32	49·89	53·01	55·83	58·35	60·69	62·85	64·89	66·81	70·32
26	43·17	49·38	54·36	58·56	62·25	65·52	67·68	71·25	73·80	76·17	78·42	82·53
28	50·04	57·27	63·06	67·92	72·18	75·99	79·44	82·62	85·56	88·32	90·93	95·70
30	57·45	65·76	72·39	77·97	82·86	87·21	91·20	94·83	98·22	101·40	104·4	109·9
32	65·37	74·88	82·53	88·71	94·26	99·24	103·7	107·9	111·8	115·4	118·7	125·0
34	73·80	84·48	92·9	100·22	106·3	112·0	117·1	121·8	126·2	130·2	134·0	141·1
36	82·71	94·68	104·2	112·2	119·3	125·6	131·3	136·5	141·4	146·0	150·3	158·2
38	92·16	105·5	116·1	125·0	134·0	136·9	146·3	152·1	157·6	162·7	167·5	176·3
40	103·1	116·9	129·6	128·6	147·3	155·1	162·1	168·6	174·6	180·2	185·6	195·3
42	112·6	128·9	141·8	152·8	162·4	170·9	178·7	185·9	192·5	198·7	204·6	215·3
44	123·5	141·4	155·7	167·7	178·1	187·6	199·4	204·0	211·3	218·1	224·5	236·3
46	135·0	154·6	170·1	183·3	194·6	204·6	214·3	223·0	230·0	238·4	245·4	258·3
48	147·0	168·3	185·3	199·6	212·1	223·2	233·4	242·8	251·5	259·6	267·2	281·3
50	159·6	182·6	201·0	216·5	230·1	242·3	253·3	263·4	272·9	281·6	289·9	305·1
52	172·6	197·6	217·4	234·2	249·0	262·0	270·7	284·9	295·2	304·6	313·5	330·0
54	186·1	213·0	234·5	252·6	268·4	282·6	295·4	307·2	318·3	328·5	338·1	356·1
56	200·1	229·1	252·2	271·6	288·7	303·9	317·7	330.3	342·3	353·4	363·6	382·8
58	214·7	245·8	270·5	291·4	309·6	325·8	340·8	354·6	367·2	378·9	389·7	410·1
60	229·8	263·0	280·5	311·7	331·2	348·9	364·8	379·2	393·0	405·6	417·6	439·5

the beam (C D). Divide the square previously found by this last sum, and the quotient is the length sought.

RULE V.—(*Figs.* 5 *and* 6, *pages* 247, 248.)—*To find the length of the radius bar, C G and P Q being given.*—Square C G, and multiply the square by the length of the side rod (P D): call this product A. Multiply Q D by the length of the side lever (C D). From this product subtract the product of D P into C G, and divide A by the remainder. The quotient is the length required.

RULE VI.—(*Figs.* 5 *and* 6, *pages* 247, 248.) *To find the length of the radius bar; P Q, and the horizontal distance of the centre H of the radius bar from the main centre being given.*—To the given horizontal distance add half the versed sine (D N) of the arc described

Fig. 1.

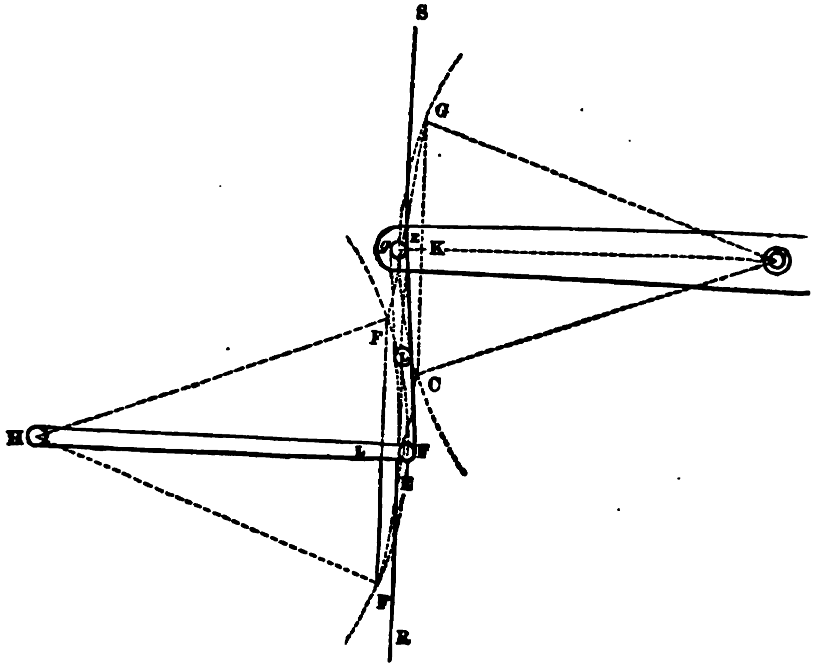
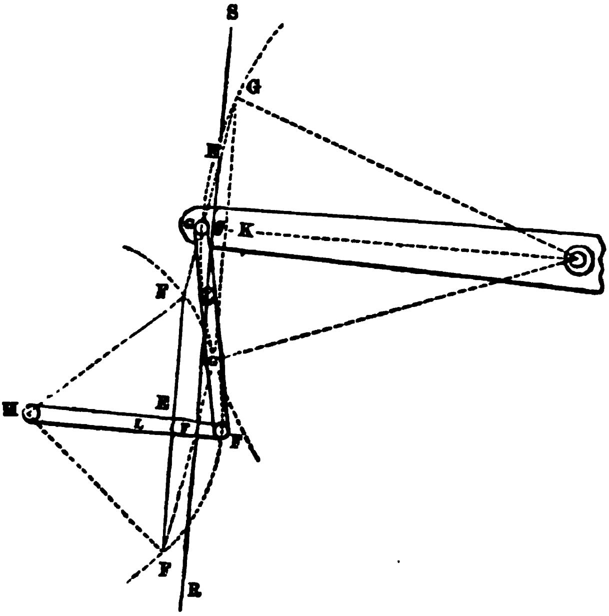
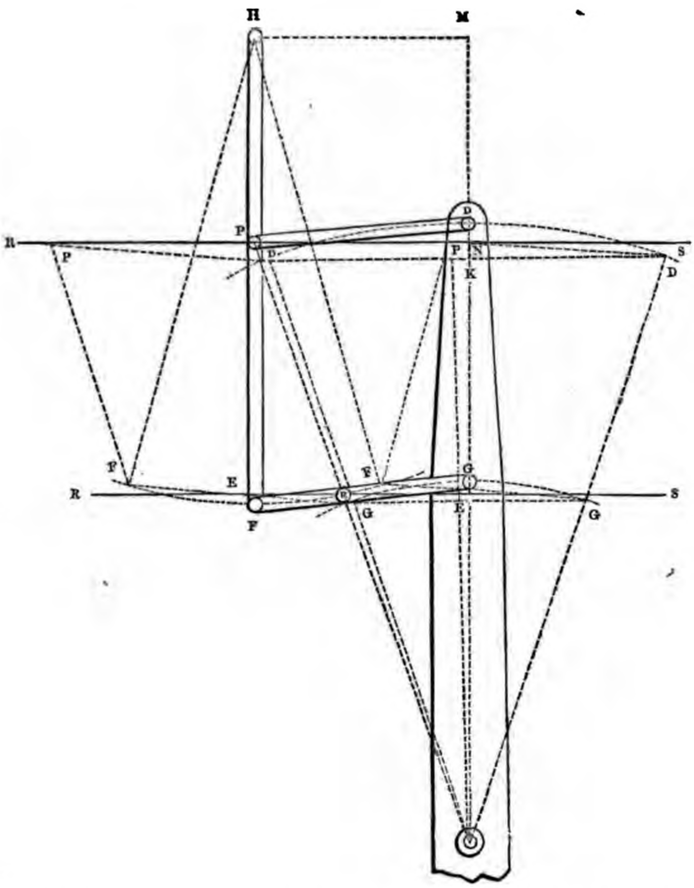


Fig. 2.



by the extremity (D) of the side lever. Square this sum and multiply the square by the length of the side rod (P D). Call this product A. Take the same horizontal distance as before added to the same half versed sine (D N), and multiply the sum by the length of the side rod (P D): to the product add the product of the length of

Fig. 8.



the side lever C D into the length of Q D, and divide A by the sum. The quotient will be the length required.

When the centre H of the radius has its position determined, rules 4 and 6 will always give the length of the radius bar F H. To get the length of C G, it will only be necessary to draw through the point F a line parallel to the side rod D P, and the point where that line cuts D C will be the position of the pin G.

In using these formulas and rules, the dimensions must all be taken in the same measure; that is, either all in feet, or all in inches; and when great accuracy is required, the corrections given in Table (A) must be added to or subtracted from the calculated length of the radius bar, according as it is less or greater than the length of C G, the part of the beam that works it.

1. **Rule 4.**—Let the horizontal distance (M C) of the centre (H)

Fig. 4.

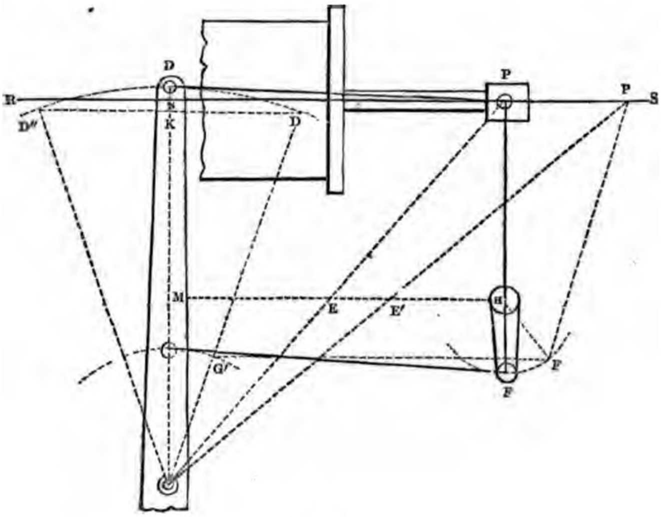
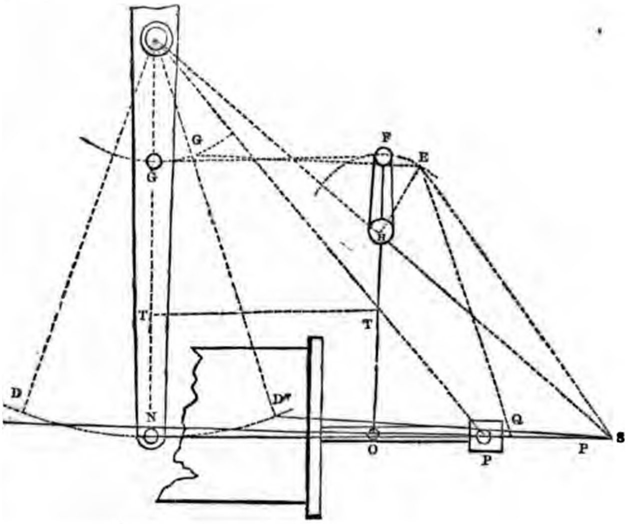

D
P
P
R
S
D''
N
K
D
M
E
E'
H
G'
F
F

Fig. 5.

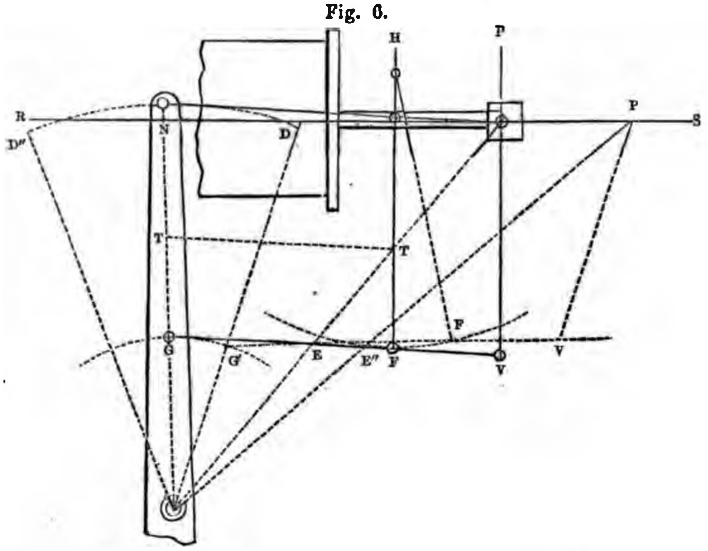

F
E
G
G
H
T
T
D
D''
N
Q
S
O
P
P

he radius bar from the main centre be equal to 51 inches; the versed sine D N = 3 inches, and D C = 126 inches; then by rule we will have

$$\frac{(51 + 3)^2}{51 + 3 + 126} = \frac{(54)^2}{180} = \frac{2916}{180} = 16 \cdot 2 \text{ inches,}$$

ch is the required length of the radius bar (F H).

Fig. 6.



2. Rule 5.—The following dimensions are those of the Red Rover steamer: $C\,G = 32 \; D\,P = 94 \; Q\,D = 74 \; C\,D = 65 \; P\,Q = 20$.

By the rule we have, $A = (32)^2 \times 94 = 96256$ and

$$\frac{96256}{74 \times 65 - 94 \times 32} = \frac{96256}{1802} = 53{\cdot}4,$$

which is the required length of the radius bar.

3. Rule 6.—Take the same data as in the last example, only supposing that C G is not given, and that the centre H is fixed at a horizontal distance from the main centre, equal to 83·5 inches. Then the half versed sine of the arc D′ D D″ will be about 2 inches, and we will have by the rule

$$A = (83{\cdot}5 + 2)^2 \times 94 = 705963{\cdot}5 \text{ and}$$

$$\frac{A}{85{\cdot}5 \times 94 + 65 \times 74} = \frac{705963{\cdot}5}{1284{\cdot}7} = 54{\cdot}8 \text{ inches,}$$

the required length of the radius bar in this case.

Table (A).

This column gives $\frac{F\,H}{C\,G}$ when C G is the greater, and $\frac{C\,G}{F\,H}$ when F H is the greater.	Correction to be added to or subtracted from the calculated length of the radius bar, in decimal parts of its calculated length.
1·0	0
·9	·0034
·8	·0075
·7	·0163
·6	·0270
·5	·0452
·4	·0817

In both of the last two examples $\frac{\text{C G}}{\text{H F}} = \cdot 6$ nearly. The correction found by Table (A), therefore, would be $54 \times \cdot 027 = 1\cdot 458$ inches, which must be subtracted from the lengths already found for the radius bar, because it is longer than C G. The corrected lengths will therefore be

In example 2....................F H = 51·94 inches.
In example 3....................F H = 53·34 inches.

RULE.—*To find the depth of the main beam at the centre.*—Divide the length in inches from the centre of motion to the point where the piston rod is attached, by the diameter of the cylinder in inches; multiply the quotient by the maximum pressure in pounds per square inch of the steam in the boiler; divide the product by 202 for cast iron, and 236 for malleable iron: in either case, the cube root of the quotient multiplied by the diameter of the cylinder in inches gives the depth in inches of the beam at the centre of motion. *To find the breadth at the centre.*—Divide the depth in inches by 16; the quotient is the breadth in inches.

An engine beam is three times the diameter of the cylinder, from the centre to the point where the piston rod acts on it; the force of the steam in the boiler when about to force open the safety valve is 10 lbs. per square inch. Required the depth and breadth when the beam is of cast iron.

In this case $n = 3$, and $P = 10$, and therefore

$$d = D\left\{\frac{30}{202}\right\}^{\frac{1}{3}} = \cdot 53\ D.$$

$$\text{The breadth} = \frac{\cdot 53}{16}\ D = \cdot 03\ D.$$

It will be observed that our rule gives the least value to the depth. In actual practice, however, it is necessary to make allowance for accidents, or for faultiness in the materials. This may be done by making the depth greater than that determined by the rule; or, perhaps more properly, by taking the pressure of the steam much greater than it can ever possibly be. As for the dimensions of the other parts of the beam, it is obvious that they ought to diminish towards the extremities; for the power of a beam to resist a cross strain varies inversely as its length. The dimensions may be determined from the formula $f\,b\,d^2 = 6\ W\ l$.

To apply the formula to cranks, we may assume the depth at the shaft to be equal to n times the diameter of the shaft; hence, if $m \times D$ be the diameter of the shaft, the depth of the crank will be $n \times m \times D$. Substituting this in the formula $f\,b\,d^2 = 6\ W\,l$, and it becomes $f\,b \times n^2 \times m^2 \times D^2 = 6\ W\ l$. Now, as before, $W = \cdot 7854 \times P \times D^2$, so that the formula becomes $f \times b \times n^2 \times m^2 = 4\cdot 7124 \times P \times l$. The value of n is arbitrary. In practice it may be made equal to $1\frac{1}{2}$ or $1\cdot 5$. Taking this value, then, for cast iron, the formula becomes $15300 \times b \times \frac{1}{4} \times m^2 = 4{\cdot}7124 \times P \times l$, or $7305\ m^2 b = P\,l$; but if L denote the length of the crank in feet, the formula becomes $609\ m^2\ b = P\,L$, and $\therefore\ b = P \times L \div 609\ m^2$. This formula may be put into the form of a rule, thus:—

RULE.—*To find the breadth at the shaft when the depth is equal to* $1\frac{1}{2}$ *times the diameter of the shaft.*—Divide the square of the diameter of the shaft in inches by the square of the diameter of the cylinder; multiply the quotient by 609, and reserve the product for a divisor; multiply the greatest elastic force of the steam in lbs. per square inch by the length of the crank in feet, and divide the product by the reserved divisor: the quotient is the breadth of the crank at the shaft.

A crank shaft is $\frac{1}{4}$ the diameter of the cylinder; the greatest possible force of the steam in the boiler is 20 lbs. per square inch; and the length of the shaft is 3 feet. Required the breadth of the crank at the shaft when its depth is equal to $1\frac{1}{2}$ times the diameter of the shaft.

In this case $m = \frac{1}{4}$, so that the reserved divisor $- \frac{609}{16} = 38$: again, elastic force of steam in lbs. per square inch = 20 lbs.; hence width of crank $= \frac{3 \times 20}{38} = 1{\cdot}6$ inches nearly.

RULE.—*To find the diameter of a revolving shaft.*—Form a reserved divisor thus: multiply the number of revolutions which the shaft makes for each double stroke of the piston by the number 1222 for cast iron, and the number 1376 for malleable iron. Then divide the radius of the crank, or the radius of the wheel, by the diameter of the cylinder; multiply the quotient by the greatest pressure of the steam in the boiler expressed in lbs. per square inch; divide the product by the reserved divisor; extract the cube root of the quotient, and multiply the result by the diameter of the cylinder in inches. The product is the diameter of the shaft in inches.

STRENGTH OF RODS WHEN THE STRAIN IS WHOLLY TENSILE; SUCH AS THE PISTON ROD OF SINGLE ACTING ENGINES, PUMP RODS, ETC.

RULE.—*To find the diameter of a rod exposed to a tensile force only.*—Multiply the diameter of the piston in inches by the square root of the greatest elastic force of the steam in the boiler estimated in lbs. per square inch; the product, divided by 95, is the diameter of the rod in inches.

Required the diameter of the transverse section of a piston rod in a single acting engine, when the diameter of the cylinder is 50 inches, and the greatest possible force of the steam in the boiler is 16 lbs. per square inch. Here, according to the formula,

$$d = \frac{50}{95} \sqrt{16} = \frac{200}{95} = 2{\cdot}1 \text{ inches.}$$

Rule.—*To find the strength of rods alternately extended and compressed, such as the piston rods of double acting engines.*—Multiply the diameter of the piston in inches by the square root of the maximum pressure of the steam in lbs. per square inch; divide the product by

47 for cast iron,
50 for malleable iron.

This rule applies to the piston rods of double acting engines, parallel motion rods, air-pump and force-pump rods, and the like. The rule may also be applied to determine the strength of connecting rods, by taking, instead of P, a number P′, such that P′ × sine of the greatest angle which the connecting rod makes with the direction = P.

Supposing the greatest force of the steam in the boiler to be 16 lbs. per square inch, and the diameter of the cylinder 50 inches; required the diameter of the piston rod, supposing the engine to be double acting. In this case

$$\text{for cast iron } d = \frac{D}{47}\sqrt{P} = \frac{50 \times 4}{47} = 5 \text{ inches nearly;}$$

$$\text{for malleable iron } d = \frac{D}{50}\sqrt{P} = 4 \text{ inches.}$$

The pressure, however, is always taken in practice at more than 16 lbs. If the pressure be taken at 25 lbs., the diameter of a malleable iron piston rod will be 5 inches, which is the usual proportion. Piston rods are never made of cast iron, but air-pump rods are sometimes made of brass, and the connecting rods of land engines are cast iron in most cases.

FORMULAS FOR THE STRENGTH OF VARIOUS PARTS OF MARINE ENGINES.

The following general rules give the dimensions proper for the parts of marine engines, and we shall recapitulate, with all possible brevity, the data upon which the denominations rest.

Let pressure of the steam in boiler = p lbs. per square inch,
Diameter of cylinder = D inches,
Length of stroke = 2 R inches.

The vacuum below the piston is never complete, so that there always remains a vapour of steam possessing a certain elasticity. We may suppose this vapour to be able to balance the weight of the piston. Hence the entire pressure on the square inch of piston in lbs. = p + pressure of atmosphere = $15 + p$. We shall substitute P for $15 + p$. Hence

$$\text{Entire pressure on piston in lbs.} = \cdot 7854 \times (15 + p) \times D^2$$
$$= \cdot 7854 \times P \times D^2.$$

The dimensions of the paddle-shaft journal may be found from the following formulas, which are calculated so that the strain in ordinary working = $\frac{2}{9}$ elastic force.

$$\text{Diameter of paddle-shaft journal} = \cdot 08264 \{R \times P \times D^2\}^{\frac{1}{3}}$$
$$\text{Length of ditto} = 1\tfrac{1}{4} \times \text{diameter.}$$

The dimensions of the several parts of the crank may be found from the following formulas, which are calculated so that the strain in ordinary working = one-half the elastic force; and when one paddle is suddenly brought up, the strain at shaft end of crank = $\frac{2}{3}$ elastic force, the strain at pin end of crank = elastic force.

Exterior diameter of large eye = diameter of paddle-shaft +

$$\left\{\frac{D\left[P \times 1{\cdot}561 \times R^2 + {\cdot}00494 \times D^2 \times P^2\right]^{\frac{1}{2}}}{75{\cdot}59 \times \sqrt{R}}\right\}^{\frac{2}{3}}$$

Length of ditto = diameter of paddle shaft.

Exterior diameter of small eye = diameter of crank pin + $\cdot02521 \times \sqrt{P} \times D$.

Length of ditto = $\cdot0375 \times \sqrt{P} \times D$.

Thickness of web at paddle centre =

$$\left\{\frac{D^2 \times P \times \sqrt{\{1{\cdot}561 \times R^2 + {\cdot}00494 \times D^2 \times P\}}}{9000}\right\}^{\frac{1}{3}}$$

Breadth of ditto = 2 × thickness.

Thickness of web at pin centre — $\cdot022 \times \sqrt{P} \times D$.

Breadth of ditto = $\frac{3}{2}$ × thickness.

As these formulas are rather complicated, we may show what they become when $p = 10$ or $P = 25$.

Exterior diameter of large eye = diameter of paddle shaft +

$$\left\{\frac{D\sqrt{(1{\cdot}561 \times R^2 + {\cdot}1235 \times D^2)}}{15{\cdot}12 \times \sqrt{R}}\right\}^{\frac{2}{3}}$$

Length of ditto = diameter of paddle shaft.

Exterior diameter of small eye = equal diameter of crank pin + $\cdot126 \times D$.

Length of ditto = $\cdot1875 \times D$.

Thickness of web at pin centre = $\cdot11 \times D$.

Breadth of ditto = $\frac{3}{2}$ × thickness of web.

The dimensions of the crank pin journal may be found from the following formulas, which are calculated so that strain when bearing at outer end = elastic force, and in ordinary working strain = one-third of elastic force.

Diameter of crank-pin journal = $\cdot02836 \times \sqrt{P} \times D$.

Length of ditto = $\frac{9}{8}$ × diameter.

The dimensions of the several parts of the cross head may be found from the following formulas, in which we have assumed, for the purpose of calculation, the length = $1{\cdot}4 \times D$. The formulas have been calculated so as to give the strain of web = $\frac{1}{2{\cdot}225} \times$ elastic force; strain of journal in ordinary working = $\frac{1}{2{\cdot}33} \times$ elastic force, and when bearing at outer end = $\frac{1}{1{\cdot}165} \times$ elastic force.

Exterior diameter of eye = diameter of hole + $\cdot 02827 \times P^{\frac{1}{3}} \times D.$

Depth of ditto = $\cdot 0979 \times P^{\frac{1}{3}} \times D.$

Diameter of journal = $\cdot 01716 \times \sqrt{P} \times D.$

Length of ditto = $\frac{9}{8}$ diameter of journal.

Thickness of web at middle = $\cdot 0245 \times P^{\frac{1}{3}} \times D.$

Breadth of ditto = $\cdot 09178 \times P^{\frac{1}{3}} \times D.$

Thickness of web at journal = $\cdot 0122 \times P^{\frac{1}{2}} \times D.$

Breadth of ditto = $\cdot 0203 \times P^{\frac{1}{2}} \times D.$

The dimensions of the several parts of the piston rod may be found from the following formulas, which are calculated so that the strain of piston rod = $\frac{1}{7}$ elastic force.

Diameter of the piston rod = $\dfrac{\sqrt{P} \times D.}{50}$

Length of part in piston = $\cdot 04 \times D \times P.$

Major diameter of part in crosshead = $\cdot 019 \times \sqrt{P} \times D.$

Minor diameter of ditto = $\cdot 018 \times \sqrt{P} \times D.$

Major diameter of part in piston = $\cdot 028 \times \sqrt{P} \times D.$

Minor diameter of ditto = $\cdot 023 \times \sqrt{P} \times D.$

Depth of gibs and cutter through crosshead = $\cdot 0358 \times P^{\frac{1}{3}} \times D.$

Thickness of ditto = $\cdot 007 \times P^{\frac{1}{3}} \times D.$

Depth of cutter through piston = $\cdot 017 \times \sqrt{P} \times D.$

Thickness of ditto = $\cdot 007 \times P^{\frac{1}{2}} \times D.$

The dimensions of the several parts of the connecting rod may be found from the following formulas, which are calculated so that the strain of the connecting rod and the strain of the strap are both equal to one-sixth of the elastic force.

Diameter of connecting rod at ends = $\cdot 019 \times P^{\frac{1}{2}} \times D.$

Diameter of ditto at middle = $\{1 + \cdot 0035 \times \text{length in inches}\} \times \cdot 019 \times \sqrt{P} \times D.$

Major diameter of part in crosstail = $\cdot 0196 \times P^{\frac{1}{2}} \times D.$

Minor ditto = $\cdot 018 \times P^{\frac{1}{2}} \times D.$

Breadth of butt = $\cdot 0313 \times P^{\frac{1}{2}} \times D.$

Thickness of ditto = $\cdot 025 \times P^{\frac{1}{2}} \times D.$

Mean thickness of strap at cutter = $\cdot 00854 \times \sqrt{P} \times D.$

Ditto above cutter = $\cdot 00634 \times \sqrt{P} \times D.$

Distance of cutter from end of strap = $\cdot 0097 \times \sqrt{P} \times D.$

Breadth of gibs and cutter through crosstail = $\cdot 0358 \times P^{\frac{1}{3}} \times D.$

Breadth of gibs and cutter through butt = $\cdot 022 \times P^{\frac{1}{2}} \times D.$

Thickness of ditto = $\cdot 00564 \times P^{\frac{1}{2}} \times D.$

The dimensions of the several parts of the side rods may be found from the following formulas, which are calculated so as to make the strain of the side rod = one-sixth of elastic force, and the strains of strap and cutter = one-fifth of elastic force.

Diameter of cylinder side rods at ends = $\cdot 0129 \times P^{\frac{1}{2}} \times D$.
Diameter of ditto at middle = $(1 + \cdot 0035 \times \text{length in inches})$.

$$\times \cdot 0129 \times P^{\frac{1}{2}} \times D.$$

Breadth of butt = $\cdot 0154 \times P^{\frac{1}{2}} \times D$.

Thickness of ditto = $\cdot 0122 \times P^{\frac{1}{2}} \times D$.

Diameter of journal at top end of side rod = $\cdot 01716 \times P^{\frac{1}{2}} \times D$.
Length of journal at top end = $\frac{9}{8}$ diameter.

Diameter of journal at bottom end = $\cdot 014 \times P^{\frac{1}{2}} \times D$.

Length of ditto = $\cdot 0152 \times P^{\frac{1}{2}} \times D$.

Mean thickness of strap at cutter = $\cdot 00643 \times P^{\frac{1}{2}} \times D$.

Ditto below cutter = $\cdot 0047 \times P^{\frac{1}{2}} \times D$.

Breadth of gibs and cutter = $\cdot 016 \times P^{\frac{1}{2}} \times D$.

Thickness of ditto = $\cdot 0033 \times P^{\frac{1}{2}} \times D$.

The dimensions of the main centre journal may be found from the following formulas, which are calculated so as to make the strain in ordinary working = one half elastic force.

Diameter of main centre journal = $\cdot 0367 \times P^2 \times D$.
Length of ditto = $\frac{3}{2} \times$ diameter.

The dimensions of the several parts of the air-pump may be found from the corresponding formulas given above, by taking for D another number *d* the diameter of air-pump.

DIMENSIONS OF THE SEVERAL PARTS OF FURNACES AND BOILERS.

Perhaps in none of the parts of a steam engine does the practice of engineers vary more than in those connected with furnaces and boilers. There are, no doubt, certain proportions for these, as well as for the others, which produce the maximum amount of useful effect for particular given purposes; but the determination of these proportions, from theoretical considerations, has hitherto been attended with insuperable difficulties, arising principally from our imperfect knowledge of the laws of combustion of fuel, and of the laws according to which caloric is imparted to the water in the boiler. In giving, therefore, the following proportions for the different parts, we desire to have it understood that we do not affirm them to be the best, absolutely considered; we give them only as the average practice of the best modern constructors. In most of the cases we have given the average value per nominal horse power. It is well known that the term horse power is a conventional unit for measuring the size of steam engines, just as a foot or a mile is a unit for the measurement of extension. There is this difference, however, in the two cases, that whereas the length of a foot is fixed definitively, and is known to every one, the dimensions proper to an engine horse power differ in the practice of every different maker: and the same kind of confusion is thereby introduced into engineering as if one person were to make his foot-rule eleven inches long, and another thirteen inches. It signifies very little what a horse power is defined to be; but when once defined, the measurement should be kept inviolable. The question now arises, what standard ought to be the accepted one. For our present purpose, it is necessary to connect by a formula the three quantities, nominal horses' power, length of stroke, and diameter of cylinder. With this intention,

Let S = length of stroke in feet,
d = diameter of cylinder in inches;

Then the nominal horse power $= \dfrac{d^2 \times \sqrt[3]{S}}{47}$ nearly.

I. *Area of Fire Grate.*—The average practice is to give ·55 square feet for each nominal horse power. Hence the following rule:

Rule 1.—*To find the area of the fire grate.*—Multiply the number of horses power by ·55; the product is the area of the fire grate in square feet.

Required the total area of the fire grate for an engine of 400 horse power. Here total area of fire grate in square feet = $400 \times \cdot 55 = 220$.

A rule may also be found for expressing the area of the fire grate in terms of the length of stroke and the diameter of the cylinder. For this purpose we have,

$$\text{total area of fire grate} = \frac{\cdot 55 \times d^2 \times \sqrt[3]{S}}{47} \text{ feet} = \frac{d^2 \times \sqrt[3]{S}}{86} \text{ feet.}$$

This formula expressed in words gives the following rule.

Rule 2.—*To find the area of fire grate.*—Multiply the cube root of the length of stroke in feet by the square of the diameter in inches; divide the product by 86; the quotient is the area of fire grate in square feet.

Required the total area of the fire grate for an engine whose stroke = 8 feet, and diameter of cylinder = 50 inches.

Here, according to the rule,

$$\text{total area of fire grate in square feet} = \frac{50^2 \times \sqrt[3]{8}}{86} = \frac{2500 \times 2}{86} = \frac{5000}{86} = 59 \text{ nearly.}$$

In order to work this example by the first rule, we find the nominal horse power of the engine whose dimensions we have specified is 104·3; hence,

total area of fire grate in square feet = $106 \cdot 4 \times \cdot 55 = 58 \cdot 5$.

With regard to these rules we may remark, not only that they are founded on practice, and therefore empyrical, but they are only applicable to large engines. When an engine is very small, it requires a much larger area of fire grate in proportion to its size than a larger one. This depends upon the necessity of having a certain amount of fire grate for the proper combustion of the coal.

II. *Length of Furnace.*—The length of the furnace differs considerably, even in the practice of the same engineer. Indeed, all the dimensions of the furnace depend to a certain extent upon the peculiarity of its position. From the difficulty of firing long furnaces efficiently, it has been found more beneficial to restrict the length of the furnace to about six feet than to employ furnaces of greater length.

III. *Height of Furnace above Bars.*—This dimension is variable, but it is a common practice to make the height about two feet.

IV. *Capacity of Furnace Chamber above Bars.*—The average per horse power may be taken at 1·17 feet. Hence the following rule:

Rule.—*To find the capacity of furnace chamber above bars.*—Multiply the number of nominal horses power by 1·17; the product is the capacity of furnace chambers above bars in cubic feet.

V. *Areas of Flues or Tubes in smallest part.*—The average value of the area per horse power is 11·2 sq. in. Hence we have the following rule:

Rule.—*To find the total area of the flues or tubes in smallest part.*—Multiply the number of horse power by 11·2; the product is the total area in square inches of flues or tubes in smallest part.

Required total area of flues or tubes for the boiler of a steam engine when the horse power = 400.

For this example we have, according to the rule,

$$\text{Total area in square inches} = 400 \times 11{\cdot}2 = 4480.$$

We may also find a very convenient rule expressed in terms of the stroke and the diameter of cylinder. Thus,

$$\text{Total area of tubes or flues in square inches} = \frac{11{\cdot}2 \times d^2 \times \sqrt[3]{\text{S}}}{47}$$

$$= \frac{d^2 \times \sqrt[3]{\text{S}}}{4}.$$

VI. *Effective Heating Surface.*—The effective heating surface of flue boilers is the whole of furnace surface above bars, the whole of tops of flues, half the sides of flues, and none of the bottoms; hence the effective flue surface is about half the total flue surface. In tubular boilers, however, the whole of the tube surface is reckoned effective surface.

EFFECTIVE HEATING SURFACE OF FLUE BOILERS.

Rule 1.—*To find the effective heating surface of marine flue boilers of large size.*—Multiply the number of nominal horse power by 5; the product is the area of effective heating surface in square feet.

Required the effective heating surface of an engine of 400 nominal horse power.

In this case, according to the rule, effective heating surface in square feet $= 400 \times 5 = 2000$.

The effective heating surface may be expressed in terms of the length of stroke and the diameter of the cylinder.

RULE 2.—*To find the total effective heating surface of marine flue boilers.*—Multiply the square of the diameter of cylinder in inches by the cube root of the length of stroke in feet; divide the product by 10: the quotient expresses the number of square feet of effective heating surface.

Required the amount of effective heating surface for an engine whose stroke = 8 ft., and diameter of cylinder = 50 inches.

Here, according to Rule 2, effective heating surface in square feet

$$= \frac{50^2 \times \sqrt[3]{8}}{10} = \frac{2500 \times 2}{10} = \frac{5000}{10} = 500.$$

To solve this example according to the first rule, we have the nominal horse power of the engine equal to 106·4. Hence, according to Rule 2, total effective heating surface in square feet $= 106{\cdot}4 \times 4{\cdot}92 = 523\frac{1}{2}$.

EFFECTIVE HEATING SURFACE OF TUBULAR BOILERS.

The effective heating surface of tubular boilers is about equal to the total heating surface of flue boilers, or is double the effective surface; but then the total tube surface is reckoned effective surface.

It appears that the total heating surface of flue and tubular marine boilers is about the same, namely, about 10 square feet per horse power.

VII. *Area of Chimney.*—RULE 1.—*To find the area of chimney.*—Multiply the number of nominal horse power by 10·23; the product is the area of chimney in square inches.

Required the area of the chimney for an engine of 400 nominal horse power.

In this example we have, according to the rule,

area of chimney in square inches $= 400 \times 10{\cdot}23 = 4092$.

We may also find a rule for connecting together the area of the chimney, the length of the stroke, and the diameter of the cylinder.

RULE 2.—*To find the area of the chimney.*—Multiply the square of the diameter expressed in inches by the cube root of the stroke expressed in feet; divide the product by the number 5; the quotient expresses the number of square inches in the area of chimney.

Required the area of the chimney for an engine whose stroke = 8 feet, and diameter of cylinder = 50 inches.

We have in this example from the rule,

area of chimney in square inches $= \frac{50^2 \times \sqrt[3]{8}}{5} = \frac{2500 \times 2}{5} =$ 1000.

To work this example according to the first rule, we find, that the nominal horse power of this engine is 104·6: hence,

$$\text{area of chimney in square inches} = 104{\cdot}6 \times 10{\cdot}23 = 1070.$$

The latter value is greater than the former one by 70 inches. This difference arises from our taking too great a divisor in Rule 2. Either of the values, however, is near enough for all practical purposes.

VIII. *Water in Boiler.*—The quantity of water in the boiler differs not only for different boilers, but differs even for the same boiler at different times. It may be useful, however, to know the average quantity of water in the boiler for an engine of a given horse power.

Rule 1.—*To determine the average quantity of water in the boiler.*—Multiply the number of horse power by 5; the product expresses the cubic feet of water usually in the boiler.

This rule may be so modified as to make it depend upon the stroke and diameter of the cylinder of engine.

Rule 2.—*To determine the cubic feet of water usually in the boiler.*—Multiply together the cube root of the stroke in feet, the square of the diameter of the cylinder in inches, and the number 5; divide the continual product by 47; the quotient expresses the cubic feet of water usually in the boiler.

Required the usual quantity of water in the boilers of an engine whose stroke = 8 feet, and diameter of cylinder 50 inches.

Here we have from the rule,

$$\text{cubic feet of water in boiler} = \frac{5 \times 50^2 \times \sqrt[3]{8}}{47} = \frac{5 \times 2500 \times 2}{47}$$

$$= \frac{25000}{47} = 532 \text{ nearly.}$$

The engine, with the dimensions we have specified, is of 106·4 nominal horse power. Hence, according to Rule 1,

$$\text{cubic feet of water in boiler} = 106{\cdot}4 \times 5 = 532.$$

IX. *Area of Water Level.*—Rule 1.—*To find the area of water level.*—The area of water level contains the same number of square feet as there are units in the number expressing the nominal horse power of the engine.

Required the area of water level for an engine of 200 nominal horse power. According to the rule, the answer is 200 square feet.

We add a rule for finding the area of water level when the diameter of cylinder and the length of stroke is given.

Rule 2.—*To find the area of water level.*—Multiply the square of the diameter in inches by the cube root of the stroke in feet; divide the product by 47; the quotient expresses the number of square feet in the area of water level.

Required the area of the water level for an engine whose stroke is 8 feet, and diameter of cylinder 50 inches.

In this case, according to the rule,

$$\text{area of water level in square feet} = \frac{50^2 \times \sqrt[3]{8}}{47} = 106.$$

X. *Steam Room.*—It is obvious that the steam room, like the quantity of water, is an extremely variable quantity, differing, not only for different boilers, but even in the same boiler at different times. It is desirable, however, to know the content of that part of the boiler usually filled with steam.

RULE 1.—*To determine the average quantity of steam room.*—Multiply the number expressing the nominal horse power by 3; the product expresses the average number of cubic feet of steam room.

Required the average capacity of steam room for an engine of 460 nominal horse power.

According to the rule,

Average capacity of steam room = 460×3 cubic feet = 1380 cubic feet.

This rule may be so modified as to apply when the length of stroke and diameter of cylinder are given.

RULE 2.—Multiply the square of the diameter of the cylinder in inches by the cube root of the stroke in feet; divide the product by 15; the quotient expresses the number of cubic feet of steam room.

Required the average capacity of steam room for an engine whose stroke is 8 feet, and diameter of cylinder 5 inches.

In this case, according to the rule,

$$\text{Steam room in cubic feet} = \frac{50^2 \times \sqrt[3]{8}}{15} = \frac{2500 \times 2}{15} = \frac{5000}{15} = 333\tfrac{1}{3}.$$

We find that the nominal horse power of this engine is 106·4; hence, according to Rule 1,

average steam room in cubic feet = $106{\cdot}4 \times 3 = 320$ nearly.

Before leaving these rules, we would again repeat that they ought not to be considered as rules founded upon considerations for giving the maximum effect from the combustion of a given amount of fuel; and consequently the engineer ought not to consider them as invariable, but merely to be followed as far as circumstances will permit. We give them, indeed, as the medium value of the very variable practice of several well-known constructors; consequently, although the proportions given by the rules may not be the best possible for producing the most useful effect, still the engineer who is guided by them is sure not to be very far from the common practice of most of our best engineers. It has often been lamented that the methods used by different engine makers for estimating the nominal powers of their engines have been so various that we can form no real estimate of the dimensions of the engine, from its reputed nominal horse power, unless we know its maker; but the same confusion exists, also, to some extent, in the construction of boilers. Indeed, many things may be mentioned, which have hitherto operated as a barrier to the practical application of any standard of engine power for proportioning the different parts of the boiler and furnace. The magnitude of furnace and the extent of heating surface necessary to produce any required rate of evaporation in the boiler are indeed known, yet each engine-maker has his own rule in these matters, and which he seems to think preferable to all others, and there are various circumstances influencing the result which render facts incomparable unless those circumstances are the same. Thus the circumstances that govern the rate of evaporation, as influenced by different degrees of draught, may be regarded as but imperfectly known. And, supposing the difficulty of ascertaining this rate of evaporation were surmounted, there would still remain some difficulty in ascertaining the amount of power absorbed by the condensation of the steam on its passage to the cylinder—the imperfect condensation of the same steam after it has worked the piston—the friction of the various moving parts of the machinery—and, especially, the difference of effect of these losses of power in engines constructed on different scales of magnitude. Practice must often vary, to a certain extent, in the construction of the different parts of the boiler and furnace of an engine; for, independently of the difficulty of solving the general problem in engineering, the determination of the maximum effect with the minimum of means, practice would still require to vary according as in any particular case the desired minimum of means was that of weight, or bulk, or expense of material. Again, in estimating the proper proportions for a boiler and its appendages, reference ought to be made to the distinction between the "power" or "effect" of the boiler, and its "duty." This is a distinction to be considered also in the engine itself. The power of an engine has reference to the time it takes to produce a certain mechanical effect without reference to the amount of fuel consumed; and, on the other hand, the duty of an engine has reference to the amount of mechanical effect produced by a certain consumption of fuel, and is independent of the time it takes to produce that effect. In expressing the duty of engines, it would have prevented much needless confusion if the duty of the boiler had been entirely separated from that of the engine, as, indeed, they are two very distinct things. The duty performed by ordinary land rotative steam engines is—

One horse power exerted by 10 lbs. of fuel an hour; or,
Quarter of a million of lbs. raised 1 foot high by 1 lb. of coal; or,
Twenty millions of lbs. raised one foot by each bushel of coals.

Though in the best class of rotative engines the consumption is not above half of this amount.

The constant aim of different engine makers is to increase the amount of the duty; that is, to make 10 lbs. of fuel exert a greater effect than one horse power; or, in other words, to make 1 lb. of coal raise more than a quarter of a million of lbs. one foot high. To a great extent they have been successful in this. They have caused 5 lbs. of coal to exert the force of one horse power, and even in some cases as little as 3½ lbs.; but in these latter cases the economy is due chiefly to expansive action. In some of the engines, however, working with a consumption of 10 lbs. of coal per nominal horse hower per hour, the power really exerted amounts to much more than that represented by 33,000 lbs. lifted one foot high in the minute for each horse power. Some engines lift 56,000 lbs. one foot high in the minute by each horse power, with a consumption of 10 lbs. of coal per horse power per hour; and even this performance has been somewhat exceeded without a recourse to expansive action. In all modern engines the actual performance much exceeds the nominal power; and reference must be had to this circumstance in contrasting the duty of different engines.

MECHANICAL POWER OF STEAM.

We may here give a table of some of the properties of steam, and of its mechanical effects at different pressures. This table may help to solve many problems respecting the mechanical effect of steam, usually requiring much laborious calculation.

Pressures.		Temperature in degrees Fahren.	Weight of a Cubic Foot Steam.	Velocity of Exit.	Mechanical Effect in Horse Power of 1 Lb. of Steam.							
					Without Condensation. Expansion.				Condensation. Expansion.			
Atmosphere.	Lbs. per Sq. Inch.				0	½	⅓	¼	0	½	⅓	¼
1·00	14·70	212·00	0·0364	0	0	32·4	95·2	170·5	91·3	150·1	178·6	104·6
1·25	18·38	223·88	0·0440	873	21·5	10·1	32·3	87·4	95·9	158·7	190·6	209·9
1·50	22·05	234·32	0·0529	1135	36·4	39·3	10·8	30·6	99·3	165·2	199·6	221·1
1·75	25·72	242·78	0·0609	1295	47·4	60·8	42·5	11·1	102·0	170·0	206·2	229·5
2·00	29·40	250·79	0·0688	1407	55·9	77·5	67·0	43·2	104·3	174·2	212·0	236·5
2·25	33·08	257·90	0·0766	1491	62·8	90·9	86·5	68·8	106·2	177·7	216·7	242·4
2·50	36·75	263·93	0·0344	1556	68·4	101·8	102·4	89·6	107·7	180·5	220·5	247·1
2·75	40·42	269·87	0·0921	1608	73·1	111·0	115·8	107·1	109·3	183·2	224·2	251·6
3·00	44·10	275·00	0·0998	1652	71·1	118·8	127·1	121·9	110·6	185·4	227·7	255·2
3·35	47·78	279·86	0·1073	1690	80·7	125·6	137·1	136·7	111·7	187·6	230·0	258·7
3·50	51·45	284·68	0·1148	1722	83·8	131·5	145·6	145·8	112·7	189·4	232·4	261·6
3·75	55·12	288·66	0·1225	1750	86·5	136·8	153·2	155·6	113·7	190·1	234·7	264·4
4·00	58·18	292·91	0·1208	1774	89·0	141·5	160	164·5	114·6	192·8	236·9	267·0
4·50	66·15	300·27	0·1445	1816	93·2	149·8	171·5	179·4	116·2	195·6	240·5	271·4
5·00	73·50	307·94	0·1590	1850	96·8	156·5	181·6	192·0	117·7	198·3	244·1	275·6
6·00	88·20	320·00	0·1878	1904	102·5	167·2	196·5	211·4	120·2	202·6	249·7	282·2
7·00	102·90	331·56	0·2159	1945	107·0	175·6	208·4	226·5	122·4	206·4	254·6	288·1
8·00	117·60	340·83	0·2136	1078	110·6	182·4	217·9	238·4	124·3	209	258·8	292·1
9·00	132·30	351·32	0·2708	2006	113·7	188·2	225·9	248·5	126·0	212	262·7	293·6
10·00	147·00	359·00	0·2977	2029	116·3	193·0	232·5	256·7	127·5	215	266·0	301·4
12·50	183·75	377·42	0·3642	2074	121·5	202·5	245·5	273·0	130·7	220	272·9	309·5
15·00	220·50	392·90	0·4288	2109	125·7	210·0	255·6	285·4	133·4	225	278·9	316·4
17·50	257·25	406·40	0·4924	2136	129·0	216·0	263·6	295·2	135·7	229	283·9	322·2
20	294·00	418·56	0·5549	2159	131·8	221·0	270·3	305·3	137·8	233	288·3	327·2
25	367·50	429·34	0·6775	2196	136·3	229·1	281·0	316·2	141·2	238	295·7	335·8
30	441·00	457·16	0·7970	2226	140·0	235·6	289·5	326·4	144·2	244	302·0	343·1

It is quite clear that although there is no theoretical limit to the benefit derivable from expansion, there must be a limit in practice, arising from the friction incidental to the use of very large cylinders, the magnitude of the deduction due to uncondensed vapour when the steam is of a very low pressure, and other circumstances which it is needless to relate. It is clear, too, that while the efficiency of the steam is increased by expansive action, the efficiency of the engine is diminished, unless the pressure of the steam or the speed of the piston be increased correspondingly; and that an engine of any given size will not exert the same power if made to operate expansively without any other alteration that would have been realized if the engine had been worked with the full pressure of the steam. In the Cornish engines, which work with steam of 40 lbs. on the inch, the steam is cut off at one-twelfth of the stroke; but if the steam were cut off at one-twelfth of the stroke in engines employing a very low pressure, it would probably be found that there would be a loss rather than a gain from carrying the expansion so far, as the benefit might be more than neutralized by the friction incidental to the use of so large a cylinder as would be necessary to accomplish this expansion; and unless the vacuum were a very good one, there would be but little difference between the pressure of the steam at the end of the stroke and the pressure of the vapour in the condenser, so that the urging force might not at that point be sufficient to overcome the friction. In practice, therefore, in particular cases, expansion may be carried too far, though theoretically the amount of the benefit increases with the amount of the expansion.

We must here introduce a simple practical rule to enable those who may not be familiar with mathematical symbols to determine the amount of benefit due to any particular measure of expansion. When expansion is performed by an expansion valve, it is an easy thing to ascertain at what point of the stroke the valve is shut by the cam, and where expansion is performed by the slide valve the amount of expansion is easily determinable when the lap and stroke of the valve are known.

RULE.—*To find the Increase of Efficiency arising from working Steam expansively.*—Divide the total length of the stroke by the distance (which call 1) through which the piston moves before the steam is cut off. The hyperbolic logarithm of the whole stroke expressed in terms of the part of the stroke performed with the full pressure of steam, represents the increase of efficiency due to expansion.

Suppose that the pressure of the steam working an engine is 45 lbs. on the square inch above the atmosphere, and that the steam is cut off at one-fourth of the stroke; what is the increase of efficiency due to this measure of expansion?

If one-fourth be reckoned as 1, then four-fourths must be taken as 4, and the hyperbolic logarithm of 4 will be found to be 1·386, which is the increase of efficiency. The total efficiency of the quantity of steam expended during a stroke, therefore, which without expansion would have been 1, becomes 2·386 when expanded into 4 times its bulk, or, in round numbers, 2·4.

Let the pressure of the steam be the same as in the last example, and let the steam be cut off at half-stroke: what, then, is the increase of efficiency?

Here half the stroke is to be reckoned as 1, and the whole stroke has therefore to be reckoned as 2. The hyperbolic logarithm of 2 is ·693, which is the increase of efficiency, and the total efficiency of the stroke is 1·693, or 1·7.

We may here give a table to illustrate the mechanical effect of steam under varying circumstances. The table shows the me-

Total pressure in lbs. per Square Inch.	Corresponding Temperature.	Volume of Steam compared with Water.	Mechanical effect of Cubic Inch of Water.	Total pressure in lbs. per Square Inch.	Corresponding Temperature.	Volume of Steam compared with Water.	Mechanical effect of Cubic Inch of Water.
1	103	20·868	1739	51	284	544	2312
2	126	10·874	1812	52	286	534	2316
3	141	7437	1859	53	287	525	2320
4	152	5685	1895	54	288	516	2324
5	161	4617	1924	55	289	508	2327
6	169	3897	1948	56	290½	500	2331
7	176	3376	1969	57	292	492	2335
8	182	2983	1989	58	293	484	2339
9	187	2674	2006	59	294	477	2343
10	192	2426	2022	60	296	470	2347
11	197	2221	2036	61	297	463	2351
12	201	2050	2050	62	298	456	2355
13	205	1904	2063	63	299	449	2359
14	209	1778	2074	64	300	443	2362
15	213	1669	2086	65	301	437	2365
16	216	1573	2097	66	302	431	2369
17	220	1488	2107	67	303	425	2372
18	223	1411	2117	68	304	419	2375
19	226	1343	2126	69	305	414	2378
20	228	1281	2135	70	306	408	2382
21	231	1225	2144	71	307	403	2385
22	234	1174	2152	72	308	398	2388
23	236	1127	2160	73	309	393	2391
24	239	1084	2168	74	310	388	2394
25	241	1044	2175	75	311	383	2397
26	243	1007	2182	76	312	379	2400
27	245	973	2189	77	313	374	2403
28	248	941	2196	78	314	370	2405
29	250	911	2202	79	315	366	2408
30	252	883	2209	80	316	362	2411
31	254	857	2215	81	317	358	2414
32	255	833	2221	82	318	354	2417
33	257	810	2226	83	318	350	2419
34	259	788	2232	84	319	346	2422
35	261	767	2238	85	320	342	2425
36	263	748	2243	86	321	339	2427
37	264	729	2248	87	322	335	2430
38	266	712	2253	88	323	332	2432
39	267	695	2259	89	323	328	2435
40	269	679	2264	90	324	325	2438
41	271	664	2268	91	325	322	2440
42	272	649	2273	92	326	319	2443
43	274	635	2278	93	327	316	2445
44	275	622	2282	94	327	313	2448
45	276	610	2287	95	328	310	2450
46	278	598	2291	96	329	307	2453
47	279	586	2296	97	330	304	2455
48	280	575	2300	98	330	301	2457
49	282	564	2304	99	331	298	2460
50	283	554	2308	100	332	295	2462

chanical effect of the steam generated from a cubic inch of water. Our formula gives the effect of a cubic foot of water; but it can be modified to give the effect of the steam of a cubic inch by dividing by 1728. In this manner we find, for the mechanical effect of the steam of a cubic inch of water, about 3 $(459 + t)$ lbs. raised one foot high. The table shows that the mechanical effect increases with the temperature. The increase is very rapid for temperatures below 212°; but for temperatures above this the increase is less; and for the temperatures used in practice we may consider, without any material error, the mechanical effect as constant.

INDICATOR.

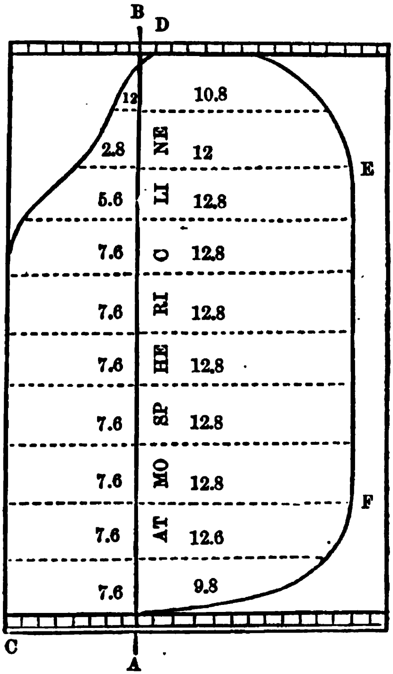
An instrument for ascertaining the amount of the pressure of steam and the state of the vacuum throughout the stroke of a steam engine. Fitzgerald and Neucumn long employed an instrument of this kind, the nature of which was for a long time not generally known. Boulton and Watt used an instrument acting upon the same principle and equally accurate; but much more portable. In peculiarity of construction it is simply a small cylinder truly bored, and into which a piston is inserted and loaded by a spring of suitable elasticity to the graduated scale thereon attached.

The action of an indicator is that of describing, on a piece of paper attached, a diagram or figure approximating more or less to that of a rectangle, varying of course with the merits or demerits of the engine's productive effect. The breadth or height of the diagram is the sum of the force of the steam and extent of the vacuum; the length being the amount of revolution given to the paper during the piston's performance of its stroke.

To render the indicator applicable, it is commonly screwed into the cylinder cover, and the motion to the paper obtained by means of a sufficient length of small twine attached to one of the radius bars; but such application cannot always be conveniently effected, more especially in engines on the marine principle; hence, other parts of such engines, and other means whereby to effect a proper degree of motion, must unavoidably be resorted to. In those of direct action the crosshead is the only convenient place of attachment; but because the length of the engine's stroke is considerably more than the movement required for the paper on the indicator, it is necessary to introduce a pulley and axle, by which means the various movements are qualified to suit each other.

When the indicator is fixed and the movement for the paper properly adjusted, allow the engine to make a few revolutions previous to opening the cock; by which means a horizontal line will be described upon the paper by the pencil attached, and denominated the atmospheric line, because it distinguishes between the effect of the steam and that of the vacuum. Open the cock, and if the engine be upon the descending stroke, the steam will instantly raise the piston of the indicator, and, by the motion of the paper with the pencil pressing thereon, the top side of the diagram will be formed.

At the termination of the stroke and immediately previous to its return, the piston of the indicator is pressed down by the surrounding atmosphere, consequently the bottom side of the diagram is described, and by the time the engine is about to make another descending stroke, the piston of the indicator is where it first started from, the diagram being completed; hence is delineated the mean elastic action of the steam above that of the atmospheric line, and also the mean extent of the vacuum underneath it.



But in order to elucidate more clearly by example, take the following diagram, taken from a marine engine, the steam being cut off after the piston had passed through two-thirds of its stroke, the graduated scale on the indicator, tenths of an inch, as shown at each end of the diagram annexed.

Previous to the cock being opened, the atmospheric line AB was formed, and, when opened, the pencil was instantly raised by the action of the steam on the piston to C, or what is generally termed the *starting corner;* by the movement of the paper and at the termination of the stroke the line CD was formed, showing the force of the steam and extent of expansion; from D to E show the moments of eduction; from E to F the quality of the vacuum; and from F to A the lead or advance of the valve; thus every change in the engine is exhibited, and every deviation from a rectangle, except that of *expansion* and *lead* of the valve show the extent of proportionate defect. Expansion produces apparently a defective diagram, but in reality such is not the case, because the diminished power of the engine is more than compensated by the saving in steam. Also the lead of the valve produces an apparent defect, but a certain amount must be given, as being found advantageous to the working of the engine, but the steam and eduction corners ought to be as square as possible; any rounding on the steam corner shows a defect from want of lead; and rounding on the eduction corner that of the passages or apertures being too small.

RULE.—*To compute the power of an Engine from the Indicator Diagram.*—Divide the diagram in the direction of its length into any convenient number of equal parts, through which draw lines at right angles to the atmospheric line, add together the lengths of all the spaces taken in measurements corresponding with the scale on the indicator, divide the sum by the number of spaces, and the quotient is the mean effective pressure on the piston in lbs. per square inch.

Let the result of the preceding diagram be taken as an example. Then, the whole sum of vacuum spaces = 1220 ÷ 10 = 12·2 lbs. mean effect obtained by the vacuum; and in a similar manner the mean effective pressure of steam is found to be 6·28 lbs., hence the total effective force = 18·48 lbs. per square inch. And supposing 2·5 lbs. per square inch be absorbed by friction, What is the actual power of the engine, the cylinder's diameter being 32 inches, and the velocity of the piston 226 feet per minute?

$$18{\cdot}48 - 2{\cdot}5 = 15{\cdot}98 \text{ lbs. per square inch of net available force.}$$

$$\text{Then } \frac{32^2 \times {\cdot}7854 \times 15{\cdot}98 \times 226}{33000} = 88 \text{ horses power.}$$

The line under the diagram and parallel to the atmospheric line is $\frac{15}{10}$ths distant, and represents the perfect vacuum line, the space between showing the amount of force with which the uncondensed steam or vapour resists the ascent or descent of the piston at every part of the stroke.

As the mean pressure of the atmosphere is 15 lbs. per square inch, and the mean specific gravity of mercury 13560, or 2·037 cubic inches equal 1 lb., it will of course rise in the barometer attached to the condenser about 2 inches for every lb. effect of vacuum, and as a pure vacuum would be indicated by 30 inches of mercury, the distance between the two lines shows whether there is or is not any amount of defect, as sometimes there is a considerable difference in extent of vacuum in the cylinder to that in the condenser.

To estimate by means of an indicator the amount of effective power produced by a steam engine.—Multiply the area of the piston in square inches by the average force of the steam in lbs. and by the velocity of the piston in feet per minute; divide the product by 33,000, and $\frac{7}{10}$ths of the quotient equal the effective power.

Suppose an engine with a cylinder of 37½ inches diameter, a stroke of 7 feet, and making 17 revolutions per minute, or 238 feet velocity, and the average indicated pressure of the steam 16·73 lbs. per square inch; required the effective power.

$$\frac{\text{Area} = 1104{\cdot}4687 \text{ inches} \times 16{\cdot}73 \text{ lbs.} \times 238 \text{ feet}}{33000}$$

$$= \frac{133{\cdot}26 \times 7}{10} = 93{\cdot}282 \text{ horse power.}$$

To determine the proper velocity for the piston of a steam engine.—Multiply the logarithm of the *n*th part of the stroke at which the steam is cut off by 2·3, and to the product of which add ·7. Multiply the sum by the distance in feet the piston has travelled when the steam is cut off, and 120 times the square root of the product equal the proper velocity for the piston in feet per minute.

WEIGHT COMBINED WITH MASS, VELOCITY, FORCE, AND WORK DONE.

CALCULATIONS ON THE PRINCIPLE OF VIS VIVA.—MATERIALS EMPLOYED IN THE CONSTRUCTION OF MACHINES.—STRENGTH OF MATERIALS, THEIR PROPERTIES.—TORSION, DEFLEXION, ELASTICITY, TENACITIES, COMPRESSIONS, ETC.—FRICTION OF REST AND OF MOTION, COEFFICIENTS OF ALL SORTS OF MOTION.—BANDS.—ROPES.—WHEELS.—HYDRAULICS.—NEW TABLES FOR THE MOTION AND FRICTION OF WATER.—WATER-WHEELS.—WINDMILLS, ETC.

1. Suppose a body resting on a perfectly smooth table, and, when in motion, to present no impediment to the body in its course, but merely to counteract the force of gravity upon it; if this body weighing 800 lbs. be pressed by the force of 30 lbs. acting horizontally and continuously, the motion under such circumstances will be uniformly accelerated: what is the acceleration?

$$\frac{30}{800} \times 32{\cdot}2 = 1{\cdot}2075 \text{ feet the second.}$$

2. What force is necessary to move the above-mentioned heavy body, with a 23 feet acceleration, under the same circumstances?

$$\frac{23}{32{\cdot}2} \times 800 = 57{\cdot}14285 \text{ lbs.}$$

The second of these examples illustrates the principle that the force which impels a body with a certain acceleration is equal to the weight of the body multiplied by the ratio of its acceleration to that of gravity. The first illustrates the reverse, namely, the acceleration with which a body is moved forward with a given force, is equal to the acceleration of gravity multiplied by the ratio of the force to the weight.

3. A railway car, weighing 1120 lbs., moves with a 5 feet velocity upon horizontal rails, which, let us suppose, offer no impediment to the motion, and is constantly pushed by an invariable force of 50 lbs. during 20 seconds: with what velocity is it moving at the end of the 20th second, or at the beginning of the 21st second?

$$5 + 32{\cdot}2 \times \frac{50}{1120} \times 20 = 33{\cdot}75, \text{ the velocity.}$$

4. A carriage, circumstanced as in the last question, weighs 4000 lbs.; its initial velocity is 30 feet the second, and its terminal velocity is 70 feet: with which force is the body impelled, supposing it to be in motion 20 seconds?

$$\frac{(70 - 30) \times 4000}{32{\cdot}2 \times 20} = 242{\cdot}17 \text{ lbs.}$$

We have before noticed that the weight (W), divided by 32·2, or (*g*), gives the *mass;* that is,

$$\frac{\text{Weight}}{g} = \text{mass},$$

$$\text{And, force} = \text{mass} \times \text{acceleration}.$$

5. Suppose a railway carriage, weighing 6440 lbs., moves on a horizontal plane offering no impediment, and is uniformly accelerated 4 feet the second, what continuous force is applied?

$$\frac{6440}{32{\cdot}2} = 200 \text{ lbs. mass.}$$

$$200 \times 4 = 800 \text{ lbs., the force applied.}$$

By the four succeeding formulas, all questions may be answered that may be proposed relative to the rectilinear motions of bodies by a constant force.

For uniformly accelerated motions:

$$v = a + 32{\cdot}2 \frac{\text{F}}{\text{W}} \times t;$$

$$s = at + 16{\cdot}1 \frac{\text{F}}{\text{W}} \times t^2.$$

For uniformly retarded motions:

$$v = a - 32{\cdot}2 \frac{\text{F}}{\text{W}} \times t;$$

$$s = at - 16{\cdot}1 \times \frac{\text{F}}{\text{W}} \times t^2;$$

t = the time in seconds, W = the weight in lbs., F = the force in lbs., a = the initial velocity, and v = the terminal velocity.

6. A sleigh, weighing 2000 lbs., going at the rate of 20 feet a second, has to overcome by its motion a friction of 30 lbs.: what velocity has it after 10 seconds, and what distance has it described?

$$20 - 32{\cdot}2 \times \frac{30}{2000} \times 10 = 15{\cdot}17 \text{ feet velocity.}$$

$$20 \times 10 - 16{\cdot}1 \times \frac{30}{2000} \times (10)^2 = 175{\cdot}85 \text{ feet, distance described.}$$

7. In order to find the mechanical work which a draught-horse performs in drawing a carriage, an instrument called a dynamometer, or measure of force, is thus used: it is put into communication on one side of the carriage, and on the other with the traces of the horse, and the force is observed from time to time. Let 126 lbs. be the initial force; after 40 feet is described, let 130 lbs. be the force given by the dynamometer; after 40 feet more is described, let 129 lbs. be the force; after 40 feet more is passed over, let 140 lbs. be the force; and let the next two spaces of 40 feet give forces of 130 and 120 lbs. respectively. What is the mechanical work done?

126 initial force.
120 terminal force.
2) 246
123 mean.

$$\frac{123 + 130 + 129 + 140 + 130}{5} = 130{\cdot}4$$

$$130{\cdot}4 \times 40 \times 5 = 26080 \text{ units of work.}$$

The following rule, usually given to find the areas of irregular figures, may be applied where great accuracy is required.

RULE.—To the sum of the first and last, or extreme ordinates, add four times the sum of the 2d, 4th, 6th, or even ordinates, and twice the sum of the 3d, 5th, 7th, &c., or odd ordinates, not including the extreme ones; the result multiplied by $\frac{1}{3}$ the ordinates' equidistance will be the sum.

$$\begin{array}{r} 126 \\ \underline{120} \\ \underline{246} \end{array} \text{ sum of first and last.}$$

$$246 + 4 \times 130 + 2 \times 129 + 4 \times 140 + 2 \times 130 = 1844.$$

$\frac{1844 \times 40}{3} = 24586{\cdot}66$ units of work or pounds raised one foot high. This rule of equidistant ordinates is of great use in the art of ship-building. This application we shall introduce in the proper place.

8. How many units of work are necessary to impart to a carriage of 3000 lbs. weight, resting on a perfectly smooth railroad, a velocity of 100 feet?

$$\frac{(100)^2}{2 \times 32{\cdot}2} \times 3000 = 465838{\cdot}2 \text{ units.}$$

A unit of work is that labour which is equal to the raising of a pound through the space of one foot. A unit of work is done when one pound pressure is exerted through a space of one foot, no matter in what direction that space may lie.

Kane Fitzgerald, the first that made steam turn a crank, and patented it, and the fly-wheel to regulate its motion, estimated that a horse could perform 33000 units of work in a minute, that is, raise 33000 lbs. one foot high in a minute. To perform 465838·2 units of work in 10 minutes would require the application 1·4116 horse power.

9. What work is done by a force, acting upon another carriage, under the same circumstances, weighing 5000 lbs., which transforms the velocity from 30 to 50 feet?

$\frac{(30)^2}{64{\cdot}4} = 13{\cdot}9907$, the height due to 30 feet velocity.

$\frac{(50)^2}{64{\cdot}4} = 38{\cdot}8043$, the height due to 50 feet velocity.

$$\begin{array}{lr} \text{From} & 38{\cdot}8043 \\ \text{Take} & \underline{13{\cdot}9907} \\ & 24{\cdot}8136 \\ & \underline{5000} \\ & \underline{124068{\cdot}0000} \end{array}$$

∴ 124068 are the units of work, and just so much work will the carriage perform if a resistance be opposed to it, and it be gradually brought from a 50 feet velocity to a 30 feet velocity.

The following is without doubt a very simple formula, but the most useful one in mechanics; by it we have solved the last two questions:

$$Fs = (H - h)\,W.$$

This simple formula involves the principle technically termed the principle of VIS VIVA, or LIVING FORCES. H is the height due to one velocity, say v or $H = \frac{v^2}{2g}$ and h, the height due to another a, or $h = \frac{a^2}{2g}$. The weight of the mass $= W$; the force F, and the space s.

To express this principle in words, we may say, that the working power (Fs) which a mass either acquires when it passes from a lesser velocity (a) to a greater velocity (v), or produces when it is compelled to pass from a greater velocity (v) into a less (a), is always equal to the product of the weight of the mass and the difference of the heights due to the velocities.

When we know the *units of work*, and the distance in which the change of velocity goes on, the force is easily found; and when the force is known, the distance is readily determined. Suppose, in the last example, that the change of velocity from 30 to 50 feet took place in a distance of 300 feet, then

$\frac{124068}{300} = 413{\cdot}56$ lbs. = F, the force constantly applied during 300 feet.

10. If a sleigh, weighing 2000 lbs., after describing a distance of 250 feet, has completely lost a velocity of 100 feet, what constant resistance does the friction offer?

Since the terminal velocity = 0, the height due to it = 0, hence

$$\frac{(100)^2}{64{\cdot}4} \times \frac{2000}{250} = 1242{\cdot}2352 \text{ lbs.}$$

We have been calculating upon the principle of *vis viva;* but the product of the *mass* and the square of the velocity, without attaching to it any definite idea, is termed the *vis viva*, or living force.

11. A body weighing 2300 lbs. moves with a velocity of 20 feet the second, required the *vis viva?*

$$\frac{2300}{32{\cdot}2} = 71{\cdot}42857 \text{ lbs., mass.}$$

$$71{\cdot}42857 \times (20)^2 = 28571{\cdot}428, \text{ the amount of } vis\ viva.$$

Hence, if a *mass* enters from a velocity a, into another v, the unit of work done is equal to half the difference of the *vis viva*, at the commencement and end of the change of velocity.

For if the *mass* be put = M, and W the weight, Then $M = \frac{W}{g}$, and the *vis viva* to velocity $a = Ma^2 = \frac{Wa^2}{g}$; and the *vis viva* to velocity $v = Mv^2 = \frac{Wv^2}{g}$.

Then $\frac{1}{2}\left\{\frac{Wv^2}{g} - \frac{Wa^2}{g}\right\} = \left(\frac{v^2}{2g} - \frac{a^2}{2g}\right) \times W = (H - h)\,W$, for $\frac{v^2}{2g}$ and $\frac{a^2}{2g}$, give the heights due to the velocities v and a, respectively. The useful formula

$$Fs = (H - h)\,W,$$

before given, page 270, may be applied to variable as well as to constant forces, if, instead of the constant force F, the mean value of the force be applied.

STRENGTH OF MATERIALS.

ON MATERIAL EMPLOYED IN THE CONSTRUCTION OF MACHINES.

In theoretical mechanics, we deal with imaginary quantities, which are perfect in all their properties; they are perfectly hard, and perfectly elastic; devoid of weight in statics and of friction in dynamics. In practical mechanics, we deal with real material objects, among which we find none which are perfectly hard, and none, except gaseous bodies, which are perfectly elastic; all have weight, and experience resistance in dynamical action. Practical mechanics is the science of automatic labour, and its objects are machines and their applications to the transmission, modification, and regulation of motive power. In this it takes as a basis the theoretical deductions of pure mechanics, but superadds to the formulæ of the mathematician a multitude of facts deduced from observation, and experimentally elaborates a new code of laws suited to the varied conditions to be fulfilled in the economy of the industrial arts.

In reference to the structure of machines, it is to be observed that however simple or complex the machine may be, it is of importance that its parts combine lightness with strength, and rigidity with uniformity of action; and that it communicates the power without shocks and sudden changes of motion, by which the passive resistances may be increased and the effect of the engine diminished.

To adjust properly the disposition and arrangement of the individual members of a machine, implies an exact knowledge and estimate of the amount of strain to which they are respectively subject in the working of the machine; and this skill, when exercised in conjunction with an intimate acquaintance with the nature of the materials of which the parts are themselves composed, must contribute to the production of a machine possessing the highest amount of capability attainable with the given conditions.

Materials.—The material most commonly employed in the construction of machinery is iron, in the two states of *cast* and *wrought* or *forged* iron; and of these, there are several varieties of quality. It becomes therefore a problem of much practical importance to determine, at least approximately, the capabilities of the particular material employed, to resist permanent alteration in the directions in which they are subjected to strain in the reception and transmission of the motive power.

To indicate briefly the fundamental conditions which determine the capability of a given weight and form of material to resist a given force, it must, in the first place, be observed, that rupture may take place either by tension or by compression in the direction of the length. To the former condition of strain is opposed the *tenacity* of the material; to the other is opposed the resistance to the *crushing of its substance.* Rupture, by *transverse strain*, is opposed both by the tenacity of the material and its capability to withstand compression together of its particles. Lastly, the bar may be ruptured by *torsion.* Mr. Oliver Byrne, the author of the present work, in his New Theory of the Strength of Materials has pointed out new elements of much importance.

The capabilities of a material to resist extension and compression are often different. Thus, the soft gray variety of cast iron offers a greater resistance to a force of extension than the white variety in a ratio of nearly eight to five; but the last offers the greatest resistance to a compressing force.

The resistance of cast iron to rupture by extension varies from 6 to 9 tons upon the square inch; and that to rupture by compression, from 36 to 65 tons. The resistance to extension of the best forged iron may be reckoned at 25 tons per inch; but the corresponding resistance to compression, although not satisfactorily ascertained, is generally considered to be greatly less than that of cast iron. Roudelet makes it 31½ tons on the square inch. Cast iron (and even wood) is therefore to be preferred for vertical supports.

The forces resisting rupture are as the areas of the sections of rupture, the material being the same; this principle holds not only in respect of iron, but also of wood. Many inquiries have been instituted to determine the commonly received principle, that the strength of rectangular beams of the same width to resist rupture by transverse strain is as the *squares of the depths* of the beams.

In these respects the experiments, although valuable on account of their extent and the care with which they were conducted, possess little novelty; but in directing attention to the elastic properties of the materials experimented upon, it was found that the received doctrine of relation between the limit of elasticity and weight requires modification. The common assumption is, that the destruction of the elastic properties of a material, that is, the displacement beyond the elastic limit, does not manifest itself until the load exceeds *one-third of the breaking weight.* It was found, however, on the contrary, that its effect was produced and manifested in a permanent *set* of the material when the load did not exceed *one-sixteenth* of that necessary to produce rupture. Thus a bar of one inch square, supported between props 4½ feet apart, did not break till loaded with 496 lbs. but showed a permanent deflection or *set* when loaded with 16 lbs. In other cases, loads of 7 lbs. and 14 lbs. were found to produce permanent sets when the breaking weights were respectively 364 lbs. and 1120 lbs. These sets were therefore given by $\frac{1}{52}$d and $\frac{1}{80}$th of the breaking weights.

Since these results were obtained, it has been found that *time* and the *weight* of the material itself are sufficient to effect a permanent deflection in a beam supported between props, so that there would seem to be no such limits in respect to transverse strain as those known by the name of elastic limits, and consequently the principle of loading a beam within the elastic limit has no foundation in practice. The beam yields continually to the load, but with an exceedingly slow progression, until the load approximates to the breaking weight, when rupture speedily succeeds to a rapid deflection.

As respects the effect of tension and compression by transverse strain, it was ascertained by a very ingenious experiment that equal loads produced equal deflections in both cases.

Another most important principle developed by experiments, is that respecting the compression of supporting columns of different heights. When the height of the column exceeded a certain limit, it was found that the crushing force became constant, and did not increase as the height of the column increased, until it reached another limit at which it began to yield, not strictly by crushing, but by the bending of the material. The first limit was found to be a height of little less than three times the radius of the column; and the second double that height, or about six times the radius of the column. In columns of different heights between these limits, having equal diameters, the force producing rupture by compression was nearly constant. When the column was *less* than the lower limit, the crushing force became *greater*, and when it was greater than the higher limit, the crushing force became *less*. It was further found that in all cases, where the height of the column was exactly above the limits of three times the radius, the section of rupture was a plane inclined at nearly the same constant angle of 55 degrees to the axis of the column. These facts mutually explain each other; for in every height of column above the limit, the section of rupture being a plane at the same angle to the axis of the column, must of necessity be a plane of the same size, and therefore in each case the cohesion of the same number of particles must be overcome in producing rupture. And further, the same number of particles being to be overcome under the same circumstances for every different height, the same force will be required to overcome that amount of cohesion, until at double the height (three diameters) the column begins to bend under its load. This height being surpassed, it follows that a pressure which becomes continually less as the length of the column is increased, will be sufficient to break it.

This property, moreover, is not confined to cast iron; the experiments of M. Rondelet show that with columns of wrought iron, wood, and stone, similar results are obtained.

From these facts then, it appears that if supporting columns be taken of different diameters, and of heights so great as not to allow of their bending, yet sufficiently high to allow of a complete separation of the planes of fracture, that is, of heights intermediate to three times and six times their radius, then will their strengths be as the number of particles in their planes of fracture; and the planes of fracture being inclined at equal angles to the axes of the columns, their areas will be as the transverse sections of the columns, and consequently the *strengths* of the columns will be as their transverse sections respectively. Taking the mean of three experiments upon a column ¼ inch diameter, the crushing force was 6426 lbs.; whilst the mean of four experiments, conducted in exactly the same manner, upon a column of ⅜ of an inch diameter, gave 14542 lbs. The diameters of the columns being 2 to 3, the areas of transverse section were therefore 4 to 9, which is very nearly the ratio of the crushing weights.

When the length of the column is so great that its fracture is produced wholly by bending of its material, the limit has been fixed for columns of cast iron, at 30 times the diameter when the ends are flat, and 15 times the diameter when the ends are rounded. In shorter columns, fracture takes place partly by crushing and partly by bending of the material. When the column is enlarged in the middle of its length from one and a half to two times the diameter of the ends, the strength was found by the same experimenter to be greater by one-seventh than in solid columns containing the same quantity of iron, in the same length, with their extremities rounded; and stronger by an eighth or a ninth when the extremities were flat and rendered immovable by disks.

The following formulas give the absolute strength of cylindrical columns to sustain pressure in the direction of their length. In these formulas

D = the external diameter of the column in inches.
d = the internal diameter of hollow columns in inches.
L = the length of the column in feet.
W = the breaking weight in tons.

Character of the column.	Length of the column exceeding 15 times its diameter.	Length of the column exceeding 30 times its diameter.
	Both ends rounded.	Both ends flat.
Solid cylindrical column of cast iron,	$W = 14{\cdot}9 \frac{D^{3{\cdot}76}}{L^{1{\cdot}7}}$	$W = 44{\cdot}16 \frac{D^{3{\cdot}55}}{L^{1{\cdot}7}}$
Hollow cylindrical column of cast iron,	$W = 13 \frac{D^{3{\cdot}76} - d^{3{\cdot}76}}{L^{1{\cdot}7}}$	$W = 44{\cdot}34 \frac{D^{3{\cdot}55} - d^{3{\cdot}55}}{L^{1{\cdot}7}}$
Solid cylindrical column of wrought iron,	$W = 42{\cdot}8 \frac{D^{3{\cdot}76}}{L^{2}}$	$W = 133{\cdot}75 \frac{D^{3{\cdot}55}}{L^{2}}$

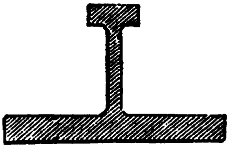
For shorter columns, if W′ represent the weight in tons which would break the column by bending alone, as given by the preceding formulas, and W″ the weight in tons which would crush the column without bending it, as determined from the subjoined table, then the absolute breaking weight of the column W, is represented in tons by the formula,

$$W = \frac{W' \times W''}{W' + W''}$$

These rules require the use of logarithms in their applications.

When a beam is deflected by transverse strain, the material on that side of it on which it sustains the strain is *compressed*, and the material on the opposite side is *extended*. The imaginary surface at which the compression terminates and the extension begins—at which there is supposed to be neither extension nor compression—is termed the *neutral axis* of the beam. What constitutes the strength of a beam is its resistance to compression on the one side and to extension on the other side of that axis—the forces acting about the line of axis like antagonist force at the two extremities of a lever, so that if either of them yield, the beam will be broken. It becomes, however, a question of importance to determine the relation of these forces; in other words, to determine whether the beam of given form and material will yield first to compression or to extension. This point is settled by reference to the columns of the subsequent table, page 280, in which it will be observed that the *metals* require a much greater force to crush them than to tear them asunder, and that the *woods* require a much smaller force.

There is also another consideration which must not be overlooked. Bearing in mind the condition of antagonism of the forces, it is obvious, that the further these forces are placed from the neutral axis, that is, from the fulcrum of their leverage, the greater must be their effect. In other words, all the material resisting compression will produce its greatest effect when collected the farthest possible from the neutral axis at the top of the beam; and, in like manner, all the material resisting extension will produce its greatest effect when similarly disposed at the bottom of the beam. We are thus directed to the first general principle of the distribution of the material into two flanges—one forming the top and the other the bottom of the beam—joined by a comparatively slender rib. Associating with this principle the relation of the forces of extension and compression of the material employed, we arrive at a form of beam in which the material is so distributed, that at the instant it is about to break by extension on the one side, it is about to break by compression on the other, and consequently is of the strongest form. Thus, supposing that it is required to determine that form in a girder of cast iron: the ratio of the crushing force of that metal to the force of extension may be taken generally as 6½ to 1, which is therefore also the ratio of the lower to the upper flange, as in the annexed sectional diagram.



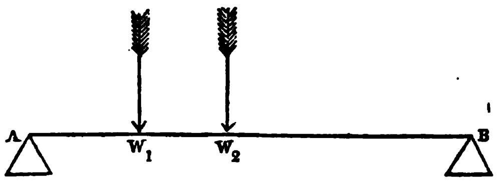
A series of nine castings were made, gradually increasing the lower flange at the expense of the upper one, and in the first eight experiments the beam broke by the tearing asunder of the lower flange; and in the last experiment the beam yielded by the crushing of the upper flange. In the eight experiments the upper flange was therefore the weakest, and in the ninth the strongest, so that the form of maximum strength was intermediate, and very closely allied to that form of beam employed in the last experiment, which was greatly the strongest. The circumstances of these experiments are contained in the following table.

No. of experiments.	Ratio of surfaces of compression and extension.	Area of cross sections in sq. inches.	Strength per sq. inch of sections in lbs.
1	1 to 1	2·82	2368
2	1 to 2	2·87	2567
3	1 to 4	3·02	2737
4	1 to 4½	3·37	3183
5	1 to 4	4·50	3214
6	1 to 5½	5·00	3346
7	1 to 3⅙	4·628	3246
8	1 to 4·3	5·86	3317
9	1 to 6·1	6·4	4075

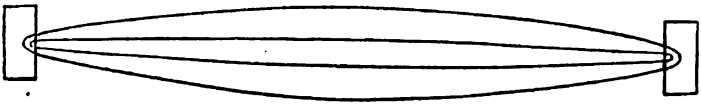
To determine the weight necessary to *break* beams cast according to the form described:

Multiply the area of the section of the lower flange by the depth of the beam, and divide the product by the distance between the two points on which the beam is supported: this quotient multiplied by 536 when the beams are cast erect, and by 514 when they are cast horizontally, will give the breaking weight in cwts.

From this it is not to be inferred that the beam ought to have the same transverse section throughout its length. On the contrary, it is clear that the section ought to have a definite relation to the leverage at which the load acts. From a mathematical consideration of the conditions, it indeed appears that the effect of a given load to break the beam varies when it is placed over different points of it, as the products of the distances of these points from the points of support of the beam. Thus the effect of a weight placed at the point W_1 is to the effect of the same weight acting upon the point W_2, as the product $AW_1 \times W_1 B$ is to the product $AW_2 \times W_2 B$; the points of support being at A and B. Since then the effect of a weight increases as it approaches the middle of the length of the beam, at which it is a maximum, it is plain that the beam does not require to have the same transverse section near to its extremities as in the middle; and, guided by the principle stated, it is easy to perceive that its strength at different points should in strictness vary as the products of the distances of these points from the points of support. By



taking this law as a fundamental condition in the distribution of the strength of a beam, whose load we may conceive to be accumulated at the middle of its length, we arrive at the strongest form which can be attained under given circumstances, with a given amount of material; we arrive at that form which renders the beam equally liable to rupture at every point. Now this form of maximum strength may be attained in two ways; either by varying the depth of the beam according to the law stated, or by preserving the depth everywhere the same, and varying the dimensions of the upper and lower flanges according to the same law. The conditions are manifestly identical. We may therefore assume generally the condition that the section is *rectangular*, and that the thickness of the flanges is constant; then the outline determined by the law in question, in the one case of the *elevation* of the beam and in the other of the *plan* of the flanges, is the geometrical curve called a parabola—rather, two parabolas joined base to base at the middle between the points of support. The annexed diagram represents the plan of a cast-iron girder according to this form, the depth being uniform throughout. Both flanges are of the same form, but the dimensions of the upper one are such as to give it only a sixth of the strength of the other.



This, it will be observed, is also the form, considered as an elevation, of the beam of a steam engine, which good taste and regard to economy of material have rendered common.

It must, however, be borne in mind, that in the actual practice of construction, materials cannot with safety be subjected to forces approaching to those which produce rupture. In machinery especially, they are liable to various and accidental pressures, besides those of a permanent kind, for which allowance must be made. The engineer must therefore in his practice depend much on experience and consideration of the species of work which the engine is designed to perform. If the engine be intended for spinning, pumping, blowing, or other *regular* work, the material may be subjected to pressures approaching two-thirds of that which would actually produce rupture; but in engines employed to drive bone-mills, stampers, breaking-down rollers, and the like, double that strength will often be found insufficient. In cases of that nature, experience is a better guide than theory.

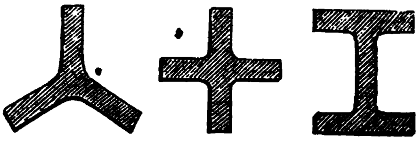
It is also to be remarked that we are often obliged to depart from the form of strength which the calculation gives, on account of the partial strains which would be put upon some of the parts of a casting, in consequence of unequal cooling of the metal when the thicknesses are unequal. An expert founder can often reduce the irregular contractions which thus result; but, even under the best management, fracture is not unfrequently produced by irregularity of cooling, and it is at all times better to avoid the danger entirely, than to endeavour to obviate it by artifice. For this reason, the parts of a casting ought to be as nearly as possible of such thickness as to cool and contract regularly, and by that means all partial strain of the parts will be avoided.

With respect to design, it is also to be remarked, that mere theoretical properties of parts will not, under all the varieties of circumstances which arise in the working of a machine, insure that exact adjustment of material and propriety of form so much desired in constructive mechanics. Every design ought to take for its basis the mathematical conditions involved, and it would, perhaps, be impossible to arrive at the best forms and proportions by any more direct mode of calculation; but it is necessary to superadd to the mathematical demonstration, the exercise of a well-matured judgment, to secure that degree of adjustment and arrangement of parts in which the merits of a good design mainly consist. A purely theoretical engine would look strangely deficient to the practised eye of the engineer; and the merely theoretical contriver would speedily find himself lost, should he venture beyond his construction on paper. His nice calculations of the "work to be performed," of the *vis viva* of the mechanical organs of his machine, and of the *modulus of elasticity* of his material, would, in practice, alike deceive him.

The first consideration in the design of a machine is the quantity of work which each part has to perform—in other words, the forces, active and inactive, which it has to resist; the direction of the forces in relation to the cross-section and points of support; the velocity, and the changes of velocity to which the moving parts are subject. The calculations necessary to obtain these must not be confined to theory alone; neither should they be entirely deduced by "rule of thumb;" by the first mode the strength would, in all probability, be deficient from deficiency of material, and by the second *rule* the material would be injudiciously disposed; weight would be added often where least needed, merely from the determination to avoid fracture, and in consequence of a want of knowledge respecting the true forms best adapted to give strength.

To the following general principles, in practice, there are but few real exceptions:

I. *Direct Strain.*—To this a straight line must be opposed, and if the part be of considerable length, vibration ought to be counteracted by intersection of planes, (technically *feathers,*) as represented in the annexed diagrams, or some such form, consistent with the purpose for which the part is intended.



II. *Transverse Strain.*—To this a parabolic form of section must be opposed, or some simple figure including the parabolic form. For economy of material, the vertex of the curve ought to be at the point where the force is applied; and when the strain passes alternately from one side of the part to the other, the curve ought to be on both sides, as in the beam of a steam engine.

When a loaded piece is supported at one end only, if the breadth be everywhere the same, the form of equal strength is a triangle; but, if the section be a circle, then the solid will be that generated by the revolution of a semi-parabola about its longer axis. In practice, it will, however, be sufficient to employ the frustum of a cone, of which, in the case of cast iron, the diameter at the unsupported end is one-third of the diameter at the fixed end.

III. *Torsion.*—The section most commonly opposed to torsion is a circle; and, if the strain be applied to a cylinder, it is obvious the rupture must first take place at the surface, where the torsion is greatest, and that the further the material is placed from the neutral axis, the greater must be its power of resistance; and hence, the amount of materials being the same, a shaft is stronger when made hollow than if it were made solid.

It ought not, however, to be supposed that the circle is the only figure which gives an axis the property of offering, in every direction, the same resistance to flexure. On the contrary, a square section gives the same resistance in the direction of its sides, and of its diagonals; and, indeed, in every direction the resistance is equal. This is, moreover, the case with a great number of other figures, which may be formed by combining the circle and the square in a symmetrical manner; and hence, if the axis, strengthened by salient sides, as in feathered shafts, do not answer as well as cylindrical ones, it must arise from their not being so well disposed to resist torsion, and not from any irregularities of flexure about the axis inherent in the particular form of section.

This subject has been investigated with much care, and, according to M. Cauchy, the modulus of rupture by torsion, T, is connected with the modulus of rupture by transverse strain S, by the simple analogy $T = \frac{4}{5} S$.

The forms of all the parts of a machine, in whatever situation and under every variety of circumstances, may be deduced from these simple figures; and, if the calculations of their dimensions be correctly determined, the parts will not only possess the requisite degree of strength, but they will also accord with the general principles of good taste.

In arranging the details of a machine, two circumstances ought to be taken into consideration. The first is, that the parts subject to wear and influenced by strain, should be capable of adjustment; the second is, that every part should, in relation to the work it has to perform, be equally strong, and present to the eye a figure that is consistent with its degree of action. Theory, practice, and taste must all combine to produce such a combination. No formal law can be expressed, either by words or figures, by which a certain contour should be preferred to another; both may be equally strong and equally correct in reference to theory; custom, then, must be appealed to as the guide.

TABLES OF THE MECHANICAL PROPERTIES OF THE MATERIALS MOST COMMONLY EMPLOYED IN THE CONSTRUCTION OF MACHINES AND FRAMINGS.

NAMES.	Specific Gravity.	Weight of 1 cubic ft. in lbs.	Tenacity per square inch in lbs.	Crushing force per sq. in. in lbs.	Modulus of elasticity in lbs.	Mod. of rupture in lbs.	Crushing force to tenacity.
TABLE I.—*Mechanical Properties of the Common Metals.*							
Brass (cast)	8·399	525·00	17968	10304	8930000	——	0·573 : 1
Copper (cast)	8·607	537·93	19072	——	——	——	——
ditto (sheet)	8·785	549·06	——	——	——	——	——
ditto (wire-drawn)	8·878	560·00	61228	——	——	——	——
ditto (in bolts)	——	——	48000	——	——	——	——
Iron (English wrought)	7·700	481·20	20½ tons	——	24920000	——	——
ditto (in bars)	7·760 7·800	475·50 487·00	—— 25½ tons	——	——	——	——
ditto (hammered)	——	——	30 tons	——	——	——	——
ditto (Russian) in bars	——	——	27 tons	——	——	——	——
ditto (Swedish) in bars	——	——	32 tons	——	——	——	——
ditto (English) in wire, 10th inch diam.	——	——	36 to 43 tons	——	——	——	——
ditto (Russian) in wire, 1-20th to 1-30th inch diameter	——	——	60 to 91 tons	——	——	——	——
ditto (rolled in sheets and cut lengthwise)	——	——	14 tons	——	——	——	——
ditto cut crosswise	——	——	18 tons	——	——	——	——
ditto in chains, oval links, 6 inches clear, iron ½ inch diameter	——	——	21½ tons	——	——	——	——
ditto (Brunton's) with stay cross link	——	——	25 tons	——	——	——	——
Cast-iron (Old Park)	——	——	——	——	18014400	48240	——
ditto (Adolphi)	——	——	——	——	18353300	45360	——
ditto (Alfreton)	——	——	——	——	17686400	44046	——
ditto (scrap)	——	——	——	——	18032000	45828	——
ditto (Carron, No. 2) hot blast	7·046	440·37	13505	108540	16085000	37503	8·037 : 1
ditto (do. do.) cold blast	7·066	441·62	16683	106375	17270500	38556	6·376 : 1
ditto (do. No. 3)	7·094	443·37	14200	115442	16246966	33980	8·129 : 1
ditto (do. do.) hot blast	7·056	441·00	17755	133440	17873100	42120	7·515 : 1
ditto (Devon, No. 3) cold blast	7·295	455·93	——	——	22907700	36288	——
ditto (do. do.) hot blast	7·229	451·81	21907	145435	22473650	43497	——
ditto (Buffrey, No. 1) cold blast	7·079	442·43	17466	93366	15381200	37503	5·346 : 1
ditto (do. do.) hot blast	6·998	437·37	13434	86397	13730500	35316	6·431 : 1
ditto (Coed-Talon, No. 2) cold blast	6·955	434·06	18855	81770	14313500	33104	4·337 : 1
ditto (do. do.) hot blast	6·968	435·50	16676	82739	14322500	33145	4·961 : 1
ditto (do. No. 3) cold blast	7·104	449·62	——	——	17102000	43541	——
ditto (do. do.) hot blast	6·970	435·62	——	——	14707900	40159	——
ditto (Milton, No. 1) hot blast	6·976	436·00	——	——	11974500	28552	——
ditto (Muirkirk, No. 1) cold blast	7·113	444·56	——	——	14003550	35923	——
ditto (do. do.) hot blast	6·953	434·56	——	——	13294400	33850	——
ditto (Elsicar, No. 1) cold blast	7·030	439·37	——	——	13981000	34862	——
Lead (English cast)	11·446	717·45	1824	——	720000	——	——
ditto (milled-sheet)	11·407	712·93	3328	——	——	——	——
ditto (wire-drawn	11·317	705·12	2581	——	——	——	——
Silver (standard)	10·312	644·50	40902	——	——	——	——
Mercury (at 32°)	13·619	851·18	——	——	——	——	——
ditto (at 60°)	13·580	848·75	——	——	——	——	——
Steel (soft)	7·780	486·25	120000	——	——	——	——
ditto (razor-tempered)	7·840	490·00	150000	——	29000000	——	——
Tin (cast)	7·291	455·68	5322	——	4608000	——	——
Zinc (cast)	7·028	439·25	——	——	13680000	——	——
ditto (rolled	7·215	450·9	——	——	——	——	——
TABLE II.—*Principal Woods.*							
Acacia (English)	0·71	44·37	16000	——	1152000	11202	——
Beech, New	0·854	53·37	15784	7733	13536000	93363	
Beech, Dry	0·690	43·12	17850	9363			0·55 : 1
Birch, Common	0·792	49·50	15000	6402	1562400	10920	0·43 : 1
Birch, American	0·648	40·50	——	11633	1257600	9624	——
Deal, Christiania middle	0·698	43·62	12400	——	1672000	9864	——
Deal, Memel middle	0·590	36·87	——	——	1535200	10386	——
Deal, Norway spruce	0·340	21·25	17600	——	——	——	——
Deal, English	0·470	29·37	7000	——	——	——	——
Elm (seasoned)	0·588	36·75	13489	10331	699840	6078	0·79 : 1
Fir, New England	0·553	34·56	——	——	2191200	6612	——
Fir, Riga	0·753	47·06	12000	6000	——	——	0·50 : 1
Larch (seasoned)	0·522	32·62	10220	5568	1052800	6894	0·55 : 1
Lignum-vitæ	1·220	76·25	11800	——	——	——	——
Mahogany (Spanish)	0·800	50·00	16500	8198	——	——	0·50 : 1
Oak, English	0·934	58·37	17300	4684 wet 9504 dry	1451200	10032	0·28 : 1 0·57 : 1
Oak, Canadian	0·872	54·50	10253	4231 wet 9509 dry	2148800	10596	0·42 : 1 0·95 : 1
Oak, Dantzic	0·756	47·24	12780	——	1191200	8748	——
Pine, Pitch	0·660	41·25	7818	——	1225600	9792	——
Pine, Red	0·657	41·06	——	5375	1840000	8946	——
Pine, Yellow	0·461	28·81	——	5445	1600000	——	——
Plane-tree	0·64	40·00	11700	——	——	——	——
Poplar	0·383	23·93	7200	3107 wet 5124 dry	——	——	0·43 : 1 0·74 : 1
Teak (dry)	0·657	41·06	15000	12101	2414400	14772	0·81 : 1
Willow (dry)	0·390	24·37	14000	——	——	——	——
Yew (Spanish)	0·807	50·43	8000	——	——	——	——

THE COHESIVE STRENGTH OF BODIES.

The following TABLE *contains the result of experiments on the cohesive strength of various bodies in avoirdupois pounds; also, one-third of the ultimate strength of each body, this being considered sufficient, in most cases, for a permanent load:*

Names of Bodies.	Square Bar.	One-third.	Round Bar.	One-third.
WOODS.	*lbs.*	*lbs.*	*lbs.*	*lbs.*
Boxwood	20000	6667	15708	5236
Ash	17000	5667	13357	4452
Teak	15000	5000	11781	3927
Fir	12000	4000	9424	3141
Beach	11500	3866	9032	3011
Oak	11000	3667	8639	2880
METALS.				
Cast iron	18656	6219	14652	4884
English wrought iron	55872	18624	43881	14627
Swedish do. do.	72064	24021	56599	18866
Blistered steel	133152	44384	104577	34859
Shear do.	124400	41366	97703	32568
Cast do.	134256	44752	105454	35151
Cast copper	19072	6357	14979	4993
Wrought do.	33792	11264	26540	8827
Yellow brass	17968	5989	14112	4704
Cast tin	4736	1579	3719	1239
Cast lead	1824	608	1432	477

PROBLEM I.

RULE.—*To find the ultimate cohesive strength of square, round, and rectangular bars, of any of the various bodies, as specified in the table.*—Multiply the strength of an inch bar, (as in the table,) of the body required, by the cross sectional area of square and rectangular bars, or by the square of the diameter of round bars; and the product will be the ultimate cohesive strength.

A bar of cast iron being 1½ inches square, required its cohesive power.

$$1{\cdot}5 \times 1{\cdot}5 \times 18656 = 41976 \text{ lbs.}$$

Required the cohesive force of a bar of English wrought iron, 2 inches broad, and ⅜ of an inch in thickness.

$$2 \times {\cdot}375 \times 55872 = 41904 \text{ lbs.}$$

Required the ultimate cohesive strength of a round bar of wrought copper ¾ of an inch in diameter.

$${\cdot}75^2 \times 26540 = 14928{\cdot}75 \text{ lbs.}$$

PROBLEM II.

RULE.—*The weight of a body being given, to find the cross sectional dimensions of a bar or rod capable of sustaining that weight.*—For square and round bars, divide the weight given by one-third of the cohesive strength of an inch bar, (as specified in the table,) and the square root of the quotient will be the side of the square, or diameter of the bar in inches.

And if rectangular, divide the quotient by the breadth, and the result will be the thickness.

What must be the side of a square bar of Swedish iron to sustain a permanent weight of 18000 lbs?

$$\sqrt{\frac{18000}{24021}} = \cdot 86\text{, or nearly }\tfrac{7}{8}\text{ of an inch square.}$$

Required the diameter of a round rod of cast copper to carry a weight of 6800 lbs.

$$\sqrt{\frac{6800}{4993}} = 1.16\text{ inches diameter.}$$

A bar of English wrought iron is to be applied to carry a weight of 2760 lbs.; required the thickness, the breadth being two inches.

$$\frac{2760}{18624} = \cdot 142 \div 2 = \cdot 071\text{ of an inch in thickness.}$$

A Table *showing the circumference of a rope equal to a chain made of iron of a given diameter, and the weight in tons that each is proved to carry; also, the weight of a foot of chain made from iron of that dimension.*

Ropes. Cir. in Ins.	Chains. Diam. in Inches.	Proved to carry in tons.	Weight of a lineal foot in lbs. Avr.
3	¼ and $\frac{1}{16}$	1	1·08
4	⅜	2	1·5
4¾	⅜ and $\frac{1}{16}$	3	2
5¼	½	4	2·7
6	½ and $\frac{1}{16}$	5	3·3
6½	⅝	6	4
7	⅝ and $\frac{1}{16}$	8	4·6
7½	¾	9¾	5·5
8	¾ and $\frac{1}{16}$	11¼	6·1
9	⅞	13	7·2
9½	⅞ and $\frac{1}{16}$	15	8·4
10½	1 inch.	18	9·4

ON THE TRANSVERSE STRENGTH OF BODIES.

The *tranverse strength* of a body is that power which it exerts in opposing any force acting in a perpendicular direction to its length, as in the case of beams, levers, &c., for the fundamental principles of which observe the following:—

That the transverse strength of beams, &c. is inversely as their lengths, and directly as their breadths, and square of their depths, and, if cylindrical, as the cubes of their diameters; that is, if a beam 6 feet long, 2 inches broad, and 4 inches deep, can carry 2000 lbs., another beam of the same material, 12 feet long, 2 inches broad, and 4 inches deep, will only carry 1000, being inversely as their lengths. Again, if a beam 6 feet long, 2 inches broad, and 4 inches deep, can support a weight of 2000 lbs., another beam of the same material, 6 feet long, 4 inches broad, and 4 inches deep, will support double that weight, being directly as their breadths; —but a beam of that material, 6 feet long, 2 inches broad, and 8 inches deep, will sustain a weight of 8000 lbs.; being as the square of their depths.

From a mean of experiments made, to ascertain the transverse strength of various bodies, it appears that the ultimate strength of an inch square, and an inch round bar of each, 1 foot long, loaded in the middle, and lying loose at both ends, is nearly as follows, in lbs. avoirdupois.

Names of Bodies.	Square Bar.	One-third.	Round Bar.	One-third.
Oak	800	267	628	209
Ash	1137	379	893	298
Elm	569	139	447	149
Pitch pine	916	305	719	239
Deal	566	188	444	148
Cast iron	2580	860	2026	675
Wrought iron	4013	1338	3152	1050

PROBLEM I.

RULE.—*To find the ultimate transverse strength of any rectangular beam, supported at both ends, and loaded in the middle; or supported in the middle, and loaded at both ends; also, when the weight is between the middle and the end; likewise when fixed at one end and loaded at the other.*—Multiply the strength of an inch square bar, 1 foot long, (as in the table,) by the breadth, and square of the depth in inches, and divide the product by the length in feet; the quotient will be the weight in lbs. avoirdupois.

What weight will break a beam of oak 4 inches broad, 8 inches deep, and 20 feet between the supports?

$$\frac{800 \times 4 \times 8^2}{20} = 10240 \text{ lbs.}$$

When a beam is supported in the middle, and loaded at each end, it will bear the same weight as when supported at both ends and loaded in the middle; that is, each end will bear half the weight.

When the weight is not situated in the middle of the beam, but placed somewhere between the middle and the end, multiply twice the length of the long end by twice the length of the short end, and divide the product by the whole length of the beam; the quotient will be the effectual length.

Required the ultimate transverse strength of a pitch pine plank 24 feet long, 3 inches broad, 7 inches deep, and the weight placed 8 feet from one end.

$$\frac{32 \times 16}{24} = 21{\cdot}3 \text{ effective length.}$$

$$\frac{916 \times 3 \times 7^2}{21{\cdot}3} = 6321 \text{ lbs.}$$

Again, when a beam is fixed at one end and loaded at the other, it will only bear $\frac{1}{4}$ of the weight as when supported at both ends and loaded in the middle.

What is the weight requisite to break a deal beam 6 inches broad, 9 inches deep, and projecting 12 feet from the wall?

$$\frac{566 \times 6 \times 9^2}{12} = 22923 \div 4 = 5730 \cdot 7 \text{ lbs.}$$

The same rules apply as well to beams of a cylindrical form, with this exception, that the strength of a round bar (as in the table) is multiplied by the cube of the diameter, in place of the breadth, and square of the depth.

Required the ultimate transverse strength of a solid cylinder of cast iron 12 feet long and 5 inches diameter.

$$\frac{2026 \times 5^3}{12} = 21104 \text{ lbs.}$$

What is the ultimate transverse strength of a hollow shaft of cast iron 12 feet long, 8 inches diameter outside, and containing the same cross sectional area as a solid cylinder 5 inches diameter?

$$\sqrt{8^2 - 5^2} = 6 \cdot 24, \text{ and } 8^3 - 6 \cdot 24^3 = 269.$$

$$\text{Then, } \frac{2026 \times 269}{12} = 45416 \text{ lbs.}$$

When a beam is fixed at both ends, and loaded in the middle, it will bear one-half more than it will when loose at both ends.

And if a beam is loose at both ends, and the weight laid uniformly along its length, it will bear double; but if fixed at both ends, and the weight laid uniformly along its length, it will bear triple the weight.

PROBLEM II.

RULE.—*To find the breadth or depth of beams intended to support a permanent weight.*—Multiply the length between the supports, in feet, by the weight to be supported in lbs., and divide the product by one-third of the ultimate strength of an inch bar, (as in the table,) multiplied by the square of the depth; the quotient will be the breadth, or, multiplied by the breadth, the quotient will be the square of the depth, both in inches.

Required the breadth of a cast iron beam 16 feet long, 7 inches deep, and to support a weight of 4 tons in the middle.

$$4 \text{ tons} = 8960 \text{ lbs. and } \frac{8960 \times 16}{860 \times 7^2} = 3 \cdot 4 \text{ inches.}$$

What must be the depth of a cast iron beam 3·4 inches broad, 16 feet long, and to bear a permanent weight of four tons in the middle?

$$\sqrt{\frac{8960 \times 16}{860 \times 3 \cdot 4}} = 7 \text{ inches.}$$

When a beam is fixed at both ends, the divisor must be multiplied by 1·5, on account of it being capable of bearing one-half more.

When a beam is loaded uniformly throughout, and loose at both ends, the divisor must be multiplied by 2, because it will bear double the weight.

If a beam is fast at both ends, and loaded uniformly throughout, the divisor must be multipled by 3, on account that it will bear triple the weight.

Required the breadth of an oak beam 20 feet long, 12 inches deep, made fast at both ends, and to be capable of supporting a weight of 12 tons in the middle.

$$12 \text{ tons} = 26880 \text{ lbs., and } \frac{26880 \times 20}{266 \times 12^2 \times 1{\cdot}5} = 9{\cdot}7 \text{ inches.}$$

Again, when a beam is fixed at one end, and loaded at the other, the divisor must be multiplied by ·25; because it will only bear one-fourth of the weight.

Required the depth of a beam of ash 6 inches broad, 9 feet projecting from the wall, and to carry a weight of 47 cwt.

$$47 \text{ cwt.} = 5264 \text{ lbs., and } \sqrt{\frac{5264 \times 9}{379 \times 6 \times {\cdot}25}} = 9{\cdot}12 \text{ inches deep.}$$

And when the weight is not placed in the middle of a beam, the effective length must be found as in Problem I.

Required the depth of a deal beam 20 feet long, and to support a weight of 63 cwt. 6 feet from one end.

$$\frac{28 \times 12}{20} = 16{\cdot}8 \text{ effective length of beam, and}$$

$$63 \text{ cwt.} = 7056 \text{ lbs.; hence}$$

$$\sqrt{\frac{7056 \times 16{\cdot}8}{188 \times 6}} = 10{\cdot}24 \text{ inches deep.}$$

Beams or shafts exposed to lateral pressure are subject to all the foregoing rules, but in the case of water-wheel shafts, &c., some allowances must be made for wear; then the divisor may be changed from 675 to 600 for cast iron.

Required the diameter of bearings for a water-wheel shaft 12 feet long, to carry a weight of 10 tons in the middle.

$$10 \text{ tons} = 22400 \text{ lbs., and}$$

$$\frac{22400}{600} = \sqrt[3]{448} = 7{\cdot}65 \text{ inches diameter.}$$

And when the weight is equally distributed along its length, the cube root of half the quotient will be the diameter, thus:

$$\frac{448}{2} = \sqrt[3]{224} = 6{\cdot}07 \text{ inches diameter.}$$

Required the diameter of a solid cylinder of cast iron, for the shaft of a crane, to be capable of sustaining a weight of 10 tons;

one end of the shaft to be made fast in the ground, the other to project 6½ feet; and the effective leverage of the jib as 1¾ to 1.

10 tons = 22400 lbs., and

$$\frac{22400 \times 6{\cdot}5 \times 1{\cdot}75}{675 \times {\cdot}25} = 1509$$

And $\sqrt[3]{1509} = 11{\cdot}47$ inches diameter.

The strength of cast iron to wrought iron, in this direction, is as 9 is to 14 nearly; hence, if wrought iron is taken in place of cast iron in the last example, what must be its diameter?

$$\sqrt[3]{\frac{1509 \times 9}{14}} = 9{\cdot}89 \text{ inches diameter.}$$

ON TORSION OR TWISTING.

The strength of bodies to resist *torsion*, or wrenching asunder, is directly as the cubes of their diameters; or, if square, as the cube of one side; and inversely as the force applied multiplied into the length of the lever.

Hence the rule.—1. Multiply the strength of an inch bar, by experiment, (as in the following table,) by the cube of the diameter, or of one side in inches; and divide by the radius of the wheel, or length of the lever also in inches; and the quotient will be the ultimate strength of the shaft or bar, in lbs. avoirdupois.

2.—Multiply the force applied in pounds by the length of the lever in inches, and divide the product by one-third of the ultimate strength of an inch bar, (as in the table,) and the cube root of the quotient will be the diameter, or side of a square bar in inches; that is, capable of resisting that force permanently.

The following TABLE *contains the result of experiments on inch bars, of various metals, in lbs. avoirdupois.*

Names of Bodies.	Round Bar.	One-third.	Square Bar.	One-third.
Cast iron	11943	3981	15206	5069
English wrought iron	12063	4021	15360	5120
Swedish do. do.	11400	3800	14592	4864
Blistered steel	20025	6675	25497	8499
Shear do	20508	6836	26112	8704
Cast do	21111	7037	26880	8960
Yellow brass	5549	1850	7065	2355
Cast copper	4825	1608	6144	2048
Tin	1688	563	2150	717
Lead	1206	402	1536	512

What weight, applied on the end of a 5 feet lever, will wrench asunder a 3 inch round bar of cast iron?

$$\frac{11943 \times 3^3}{60} = 5374 \text{ lbs. avoirdupois.}$$

Required the side of a square bar of wrought iron, capable of resisting the twist of 600 lbs. on the end of a lever 8 feet long.

$$\sqrt[3]{\frac{600 \times 96}{5120}} = 2\tfrac{1}{4} \text{ inches.}$$

In the case of revolving shafts for machinery, &c., the strength is directly as the cubes of their diameters, and revolutions, and inversely as the resistance they have to overcome; hence,

From *practice*, we find that a 40 horse power steam engine, making 25 revolutions per minute, requires a shaft (*if made of wrought-iron*) to be 8 inches diameter: now, the cube of 8, multiplied by 25, and divided by 40 = 320; which serves as a constant multiplier for all others in the same proportion.

What must be the diameter of a wrought iron shaft for an engine of 65 horse power, making 23 revolutions per minute?

$$\sqrt[3]{\frac{65 \times 320}{23}} = 9{\cdot}67 \text{ inches diameter.}$$

James Glenie, the mathematician, gives 400 as a constant multiplier for cast iron shafts that are intended for first movers in machinery;

200 for second movers; and

100 for shafts connecting smaller machinery, &c.

The velocity of a 30 horse power steam engine is intended to be 19 revolutions per minute. Required the diameter of bearings for the fly-wheel shaft.

$$\sqrt[3]{\frac{400 \times 30}{19}} = 8{\cdot}579 \text{ inches diameter.}$$

Required the diameter of the bearings of shafts, as second movers from a 30 horse engine; their velocity being 36 revolutions per minute.

$$\sqrt[3]{\frac{200 \times 30}{36}} = 5{\cdot}5 \text{ inches diameter.}$$

When shafting is intended to be of wrought iron, use 160 as the multiplier for second movers; and 80 for shafts connecting smaller machinery.

TABLE *of the Proportionate Length of Bearings, or Journals for Shafts of various diameters.*

Dia. in Inches.	Len. in Inches.	Dia. in Inches.	Len. in Inches.
1	1¾	6½	8¾
1½	2¼	7	9⅜
2	3	7½	10
2¼	3¼	8	10¾
2½	3½	8½	11⅜
3	4¼	9	12
3½	4⅞	9½	12¾
4	5½	10	13¼
4½	6⅛	10½	14
5	6¾	11	14½
5½	7½	11½	15¼
6	8¼	12	16

Tenacities, Resistances to Compression, and other Properties of the common Materials of Construction.

Names of Bodies.	Absolute.		Compared with Cast Iron.		
	Tenacity in lbs. per sq. inch.	Resistance to compression in lbs. per sq. in.	Its strength is	Its extensibility is	Its stiffness is
Ash	14130	—	0·23	2·6	0·089
Beech	12225	8548	0·15	2·1	0·073
Brass	17968	10304	0·435	0·9	0·49
Brick	275	562	—	—	—
Cast iron	18434	86397	1·000	1·0	1·000
Copper (wrought)	33000	—	—	—	—
Elm	9720	1033	0·21	2·9	0·073
Fir, or Pine, white	12346	2028	0·23	2·4	0·1
— — red	11800	5375	0·8	2·4	0·1
— — yellow	11835	5445	0·25	2·9	0·087
Granite, Aberdeen	—	10910	—	—	—
Gun-metal (copper 8, and tin 1)	35838	—	0·65	1·25	0·535
Malleable iron	56000	—	1·12	0·86	1·3
Larch	12240	5568	0·136	2·3	0·058
Lead	1824	—	0·096	2·5	0·0385
Mahogany, Honduras	11475	8000	0·24	2·9	0·487
Marble	551	6060	—	—	—
Oak	11880	9504	0·25	2·8	0·093
Rope (1 in. in circum.)	200	—	—	—	—
Steel	128000	—	—	—	—
Stone, Bath	478	—	—	—	—
— Craigleith	772	5490	—	—	—
— Dundee	2661	6630	—	—	—
— Portland	857	3729	—	—	—
Tin (cast)	4736	—	0·182	0·75	0·25
Zinc (sheet)	9120	—	0·365	0·5	0·76

Comparative Strength and Weight of Ropes and Chains.

Circum. of rope in inches.	Weight per fathom in lbs.	Diameter of chain in inches.	Weight per fathom in lbs.	Proof strength in tons & cwt.	Circum. of rope in inches.	Weight per fathom in lbs.	Diameter of chain in inches.	Weight per fathom in lbs.	Proof strength in tons & cwt.
$3\frac{1}{2}$	$2\frac{3}{4}$	$\frac{5}{16}$	$5\frac{1}{2}$	$1\ \ 5\frac{1}{2}$	10	23	$\frac{7}{8}$	43	10 0
$4\frac{1}{4}$	$4\frac{3}{4}$	$\frac{3}{8}$	8	$1\ \ 16\frac{3}{4}$	$10\frac{3}{4}$	28	$\frac{15}{16}$	49	11 11
5	$5\frac{3}{4}$	$\frac{7}{16}$	$10\frac{1}{2}$	2 10	$11\frac{1}{2}$	$30\frac{1}{2}$	1in.	56	13 8
$5\frac{3}{4}$	7	$\frac{1}{2}$	14	$3\ \ 5\frac{1}{2}$	$12\frac{1}{4}$	36	$1\frac{1}{16}$	63	14 18
$6\frac{1}{2}$	$9\frac{3}{4}$	$\frac{9}{16}$	18	$4\ \ 3\frac{1}{2}$	13	39	$1\frac{1}{8}$	71	16 14
7	$11\frac{1}{4}$	$\frac{5}{8}$	22	5 2	$13\frac{3}{4}$	45	$1\frac{3}{16}$	79	18 11
8	15	$\frac{11}{16}$	27	$6\ \ 4\frac{1}{2}$	$14\frac{1}{2}$	$48\frac{1}{2}$	$1\frac{1}{4}$	87	20 8
$8\frac{3}{4}$	19	$\frac{3}{4}$	32	7 7	$15\frac{1}{4}$	56	$1\frac{5}{16}$	96	22 13
$9\frac{1}{2}$	21	$\frac{13}{16}$	37	$8\ \ 13\frac{1}{2}$	16	60	$1\frac{3}{8}$	106	24 18

It must be understood and also borne in mind, that in estimating the amount of tensile strain to which a body is subjected, the weight of the body itself must also be taken into account; for according to its position so may it approximate to its whole weight, in tending to produce tension within itself; as in the almost constant application of ropes and chains to great depths, considerable heights, &c.

Alloys that are of greater Tenacity than the sum of their Constituents, as determined by the Experiments of Muschenbroek.

Swedish copper 6 parts, Malacca tin 1—tenacity per square inch	64,000 lbs.
Chili copper 6 parts, Malacca tin 1	60,000
Japan copper 5 parts, Banca tin 1	57,000
Anglesea copper 6 parts, Cornish tin 1	41,000
Common block tin 4, lead 1, zinc 1	13,000
Malacca tin 4, regulus of antimony 1	12,000
Block tin 3, lead 1	10,200
Block tin 8, zinc 1	10,000
Lead 1, zinc 1	4,500

TABLE *of Data, containing the Results of Experiments on the Elasticity and Strength of various Species of Timber.*

Species of Timber.	Value of E.	Value of S.	Species of Timber.	Value of E.	Value of S.
Teak	174·7	2462	Elm	50·64	1013
Poona	122·26	2221	Pitch pine	88·68	1632
English oak	105	1672	Red pine	133	1341
Canadian do	155·5	1766	New England fir	158·5	1102
Dantzic do	86·2	1457	Riga fir	90	1100
Adriatic do	70·5	1383	Mar Forest do.	63	1200
Ash	119	2026	Larch	76	900
Beech	98	1556	Norway spruce	105·47	1474

RULE.—*To find the dimensions of a beam capable of sustaining a given weight, with a given degree of deflection, when supported at both ends.*—Multiply the weight to be supported in lbs. by the cube of the length in feet; divide the product by 32 times the tabular value of E, multiplied into the given deflection in inches, and the quotient is the breadth multiplied by the cube of the depth in inches.

When the beam is intended to be square, then the fourth root of the quotient is the breadth and depth required.

If the beam is to be cylindrical, multiply the quotient by 1·7, and the fourth root of the product is the diameter.

The distance between the supports of a beam of Riga fir is 16 feet, and the weight it must be capable of sustaining in the middle of its length is 8000 lbs., with a deflection of not more than ¾ of an inch; what must be the depth of the beam, supposing the breadth 8 inches?

$$\frac{16 \times 8000}{90 \times 32 \times \cdot75} = 15175 \div 8 = \sqrt[4]{1897} = 12\cdot35 \text{ in. the depth.}$$

RULE.—*To determine the absolute strength of a rectangular beam of timber when supported at both ends, and loaded in the middle of its length, as beams in general ought to be calculated to, so that they may be rendered capable of withstanding all accidental cases of emergency.*—Multiply the tabular value of S by four times the depth of the beam in inches, and by the area of the cross section in inches; divide the product by the distance between the supports in inches, and the quotient will be the absolute strength of the beam in lbs.

If the beam be not laid horizontally, the distance between the supports, for calculation, must be the horizontal distance.

One-fourth of the weight obtained by the rule is the greatest weight that ought to be applied in practice as permanent load.

If the load is to be applied at any other point than the middle, then the strength will be, as the product of the two distances is to the square of half the length of the beam between the supports; or, twice the distance from one end, multiplied by twice from the other, and divided by the whole length, equal the effective length of the beam.

In a building 18 feet in width, an engine boiler of 5½ tons is to be fixed, the centre of which to be 7 feet from the wall; and having two pieces of red pine 10 inches by 6, which I can lay across the two walls for the purpose of slinging it at each end,—may I with sufficient confidence apply them, so as to effect this object?

$$\frac{2240 \times 5{\cdot}5}{2} = 6160 \text{ lbs. to carry at each end.}$$

And 18 feet − 7 = 11, double each, or 14 and 22, then $\frac{14 \times 22}{18}$ = 17 feet, or 204 inches, effective length of beam.

Tabular value of S, red pine = $\frac{1341 \times 4 \times 10 \times 60}{204} = 15776$ lbs., the absolute strength of each piece of timber at that point.

Rule.—*To determine the dimensions of a rectangular beam capable of supporting a required weight, with a given degree of deflection, when fixed at one end.*—Divide the weight to be supported, in lbs., by the tabular value of E, multiplied by the breadth and deflection, both in inches; and the cube root of the quotient, multiplied by the length in feet, equal the depth required in inches.

A beam of ash is intended to bear a load of 700 lbs. at its extremity; its length being 5 feet, its breadth 4 inches, and the deflection not to exceed ½ an inch.

Tabular value of E = 119 × 4 × ·5 = 238, the divisor; then $700 \div 238 = \sqrt[3]{2{\cdot}94} \times 5 = 7{\cdot}25$ inches, depth of the beam.

Rule.—*To find the absolute strength of a rectangular beam, when fixed at one end, and loaded at the other.*—Multiply the value of S by the depth of the beam, and by the area of its section, both in inches; divide the product by the leverage in inches, and the quotient equal the absolute strength of the beam in lbs.

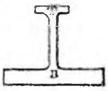
A beam of Riga fir, 12 inches by 4½, and projecting 6½ feet from the wall; what is the greatest weight it will support at the extremity of its length?

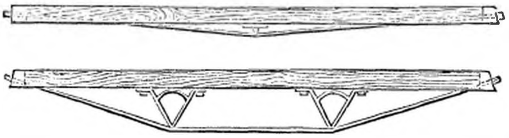
Tabular value of S = 1100

12 × 4·5 = 54 sectional area,

$$\text{Then, } \frac{1100 \times 12 \times 54}{78} = 9138{\cdot}4 \text{ lbs.}$$

When fracture of a beam is produced by vertical pressure, the fibres of the lower section of fracture are separated by extension, whilst at the same time those of the upper portion are destroyed by compression; hence exists a point in section where neither the one nor the other takes place, and which is distinguished as the point of neutral axis. Therefore, by the law of fracture thus established, and proper data of tenacity and compression given, as in the Table (p. 281), we are enabled to form metal beams of strongest section with the least possible material: thus, in cast iron the resistance to compression is nearly as 6½ to 1 of tenacity; consequently a beam of cast iron, to be of strongest section, must be of the form TB, and a parabola in the direction of its length, the quantity of material in the bottom flange being about 6½ times that of the upper: but such is not the case with beams of timber; for although the tenacity of timber be on an average twice that of its resistance to compression, its flexibility is so great, that any considerable length of beam, where columns cannot be situated to its support, requires to be strengthened or trussed by iron rods, as in the following manner:





And these applications of principle not only tend to diminish deflection, but the required purpose is also more effectively attained, and that by lighter pieces of timber.

Rule.—*To ascertain the absolute strength of a cast iron beam of the preceding form, or that of strongest section.*—Multiply the sectional area of the bottom flange in inches by the depth of the beam in inches, and divide the product by the distance between the supports also in inches; and 514 times the quotient equal the absolute strength of the beam in cwts.

The strongest form in which any given quantity of matter can be disposed is that of a hollow cylinder; and it has been demonstrated that the maximum of strength is obtained in cast iron, when the thickness of the annulus or ring amounts to ⅕th of the cylinder's external diameter; the relative strength of a solid to that of a hollow cylinder being as the diameters of their sections.

The following table shows the greatest weight that ever ought to be laid upon a beam for permanent load, and if there be any liability to jerks, &c., ample allowance must be made; also, the weight of the beam itself must be included.

Rule.—*To find the weight of a cast iron beam of given dimensions.*—Multiply the sectional area in inches by the length in feet, and by 3·2, the product equal the weight in lbs.

Required the weight of a uniform rectangular beam of cast iron, 16 feet in length, 11 inches in breadth, and 1½ inch in thickness.

$$11 \times 1{\cdot}5 \times 16 \times 3{\cdot}2 = 844{\cdot}8 \text{ lbs.}$$

A TABLE *showing the Weight or Pressure a Beam of Cast Iron, 1 inch in breadth, will sustain without destroying its elastic force, when it is supported at each end, and loaded in the middle of its length, and also the deflection in the middle which that weight will produce.*

Length.	6 feet.		7 feet.		8 feet.		9 feet.		10 feet.	
Depth in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.
3	1278	·24	1089	·33	954	·426	855	·54	765	·66
3½	1739	·205	1482	·28	1298	·365	1164	·46	1041	·57
4	2272	·18	1936	·245	1700	·32	1520	·405	1360	·5
4½	2875	·16	2450	·217	2146	·284	1924	·36	1721	·443
5	3560	·144	3050	·196	2650	·256	2375	·32	2125	·4
6	5112	·12	4356	·163	3816	·213	3420	·27	3060	·33
7	6958	·103	5929	·14	5194	·183	4655	·23	4165	·29
8	9088	·09	7744	·123	6784	·16	6080	·203	5440	·25
9	—	—	9801	·109	8586	·142	7695	·18	6885	·22
10	—	—	12100	·098	10600	·128	9500	·162	8500	·2
11	—	—	—	—	12826	·117	11495	·15	10285	·18
12	—	—	—	—	15264	·107	13680	·135	12240	·17
13	—	—	—	—	—	—	16100	·125	14400	·154
14	—	—	—	—	—	—	18600	·115	16700	·143
	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.	
6	2548	·48	2184	·65	1912	·85	1699	1·08	1530	1·34
7	3471	·41	2975	·58	2603	·73	2314	·93	2082	1·14
8	4532	·36	3884	·49	3396	·64	3020	·81	2720	1·00
9	5733	·32	4914	·44	4302	·57	3825	·72	3438	·89
10	7083	·28	6071	·39	5312	·51	4722	·64	4250	·8
11	8570	·26	7346	·36	6428	·47	5714	·59	5142	·73
12	10192	·24	8736	·33	7648	·43	6796	·54	6120	·67
13	11971	·22	10260	·31	8978	·39	7980	·49	7182	·61
14	13883	·21	11900	·28	10412	·36	9255	·46	8330	·57
15	15937	·19	13660	·26	11952	·34	10624	·43	9562	·53
16	18128	·18	15536	·24	13584	·32	12080	·40	10880	·5
17	20500	·17	17500	·23	15353	·3	13647	·38	12282	·47
18	22932	·16	19656	·21	17208	·28	15700	·36	13752	·44

Resistance of Bodies to Flexure by Vertical Pressure.—When a piece of timber is employed as a column or support, its tendency to yielding by compression is different according to the proportion between its length and area of its cross section; and supposing the form that of a cylinder whose length is less than seven or eight times its diameter, it is impossible to bend it by any force applied longitudinally, as it will be destroyed by splitting before that bending can take place; but when the length exceeds this, the column will bend under a certain load, and be ultimately destroyed by a similar kind of action to that which has place in the transverse strain.

Columns of cast iron and of other bodies are also similarly circumstanced.

When the length of a cast iron column with flat ends equals about thirty times its diameter, fracture will be produced wholly by bending of the material;—when of less length, fracture takes place partly by crushing and partly by bending: but, when the column is enlarged in the middle of its length from one and a half to twice its diameter at the ends, by being cast hollow, the strength is greater by $\frac{1}{7}$th than in a solid column containing the same quantity of material.

Rule.—*To determine the dimensions of a support or column to bear without sensible curvature a given pressure in the direction of its axis.*—Multiply the pressure to be supported in lbs. by the square of the column's length in feet, and divide the product by twenty times the tabular value of E; and the quotient will be equal to the breadth multiplied by the cube of the least thickness, both being expressed in inches.

When the pillar or support is a square, its side will be the fourth root of the quotient.

If the pillar or column be a cylinder, multiply the tabular value of E by 12, and the fourth root of the quotient equal the diameter.

What should be the least dimensions of an oak support, to bear a weight of 2240 lbs. without sensible flexure, its breadth being 3 inches, and its length 5 feet?

Tabular value of E = 105, and $\frac{2240 \times 5^2}{20 \times 105 \times 3} = \sqrt[3]{8\cdot888} =$ 2·05 inches.

Required the side of a square piece of Riga fir, 9 feet in length, to bear a permanent weight of 6000 lbs.

Tabular value of E = 96, and $\frac{6000 \times 9^2}{20 \times 96} = \sqrt[4]{253} =$ 4 inches nearly.

Dimensions of Cylindrical Columns of Cast Iron to sustain a given load or pressure with safety.

Diameter in inches.	Length or height in feet.										
	4	6	8	10	12	14	16	18	20	22	24
	Weight or load in cwts.										
2	72	60	49	40	32	26	22	18	15	13	11
2½	119	105	91	77	65	55	47	40	34	29	25
3	178	163	145	128	111	97	84	73	64	56	49
3½	247	232	214	191	172	156	135	119	106	94	83
4	326	310	288	266	242	220	198	178	160	144	130
4½	418	400	379	354	327	301	275	251	229	208	189
5	522	501	479	452	427	394	365	337	310	285	262
6	607	592	573	550	525	497	469	440	413	386	360
7	1032	1013	989	959	924	887	848	808	765	725	686
8	1333	1315	1289	1259	1224	1185	1142	1097	1052	1005	959
9	1716	1697	1672	1640	1603	1561	1515	1467	1416	1364	1311
10	2119	2100	2077	2045	2007	1964	1916	1865	1811	1755	1697
11	2570	2550	2520	2490	2450	2410	2358	2305	2248	2189	2127
12	3050	3040	3020	2970	2930	2900	2830	2780	2730	2670	2600

Practical utility of the preceding Table.—Wanting to support the front of a building with cast iron columns 18 feet in length, 8 inches in diameter, and the metal 1 inch in thickness; what weight may

I confidently expect each column capable of supporting without tendency to deflection?

Opposite 8 inches diameter and under 18 feet = 1097
Also opposite 6 in. diameter and under 18 feet = 440
= 657 cwts.

The strength of cast iron as a column being = 1·0000
— steel — = 2·518
— wrought iron — = 1·745
— oak (Dantzic) — = ·1088
— red deal — = ·0785

Elasticity of torsion, or resistance of bodies to twisting.—The angle of flexure by torsion is as the length and extensibility of the body directly, and inversely as the diameter; hence, the length of a bar or shaft being given, the power, and the leverage the power acts with, being known, and also the number of degrees of torsion that will not affect the action of the machine, to determine the diameter in cast iron with a given angle of flexure.

RULE.—Multiply the power in lbs. by the length of the shaft in feet, and by the leverage in feet; divide the product by fifty-five times the number of degrees in the angle of torsion, and the fourth root of the quotient equal the shaft's diameter in inches.

Required the diameters for a series of shafts 35 feet in length, and to transmit a power equal to 1245 lbs., acting at the circumference of a wheel $2\frac{1}{2}$ feet radius, so that the twist of the shafts on the application of the power may not exceed one degree.

$$\frac{1245 \times 35 \times 2\cdot5}{55 \times 1} = \sqrt[4]{1981} = 6\cdot67 \text{ inches in diameter.}$$

Relative strength of metals to resist torsion.

Cast iron............	= 1	Swedish bar iron ...	= 1·05
Copper..............	= ·48	English do.	= 1·12
Yellow brass........	= ·511	Shear steel...........	= 1·96
Gun-metal..........	= ·55	Cast do.............	= 2·1

DEFLEXION OF RECTANGULAR BEAMS.

RULE.—*To ascertain the amount of deflexion of a uniform beam of cast iron, supported at both ends, and loaded in the middle to the extent of its elastic force.*—Multiply the square of the length in feet by ·02, and the product divided by the depth in inches equal the deflexion.

Required the deflection of a cast iron beam 18 feet long between the supports, 12·8 inches deep, 2·56 inches in breadth, and bearing a weight of 20,000 lbs. in the middle of its length.

$$\frac{18^2 \times \cdot02}{12\cdot8} = \cdot506 \text{ inches from a straight line in the middle.}$$

For beams of a similar description, loaded uniformly, the rule is the same, only multiply by ·025 in place of ·02.

RULE.—*To find the deflection of a beam when fixed at one end* *and loaded at the other.*—Divide the length in feet of the fixed part of the beam by the length in feet of the part which yields to the force, and add 1 to the quotient; then multiply the square of the length in feet by the quotient so increased, and also by ·13; divide this product by the middle depth in inches, and the quotient will be the deflection, in inches also.

Multiply the deflection so obtained for cast iron by ·86, the product equal the deflection for wrought iron; for oak, multiply by 2·8; and for fir, 2·4.

A TABLE *of the Depths of Square Beams or Bars of Cast Iron, calculated to support from* 1 *Cwt. to* 14 *Tons in the Middle, the Deflection not to exceed* $\frac{1}{40}$*th of an Inch for each Foot in Length.*

Lengths in Feet		4	6	8	10	12	14	16	18	20	22	24	26	28	30
Weight in cwt.	Weight in lbs.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.
		In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1 cwt.	112	1·2	1·4	1·7	1·9	2·0	2·2	2·4	2·5	2·6	2·7	2·9	3·0	3·1	3·2
2	124	1·4	1·7	2·0	2·2	2·4	2·6	2·8	3·0	3·1	3·3	3·4	3·6	3·7	3·8
3	336	1·6	1·9	2·2	2·4	2·7	2·9	3·1	3·3	3·4	3·6	3·8	3·9	4·1	4·2
4	448	1·7	2·0	2·4	2·6	2·9	3·1	3·3	3·5	3·7	3·9	4·0	4·2	4·3	4·5
5	560	1·8	2·2	2·5	2·8	3·0	3·3	3·5	3·7	3·9	4·1	4·3	4·4	4·6	4·8
6	672	1·8	2·2	2·6	2·9	3·2	3·4	3·7	3·9	4·1	4·3	4·5	4·6	4·8	5·0
7	784	1·9	2·3	2·7	3·0	3·3	3·6	3·8	4·1	4·2	4·4	4·6	4·8	5·0	5·2
8	896	2·0	2·4	2·8	3·1	3·4	3·7	3·9	4·2	4·4	4·6	4·8	5·0	5·2	5·4
9	1,008	2·0	2·5	2·9	3·2	3·5	3·8	4·0	4·3	4·5	4·7	4·9	5·1	5·3	5·5
10	1,120	2·1	2·6	3·0	3·3	3·6	3·9	4·2	4·4	4·7	4·9	5·2	5·3	5·4	5·7
11	1,232	2·1	2·6	3·0	3·4	3·7	4·0	4·3	4·5	4·8	5·0	5·3	5·4	5·6	5·8
12	1,344	2·2	2·7	3·1	3·5	3·8	4·1	4·4	4·7	4·9	5·1	5·3	5·5	5·7	5·9
13	1,456	2·2	2·7	3·1	3·5	3·8	4·2	4·4	4·7	4·9	5·2	5·4	5·6	5·9	6·0
14	1,568	2·3	2·8	3·2	3·6	3·9	4·2	4·5	4·8	5·0	5·3	5·5	5·7	6·0	6·1
15	1,680	2·3	2·8	3·2	3·6	4·0	4·3	4·6	4·9	5·2	5·4	5·6	5·8	6·1	6·2
16	1,792	2·4	2·9	3·3	3·7	4·0	4·4	4·7	5·0	5·2	5·5	5·7	5·9	6·2	6·4
17	1,904	2·4	2·9	3·4	3·8	4·1	4·4	4·7	5·0	5·3	5·5	5·8	6·0	6·2	6·5
18	2,016	2·4	3·0	3·4	3·8	4·2	4·5	4·8	5·1	5·4	5·6	5·9	6·1	6·4	6·6
19	2,128	2·5	3·0	3·5	3·9	4·2	4·6	4·9	5·2	5·4	5·7	6·0	6·2	6·5	6·7
1 ton.	2,240	2·5	3·0	3·5	3·9	4·3	4·6	4·9	5·2	5·5	5·8	6·0	6·3	6·5	6·8
1¼	2,800	2·6	3·2	3·7	4·1	4·5	4·9	5·2	5·5	5·8	6·1	6·4	6·6	6·9	7·2
1½	3,360	2·8	3·4	3·9	4·3	4·7	5·1	5·5	5·8	6·1	6·4	6·7	7·0	7·2	7·5
1¾	3,920	2·9	3·5	4·0	4·5	4·9	5·3	5·7	6·0	6·3	6·7	6·9	7·2	7·5	7·7
2	4,480	2·9	3·5	4·1	4·7	5·1	5·5	5·9	6·2	6·5	6·8	7·2	7·6	7·7	8·0
2½	5,600	3·1	3·8	4·4	4·9	5·5	5·8	6·2	6·6	6·9	7·3	7·6	7·9	8·2	8·5
3	6,720	3·3	4·0	4·6	5·1	5·7	6·1	6·5	6·9	7·3	7·6	7·9	8·3	8·6	8·9
3½	7,840	3·4	4·1	4·8	5·3	5·8	6·3	6·7	7·1	7·5	7·9	8·2	8·6	8·9	9·2
4	8.960	3·5	4·3	4·9	5·5	6·0	6·5	7·0	7·4	7·8	8·2	8·5	8·9	9·2	9·5
4½	10,080	—	4·4	5·1	5·7	6·2	6·7	7·2	7·6	8·0	8·4	8·8	9·1	9·5	9·8
5	11,200	—	4·5	5·2	5·8	6·4	6·9	7·4	7·8	8·2	8·6	9·0	9·4	9·7	10·1
6	13,440	—	—	5·5	6·1	6·7	7·2	7·7	8·2	8·6	9·0	9·4	9·8	10·2	10·5
7	15,680	—	—	5·7	6·3	6·9	7·5	8·0	8·5	8·9	9·4	9·8	10·2	10·6	11·0
8	17,920	—	—	5·9	6·6	7·2	7·8	8·3	8·8	9·3	9·7	10·1	10·6	10·9	11·3
9	20,160	—	—	6·0	6·8	7·4	8·0	8·5	9·0	9·5	10·0	10·4	10·9	11·3	11·7
10	22,400	—	—	—	6·9	7·6	8·2	8·8	9·3	9·8	10·3	10·7	11·2	11·6	12·0
11	24,640	—	—	—	7·1	7·8	8·4	9·0	9·5	10·0	10·5	11·0	11·5	11·9	12·3
12	26,880	—	—	—	7·2	7·9	8·6	9·2	9·7	10·2	10·8	11·2	11·7	12·1	12·5
13	29,120	—	—	—	7·4	8·1	8·8	9·4	9·9	10·4	11·0	11·5	11·9	12·4	12·8
14	31,360	—	—	—	7·5	8·3	8·9	9·5	10·1	10·6	11·1	11·7	12·1	12·6	13·0
Deflection in inches		·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75

Lengths in Feet		10	12	14	16	18	20	22	24	26	28	30	32	34	36
15	33,600	7·7	8·4	9·1	9·7	10·3	10·8	11·4	11·9	12·3	12·8	13·2	13·7	14·1	14·5
16	35,840	7·8	8·5	9·2	9·8	10·4	11·0	11·5	12·0	12·5	13·0	13·5	13·9	14·3	14·7
17	38,080	7·9	8·7	9·4	10·0	10·6	11·2	11·7	12·2	12·7	13·2	13·7	14·1	14·5	14·9
18	40.320	8·0	8·8	9·5	10·1	10·8	11·3	11·9	12·4	12·9	13·4	13·9	14·3	14·7	15·1
19	42,560	8·1	8·9	9·6	10·3	10·9	11·5	12·2	12·6	13·1	13·6	14·1	14·5	15·0	15·4
20	44,800	—	9·0	9·7	10·4	11·0	11·6	12·5	12·7	13·2	13·8	14·2	14·7	15·1	15·6
22	49,280	—	9·2	10·0	10·7	11·3	11·9	12·8	13·0	13·6	14·1	14·6	15·1	15·5	15·9
24	53,760	—	9·4	10·2	10·9	11·5	12·2	13·0	13·4	13·9	14·4	14·9	15·4	15·9	16·3
26	58,240	—	9·6	10·4	11·1	11·8	12·4	13·3	13·6	14·2	14·7	15·2	15·7	16·2	16·7
28	62,720	—	9·8	10·6	11·4	12·0	12·7	13·5	13·9	14·4	15·0	15·5	16·0	16·5	17·0
Deflection in inches		·25	·3	·35	·4	·45	·5	·55	·6	·66	·7	·75	·8	·85	·9

Lengths in Feet		14	16	18	20	22	24	26	28	30	32	34	36	38	40
Weight in tons.	Weight in lbs.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.
		In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
30	67,200	10·8	11·5	12·2	12·9	13·5	14·1	14·7	15·2	15·7	16·3	16·8	17·3	17·7	18·2
32	71,680	11·0	11·7	12·4	13·1	13·7	14·3	14·9	15·6	16·0	16·5	17·0	17·5	18·0	18·5
34	76,160	11·1	11·9	12·6	13·3	13·9	14·5	15·1	15·7	16·2	16·8	17·3	17·8	18·3	18·8
36	80,640	11·3	12·0	12·8	13·4	14·1	14·7	15·3	15·9	16·5	17·0	17·5	18·0	18·5	19·0
38	85,120	11·4	12·2	13·0	13·6	14·3	14·9	15·5	16·1	16·7	17·2	17·8	18·3	18·8	19·3
40	89,600	—	12·4	13·1	13·8	14·5	15·1	15·7	16·4	16·9	17·5	18·0	18·5	19·1	19·5
42	94,080	—	12·5	13·3	14·0	14·7	15·3	15·9	16·5	17·1	17·7	18·2	18·7	19·3	19·8
44	98,560	—	12·7	13·5	14·2	14·9	15·5	16·1	16·8	17·4	17·9	18·5	19·0	19·5	20·0
46	103,040	—	12·8	13·6	14·3	15·0	15·7	16·3	17·0	17·6	18·1	18·7	19·2	19·8	20·3
48	107,520	—	13·0	13·7	14·5	15·2	15·9	16·5	17·1	17·7	18·3	18·8	19·4	20·0	20·5
50	112,000	—	—	13·8	14·6	15·3	16·0	16·6	17·3	17·9	18·5	19·0	19·6	20·1	20·7
52	116,480	—	—	14·0	14·7	15·5	16·2	16·8	17·5	18·1	18·7	19·2	19·8	20·3	21·0
54	120,960	—	—	14·1	14·9	15·7	16·3	17·0	17·6	18·2	18·8	19·4	19·9	20·5	21·1
56	125,440	—	—	14·3	15·0	15·8	16·5	17·1	17·8	18·4	19·0	19·6	20·1	20·7	21·3
58	129,920	—	—	14·4	15·1	15·9	16·6	17·3	17·9	18·5	19·2	19·7	20·3	20·9	21·4
60	134,400	—	—	14·5	15·3	16·0	16·7	17·4	18·1	18·7	19·3	19·9	20·5	21·1	21·6
Deflection in inches		·35	·4	·45	·5	·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0

Examples illustrative of the Table.—1. To find the depth of a rectangular bar of cast iron to support a weight of 10 tons in the middle of its length, the deflection not to exceed $\frac{1}{40}$ of an inch per foot in length, and its length 20 feet, also let the depth be 6 times the breadth.

Opposite 6 times the weight and under 20 feet in length is 15·3 inches, the depth, and $\frac{1}{6}$ of 15·3 = 2·6 inches, the breadth.

2. To find the diameter for a cast iron shaft or solid cylinder that will bear a given pressure, the flexure in the middle not to exceed $\frac{1}{40}$th of an inch for each foot of its length, the distance of the bearings being 20 feet, and the pressure on the middle equals 10 tons.

Constant multiplier 1·7 for round shafts, then 10 × 1·7 = 17. And opposite 17 tons and under 20 feet is 11·2 inches for the diameter.

But half that flexure is quite enough for revolving shafts: hence 17 × 2 = 34 tons, and opposite 34 tons is 13·3 inches for the diameter.

3. A body 256 lbs. weight, presses against its horizontal support, so that it requires the force of 52 lbs. to overcome its friction; if the body be increased to 8750 lbs., what force will cause it to pass from a state of rest to one of motion?

$\frac{52}{256}$ = ·203125 = , in this case, the *coefficient of friction;*

∴ 8750 × 203125 = 1777·34375 lbs., the force required.

This calculation is based upon the law, that friction is proportional to the normal pressure between the rubbing surfaces. Twice the pressure gives twice the friction; three times the pressure gives three times the friction; and so on. With light pressures, this law may not hold, but then it is to be attributed to the proportionately greater effect of adhesion.

4. If a sleigh, weighing 250 lbs., requires a force of 28 lbs. to draw it along; when 1120 lbs. are placed in it, required the units of work expended to move the whole 350 feet?

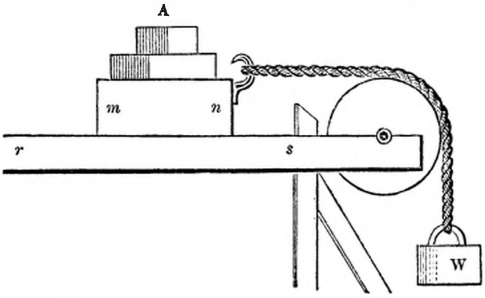
$$\frac{28}{250} = \cdot 112, \text{ the coefficient of friction.}$$

Then $(1120 + 250) \times \cdot 112 = 153 \cdot 44$ lbs., the force required to move the whole.

$$\therefore 153 \cdot 44 \times 350 = 53704, \text{ the units of work required.}$$

A UNIT OF WORK is the labour which is equal to that of raising one pound a foot high. It is supposed that a horse can perform 33000 units of work in a minute.

It may also be remarked that friction is independent of the extent of the surfaces in contact, except with trifling pressures and large surfaces, which is on account of the effect of adhesion. The friction of motion is independent of velocity, and is generally less than that of quiescence.



5. Required the coefficient of friction, for *a sliding motion*, of cast iron upon wrought, lubricated with Devlin's oil, and under the following circumstances: the load A, and sledge *nm*, weighs 8420 lbs., and requires a weight W, of 1200 lbs. to cause it to pass from a state of rest into one of motion: the sledge and load pass over 22 feet on the horizontal way *rs*, in 8 seconds.

In this case the coefficient of sliding motion will be

$$\frac{1200}{8420} - \frac{1200 + 8420}{8420} \times \frac{2 \times 22}{g \times 8^2},$$

in which $g = 32 \cdot 2$ feet; the acceleration of the free descent of bodies brought about by gravity. The above expression becomes

$$142515 - 1 \cdot 142515 \times \frac{44}{2060 \cdot 8} = \cdot 118121.$$

Hence the coefficient of the friction of motion is ·118121, and the coefficient of the friction of quiescence is ·142515.

OF FRICTION, OR RESISTANCE TO MOTION IN BODIES ROLLING OR RUBBING ON EACH OTHER.

In the years 1831, 1832, and 1833, a very extensive set of experiments were made at Metz, by M. Morin, under the sanction of the French government, to determine as nearly as possible the laws of friction; and by which the following were fully established:

1. When no unguent is interposed, the friction of any two surfaces (whether of quiescence or of motion) is directly proportional to the force with which they are pressed perpendicularly together; so that for any two given surfaces of contact there is a constant ratio of the friction to the perpendicular pressure of the one surface upon the other. Whilst this ratio is thus the same for the same

surfaces of contact, it is different for different surfaces of contact. The particular value of it in respect to any two given surfaces of contact is called the coefficient of friction in respect to those surfaces.

2. When no unguent is interposed, the amount of the friction is, in every case, wholly independent of the extent of the surfaces of contact; so that, the force with which two surfaces are pressed together being the same, their friction is the same, whatever may be the extent of their surfaces of contact.

3. That the friction of motion is wholly independent of the velocity of the motion.

4. That where unguents are interposed, the coefficient of friction depends upon the nature of the unguent, and upon the greater or less abundance of the supply. In respect to the supply of the unguent, there are two extreme cases, that in which the surfaces of contact are but slightly rubbed with the unctuous matter, as, for instance, with an oiled or greasy cloth, and that in which a continuous stratum of unguent remains continually interposed between the moving surfaces; and in this state the amount of friction is found to be dependent rather upon the nature of the unguent than upon that of the surfaces of contact. M. Morin found that with unguents (hog's lard and olive oil) interposed in a continuous stratum between surfaces of wood on metal, wood on wood, metal on wood, and metal on metal, when in motion, have all of them very near the same coefficient of friction, being in all cases included between ·07 and ·08.

The coefficient for the unguent tallow is the same, except in that of metals upon metals. This unguent appears to be less suited for metallic substances than the others, and gives for the mean value of its coefficient, under the same circumstances, ·10. Hence, it is evident, that where the extent of the surface sustaining a given pressure is so great as to make the pressure less than that which corresponds to a state of perfect separation, this greater extent of surface tends to increase the friction by reason of that adhesiveness of the unguent, dependent upon its greater or less viscosity, whose effect is proportional to the extent of the surfaces between which it is interposed.

It was found, from a mean of experiments with different unguents on axles, in motion and under different pressures, that, with the unguent tallow, under a pressure of from 1 to 5 cwt., the friction did not exceed $\frac{1}{39}$th of the whole pressure; when soft soap was applied, it became $\frac{1}{34}$th; and with the softer unguents applied, such as oil, hog's lard, &c., the ratio of the friction to the pressure increased; but with the harder unguents, as soft soap, tallow, and anti-attrition composition, the friction considerably diminished; consequently, to render an unguent of proper efficiency, the nature of the unguent must be measured by the pressure or weight tending to force the surfaces together.

TABLE *of the Results of Experiments on the Friction of Unctuous Surfaces.* By M. MORIN.

Surfaces of Contact.	Coefficients of Friction.	
	Friction of Motion.	Friction of Quiescence.
Oak upon oak, the fibres being parallel to the motion	0·018	0·390
Ditto, the fibres of the moving body being perpendicular to the motion	0·143	0·314
Oak upon elm, fibres parallel	0·136	
Elm upon oak, do	0·119	0·420
Beech upon oak, do	0·330	
Elm upon elm, do	0·140	
Wrought iron upon elm, do	0·138	
Ditto upon wrought iron, do	0·177	
Ditto upon cast iron, do	...	0·118
Cast iron upon wrought iron, do	0·143	
Wrought iron upon brass, do	0·160	
Brass upon wrought iron, do	0·166	
Cast iron upon oak, do	0·107	0·100
Ditto upon elm, do., the unguent being tallow	0·125	
Ditto, do., the unguent being hog's lard and black lead	0·137	
Elm upon cast iron	0·135	0·098
Cast iron upon cast iron	0·144	
Ditto upon brass	0·132	
Brass upon cast iron	0·107	
Ditto upon brass	0·134	0·164
Copper upon oak	0·100	
Yellow copper upon cast iron	0·115	
Leather (ox-hide), well tanned, upon cast iron, wetted	0·229	0·267
Ditto upon brass, wetted	0·244	

In these experiments, the surfaces, after having been smeared with an unguent, were wiped, so that no interposing layer of the unguent prevented intimate contact.

TABLE *of the Results of Experiments on Friction, with Unguents interposed.* By M. MORIN.

Surfaces of Contact.	Coefficients of Friction.		Unguents.
	Friction of Motion.	Friction of Quiescence.	
Oak upon oak, fibres parallel	0·164	0·440	Dry soap.
Do. do	0·075	0·164	Tallow.
Do. do	0·067	...	Hog's lard.
Do., fibres perpendicular	0·083	0·254	Tallow.
Do. do	0·072	...	Hog's lard.
Do. do	0·250	...	Water.
Do. upon elm, fibres parallel	0·136	...	Dry soap.
Do. do	0·073	0·178	Tallow.
Do. do	0·066	...	Hog's lard.
Do. upon cast iron	0·080	...	Tallow.
Do. upon wrought iron	0·098	...	Tallow.
Beech upon oak, fibres parallel	0·055	...	Tallow.
Elm upon oak, do	0·137	0·411	Dry soap.
Do. do	0·170	0·142	Tallow.
Do. do	0·060	...	Hog's lard.
Elm upon elm, do	0·139	0·217	Dry soap.
Do. upon cast iron	0·066	...	Tallow.
Wrought iron upon oak, fibres parallel	0·256	0·649	Greased and saturated with water.
Do. do	0·214	...	Dry soap.

Surfaces of Contact.	Coefficients of Friction.		Unguents.
	Friction of Motion.	Friction of Quiescence.	
Wrought iron upon oak, fibres parallel...	0·085	0·108	Tallow.
Do. upon elm, do...	0·078	...	Tallow.
Do. do...	0·076	...	Hog's lard.
Do. do...	0·055	...	Olive oil.
Do. upon cast iron, do...	0·103	...	Tallow.
Do. do...	0·076	...	Hog's lard.
Do. do...	0·066	0·100	Olive oil.
Do. upon wrought iron, do...	0·082	...	Tallow.
Do. do...	0·081	...	Hog's lard.
Do. do...	0·070	0·115	Olive oil.
Wrought iron upon brass, do...	0·103	...	Tallow.
Do. do...	0·075	...	Hog's lard.
Do. do...	0·078	...	Olive oil.
Cast iron upon oak, do...	0·189	...	Dry soap.
Do. do...	0·218	0·646	Greased and saturated with water.
Do. do...	0·078	0·100	Tallow.
Do. do...	0·075	...	Hog's lard.
Do. do...	0·075	0·100	Olive oil.
Do. upon elm, do...	0·077	...	Tallow.
Do. do...	0·061	...	Olive oil.
Do. do...	0·091	...	Hog's lard and plumbago.
Do. upon wrought iron...	...	0·100	Tallow.
Do. upon cast iron...	0·314	...	Water.
Do. do...	0·197	...	Soap.
Do. do...	0·100	0·100	Tallow.
Do. do...	0·070	0·100	Hog's lard.
Do. do...	0·064	...	Olive oil.
Do. do...	0·055	...	Hog's lard and plumbago.
Do. upon brass...	0·103	...	Tallow.
Do. do...	0·075	...	Hog's lard.
Do. do...	0·078	...	Olive oil.
Copper upon oak, fibres parallel	0·069	0·100	Tallow.
Yellow copper upon cast iron...	0·072	0·103	Tallow.
Do. do...	0·068	...	Hog's lard.
Do. do..	0·066	...	Olive oil.
Brass upon cast iron...	0·086	0·106	Tallow.
Do. do...	0·077	...	Olive oil.
Do. upon wrought iron...	0·081	...	Tallow.
Do. do...	0·089	...	Lard and plumbago.
Do. do...	0·072	...	Olive oil.
Brass upon brass...	0·058	...	Olive oil.
Steel upon cast iron...	0·105	0·108	Tallow.
Do. do...	0·081	...	Hog's lard.
Do. do...	0·079	...	Olive oil.
Do. upon wrought iron...	0·093	...	Tallow.
Do. do...	0·076	...	Hog's lard.
Do. upon brass...	0·056	...	Tallow.
Do. do...	0·053	...	Olive oil.
Do. do...	0·067	...	Lard and plumbago.
Tanned ox-hide upon cast iron...	0·365	...	Greased and saturated with water.

The extent of the surfaces in these experiments bore such a relation to the pressure as to cause them to be separated from one another throughout by an interposed stratum of the unguent.

TABLE *of the Results of Experiments on the Friction of Gudgeons or Axle-ends, in motion upon their bearings.* By M. MORIN.

Surfaces in Contact.	State of the Surfaces.	Coefficient of Friction.
Cast iron axles in cast iron bearings.	Coated with oil of olives, with hog's lard, tallow, and soft gome...........	0·07 to 0·08
	With the same and water...	0·08
	Coated with asphaltum.....	0·054
	Greasy.........................	0·14
	Greasy and wetted..........	0·14
Cast iron axles in cast iron bearings.	Coated with oil of olives, with hog's lard, tallow, and soft gome...........	0·07 to 0·08
	Greasy	0·16
	Greasy and damped.........	0·16
	Scarcely greasy...............	0·19
Wrought iron axles in cast iron bearings.	Coated with oil of olives, tallow, hog's lard, or soft gome..........	0·07 to 0·08
Wrought iron axles in brass bearings.	Coated with oil of olives, hog's lard, or tallow,	0·07 to 0·08
	Coated with hard gome.....	0·09
	Greasy and wetted...........	0·19
	Scarcely greasy...............	0·25
Iron axles in lignum vitæ bearings.	Coated with oil or hog's lard	0·11
	Greasy	0·19
Brass axles in brass bearings.	Coated with oil	0·10
	With hog's lard...............	0·09

TABLE *of Coefficients of Friction under Pressures increased continually up to limits of Abrasion.*

Pressure per Square Inch.	Coefficients of Friction.			
	Wrought Iron upon Wrought Iron.	Wrought Iron upon Cast Iron.	Steel upon Cast Iron.	Brass upon Cast Iron.
32·5 lbs.	·140	·174	·166	·157
1·66 cwts.	·250	·275	·300	·225
2·00	·271	·292	·333	·219
2·33	·285	·321	·340	·214
2·66	·297	·329	·344	·211
3·00	·312	·333	·347	·215
3·33	·350	·351	·351	·206
3·66	·376	·353	·353	·205
4·00	·395	·365	·354	·208
4·33	·403	·366	·356	·221
4·66	·409	·366	·357	·223
5·00	......	·367	·358	·233
5·33	......	·367	·359	·234
5·66	......	·367	·367	·235
6·00	......	·376	·403	·233
6·33	......	·434	......	·234
6·66	......	......	......	·235
7·00	......	......	......	·232
7·33	......	......	......	·273

Comparative friction of steam engines of different modifications, if the beam engine be taken as the standard of comparison:—

The vibrating engine..................	has a	gain of 1·1	per cent.
The direct-action engine, with slides	—	loss of 1·8	—
Ditto, with rollers.....................	—	gain of 0·8	—
Ditto, with a parallel motion.........	—	gain of 1·3	—

Excessive allowance for friction has hitherto been made in calculating the effective power of engines in general; as it is found practically, by experiments, that, where the pressure upon the piston is about 12 lbs. per square inch, the friction does not amount to more than 1½ lbs.; and also that, by experiments with an indicator on an engine of 50 horse power, the whole amount of friction did not exceed 5 horse power, or one-tenth of the whole power of the engine.

RECENT EXPERIMENTS MADE BY M. MORIN ON THE STIFFNESS OF ROPES, OR THE RESISTANCE OF ROPES TO BENDING UPON A CIRCULAR ARC.

The experiments upon which the rules and table following are founded were made by Coulomb, with an apparatus the invention of Amonton, and Coulomb himself deduced from them the following results:—

1. That the resistance to bending could be represented by an expression consisting of two terms, the one constant for each rope and each roller, which we shall designate by the letter A, and which this philosopher named the natural stiffness, because it depends on the mode of fabrication of the rope, and the degree of tension of its yarns and strands; the other, proportional to the tension, T, of the end of the rope which is being bent, and which is expressed by the product, BT, in which B is also a number constant for each rope and each roller.

2. That the resistance to bending varied inversely as the diameter of the roller.

Thus the complete resistance is represented by the expression

$$\frac{A + BT}{D},$$

where D represents the diameter of the roller.

Coulomb supposed that for tarred ropes the stiffness was proportional to the number of yarns, and M. Navier inferred, from examination of Coulomb's experiments, that the coefficients A and B were proportional to a certain power of the diameter, which depended on the extent to which the cords were worn. M. Morin, however, deems this hypothesis inadmissible, and the following is an extract from his new work, "Leçons de Mécanique Pratique," December, 1846:—

"To extend the results of the experiments of Coulomb to ropes of different diameters from those which had been experimented upon, M. Navier has allowed, very explicitly, what Coulomb had but surmised: that the coefficients, A, were proportional to a certain power of the diameter, which depended on the state of wear of the ropes; but this supposition appears to us neither borne out, nor even admissible, for it would lead to this consequence, that a worn rope of a metre diameter would have the same stiffness as a new rope, which is evidently wrong; and, besides, the comparison alone of the values of A and B shows that the power to which the diameter should be raised would not be the same for the two terms of the resistance."

Since, then, the form proposed by M. Navier for the expression of the resistance of ropes to bending cannot be admitted, it is necessary to search for another, and it appears natural to try if the factors A and B cannot be expressed for white ropes, simply according to the number of yarns in the ropes, as Coulomb has inferred for tarred ropes.

Now, dividing the values of A, obtained for each rope by M. Navier, by the number of yarns, we find for

$$n = 30 \; d = 0^{\text{m}}{\cdot}200 \;\; \text{A} = 0{\cdot}222460 \;\; \frac{\text{A}}{n} = 0{\cdot}0074153.$$

$$n = 15 \; d = 0^{\text{m}}{\cdot}144 \;\; \text{A} = 0{\cdot}063514 \;\; \frac{\text{A}}{n} = 0{\cdot}0042343.$$

$$n = 6 \; d = 0^{\text{m}}{\cdot}0088 \;\; \text{A} = 0{\cdot}010604 \;\; \frac{\text{A}}{n} = 0{\cdot}0017673.$$

It is seen from this that the number A is not simply proportional to the number of yarns.

Comparing, then, the values of the ratio $\frac{\text{A}}{n}$ corresponding to the three ropes, we find the following results:—

Number of yarns.	Values of $\frac{\text{A}}{n}$.	Differences of the numbers of yarns.	Differences of the values of $\frac{\text{A}}{n}$.	Differences of the values of $\frac{\text{A}}{n}$ for each yarn of difference.
30	0·0074153	From 30 to 15. 15 yarns	0·0031810	0·000212
15	0·0042343	— 15 to 6. 9 —	0·0024770	0·000272
6	0·0017673	— 30 to 6. 24 —	0·0056400	0·000252

Mean difference per yarn, 0·000245

It follows, from the above, that the values of A, given by the experiments, will be represented with sufficient exactness for all practical purposes by the formula

$$\begin{aligned} \text{A} &= n\left[0{\cdot}0017673 + 0{\cdot}000245\,(n - 6)\right]. \\ &= n\left[0{\cdot}0002973 + 0{\cdot}000245\,n\right]. \end{aligned}$$

An expression relating only to dry white ropes, such as were used by Coulomb in his experiments.

With regard to the number B, it appears to be proportional to the number of yarns, for we find for

$$n = 30 \quad d = 0^{m}{\cdot}0200 \quad B = 0{\cdot}009738 \quad \frac{B}{n} = 0{\cdot}0003246$$

$$n = 15 \quad d = 0^{m}{\cdot}0144 \quad B = 0{\cdot}005518 \quad \frac{B}{n} = 0{\cdot}0003678$$

$$n = 6 \quad d = 0^{m}{\cdot}0088 \quad B = 0{\cdot}002380 \quad \frac{B}{n} = 0{\cdot}0003967$$

Mean....................0·0003630

Whence

$$B = 0{\cdot}000363\, n.$$

Consequently, the results of the experiments of Coulomb on dry white ropes will be represented with sufficient exactness for practical purposes by the formula

$$K = n\,[0{\cdot}000297 + 0{\cdot}000245\, n + 0{\cdot}000363\, T] \text{ kil.}$$

which will give the resistance to bending upon a drum of a metre in diameter, or by the formula

$$R = \frac{n}{D}\,[0{\cdot}000297 + 0{\cdot}000245\, n + 0{\cdot}000363\, T] \text{ kil.}$$

for a drum of diameter D metres.

These formulas, transformed into the American scale of weights and measures, become

$$R = n\,[0{\cdot}0021508 + 0{\cdot}0017724\, n + 0{\cdot}00119096\, T] \text{ lbs.}$$

for a drum of a foot in diameter, and

$$R = \frac{n}{D}\,[0{\cdot}0021508 + 0{\cdot}0017724\, n + 0.00119096\, T] \text{ lbs.}$$

for a drum of diameter D feet.

With respect to worn ropes, the rule given by M. Navier cannot be admitted, as we have shown above, because it would give for the stiffness of a rope of a diameter equal to unity the same stiffness as for a new rope.

The experiments of Coulomb on worn ropes not being sufficiently complete, and not furnishing any precise data, it is not possible, without new researches, to give a rule for calculating the stiffness of these ropes.

TARRED ROPES.

In reducing the results of the experiments of Coulomb on tarred ropes, as we have done for white ropes, we find the following values:—

$n = 30$ yarns $A = 0{\cdot}34982$ $B = 0{\cdot}0125605$
$n = 15$ — $A = 0{\cdot}106003$ $B = 0{\cdot}006037$
$n = 6$ — $A = 0{\cdot}0212012$ $B = 0{\cdot}0025997$

which differ very slightly from those which M. Navier has given. But, if we look for the resistance corresponding to each yarn, we find

$$n = 30 \text{ yarns} \quad \frac{A}{n} = 0{\cdot}0116603 \quad \frac{B}{n} = 0{\cdot}000418683$$

$$n = 15 \quad — \quad \frac{A}{n} = 0{\cdot}0070662 \quad \frac{B}{n} = 0{\cdot}000402466$$

$$n = \ \ 6 \quad — \quad \frac{A}{n} = 0{\cdot}0035335 \quad \frac{B}{n} = 0{\cdot}000433283$$

Mean............0·000418144

We see by this that the value of B is for tarred ropes, as for white ropes, sensibly proportional to the number of yarns, but it is not so for that of A, as M. Navier has supposed.

Comparing, as we have done for white ropes, the values of $\frac{A}{n}$ corresponding to the three ropes of 30, 15, and 6 yarns, we obtain the following results:—

Number of yarns.	Values of $\frac{A}{n}$.	Differences of the number of yarns.	Differences of the values of $\frac{A}{n}$.	Differences of the values of $\frac{A}{n}$ for each yarn of difference.
30	0·0116603	From 30 to 15. 15 yarns	0·0045941	0·000306
15	0·0070662	— 15 to 6. 9 —	0·0035327	0·000392
6	0·0035335	— 60 to 6. 25 —	0·0081268	0·000339

Mean....................0·000346

It follows from this that the value of A can be represented by the formula

$$\begin{aligned} A &= n\,[0{\cdot}0035335 + 0{\cdot}000346\,(n - 6)] \\ &= n\,[0{\cdot}0014575 + 0{\cdot}000346\,n] \end{aligned}$$

and the whole resistance on a roller of diameter D metres, by

$$R = \frac{n}{D}\,[0{\cdot}0014575 + 0{\cdot}000346\,n + 0{\cdot}000418144\,T] \text{ kil.}$$

Transforming this expression to the American scale of weights and measures, we have

$$R = \frac{n}{D}\,[0{\cdot}01054412 + 0{\cdot}00250309\,n + 0{\cdot}001371889\,T] \text{ lbs.}$$

for the resistance on a roller of diameter D feet.

This expression is exactly of the same form as that which relates to white ropes, and shows that the stiffness of tarred ropes is a little greater than that of new white ropes.

In the following table, the diameters corresponding to the different numbers of yarns are calculated from the data of Coulomb, by the formulas,

$$d \text{ cent.} = \sqrt{0{\cdot}1338\,n} \text{ for dry white ropes, and}$$

$$d \text{ cent.} = \sqrt{0.186\,n} \text{ for tarred ropes,}$$

which, reduced to the American scale, become

$$d \text{ inches} = \sqrt{0{\cdot}020739\,n} \text{ for dry white ropes, and}$$

$$d \text{ inches} = \sqrt{0{\cdot}02883} \text{ for tarred ropes.}$$

NOTE.—The diameter of the rope is to be included in D; thus, with an inch rope passing round a pulley, 8 inches in diameter in the groove, the diameter of the roller is to be considered as 9 inches.

No. of yarns.	Dry White Ropes.			Tarred Ropes.		
	Diameter.	Value of the natural stiffness, A.	Value of the stiffness proportional to the tension, B.	Diameter.	Value of the natural stiffness, A.	Value of the stiffness proportional to the tension, B.
	ft.	lbs.		ft.	lbs.	
6	0·0293	0·0767120	0·0071457	0·0347	0·153376	0·00823133
9	0.0360	0·1629234	0·0107186	0·0425	0·297647	0·01234700
12	0·0416	0·2810384	0·0142915	0·0490	0·486976	0·01646267
15	0·0465	0·4310571	0·0178644	0·0548	0·721357	0·02057834
18	0·0509	0·6129795	0·0214373	0·0600	0·000795	0·02469400
21	0·0550	0·8268054	0·0250102	0·0648	1·325289	0·02880967
24	0·0588	1.0725350	0·0285831	0·0693	1·694839	0·03292534
27	0·0622	1·3501682	0·0321559	0·0735	2·109444	0·03704100
30	0·0657	1·6597051	0·0357288	0·0775	2·569105	0·04115667
33	0·0689	2·0011455	0·0393017	0·0813	3·073821	0·04527234
36	0·0720	2·3744897	0·0428746	0·0849	3·623593	0·04938800
39	0·0749	2·7797375	0·0464475	6·0884	4·218416	0·05350367
42	0·0778	3·2168858·	0·0500203	0·0917	4·858304	0·05761934
45	0·0805	3·6859438	0·0535932	0·0949	5·543242	0·06173501
48	0·0831	4·1869024	0·0571661	0·0980	6·273237	0·06585067
51	0·0857	4·7197647	0·0607390	0·1010	7·048287	0·06996634
54	0·0882	5·2845306	0·0643119	0·1040	7·868393	0·07408201
57	0·0908	5·8812001	0·0678847	0·1070	8·733554	0·07819767
60	0·0926	6·5097733	0·0714576	0·1099	9·643771	0·08231334
n	$\sqrt{0{\cdot}000144n}$	$\left\{\begin{array}{l}0{\cdot}0021503n \\ +0{\cdot}0017724n\frac{2}{n}\end{array}\right.$	$0{\cdot}00119099n$	$\sqrt{0{\cdot}00020n}$	$\left\{\begin{array}{l}0{\cdot}01054412n \\ +0{\cdot}00250309n\frac{2}{n}\end{array}\right.$	$0{\cdot}001371889n$

Application of the preceding Tables or Formulas.

To find the stiffness of a rope of a given diameter or number of yarns, we must first obtain from the table, or by the formulas, the values of the quantities A and B corresponding to these given quantities, and knowing the tension, T, of the end to be wound up, we shall have its resistance to bending on a drum of a foot in diameter, by the formula

$$R = A + BT.$$

Then, dividing this quantity by the diameter of the roller or pulley round which the rope is actually to be bent, we shall have the resistance to bending on this roller.

What is the stiffness of a dry white rope, in good condition, of 60 yarns, or ·0928 diameter, which passes over a pulley of 6 inches diameter in the groove, under a tension of 1000 lbs.? The table gives for a dry white rope of 60 yarns, in good condition, bent upon a drum of a foot in diameter,

$$A = 0{\cdot}50977 \quad B = 0{\cdot}0714576$$

and we have $D = 0{\cdot}5 + 0{\cdot}0928$; and consequently,

$$R = \frac{6{\cdot}50977 + 0{\cdot}0714576 \times 1000}{0{\cdot}5928} = 128 \text{ lbs.}$$

The whole resistance to be overcome, not including the friction on the axis, is then

$$Q + R = 1000 + 128 = 1128 \text{ lbs.}$$

The stiffness in this case augments the resistance by more than one-eighth of its value.

FURTHER RECENT EXPERIMENTS MADE BY M. MORIN, ON THE TRACTION OF CARRIAGES, AND THE DESTRUCTIVE EFFECTS WHICH THEY PRODUCE UPON THE ROADS.

The study of the effects which are produced when a carriage is set in motion can be divided into two distinct parts: the traction of carriages, properly so called, and their action upon the roads.

The researches relative to the traction of carriages have for their object to determine the magnitude of the effort that the motive power ought to exercise according to the weight of the load, to the diameter and breadth of the wheels, to the velocity of the carriage, and to the state of repair and nature of the roads.

The first experiments on the resistance that cylindrical bodies offer to being rolled on a level surface are due to Coulomb, who determined the resistance offered by rollers of lignum vitæ and elm, on plane oak surfaces placed horizontally.

His experiments showed that the resistance was directly proportional to the pressure, and inversely proportional to the diameter of the rollers.

If, then, P represent the pressure, and r the radius of the roller, the resistance to rolling, R, could, according to the laws of Coulomb, be expressed by the formula

$$R = A \frac{P}{r}$$

in which A would be a number, constant for each kind of ground, but varying with different kinds, and with the state of their surfaces.

The results of experiments made at Vincennes show that the law of Coulomb is approximately correct, but that the resistance increases as the width of the parts in contact diminishes.

Other experiments of the same nature have confirmed these conclusions; and we may allow, at least, as a law sufficiently exact for practical purposes, that for woods, plasters, leather, and generally for hard bodies, the resistance to rolling is nearly—

1st. Proportional to the pressure.
2d. Inversely proportional to the diameter of the wheels.
3d. Greater as the breadth of the zone in contact is smaller.

EXPERIMENTS UPON CARRIAGES TRAVELLING ON ORDINARY ROADS.

These experiments were not considered sufficient to authorize the extension of the foregoing conclusions to the motion of carriages on ordinary roads. It was necessary to operate directly on the carriages themselves, and in the usual circumstances in which they are placed. Experiments on this subject were therefore undertaken, first at Metz, in 1837 and 1838, and afterwards at Courbevoie, in 1839 and 1841, with carriages of every species; and attention was directed separately to the influence upon the magnitude of the traction, of the pressure, of the diameter of the wheels, of their breadth, of the speed, and of the state of the ground.

In heavily laden carriages, which it is most important to take into consideration, the weight of the wheels may be neglected in comparison with the total load; and the relation between the load and the traction, upon a level road, is approximately given by the equation—

$$\frac{F_1}{P_1} = \frac{2\,(A \times fr_1)}{r' \times r''} \text{ for carriages with four wheels,}$$

and $$\frac{F_1}{P_1} = \frac{A \times fr_1}{r} \text{ for carriages with two wheels,}$$

in which F_1 represents the horizontal component of the traction;

P_1 the total pressure on the ground;

r' and r'' the radii of the fore and hind wheels;

r_1 the mean radius of the boxes;

f the coefficient of friction;

and A the constant multiplier in Coulomb's formula for the resistance to rolling.

These expressions will serve us hereafter to determine, by aid of experiment, the ratio of the traction to the load for the most usual cases.

Influence of the Pressure.

To observe the influence of the pressure upon the resistance to rolling, the same carriages were made to pass with different loads over the same road in the same state.

The results of some of these experiments, made at a walking pace, are given in the following table:—

Carriages employed.	Road traversed.	Pressure.	Traction.	Ratio of the traction to the load.
		kil.	kil.	
Chariot porte corps d'artillerie.	Road from Courbevoie to Colomber, dry, in good repair, dusty.	6992	180·71	1/38·6
		6140	159·9	1/39·2
		4580	113·7	1/40·2
Chariot de roulage, without springs.	Road from Courbevoie to Bezous, solid, *hard gravel, very dry.	7126	138·9	1/51·3
		5458	115·5	1/48·9
		4450	93·2	1/47·7
		3430	68·4	1/50·2
Chariot de roulage, with springs.	Road from Colomber to Courbevoie, pitched, in ordinary repair, †muddy	1600	39·3	1/40·8
		3292	89·2	1/36·9
		4996	136·0	1/36·8
Carriages with six equal wheels.	Road from Courbevoie to Colomber, deep ruts, with muddy detritus.	3000	138·9	1/21·6
		4692	224·0	1/21·0
Two carriages with six equal wheels, hooked on, one behind the other.		6000	285·8	1/21·0
		6000	286·7	1/21·0

From the examination of this table, it appears that on ‡solid gravel and on pitched roads the resistance of carriages to traction is sensibly proportional to the pressure.

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* En gravier dur. † Pavé en état ordinaire. ‡ En empierrement solide.

We remark that the experiments made upon one and upon two six-wheeled carriages have given the same traction for a load of 6000 kilogrammes, including the vehicle, whether it was borne upon one carriage or upon two. It follows thence that the traction is, cæteris paribus and between certain limits, independent of the number of wheels.

Influence of the Diameter of the Wheels.

To observe the influence of the diameter of the wheels on the traction, carriages loaded with the same weights, having wheels with tires of the same width, and of which the diameters only were varied between very extended limits, were made to traverse the same parts of roads in the same state. Some of the results obtained are given in the following table.

These examples show that on solid roads it may be admitted as a practical law that the traction is inversely proportional to the diameters of the wheels.

Carriages employed.	Roads traversed.	Diameter of the wheels in metres.		Diameter of the wheels in English feet.		Total pressure, P_1.	Traction, F_1.	Ratio of the traction to the pressure.	Friction of the boxes on the axles.	Resistance to rolling, R.	Value of A for the French scale.	Value of A for the American scale.
		Fore wheels $2r'$	Hind wheels $2r''$	Fore wheels $2r'$	Hind wheels $2r''$							
		m.	m.			kil.	kil.		kil.	kil.		
Chariot porte corps d'artillerie.	Road from Courbevoie to Colomber, *solid gravel, dusty.	2·029	2·029	6·657	6·657	4928	81·6	1/60·	9·6	72·0	0·0148	0·04856
		1·453	1·453	4·767	4·767	4930	108·6	1/45·5	14·4	94·2	0·0139	0·04560
		0·872	0·872	2·861	2·861	4924	179·0	1/27·4	25·3	153·7	0·0137	0·04494
Porte corps d'artillerie.	†Pitched pavement of Fontainebleau.	2·029	2·029	6·657	6·657	4692	51·45	1/90·45	9·0	42·45	0·0092	0·03018
		1·453	1·453	4·767	4·767	4594	71·45	1/64·3	13·2	58·25	0·0092	0·03018
Chariot comtois.		1·110	1·358	3·642	4·455	1871	32·10	1/58·4	4·7	27·40	0·0089	0·02920
A six-wheeled carriage.		0·860	0·860	2·822	2·822	3270	81·05	1/40·4	9·7	71·35	0·0094	0·03084
The same with four wheels.		0·860	0·860	2·822	2·822	3270	78·80	1/41·5	9·7	69·10	0·0091	0·02986
Camion.		0·592	0·660	1·942	2·165	1500	52·30	1/28·8	8·8	43·50	0·0091	0·02986
Camion.		0·420	0·597	1·378	1·959	1600	68·20	1/22·4	11·6	56·60	0·0089	0·02920

Influence of the Width of the Felloes.

Experiments made upon wheels of different breadths, having the same diameter, show, 1st, that on soft ground the resistance to rolling *increases* as the width of the felloe; 2dly, on solid gravel and pitched roads, the resistance is very nearly *independent* of the width of the felloe.

Influence of the Velocity.

To investigate the influence of the velocity on the traction of carriages, the same carriages were made to traverse different roads in various conditions; and in each series of experiments the velocities, while all other circumstances remained the same, underwent successive changes from a walk to a canter.

Some of the results of these experiments are given in the following table:—

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* Empierrement solide.

† Pavé en grès.

Carriage employed.	Ground passed over.	Load.	Pace.	Rate of speed, in miles, per hour.	Traction.	Ratio of the traction to the load.
		kil.		miles.	kil.	
Apparatus upon a brass shaft.	Ground of the polygon at Metz, wet and soft.	1042	Walk........	3·13	165·0	1/6·32
			Trot	6·26	168·0	1/6·2
		1335	Walk........	2·860	215·0	1/6·21
			Trot.........	7·560	197·0	1/6·58
A sixteen-pounder carriage and piece.	Road from Metz to Montigny, solid gravel, very even and very dry.	3750	Walk........	2·820	92·	1/40·8
			*Brisk walk	3·400	92·	1/40·8
			Trot.........	5·480	102·	1/36·8
			†Canter......	8·450	121·	1/31·
Chariot des Messageries, suspended upon six springs.	Pitched road of Fontainebleau.	3288	Walk........	2·770	144·	1/22·8
		3353	*Brisk walk	3·82	153·	1/21·9
			Trot.........	5·28	161·	1/20·8
			‡Brisk trot.	8·05	183·5	1/18·3

We see, by these examples, that the traction undergoes no sensible augmentation with the increase of velocity on soft grounds; but that on solid and uneven roads it increases with an increase of velocity, and in a greater degree as the ground is more uneven, and the carriage has less spring.

To find the relation between the resistance to rolling and the velocity, the velocities were set off as abscissas, and the values of A furnished by the experiments, as ordinates; and the points thus determined were, for each series of experiments, situated very nearly upon a straight line. The value of A, then, can be represented by the expression,

$$A = a + d\,(V - 2)$$

in which a is a number constant for each particular state of each kind of ground, and which expresses the value of the number A for the velocity, $V = 2$ miles, (per hour,) which is that of a very slow walk.

d, a factor constant for each kind of ground and each sort of carriage.

The results of experiments made with a carriage of a siege train, with its piece, gave, on the Montigny road, §very good solid gravel,—

$$A = 0{\cdot}03215 \times 0{\cdot}00295\,(V - 2).$$

On the ||pitched road of Metz, $A = 0{\cdot}01936 \times 0{\cdot}08200\,(V - 2)$.

These examples are sufficient to show—

1st. That, at a walk, the resistance on a good pitched road is less than that on very good solid gravel, very dry.

2d. That, at high speeds, the resistance on the pitched road increases very rapidly with the velocity.

On rough roads the resistance increases with the velocity much more slowly, however, for carriages with springs.

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* Pas allongé. † Grand trot. ‡ Trot allongé.

§ En très bon empierrement. || Pavé en grès de Sieack.

Thus, for a chariot des Messageries Générales, on a pitched road, the experiments gave $A = 0·0117 \times 0·00361\,(V - 2)$; while, with the springs wedged so as to prevent their action, the experiments gave, for the same carriage, on a similar road, $A = 0·02723 \times 0·01312\,(V - 2)$. At a speed of nine miles per hour, the springs diminish the resistance by one-half.

The experiments further showed that, while the pitched road was inferior to a *solid gravel road when dry and in good repair, the latter lost its superiority when muddy or out of repair.

INFLUENCE OF THE INCLINATION OF THE TRACES.

The inclination of the traces, to produce the maximum effect, is given by the expression—

$$h\,f = \frac{A \times 0·96\,f\,r'}{r - 0·4\,f\,r'}$$

in which h = the height of the fore extremity of the trace above the point where it is attached to the carriage; b = the horizontal distance between these two points. r' is the radius of interior of the boxes, and r the radius of the wheel.

The inclination given by this expression for ordinary carriages is very small; and for trucks with wheels of small diameter it is much less than the construction generally permits.

It follows, from the preceding remarks, that it is advantageous to employ, for all carriages, wheels of as large a diameter as can be used, without interfering with the other essentials to the purposes to which they are to be adapted. Carts have, in this respect, the advantage over wagons; but, on the other hand, on rough roads, the thill horse, jerked about by the shafts, is soon fatigued. Now, by bringing the hind wheels as far forward as possible, and placing the load nearly over them, the wagon is, in effect, transformed into a cart; only care must be taken to place the centre of gravity of the load so far in front of the hind wheels that the wagon may not turn over in going up hill.

ON THE DESTRUCTIVE EFFECTS PRODUCED BY CARRIAGES ON THE ROADS.

If we take stones of mean diameter from $2\frac{3}{4}$ to $3\frac{1}{4}$ inches, and, on a road slightly moist and soft, place them first under the small wheels of a diligence, and then under the large wheels, we find that, in the former case, the stones, pushed forward by the small wheels, penetrate the surface, ploughing and tearing it up; while in the latter, being merely pressed and leant upon by the large wheels, they undergo no displacement.

From this simple experiment we are enabled to conclude that the wear of the roads by the wheels of carriages is greater the smaller the diameter of the wheels.

Experiments having proved that on hard grounds the traction was but slightly increased when the breadths of the wheels was

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* En empierrement.

diminished, we might also conclude that the wear of the road would be but slightly increased by diminishing the width of the felloes.

Lastly, the resistance to rolling increasing with the velocity, it was natural to think that carriages going at a trot would do more injury to the roads than those going at a walk. But springs, by diminishing the intensity of the impacts, are able to compensate, in certain proportions, for the effects of the velocity.

Experiments, made upon a grand scale, and having for their object to observe directly the destructive effects of carriages upon the roads, have confirmed these conclusions.

These experiments showed that with equal loads, on a solid gravel road, wheels of two inches breadth produced considerably more wear than those of 4½ inches, but that beyond the latter width there was scarcely any advantage, so far as the preservation of the road was concerned, in increasing the size of the tire of the wheel.

Experiments made with wheels of the same breadth, and of diameters of 2·86 ft., 4·77 ft., and 6·69 ft., showed that after the carriage of 10018·2 tons, over tracks 218·72 yards long, the track passed over by the carriage with the smallest wheels was by far the most worn; while, on that passed over by the carriage with the wheels of 6·69 ft. diameter, the wear was scarcely perceptible.

Experiments made upon two wagons exactly similar in all other respects, but one with and one without springs, showed that the wear of the roads, as well as the increase of traction, after the passage of 4577·36 tons over the same track, was sensibly the same for the carriage without springs, going at a walk of from 2·237 to 2·684 miles per hour, and for that with springs, going at a trot of from 7·158 to 8·053 miles per hour.

HYDRAULICS.

THE DISCHARGE OF WATER BY SIMPLE ORIFICES AND TUBES.

The formulas for finding the quantities of water discharged in a given time are of an extensive and complicated nature. The more important and practical results are given in the following Deductions.

When an aperture is made in the bottom or side of a vessel containing water or other homogeneous fluid, the whole of the particles of fluid in the vessel will descend in lines nearly vertical, until they arrive within three or four inches of the place of discharge, when they will acquire a direction more or less oblique, and flow directly towards the orifice.

The particles, however, that are immediately over the orifice, descend vertically through the whole distance, while those nearer to the sides of the vessel, diverted into a direction more or less oblique as they approach the orifice, move with a less velocity than the former; and thus it is that there is produced a contraction in the size of the stream immediately beyond the opening, designated the *vena contracta*, and bearing a proportion to that of the orifice of about 5 to 8, if it pass through a thin plate, or of 6 to 8, if through a short cylindrical tube. But if the tube be conical to a length equal to half its larger diameter, having the issuing diameter less than the entering diameter in the proportion of 26 to 33, the stream does not become contracted.

If the vessel be kept constantly full, there will flow from the aperture twice the quantity that the vessel is capable of containing, in the same time in which it would have emptied itself if not kept supplied.

1. How many horse-power (H. P.) is required to raise 6000 cubic feet of water the hour from a depth of 300 feet?

A cubic foot of water weighs 62·5 lbs. avoirdupois.

$\frac{6000 \times 62{\cdot}5}{60} = 6250$, the weight of water raised a minute.

$6250 \times 300 = 1875000$, the units of work each minute.

Then $\frac{1875000}{33000} = 56{\cdot}818 =$ the horse-power required.

2. What quantity of water may be discharged through a cylindrical mouth-piece 2 inches in diameter, under a head of 25 feet?

$\frac{2}{12} = \frac{1}{6}$ of a foot; ∴ the area of the cross section of the mouth-piece, in feet, is $\frac{1}{6} \times \frac{1}{6} \times {\cdot}7854 = {\cdot}021816$.

Theory gives $\cdot021816 \sqrt{2\,g \times 25}$ the cubic feet discharged each second; but experiments show that the effective discharge is 97 per cent. of this theoretical quantity: $g = 32{\cdot}2$.

Hence, $\cdot97 \times \cdot021816 \sqrt{64{\cdot}4 \times 25} = \cdot84912$, the cubic feet discharged each second.

$\cdot84912 \times 62{\cdot}5 = 53{\cdot}0688$ lbs. of water discharged each second.

Effluent water produces, by its *vis viva*, about 6 per cent. less mechanical effect than does its weight by falling from the height of the head.

3. What quantity of water flows through a circular orifice in a thin horizontal plate, 3 inches in diameter, under a head of 49 feet?

Taking the contraction of the fluid vein into account, the velocity of the discharge is about 97 per cent. of that given by theory.

The theoretic velocity is $\sqrt{2g \times 49} = 7\sqrt{6{\cdot}44} = 56{\cdot}21$.

$\cdot97 \times 56{\cdot}21 = 54{\cdot}523 =$ the velocity of the discharge.

The area of the transverse section of the contracted vein is ·64 of the transverse section of the orifice.

$\frac{3}{12} = \frac{1}{4} = \cdot25$, and $(\cdot25)^2 \times \cdot7854 = \cdot0490875 =$ area of orifice.

∴ $\cdot64 \times \cdot0490875 = \cdot031416$, the area of the transverse section of the contracted vein.

Hence, $54{\cdot}523 \times {\cdot}031416 = 1{\cdot}7129$, the cubic feet of water discharged each second. The later experiments of Poncelet, Bidone, and Lesbros give ·563 for the coefficient of contraction. Water issuing through lesser orifices give greater coefficients of contraction, and become greater for elongated rectangles, than for those which approach the form of a square.

Observations show that the result above obtained is too great; $\frac{8}{13}$ of this result are found to be very near the truth.

$$\frac{8}{13} \text{ of } 1{\cdot}7129 = 1{\cdot}0541.$$

4. What quantity of water flows through a rectangular aperture 7·87 inches broad, and 3·94 inches deep, the surface of the water being 5 feet above the upper edge; the plate through which the water flows being ·125 of an inch thick.

$$\frac{7{\cdot}87}{12} = {\cdot}65583, \text{ decimal of a foot.}$$

$$\frac{3{\cdot}94}{12} = {\cdot}32833, \text{ decimal of a foot.}$$

5· and 5·32833 are the heads of water above the uppermost and lowest horizontal surfaces.

The theoretical discharge will be

$$\frac{2}{3} \times {\cdot}65583 \sqrt{2\,g}\left((5{\cdot}328)^{\frac{3}{2}} - (5)^{\frac{3}{2}}\right) = 3{\cdot}9268 \text{ cubic feet.}$$

Table I. gives the coefficient of efflux in this case, ·615, which is found opposite 5 feet and under 4 inches; for 3·94 is nearly equal 4.

$3{\cdot}9268 \times {\cdot}615 = 2{\cdot}415$ cubic feet, the effective discharge.

5. What water is discharged through a rectangular orifice in a thin plate 6 inches broad, 3 inches deep, under a head of 9 feet measured directly over the orifice?

$$\frac{6}{12} = {\cdot}5, \text{ decimal of a foot.}$$

$$\frac{3}{12} = {\cdot}25, \text{ decimal of a foot.}$$

The theoretical discharge will be

$$\frac{2}{3} \times {\cdot}5 \sqrt{2g} \left\{ (9{\cdot}25)^{\frac{3}{2}} - (9)^{\frac{3}{2}} \right\} = 3{\cdot}033 \text{ cubic feet.}$$

Table II. gives the coefficient of efflux between ·604 and ·606; we shall take it at ·605, then

$3{\cdot}033 \times {\cdot}605 = 1{\cdot}833$ cubic feet, the effective discharge.

6. A weir ·82 feet broad, and 4·92 feet head of water, how many cubic feet are discharged each second?

The quantity will be

$$c \times {\cdot}82 \sqrt{2g\,(4{\cdot}92)^3};\; g = 32{\cdot}2;$$

TABLE I.—*The Coefficients for the Efflux through rectangular orifices in a thin vertical plate. The heads are measured where the water may be considered still.*

Head of water, or distance of the surface of the water from the upper side of the orifice in feet.	HEIGHT OF ORIFICE.					
	In. 8·	In. 4·	In. 2·	In. 1·	In. ·8	In. ·4
·1	·579	·599	·619	·634	·656	·686
·2	·582	·601	·620	·638	·654	·681
·3	·585	·603	·621	·640	·653	·676
·4	·588	·605	·622	·639	·652	·671
·5	·591	·607	·623	·637	·650	·666
·6	·594	·609	·624	·635	·649	·662
·7	·596	·611	·625	·634	·648	·659
·8	·597	·613	·623	·632	·647	·656
·9	·598	·615	·627	·631	·645	·653
1·0	·599	·616	·628	·630	·644	·650
2·0	·600	·617	·628	·628	·641	·647
3·0	·601	·617	·626	·626	·638	·644
4·0	·602	·616	·624	·623	·634	·640
5·0	·604	·615	·621	·621	·630	·635
6·0	·603	·613	·618	·618	·625	·630
7·0	·602	·611	·615	·615	·621	·625
8·0	·601	·609	·612	·613	·617	·619
9·0	·600	·606	·609	·610	·614	·613
10·0	·600	·604	·606	·608	·611	·609

TABLE II.—*The Coefficients for the Efflux through rectangular orifices in a thin vertical plate, the heads of water being measured directly over the orifice.*

Head of water, or distance of the surface of the water from the upper side of the orifice in feet.	EIGHT OF ORIFICE.					
	In. 8·	In. 4·	In. 2·	In. 1·	In. ·8	In. 4
·1	·593	·613	·637	·659	·685	·708
·2	·593	·612	·636	·656	·680	·701
·3	·593	·613	·635	·653	·676	·694
·4	·594	·614	·634	·650	·672	·687
·5	·595	·614	·633	·647	·668	·681
·6	·597	·615	·632	·644	·664	·675
·7	·598	·615	·631	·641	·660	·669
·8	·599	·616	·630	·638	·655	·663
·9	·601	·616	·629	·635	·650	·657
1·0	·603	·617	·629	·632	·644	·651
2·0	·604	·617	·626	·628	·640	·646
3·0	·605	·616	·622	·627	·636	·641
4·0	·604	·614	·618	·624	·632	·636
5·0	·604	·613	·616	·621	·628	·631
6·0	·603	·612	·613	·618	·624	·626
7·0	·603	·610	·611	·616	·620	·621
8·0	·602	·608	·609	·614	·616	·617
9·0	·601	·607	·607	·612	·613	·613
10·0	·601	·603	·606	·610	·610	·609

c is termed the coefficient of efflux, and on an average may be taken at ·4. It is found to vary from ·385 to ·444.

Then $\cdot 4 \times \cdot 82 \sqrt{(64\cdot 4)\ (4\cdot 92)^3} = 2\cdot 670033$, the cubic feet discharged each second.

7. What breadth must be given to a notch, in a thin plate, with a head of water of 9 inches, to allow 10 cubic feet to flow each second?

The breadth will be represented by

$$\frac{10}{c\sqrt{2g} \times (\cdot 75)^3} = \frac{10}{\cdot 4 \times \sqrt{64\cdot 4 \times (\cdot 75)^3}} = 4\cdot 7963 \text{ feet.}$$

Changes in the coefficients of efflux through convergent sides often present themselves in practice: they occur in dams which are inclined to the horizon.

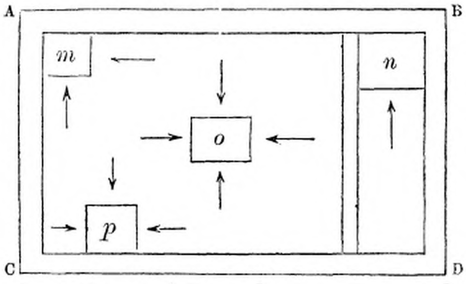
Poncelet found the coefficient ·8, when the board was inclined 45°, and the coefficient ·74 for an inclination of 63° 34′, that is for a slope of 1 for a base, and 2 for a perpendicular.

8. If a sluice board, inclined at an angle of 50°, which goes across a channel 2·25 feet broad, is drawn out ·5 feet, what quantity of water will be discharged, the surface of the water standing 4· feet above the surface of the channel, and the coefficient of efflux taken at ·78?

The height of the aperture = ·5 sin. 50° = ·3830222; 4· and $4\cdot - \cdot 3830222 = 3\cdot 6169778$, are the heads of water.

$$\therefore \frac{2}{3} \times 2\cdot 25 \times \cdot 78 \times \sqrt{2g} \left\{ (4)^{\frac{3}{2}} - (3\cdot 617)^{\frac{3}{2}} \right\} = 10\cdot 5257$$ cubic feet, the quantity discharged.

The calculations just made appertain to those cases where the water flows from all sides towards the aperture, and forms a contracted vein on every side. We shall next calculate in cases where the water flows from one or more sides to the aperture, and hence produces a stream only partially contracted. m, n, o, p, are four orifices in the bottom ABCD of a vessel; the contraction by efflux through the orifice o, in the middle of the bottom, is general, as the water can flow to it from all sides; the contraction from the efflux through m, n, p, is partial, as the water can only flow to them from one, two, or three sides. Partial contraction gives an oblique direction to the stream, and increases the quantity discharged.



9. What quantity of water is delivered through a flow 4 feet broad, and 1 foot deep, vertical aperture, at a pressure of 2 feet above the upper edge, supposing the lower edge to coincide with

the lower side of the channel, so that there is no contraction at the bottom?

The theoretical discharge will be

$$\frac{2}{3} \times \frac{4}{1} \times \sqrt{2g} \left\{ (3)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right\} = 50{\cdot}668 \text{ cubic feet.}$$

The coefficient of contraction given in the table page 315, may be taken at ·603.

I.—*Comparison of the Theoretical with the Real Discharges from an Orifice.*

Constant height of the water in the reservoir above the centre of the orifice.	Theoretical discharge through a circular orifice one inch in diameter.	Real discharge in the same time through the same orifice.	Ratio of the theoretical to the real discharge.
Paris Feet.	Cubic Inches.	Cubic Inches.	
1	4381	2722	1 to 0·62133
2	6196	3846	1 to 0·62073
3	7589	4710	1 to 0·62064
4	8763	5436	1 to 0·62034
5	9797	6075	1 to 0·62010
6	10732	6654	1 to 0·62000
7	11592	7183	1 to 0·61965
8	12392	7672	1 to 0·61911
9	13144	8135	1 to 0·61892
10	13855	8574	1 to 0·61883
11	14530	8990	1 to 0·61873
12	15180	9384	1 to 0·61819
13	15797	9764	1 to 0·61810
14	16393	10130	1 to 0·61795
15	16968	10472	1 to 0·61716

II.—*Comparison of the Theoretical with the Real Discharges from a Tube.*

Constant height of the water in the reservoir above the centre of the orifice.	Theoretical discharge through a circular orifice one inch in diameter.	Real discharge in the same time by a cylindrical tube one inch in diameter and two inches long.	Ratio of the theoretical to the real discharge.
Paris Feet.	Cubic Inches.	Cubic Inches.	
1	4381	3539	1 to 0·81781
2	6196	5002	1 to 0·80729
3	7589	6126	1 to 0·80724
4	8763	7070	1 to 0·80681
5	9797	7900	1 to 0·80638
6	10732	8654	1 to 0·80638
7	11592	9340	1 to 0·80577
8	12392	9975	1 to 0·80496
9	13144	10579	1 to 0·80485
10	13855	11151	1 to 0·80483
11	14530	11693	1 to 0·80477
12	15180	12205	1 to 0·80403
13	15797	12699	1 to 0·80390
14	16393	13177	1 to 0·80382
15	16968	13620	1 to 0·80270

THE DISCHARGE BY DIFFERENT APERTURES AND TUBES, UNDER DIFFERENT HEADS OF WATER.

The velocity of water flowing out of a horizontal aperture, is as the square root of the height of the head of the water.—That is, the pressure, and consequently the height, is as the square of the velocity; for, the quantity flowing out in any short time is as the velocity; and the force required to produce a velocity in a certain quantity of matter in a given time is also as that velocity; therefore, the force must be as the square of the velocity.

Or, supposing a very small cylindrical plate of water, immediately over the orifice, to be put in motion at each instant, by the pressure of the whole cylinder upon it, employed only in generating its velocity; this plate would be urged by a force as much greater than its own weight as the column is higher than itself, through a space shorter in the same proportion than that height. But where the forces are inversely as the spaces described, the final velocities are equal. Therefore, the velocity of the water flowing out must be equal to that of a heavy body falling from the height of the head of water; which is found, very nearly, by multiplying the square root of that height in feet by 8, for the number of feet described in a second. Thus, a head of 1 foot gives 8; a head of 9 feet, 24. This is the theoretical velocity; but, in consequence of the contraction of the stream, we must, in order to obtain the actual velocity, multiply the square root of the height, in feet, by 5 instead of 8.

The velocity of a fluid issuing from an aperture is not affected by its density being greater or less. Mercury and water issue with equal velocities at equal altitudes.

The proportion of the theoretical to the actual velocity of a fluid issuing through an opening in a thin substance, according to M. Eytelwein, is as 1 to ·619; but more recent experiments make it as 1 to ·621 up to ·645.

APPLICATION OF THE TABLES IN THE PRECEDING PAGE.

TABLE I.—*To find the quantities of water discharged by orifices of different sizes under different altitudes of the fluid in the reservoir.*

To find the quantity of fluid discharged by a circular aperture 3 inches in diameter, the constant altitude being 30 feet.

As the real discharges are in the compound ratio of the area of the apertures and the square roots of the altitudes of the water, and as the theoretical quantity of water discharged by an orifice one inch in diameter from a height of 15 feet is, by the second column of the table, 16968 cubic inches in a minute, we have this proportion: $1 \sqrt{15} : 9 \sqrt{30} :: 16968 : 215961$ cubic inches; the theoretical quantity required. This quantity being diminished in the ratio of 1 to ·62, being the ratio of the theoretical to the actual discharge, according to the fourth column of the table, gives 133896 cubic inches for the actual quantity of water discharged by the given aperture. Hence, the quantity should be rather greater, because large orifices discharge more in proportion than small ones; while it should be rather less, because the altitude of the fluid being greater than that in the table with which it is compared, the flowing vein of water becomes rather more contracted. The quantity thus found, therefore, is nearly accurate as an average.

When the orifice and altitude are less than those in the table, a few cubic inches should be deducted from the result thus derived.

The altitude of the fluid being multiplied by the coefficient 8·016 will give its theoretical velocity; and as the velocities are as the quantities discharged, the real velocity may be deducted from the theoretical by means of the foregoing results.

Table II.—*To find the quantities of water discharged by tubes of different diameter, and under different heights of water.*

To find the quantity of water discharged by a cylindrical tube, 4 inches in diameter, and 8 inches long, the constant altitude of the water in the reservoir being 25 feet.

Find, in the same manner as by the example to Table I., the theoretical quantity discharged, which is furnished by this analogy. $1\sqrt{15} : 16\sqrt{25} :: 16968 : 350490$ cubic inches, the theoretical discharge. This, diminished in the ratio of 1 to ·81 by the 4th column, will give 28473 cubic inches for the actual quantity discharged. If the tube be *shorter* than twice its diameter, the quantity discharged will be diminished, and approximate to that from a simple orifice, as shown by the production of the *vena contracta* already described.

According to Eytelwein, the proportion of the theoretical to the real discharge through tubes, is as follows:

Through the shortest tube that will cause the stream to adhere everywhere to its sides, as 1 to 0·8125.

Through short tubes, having their lengths from two to four times their diameters, as 1 to 0·82.

Through a tube projecting within the reservoir, as 1 to 0·50.

It should, however, be stated, that in the contraction of the stream the ratio is not constant. It undergoes perceptible variations by altering the form and position of the orifice, the thickness of the plate, the form of the vessel, and the velocity of the issuing fluid.

Deductions from experiments made by Bossut, Michelloti.

1. That the quantities of fluid discharged in equal times from different-sized apertures, the altitude of the fluid in the reservoir being the same, are to each other nearly as the area of the apertures.

2. That the quantities of water discharged in equal times by the same orifice under different heads of water, are nearly as the square roots of the corresponding heights of water in the reservoir above the centre of the apertures.

3. That, in general, the quantities of water discharged, in the same time, by different apertures under different heights of water in the reservoir, are to one another in the compound ratio of the areas of the apertures, and the square roots of the altitudes of the water in the reservoirs.

4. That on account of the friction, the smallest orifice discharges proportionally less water than those which are larger and of a similar figure, under the same heads of water.

5. That, from the same cause, of several orifices whose areas are equal, that which has the smallest perimeter will discharge more water than the other, under the same altitudes of water in the reservoir. Hence, circular apertures are most advantageous, as they have less rubbing surface under the same area.

6. That, in consequence of a slight augmentation which the contraction of the fluid vein undergoes, in proportion as the height of the fluid in the reservoir increases, the expenditure ought to be a little diminished.

7. That the discharge of a fluid through a cylindrical horizontal tube, the diameter and length of which are equal to one another, is the same as through a simple orifice.

8. That if the cylindrical horizontal tube be of greater length than the extent of the diameter, the discharge of water is much increased.

9. That the length of the cylindrical horizontal tube may be increased with advantage to four times the diameter of the orifice.

10. That the diameters of the apertures and altitudes of water in the reservoir being the same, the theoretic discharge through a thin aperture, which is supposed to have no contraction in the vein, the discharge through an additional cylindrical tube of greater length than the extent of its diameter, and the actual discharge through an aperture pierced in a thin substance, are to each other as the numbers 16, 13, 10.

11. That the discharges by different additional cylindrical tubes, under the same head of water, are nearly proportional to the areas of the orifices, or to the squares of the diameters of the orifices.

12. That the discharges by additional cylindrical tubes of the same diameter, under different heads of water, are nearly proportional to the square roots of the head of water.

13. That from the two preceding corollaries it follows, in general, that the discharge during the same time, by different additional tubes, and under different heads of water in the reservoir, are to one another nearly in the compound ratio of the squares of the diameters of the tubes, and the square roots of the heads of water.

The discharge of fluids by additional tubes of a conical figure, when the inner to the outer diameter of the orifice is as 33 to 26, is augmented very nearly one-seventeenth and seven-tenths more than by cylindrical tubes, if the enlargement be not carried too far.

DISCHARGE BY COMPOUND TUBES.

Deductions from the experiments of M. Venturi.

In the discharge by compound tubes, if the part of the additional tube nearest the reservoir have the form of the contracted vein, the expenditure will be the same as if the fluid were not contracted at all; and if to the smallest diameter of this cone a cylindrical pipe be attached, of the same diameter as the least section of the contracted vein, the discharge of the fluid will, in a horizontal direction, be lessened by the friction of the water against the side of the pipe; but if the same tube be applied in a vertical direction, the expenditure will be augmented, on the principle of the gravitation of falling bodies; consequently, the greater the length of pipe, the more abundant is the discharge of fluid.

If the additional compound tube have a cone applied to the opposite extremity of the pipe, the expenditure will, under the same head of water, be increased, in comparison with that through a simple orifice, in the ratio of 24 to 10.

In order to produce this singular effect, the cone nearest to the reservoir must be of the form of the contracted vein, which will increase the expenditure in the ratio of 12·1 to 10. At the other extremity of the pipe, a truncated conical tube must be applied, of which the length must be nearly nine times the smaller diameter, and its outward diameter must be 1·8 times the smaller one. This additional cone will increase the discharge in the proportion of 24 to 10. But if a great length of pipe intervene, this additional tube has little or no effect on the quantity discharged.

According to M. Venturi's experiments on the discharge of water by bent tubes, it appears that while, with a height of water in the reservoir of 32·5 inches, 4 Paris cubic feet were discharged through a cylindrical horizontal tube in the space of 45 seconds, the discharge of the same quantity through a tube of the same diameter, with a curved end, occupied 50 seconds, and through a like tube bent at right angles, 70 seconds. Therefore, in making cocks or pipes for the discharge or conveyance of water, great attention should be paid to the nature and angle of the bendings; right angles should be studiously avoided.

The interruption of the discharge by various enlargements of the diameter of the tubes having been investigated by M. Venturi, by means of a tube with a diameter of 9 lines, enlarged in several parts to a diameter of 24 lines, the retardation was found to increase nearly in proportion to the number of enlargements; the motion of the fluid, in passing into the enlarged parts, being diverted from its direct course into eddies against the sides of the enlargements. From which it may be deduced, that if the internal roughness of a pipe diminish the expenditure, the friction of the water against these asperities does not form any considerable part of the cause. A right-lined tube may have its internal surface highly polished throughout its whole length, and it may everywhere possess a diameter greater than the orifice to which it is applied; but, nevertheless, the expenditure will be greatly retarded if the pipe should have enlarged parts or swellings. It is not enough that elbows and contractions be avoided; for it may happen, by an intermediate enlargement, that the whole of the other advantage may be lost. This will be obvious from the results in the following table, deduced from experiments with tubes having various enlargements of diameter.

Head of water in inches.	Number of enlarged parts.	Seconds in which 4 cubic feet were discharged.
32·5	0	109
32·5	1	147
32·5	3	192
32·5	5	240

DISCHARGE BY CONDUIT PIPES.

On account of the friction against the sides, the less the diameter of the pipe, the less proportionally is the discharge of fluid. And, from the same cause, the greater the length of conduit pipe, the greater the diminution of the discharge. Hence, the discharges made in equal times by horizontal pipes of different lengths, but of the same diameter, and under the same altitude of water, are to one another in the inverse ratio of the square roots of the lengths. In order to have a perceptible and continuous discharge of fluid, the altitude of the water in the reservoir, above the axis of the conduit pipe, must not be less than 1⅗ inch for every 180 feet of the length of the pipe.

The ratio of the difference of discharge in pipes, 16 and 24 lines diameter respectively, may be known by comparing the ratios of Table I. with the ratios of Table II., in the following page.

The greater the angle of inclination of a conduit pipe, the greater will be the discharge in a given time; but when the angle of the conduit pipe is 6° 31′, or the depression of the lower extremity of the pipe is one-eighth or one-ninth of its length, the relative gravity of the fluid will be counterbalanced by the resistance or friction against the sides; and the discharge is then the same as by an additional horizontal tube of the same diameter.

A curvilinear pipe, the altitude of the water in the reservoir being the same, discharges less water when the flexures lie horizontally, than a rectilinear pipe of the same diameter and length.

The discharge by a curvilinear pipe of the same diameter and length, and under the same head of water, is still further diminished when the flexures lie in a vertical instead of a horizontal plane.

When there is a number of contrary flexures in a large pipe, the air sometimes lodges in the highest parts of the flexures, and greatly retards the motion of the water, unless prevented by air-holes, or stopcocks.

TABLE I.—*Comparison of the discharge by conduit pipes of different lengths,* 16 *lines in diameter, with the discharge by additional tubes inserted in the same reservoir.*—By M. BOSSUT.

Constant altitude of the Water above the centre of the aperture.	Length of the conduit pipe.	Quantity of Water discharged in a minute.		Ratio between the quantities furnished by tube and pipe.
		by additional tube, 16 lines in diameter.	by conduit pipe, 16 lines in diameter.	
Feet.	Feet.	Cubic Inches.	Cubic Inches.	
1	30	6330	2778	100 to 43·39
1	60	6330	1957	100 to 30·91
1	90	6330	1587	100 to 25·07
1	120	6330	1351	100 to 21·34
1	150	6330	1178	100 to 18·61
1	180	6330	1052	100 to 16·62
2	30	8939	4066	100 to 45·48
2	60	8939	2888	100 to 32·31
2	90	8939	2352	100 to 26·31
2	120	8939	2011	100 to 22·50
2	150	8939	1762	100 to 19·71
2	180	8939	1583	100 to 17·70

TABLE II.—*Comparison of the discharge by conduit pipes of different lengths,* 24 *lines in diameter, with the discharge by additional tubes inserted in the same reservoir.*—By M. BOSSUT.

Constant altitude of the Water above the centre of the aperture.	Length of the conduit pipe.	Quantity of Water discharged in a minute.		Ratio between the quantities furnished by tube and pipe.
		by additional tube, 24 lines in diameter.	by conduit pipe, 24 lines in diameter.	
Feet.	Feet.	Cubic Inches.	Cubic Inches.	
1	30	14243	7680	100 to 53·92
1	60	14243	5564	100 to 39·06
1	90	14243	4534	100 to 31·83
1	120	14243	3944	100 to 27·69
1	150	14243	3486	100 to 24·48
1	180	14243	3119	100 to 21·90
2	30	20112	11219	100 to 55·78
2	60	20112	8190	100 to 40·72
2	90	20112	6812	100 to 33·87
2	120	20112	5885	100 to 29·26
2	150	20112	5232	100 to 26·01
2	180	20112	4710	100 to 23·41

DISCHARGE BY WEIRS AND RECTANGULAR APERTURES.

Rectangular orifices in the side of a reservoir, extending to the surface.

The velocity varying nearly as the square root of the height, may here be represented by the ordinates of a parabola, and the quantity of water discharged by the area of the parabola, or two-thirds of that of the circumscribing rectangle. So that the quantity discharged may be found by taking two-thirds of the velocity due to the mean height, and allowing for the contraction of the stream, according to the form of the opening.

In a lake, for example, in the side of which a rectangular opening is made without any oblique lateral walls, three feet wide, and extending two feet below the surface of the water, the coefficient of the velocity, corrected for contraction, is 5·1, and the corrected mean velocity $\frac{2}{3}\sqrt{2} \times 5{\cdot}1 = 4{\cdot}8$; therefore the area being 6, the discharge of water in a second is 28·8 cubic feet, or nearly four hogsheads.

The same coefficient serves for determining the discharge over a weir of considerable breadth; and, hence, to deduce the depth or breadth requisite for the discharge of a given quantity of water. For example, a lake has a weir three feet in breadth, and the surface of the water stands at the height of five feet above it: it is required how much the weir must be widened, in order that the water may be a foot lower. Here the velocity is $\frac{2}{3}\sqrt{5} \times 5{\cdot}1$, and the quantity of water $\frac{2}{3}\sqrt{5} \times 5{\cdot}1 \times 3 \times 5$; but the velocity must be reduced to $\frac{2}{3}\sqrt{4} \times 5{\cdot}1$, and then the section will be $\dfrac{\frac{2}{3}\sqrt{5} \times 5{\cdot}1 \times 3 \times 5}{\frac{2}{3}\sqrt{4} \times 5{\cdot}1}$ $= \dfrac{\sqrt{5} \times 3 \times 5}{\sqrt{4}} = 7{\cdot}5 \times \sqrt{5}$; and the height being 4, the breadth must be $\dfrac{7{\cdot}5}{4}\sqrt{5} = 4{\cdot}19$ feet.

The discharge from reservoirs, with lateral orifices of considerable magnitude, and a constant head of water, may be found by determining the difference in the discharge by two open orifices of different heights; or, in most cases, with nearly equal accuracy, by considering the velocity due to the distance, below the surface, of the centre of gravity of the orifice.

Under the same height of water in the reservoir, the same quantity always flows in a canal, of whatever length and declivity; but in a tube, a difference in length and declivity has a great effect on the quantity of water discharged.

The velocity of water flowing in a river or stream varies at different parts of the same transverse section. It is found to be greatest where the water is deepest, at somewhat less than one-half the depth from the surface; diminishing towards the sides and shallow parts.

Resistance to bodies moving in fluids.—The deductions from the experiments of C. Colles, (who first planned the Croton Aqueduct, New York,) and others, on this intricate subject, are, as stated, thus:

1. The confirmation of the theory, that the resistance of fluids to passing bodies is as the squares of the velocities.

2. That, contrary to the received opinion, a cone will move through the water with much less resistance with its apex foremost, than with its base forward.

3. That the increasing the length of a solid, of almost any form, by the addition of a cylinder in the middle, diminishes the resistance with which it moves, provided the weight in the water remains the same.

4. That the greatest breadth of the moving body should be placed at the distance of two-fifths of the whole length from the bow, when applied to the ordinary forms in naval architecture.

5. That the bottom of a floating solid should be made triangular; as in that case it will meet with the least resistance when moving in the direction of its longest axis, and with the greatest resistance when moving with its broadside foremost.

Friction of fluids.—Some experiments have been made on this subject, with reference to the motion of bodies in water, upon a cylindrical model, 30 inches in length, 26 inches in diameter, and weighing 255 lbs. avoirdupois. The cylinder was placed in a cistern of salt water, and made to vibrate on knife-edges passing through its axis, and was deflected over to various angles by means of a weight attached to the arm of a lever. The experiments were then repeated without the water, and the following are the angles of deflection and vibration in the two cases.

In the salt water.		In the atmosphere.	
Angle of Deflection.	Angle to which it vibrated.	Angle of Deflection.	Angle to which it vibrated.
22° 30′	22° 24′	22° 30′	20° 0′
22 10	22 6	21 36	21 3
21 54	21 48	20 48	20 16
21 36	21 30	&c.	&c.
&c.	&c.		

Showing that the amplitude of vibration when oscillating in water is considerably less than when oscillating without water. In the experiments there is a falling off in the angle of 24′, or nearly half a degree. The amount of force acting on the surface of the cylinder necessary to cause the above difference was calculated; and the author thinks that it is not equally distributed on the surface of the cylinder, but that the amount on any particular part might vary as the depth. On this supposition, a constant pressure at a unit of depth is assumed, and this, multiplied by the depth of any other point of the cylinder immersed in the water, will give the pressure at that point. These forces or moments being summed by integration and equated with the sum of the moments given by the experiments, we have the value of the constant pressure at a unit of depth = ·0000469. This constant, in another experiment, the weight of the model being 197 lbs. avoirdupois, and consequently the part immersed in the water being different from that in the other experiment, was ·0000452, which differs very little from the former,—indicating the probability of the correctness of the assumption.

The drainage of water through pipes.—The experiments made under the direction of the Metropolitan Commissioners of Sewers, on the capacities of pipes for the drainage of towns, have presented some useful results for the guidance of those who have to make calculations for a similar purpose. The pipes, of various diameters, from 3 to 12 inches, were laid on a platform of 100 feet in length, the declivity of which could be varied from a horizontal level to a fall of 1 in 10. The water was admitted at the head of the pipe, and at five junctions, or tributary pipes on each side, so regulated as to keep the main pipe full.

The results were as follow:—

It was found—to mention only one result—that a line of 6-inch pipes, 100 feet long, at an inclination of 1 in 60, discharged 75 cubic feet per minute. The same experiment, repeated with the line of pipes reduced to 50 feet in length, gave very nearly the same result. Without the addition of junctions, the transverse sectional area of the stream of water near the discharging end was reduced to one-fifth of the corresponding area of the pipe, and it required a simple head of water of about 22 inches to give the same result as that accruing under the circumstances of the junctions. With regard to varying sizes and inclinations, it appears, sufficiently for practical purposes, that the squares of the discharges are as the fifth powers of the diameters; and again, that in steeper declivities than 1 in 70, the discharges are as the square roots of the inclinations; but at less declivities than 1 in 70, the ratios of the discharges diminish very rapidly, and are governed by no constant law. At a certain small declivity, the relative discharge is as the fifth root of the inclination; at a smaller declivity, it is found as the seventh root of the inclination; and so on, as it approaches the horizontal plane. This may be exemplified by the following results found by actual experiment:

Discharges of a 6-inch pipe at several inclinations.

Inclination.	Discharges in 100 feet per minute.	Inclination.	Discharges in 100 feet per minute.
1 in 60	75	1 in 320	49
1 in 80	68	1 in 400	48·5
1 in 100	63	1 in 480	48
1 in 120	59	1 in 640	47·5
1 in 160	54	1 in 800	47·2
1 in 200	52	1 in 1200	46·7
1 in 240	50	Level	46

The conclusion arrived at is, that the requisite sizes of drains and sewers can be determined (near enough for practical purposes, as an important circumstance has to be considered in providing for the deposition of solid matter, which disadvantageously alters the *form* of the aqueduct, and contracts the water-way) by taking the result of the 6-inch pipe, under the circumstances before mentioned as a *datum*, and assuming that the squares of the discharges are as the fifth powers of the diameters.

That at greater declivities than 1 in 70, the discharges are as the square roots of the inclinations.

That at less declivities than 1 in 70, the usual law will not obtain; but near approximations to the truth may be obtained by observing the relative discharges of a pipe laid at various small inclinations.

That increasing the number of junctions, at intervals, accelerates the velocity of the main stream in a ratio which increases as the square root of the inclination, and which is greater than the ratio of resistance due to a proportionable increase in the length of the aqueduct. The velocity at which the lateral streams enter the main line, is a most important circumstance governing the flow of water. In practice, these velocities are constantly variable, considered individually, and always different considered collectively, so that their united effect it is difficult to estimate. Again, the same sewer at different periods may be quite filled, but discharges in a given time very different quantities of water. It should be mentioned that in the case of the 6-inch pipe, which discharged 75 cubic feet per minute, the lateral streams had a velocity of a few feet per second, and the junctions were placed at an angle of about 35° with the main line. It is needless to say that all junctions should be made as nearly parallel with the main line as possible, otherwise the forces of the lateral currents may impede rather than maintain or accelerate the main streams.

WATER WHEELS.

THE UNDERSHOT WHEEL.

THE ratio between the power and effect of an undershot wheel is as 10 to 3·18; consequently 31·43 lbs. of water must be expended per second to produce a mechanical effect equal to that of the estimated labour of an active man.

The velocity of the periphery of the undershot wheel should be equal to half the velocity of the stream; the float-boards should be so constructed as to rise perpendicularly from the water; not more than one-half should ever be below the surface; and from 3 to 5 should be immersed at once, according to the magnitude of the wheel.

The following maxims have been deduced from experiments:—

1. The virtual or effective head of water being the same, the effect will be nearly as the quantity expended; that is, if a mill, driven by a fall of water, whose virtual head is 10 feet, and which discharges 30 cubic feet of water in a second, grind four bolls of corn in an hour; another mill having the same virtual head, but which discharges 60 cubic feet of water, will grind eight bolls of corn in an hour.

2. The expense of water being the same, the effect will be nearly as the height of the virtual or effective head.

3. The quantity of water expended being the same, the effect is nearly as the square of its velocity; that is, if a mill, driven by a

certain quantity of water, moving with the velocity of four feet per second, grind three bolls of corn in an hour; another mill, driven by the same quantity of water, moving with the velocity of five feet per second, will grind nearly $4\frac{7}{10}$ bolls in the hour, because $3 : 4\frac{7}{10} :: 4^2 : 5^2$ nearly.

4. The aperture being the same, the effect will be nearly as the cube of the velocity of the water; that is, if a mill driven by water, moving through a certain aperture, with the velocity of four feet per second, grind three bolls of corn in an hour; another mill, driven by water, moving through the same aperture with the velocity of five feet per second, will grind $5\frac{43}{50}$ bolls nearly in an hour; for as $3 : 5\frac{43}{50} :: 4^3 : 5^3$ nearly.

The height of the virtual head of water may be easily determined from the velocity of the water, for the heights are as the squares of the velocities, and, consequently, the velocities are as the square roots of the height.

To calculate the proportions of undershot wheels.—Find the perpendicular height of the fall of water above the bottom of the mill-course, and having diminished this number by one-half the depth of the water where it meets the wheel, call that *the height of the fall.*

Multiply the height of the fall, so found, by 64·348, and take the square root of the product, which will be *the velocity of the water.*

Take one-half of the velocity of the water, and it will be the velocity to be given to the float-boards, or the number of feet they must move through in a second, to produce a *maximum* effect. Divide the circumference of the wheel by the velocity of its float-boards per second, and the quotient will be the number of seconds in which the wheel revolves. Divide 60 by the quotient thus found, and the new quotient will be the number of revolutions made by the wheel in a minute.

Divide 90, the number of revolutions which a millstone, 5 feet in diameter, should make in a minute, by the number of revolutions made by the wheel in a minute, the quotient will be the number of turns the millstone ought to make for one turn of the wheel. Then, as the number of revolutions of the wheel in a minute is to the number of revolutions of the millstone in a minute, so must the number of staves in the trundle be to the number of teeth in the wheel, (the nearest in whole numbers.) Multiply the number of revolutions made by the wheel in a minute, by the number of revolutions made by the millstone for one turn of the wheel, and the product will be the number of revolutions made by the millstone in a minute.

The effect of the water wheel is a *maximum*, when its circumference moves with one-half, or, more accurately, with three-sevenths of the velocity of the stream.

THE BREAST WHEEL.

The effect of a breast wheel is equal to the effect of an under shot wheel, whose head of water is equal to the difference of level between the surface of water in the reservoir, and the part where it strikes the wheel, added to that of an overshot, whose height is equal to the difference of level between the part where it strikes the wheel and the level of the tail water.

When the fall of water is between 4 and 10 feet, a breast wheel should be erected, provided there be enough of water; an undershot should be used when the fall is below 4 feet, and an overshot wheel when the fall exceeds 10 feet. Also, when the fall exceeds 10 feet, it should be divided into two, and two breast wheels be erected upon it.

Table *for breast wheels.*

	Breadth of the float-boards.	Depth of the float-boards.	Radius of water wheel, reckoned from the extremity of float-boards.	Velocity of the wheel in a second.	Time in which the wheel performs one revolution.	Turns of the mill-stone for one of the wheels.	Force of the water upon the float-boards.	Water required in a second to turn the wheel.
	Feet.	Feet.	Feet.	Feet.	Sec.	....	lbs. avr.	Cubic ft.
1	0·17	198·6	0·75	2·18	1·92	4·80	1536	74·30
2	0·34	35·1	1·50	3·09	2·72	6·80	1084	37·15
3	0·51	12·7	2·26	3·78	3·33	8·32	886	24·77
4	0·69	6·2	3·01	4·36	3·84	9·60	762	18·57
5	0·86	3·57	3·76	4·88	4·28	10·70	680	14·86
6	1·03	2·25	4·51	5·35	4·70	11·76	626	12·38
7	1·20	1·53	5·26	5·77	5·08	12·70	581	10·61
8	1·37	1·10	6·02	6·17	5·43	13·58	543	9·29
9	1·54	0·81	6·77	6·55	5·76	14·40	512	8·26
10	1·71	0·77	7·52	6·90	6·07	15·18	486	7·43

It is evident, from the preceding table, that when the height of the fall is less than 3 feet, the depth of the float-boards is so great, and their breadth so small, that the breast wheel cannot well be employed; and, on the contrary, when the height of the fall approaches to 10 feet, the depth of the float-boards is too small in proportion to their breadth; these two extremes, therefore, must be avoided in practice. The ninth column contains the quantity of water necessary for impelling the wheel; but the total expense of water should always exceed this by the quantity, at least, which escapes between the mill-course and the sides and extremities of the float-boards.

THE OVERSHOT WHEEL.

The ratio between the power and effect of an overshot wheel, is as 10 to 6·6, when the water is delivered above the apex of the wheel, and is computed from the whole height of the fall; and as 10 to 8 when computed from the height of the wheel only; consequently, the quantity of water expended per second, to produce a mechanical effect equal to that of the aforesaid estimated labour of an active man, is, in the first instance, 15·15 lbs., and in the second instance, 12·5 lbs.

Hence, the effect of the overshot wheel, under the same circumstances of quantity and fall, is, at a medium, double that of the undershot.

The velocity of the periphery of an overshot wheel should be from $6\frac{1}{2}$ to $8\frac{1}{2}$ feet per second.

The higher the wheel is, in proportion to the whole descent, the greater will be the effect.

And from the equality of the ratio between the power and effect, subsisting where the constructions are similar, we must infer that the effects, as well as the powers, are as the quantities of water and perpendicular heights multiplied together respectively.

Working machinery by hydraulic pressure.—The vertical pressure of water, acting on a piston, for raising weights and driving machinery, is coming into use in many places where it can be advantageously applied. At Liverpool, Newcastle, Glasgow, and other places, it is applied to the working of cranes, drawing coal-wagons, and other purposes requiring continuous power. The presence of a natural fall, like that of Golway, Ireland, which can be conducted to the engine through pipes, is, of course, the most economical situation for the application of such power; in other situations, artificial power must be used to raise the water, which, even under this disadvantage, may, from its readiness and simplicity of action, be often serviceably employed. Wherever the contiguity of a steam engine would be dangerous, or otherwise objectionable, a water engine would afford the means of receiving and applying the power from any required distance, precautions being taken against the action of frost on the fluid.

Required the horse power of a centre discharging *Turbine* water wheel, the head of water being 25 feet, and the area of the opening 400 inches.

The following table shows the working horse power of both the inward and outward discharging Turbine water wheels; they are calculated to the square inch of opening.

Centre Discharging Turbine.		Outward Discharging Turbine.	Centre Discharging Turbine.		Outward Discharging Turbine.
Head.	Horse Power.	Horse Power.	Head.	Horse Power.	Horse Power.
3	·00821	·012611	22	·19523	·339972
4	·01483	·025145	23	·20787	·364182
5	·02137	·038124	24	·22315	·384615
6	·02685	·045618	25	·23667	·412013
7	·03414	·058314	26	·25125	·437519
8	·04198	·074413	27	·26482	·455698
9	·05206	·089025	28	·28135	·484427
10	·05883	·106215	29	·29563	·510833
11	·06921	·118127	30	·30817	·537721
12	·07851	·135610	31	·32316	·561425
13	·08882	·150638	32	·33617	·587148
14	·10054	·173158	33	·34823	·611013
15	·11002	·192234	34	·36154	·638174
16	·12093	·211592	35	·37123	·665164
17	·13196	·231161	36	·39874	·692156
18	·14275	·257145	37	·40118	·726148
19	·15613	·273325	38	·41762	·764115
20	·16927	·296618	39	·42156	·804479
21	·18109	·317167	40	·43718	·849814

Opposite 25 in the column marked "Head," the working horse power to the square inch is found to be ·25667, which, multiplied by 400, gives 94·668, the horse power required.

What is the working horse power of an outward discharging Turbine, under the effective head of 20 feet; the area of all the openings being 325 square inches. In the table, opposite 20, we find ·296618, then ·296618 × 325 = 96·4, the required horse power.

What is the number of revolutions a minute of an outward discharging Turbine wheel, the head being 19 feet and the diameter of the wheel 60 inches?

In the table for the outward discharging wheel, opposite 19, and under 60 inches, we find 97, the number of revolutions required.

What is the number of revolutions a minute of an inward discharging Turbine, under a head of 21 feet, the diameter being 72 inches?

In the table for the inward discharging wheel, opposite 21 feet, and under 72 inches, we find 95, the number of revolutions a minute.

These Turbine tables were calculated by the author's brother, the late John O'Byrne, C. E., who died in New York, on the 6th of April, 1851.

Outward discharging Turbine.

Head in feet.	DIAMETER IN INCHES.												
	24	30	36	42	48	54	60	66	72	78	84	90	96
3	100	80	70	60	52	42	37	35	32	30	28	27	21
4	111	89	73	63	57	49	44	41	37	34	32	30	28
5	123	100	82	71	62	55	51	45	42	38	37	33	31
6	135	109	91	78	68	62	55	50	45	42	38	37	36
7	146	118	96	84	73	65	59	53	49	47	42	40	38
8	156	125	105	90	79	71	63	57	52	49	43	42	39
9	166	133	111	95	83	75	67	61	57	50	49	45	41
10	175	140	117	100	87	79	70	64	59	55	51	47	46
11	183	147	122	105	92	81	74	67	62	57	54	49	48
12	191	156	127	110	96	85	79	70	64	59	55	53	51
13	200	159	133	115	100	89	81	73	67	62	57	55	53
14	206	166	138	118	104	92	83	75	69	64	59	57	55
15	213	171	142	122	107	95	86	78	72	66	61	58	56
16	222	177	148	126	111	98	89	82	74	69	64	59	57
17	227	182	152	131	115	101	91	83	77	71	66	62	59
18	234	187	156	134	117	105	94	85	78	73	67	63	61
19	238	193	161	138	120	107	97	88	81	74	69	64	63
20	247	197	164	141	124	110	99	90	84	76	71	66	64
21	252	202	168	145	126	114	101	92	85	78	73	68	65
22	259	208	172	149	129	115	105	94	87	80	74	69	67
23	263	212	176	151	133	119	106	96	89	84	77	72	70
24	270	216	180	155	135	120	109	98	92	85	78	74	72
25	277	222	184	158	138	123	111	101	93	86	80	76	74
26	282	226	189	161	141	125	113	103	95	87	81	78	76
27	286	229	191	165	143	129	116	105	97	88	83	79	77
28	291	233	195	167	146	130	118	107	99	91	85	80	78
29	297	237	199	170	149	132	119	109	100	92	86	81	80
30	303	241	202	174	152	135	122	111	102	94	88	82	81

Inward discharging Turbine.

Head in feet.	Diameter in Inches.												
	24	30	36	42	48	54	60	66	72	78	84	90	96
3	111	86	74	62	54	48	47	40	36	32	31	30	27
4	125	96	83	70	62	55	51	45	41	37	36	34	31
5	141	112	94	78	69	61	55	50	46	43	40	37	36
6	152	122	101	86	76	67	62	55	51	47	43	42	38
7	166	131	108	93	82	72	65	60	54	51	47	44	42
8	175	139	116	99	87	76	71	63	57	54	49	47	45
9	186	149	123	106	93	81	74	68	63	57	53	51	47
10	195	156	129	111	99	86	78	71	66	61	56	52	49
11	208	167	136	117	102	91	82	74	68	63	58	56	52
12	217	169	142	122	107	97	85	78	71	66	61	57	54
13	221	178	148	127	112	99	89	82	74	69	64	61	56
14	231	184	153	133	116	104	92	85	76	71	66	62	58
15	238	191	159	136	119	107	95	87	80	73	68	64	61
16	245	198	165	144	123	111	99	90	83	76	71	66	63
17	252	203	168	148	127	114	102	92	85	78	73	68	64
18	269	209	173	150	132	116	104	95	87	82	75	69	66
19	267	215	176	153	134	120	108	98	89	83	77	72	67
20	276	222	183	157	138	122	111	101	93	85	79	74	69
21	288	226	186	162	141	125	113	103	95	86	80	75	71
22	290	230	192	164	145	129	116	107	96	89	83	77	73
23	299	235	196	167	146	133	118	109	97	91	84	79	74
24	303	240	201	171	151	135	122	111	101	93	86	80	75
25	310	247	206	176	155	138	123	112	104	96	88	82	76
26	314	248	210	180	157	139	126	115	106	97	90	84	79
27	319	254	213	183	162	142	128	117	108	99	92	85	80
28	327	261	218	186	164	146	129	119	109	102	93	87	82
29	333	265	221	189	166	148	133	121	111	103	95	89	83
30	336	271	224	193	168	151	136	124	114	105	97	90	85

WINDMILLS.

1. The velocity of windmill sails, whether unloaded or loaded, so as to produce a maximum effect, is nearly as the velocity of the wind, their shape and position being the same.

2. The load at the maximum is nearly, but somewhat less than, as the square of the velocity of the wind, the shape and position of the sails being the same.

3. The effects of the same sails, at a maximum, are nearly, but somewhat less than, as the cubes of the velocity of the wind.

4. The load of the same sails, at the maximum, is nearly as the squares, and their effect as the cubes of their number of turns in a given time.

5. When sails are loaded so as to produce a maximum at a given velocity, and the velocity of the wind increases, the load continuing the same,—1st, the increase of effect, when the increase of the velocity of the wind is small, will be nearly as the squares of those velocities; 2dly, when the velocity of the wind is double, the effects will be nearly as 10 to 27½; but, 3dly, when the velocities compared are more than double of that when the given load produces a maximum, the effects increase nearly in the simple ratio of the velocity of the wind.

6. In sails where the figure and position are similar, and the velocity of the wind the same, the number of turns, in a given time, will be reciprocally as the radius or length of the sail.

7. The load, at a maximum, which sails of a similar figure and position will overcome, at a given distance from the centre of motion, will be as the cube of the radius.

8. The effects of sails of similar figure and position are as the square of the radius.

9. The velocity of the extremities of Dutch sails, as well as of the enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, is considerably greater than that of the wind.

The results in Table 1 are for Dutch sails, in their common position, when the radius was 30 feet. Table 2 contains the most efficient angles.

1.

Number of revolutions of wind-shaft in a minute.	Velocity of the wind in an hour.	Ratio between velocity of wind and revolutions of wind-shaft.
3	2 miles	0·666
5	4 miles	0·800
6	5 miles	0·833

2.

Parts of the radius, which is divided into six parts.	Angle with the axis.	Angle of weather.
1	72°	18°
2	71	19
3	72	18 middle
4	74	16
5	77½	12½
6	83	7

Supposing the radius of the sail to be 30 feet, then the sail will commence at $\frac{1}{6}$, or 5 feet from the axis, where the angle of inclination will be 72 degrees; at $\frac{2}{6}$, or 10 feet from the axis, the angle will be 71 degrees, and so on.

Results of Experiments on the effect of Windmill Sails in grinding corn.—By M. Coulomb.

A windmill, with four sails, measuring 72 feet from the extremity of one sail to that of the opposite one, and 6 feet 7 inches wide, or a little more, was found capable of raising 1100 lbs. avoirdupois 238 feet in a minute, and of working, on an average, eight hours in a day. This is equivalent to the work of 34 men, 30 square feet of canvas performing about the daily work of a man.

When a vertical windmill is employed to grind corn, the millstone makes 5 revolutions in the same time that the sails and the arbor make 1.

The mill does not begin to turn till the velocity of the wind is about 13 feet per second.

When the velocity of the wind is 19 feet per second, the sails make from 11 to 12 turns in a minute, and the mill will grind from 880 to 990 lbs. avoirdupois in an hour, or about 22,000 lbs. in 24 hours.

THE APPLICATION OF LOGARITHMS.

The practice of performing calculations by Logarithms is an exercise so useful to computers, that it requires a more particular explanation than could have been properly given in that part of the work allotted to Arithmetic.

A few of the various applications of logarithms, best suited to the calculations of the engineer and mechanic, have therefore been collected, and are, with other matter, given, in hopes that they will come into general use, as the certainty and accuracy of their results can be more safely relied upon and more easily obtained than with common arithmetic.

By a slight examination, the student will perceive, in some degree, the nature and effect of these calculations; and, by frequent exercise, will obtain a dexterity of operation in every case admitting of their use. He will also more readily penetrate the plans of the different devices employed in instrumental calculations, which are rendered obscure and perplexing to most practical men by their ignorance of the proper application of logarithms.

Logarithms are artificial numbers which stand for natural numbers, and are so contrived, that if the logarithm of one number be added to the logarithm of another, the sum will be the logarithm of the product of these numbers; and if the logarithm of one number be taken from the logarithm of another, the remainder is the logarithm of the latter divided by the former; and also, if the logarithm of a number be multiplied by 2, 3, 4, or 5, &c., we shall have the logarithm of the square, cube, &c., of that number; and, on the other hand, if divided by 2, 3, 4, or 5, &c., we have the logarithm of the square root, cube root, fourth root, &c., of the proposed number; so that with the aid of logarithms, multiplication and division are performed by addition and subtraction; and the raising of powers and extracting of roots are effected by multiplying or dividing by the indices of the powers and roots.

In the table at the end of this work, are given the logarithms of the natural numbers, from 1· to 1000000 by the help of differences; in large tables, only the decimal part of the logarithm is given, as the index is readily determined; for the index of the logarithm of any number greater than unity, is equal to one less than the number of figures on the left hand of the decimal point; thus,

The index of 12345· is 4·,
———— 1234·5 — 3·,
———— 123·45 — 2·,
———— 12·345 — 1·,
———— 1·2345 — 0·

The index of any decimal fraction is a negative number equal to one and the number of zeros immediately following the decimal point; thus,

The index of ·00012345 is −4· or $\bar{4}$·
———— ·0012345 is −3· or $\bar{3}$·
———— ·012345 is −2· or $\bar{2}$·
———— ·12345 is −1· or $\bar{1}$·

Because the decimal part of the logarithm is always positive, it is better to place the negative sign of the index above, instead of before it; thus, $\bar{3}$· instead of −3. For the log. of ·00012345 is better expressed by $\bar{4}$·0914911, than by −4·0914911, because only the index is negative—*i. e.*, 4 is negative and ·0914911 is positive, and may stand thus, −4· + ·0914911.

Sometimes, instead of employing negative indices, their complements to 10 are used:

for $\bar{4}$·0914911 is substituted 6·0914911
— $\bar{3}$·0914911 ———— 7·0914911
— $\bar{2}$·0914911 ———— 8·0914911
&c. &c.

When this is done, it is necessary to allow, at some subsequent stage, for the tens by which the indices have thus been increased.

It is so easy to take logarithms and their corresponding numbers out of tables of logarithms, that we need not dwell on the method of doing so, but proceed to their application.

MULTIPLICATION BY LOGARITHMS.

Take the logarithms of the factors from the table, and add them together; then the natural number answering to the sum is the product required: observing, in the addition, that what is to be carried from the decimal parts of the logarithms is always positive, and must therefore be added to the positive indices; the difference between this sum and the sum of the negative indices is the index of the logarithm of the product, to which prefix the sign of the greater.

This method will be found more convenient to those who have only a slight knowledge of logarithms, than that of using the arithmetical complements of the negative indices.

1. Multiply 37·153 by 4·086, by logarithms.

Nos.	*Logs.*
37·153	1·5699939
4·086	0·6112984
Prod. 151·8071	2·1812923

2. Multiply 112·246 by 13·958, by logarithms.

Nos.	*Logs.*
112·246	2·0501709
13·958	1·1448232
Prod. 1566·729	3·1949941

3. Multiply 46·7512 by ·3275, by logarithms.

Nos.	*Logs.*
46·7512	1·6697928
·3275	$\bar{1}$·5152113
Prod. 15·31102	1·1850041

Here the +1 that is to be carried from the decimals, cancels the —1, and consequently there remains 1 in the upper line to be set down.

4. Multiply ·37816 by ·04782, by logarithms.

Nos.	*Logs.*
·37816	$\bar{1}$·5776756
·04782	$\bar{2}$·6796096
Prod. 0·0180836	$\bar{2}$·2572852

Here the +1 that is to be carried from the decimals, destroys the —1 in the upper line, as before, and there remains the —2 to be set down.

5. Multiply 3·768, 2·053, and ·007693, together.

Nos.	*Logs.*
3·768	0·5761109
2·053	0·3123889
·007693	$\bar{3}$·8860957
Prod. ·0595108	$\bar{2}$·7745955

Here the +1 that is to be carried from the decimals, when added to —3, makes —2 to be set down.

6. Multiply 3·586, 2·1046, ·8372, and ·0294, together.

Nos.	*Logs.*
3·586	0·5546103
2·1046	0·3231696
·8372	$\bar{1}$·9228292
·0294	$\bar{2}$·4683473
Prod. ·1857618	$\bar{1}$·2689564

Here the +2 that is to be carried, cancels the —2, and there remains the —1 to be set down.

DIVISION BY LOGARITHMS.

From the logarithm of the dividend, subtract the logarithm of the divisor; the natural number answering to the remainder will be the quotient required.

Observing, that if the index of the logarithm to be subtracted is positive, it is to be counted as negative, and if negative, to be considered as positive; and if one has to be carried from the decimals, it is always negative: so that the index of the logarithm of the quotient is equal to the sum of the index of the dividend, the index of the divisor with its sign changed, and −1 when 1 is to be carried from the decimal part of the logarithms.

1. Divide 4768·2 by 36·954, by logarithms.

Nos.	*Logs.*
4768·2	3·6783545
36·954	1·5676615
Quot. 129·032	2·1106930

2. Divide 21·754 by 2·4678, by logarithms.

Nos.	*Logs.*
21·754	1·3375391
2·4678	0·3923100
Quot. 8·81514	0·9452291

3. Divide 4·6257 by ·17608, by logarithms.

Nos.	*Logs.*
4·6257	0·6651775
·17608	$\bar{1}$·2457100
Quot. 26·27045	1·4194675

Here the −1 in the lower index, is changed into +1, which is then taken for the index of the result.

4. Divide ·27684 by 5·1576, by logarithms.

Nos.	*Logs.*
·27684	$\bar{1}$·4422288
5·1576	0·7124477
Quot. ·0536761	$\bar{2}$·7297811

Here the 1 that is to be carried from the decimals, is taken as −1, and then added to −1 in the upper index, which gives −2 for the index of the result.

5. Divide 6·9875 by ·075789, by logarithms.

Nos.	*Logs.*
6·9875	0·8443218
·075789	$\bar{2}$·8796062
Quot. 92·1967	1·9647156

Here the 1 that is to be carried from the decimals, is added to −2, which makes −1, and this put down, with its sign changed, is +1.

6. Divide ·19876 by ·0012345, by logarithms.

Nos.	*Logs.*
·19876	$\bar{1}$·2983290
·0012345	$\bar{3}$·0914911
Quot. 161·0043	2·2068379

Here −3 in the lower index, is changed into +3, and this added to 1, the other index, gives +3 − 1, or 2.

PROPORTION; OR, THE RULE OF THREE, BY LOGARITHMS.

From the sum of the logarithms of the numbers to be multiplied together, take the sum of the logarithms of the divisors: the remainder is the logarithm of the term sought.

Or the same may be performed more conveniently, for any single proportion, thus:—Find the complement of the logarithm of the first term, or what it wants of 10, by beginning at the left hand and taking each of the figures from 9, except the last figure on the right, which must be taken from 10; then add this result and the logarithms of the other two figures together: the sum, abating 10 in the index, will be the logarithm of the fourth term.

1. Find a fourth proportional to 37·125, 14·768, and 135·279, by logarithms.

Log. of 37·125..........................1·5696665

Complement..............................8·4303335
Log. of 14·768..........................1·1693217
Log. of 135·279.........................2·1312304

Ans. 53·8128............................1·7308856

2. Find a fourth proportional to ·05764, ·7186, and ·34721, by logarithms.

Log. of ·05764........................ $\bar{2}$·7607240

Complement.............................11·2392760
Log. of ·7186.......................... $\bar{1}$·8564872
Log. of ·34721......................... $\bar{1}$·5405922

Ans. 4·32868........................... 0·6363554

3. Find a third proportional to 12·796 and 3·24718, by logarithms.

Log. of 12·796..........................1·1070742

Complement..............................8·8929258
Log. of 3·24718.........................0·5115064
Log. of 3·24718.........................0·5115064

Ans. ·8240216...........................$\bar{1}$·9159386

INVOLUTION; OR, THE RAISING OF POWERS, BY LOGARITHMS.

Multiply the logarithm of the given number by the index of the proposed power; then the natural number answering to the result will be the power required. Observing, if the index be negative, the index of the product will be negative; but as what is to be carried from the decimal part will be affirmative, therefore the difference is the index of the result.

1. Find the square of 2·7568, by logarithms.

Log. of 2·7568..........................0·4404053
2

Square 7·599947.........................0·8808106

2. Find the cube of 7·0851, by logarithms.

Log. of 7·0851..........................0·8503460
3

Cube 355·6625..........................2·5510380

Therefore 355·6625 is the answer.

3. Find the fifth power of ·87451, by logarithms.

Log. of ·87451..........................$\bar{1}$·9417648
5

Fifth power ·5114695 ....$\bar{1}$·7088240

Where 5 times the negative index $\bar{1}$, being −5, and +4 to carry, the index of the power is $\bar{1}$.

4. Find the 365th power of 1·0045, by logarithms.

Log. of 1·0045..........................0·0019499
365

97495
116994
58497

Power 5·148888..................Log. 0·7117135

EVOLUTION; OR, THE EXTRACTION OF ROOTS, BY LOGARITHMS.

Divide the logarithm of the given number by 2 for the square root, 3 for the cube root, &c., and the natural number answering to the result will be the root required.

But if it be a compound root, or one that consists both of a root and a power, multiply the logarithm of the given number by the numerator of the index, and divide the product by the denominator, for the logarithm of the root sought.

Observing, in either case, when the index of the logarithm is negative, and cannot be divided without a remainder, to increase it by such a number as will render it exactly divisible; and then carry the units borrowed, as so many tens, to the first figure of the decimal part, and divide the whole accordingly.

1. Find the square root of 27·465, by logarithms.

Log. of 27·465......................2) 1·4387796

Root 5·2407..............................·7193898

2. Find the cube root of 35·6415, by logarithms.

Log. of 35·6415.....................3) 1·5519560

Root 3·29093 ....·5173186

3. Find the fifth root of 7·0825, by logarithms.

Log. of 7·0825......................5) 0·8501866

Root 1·479235............................·1700373

4. Find the 365th root of 1·045, by logarithms.

Log. of 1·045......................365) 0·0191163

Root 1·000121 ....0·0000524

5. Find the value of $(\cdot001234)^{\frac{2}{3}}$, by logarithms.

Log. of ·001234.........................$\bar{3}\cdot0913152$

2

$3\,)\,6\cdot1826304$

Ans. ·00115047 ....$\bar{2}\cdot0608768$

Here the divisor 3 being contained exactly twice in the negative index —6, the index of the quotient, to be put down, will be —2.

6. Find the value of $(\cdot024554)^{\frac{3}{2}}$, by logarithms.

Log. of ·024554 ....$\bar{2}\cdot3901223$

3

$2\,)\,\bar{6}\cdot1703669$

Ans. ·00384754.........................$\bar{3}\cdot5851834$

Here, 2 not being contained exactly in —5, 1 is added to it, which gives —3 for the quotient; and the 1 that is borrowed being carried to the next figure makes 11, which, divided by 2, gives ·5851834 for the decimal part of the logarithm.

METHOD OF CALCULATING THE LOGARITHM OF ANY GIVEN NUMBER, AND THE NUMBER CORRESPONDING TO ANY GIVEN LOGARITHM. DISCOVERED BY OLIVER BYRNE, THE AUTHOR OF THE PRESENT WORK.

The succeeding numbers possess a particular property, which is worth being remembered.

log. 1·371288574238542 = 0·1371288574238542
log. 10·00000000000000 = 1·000000000000000
log. 237·5812087593221 = 2·375812087593221
log. 3550·260181586591 = 3·550260181586591
log. 46692·46832877758 = 4·669246832877758
log. 576045·6934135527 = 5·760456934135527
log. 6834720·776754357 = 6·834720776754357
log. 78974890·31398144 = 7·897489031398144
log. 895191599·8267852 = 8·951915998267839
log. 9999999999·999999 = 9·999999999999999

In these numbers, if the decimal points be changed, it is evident the logarithms corresponding can also be set down without any calculation whatever.

Thus, the log. of 137·1288574238542 = 2·1371288574238542;
the log. of 35·50260181586591 = 1·550260181586591;
log. ·002375812087593221 = $\bar{3}\cdot375812087593221$;
log. ·0008951915998267852 = $\bar{4}\cdot951915998267852$;

and so on in similar cases, since the change of the decimal point in a number can only affect the whole number of its logarithm.

These numbers whose logarithms are made up of the same digits will be found extremely useful hereafter. We shall next give a simple method of multiplying any number by any power of 11, 101, 1001, 10001, 100001, &c.

This multiplication is performed by the aid of coefficients of a binomial raised to the proposed power.

$(x+y)^1 = x + y$, the coefficients are 1, 1.
$(x+y)^2 = x^2 + 2xy + y^2$, the coefficients are 1, 2, 1.
$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, the coefficients are 1, 3, 3 1.

The coefficients of $(x+y)^4$ are 1, 4, 6, 4, 1.
— — $(x+y)^5$ — 1, 5, 10, 10, 5, 1.
— — $(x+y)^6$ — 1, 6, 15, 20, 15, 6, 1.
— — $(x+y)^7$ — 1, 7, 21, 35, 35, 21, 7, 1.
— — $(x+y)^8$ — 1, 8, 28, 56, 70, 56, 28, 8, 1.
— — $(x+y)^9$ — 1, 9, 36, 84, 126, 126, 84, 36, 9, 1.

Let it be required to multiply 54247 by $(101)^6$.

The number must be divided into periods of two figures when the multiplier is 101; into periods of three figures when the multiplier is 1001; into periods of four figures when the multiplier is 10001; and so on.

e	*d*	*c*	*b*	*a*			
	54	24	70	00	00		1
	3	25	48	20	00	*a*	6
		8	13	70	50	*b*	15
			10	84	94	*c*	20
				8	14	*d*	15
					3	*e*	6

$(54247) \times (101)^6 = 57\,58\,42\,83\,61$, true to 10 places of figures.

This operation is readily understood, since the multipliers for the 6th power are 1, 6, 15, 20, 15, 6, 1; we begin at *a*, a period in advance, and multiply by 6; then we commence at *b*, two periods in advance, and multiply by 15; at *c*, three periods in advance, and multiply by 20; at *d*, four periods in advance (counting from the right to the left), and multiply by 15; the period, *e*, should be multiplied by 6, but, as it is blank, we only set down the 3 carried from multiplying *d*, or its first figure by 6.

As it is extremely easy to operate with 1, 5, 10, 10, 5, 1, the multipliers for the 5th power, it may be more convenient first to multiply the given number by $(101)^5$, and then by $(101)^1$; because, to multiply any number by 5, we have only to affix a cipher (or suppose it affixed) and to take the half of the result.

The above example, if worked in the manner just described, will stand as follows:

$$\begin{array}{r|r|r|r|r|l}
d & c & b & a & & \\
54 & 24 & 70 & 00 & 00 & .....1 \\
2 & 71 & 23 & 50 & 00 & .....5..a \\
 & 5 & 42 & 47 & 00 & ...10..b \\
 & & 5 & 42 & 47 & ...10..c \\
 & & & 2 & 71 & .....5..d \\
 & & & & 1 & .....1 \\
\hline
(54247) \times (101)^5 = 57 & 01 & 41 & 42 & 19 & \\
 & 57 & 01 & 41 & 42 & \\
\hline
\multicolumn{5}{r}{57\ 58\ 42\ 83\ 61} & = (54247)^6 \times (101)^6.
\end{array}$$

The truth of this is readily shown by common multiplication, but the process is cumbersome. However, for the sake of comparison, we shall in this instance multiply 54247 by (101) raised to the 6th power.

$$\begin{array}{rl}
101 & \\
101 & \\
\hline
101 & \\
1010 & \\
\hline
10201 & = (101)^2. \\
101 & \\
\hline
10201 & \\
102010 & \\
\hline
1030301 & = (101)^3. \\
101 & \\
\hline
1030301 & \\
10303010 & \\
\hline
104060301 & = (101)^4. \\
101 & \\
\hline
104060401 & \\
1040604010 & \\
\hline
10510100501 & = (101)^5. \\
101 & \\
\hline
10510100501 & \\
105101005010 & \\
\hline
1061520150601 & = (101)^6. \\
54247 & \\
\hline
7430641054207 & \\
4246080602404 & \\
2123040301202 & \\
4246080602404 & \\
5307600753005 & \\
\hline
5758428360|9652447 & \text{the required product,}
\end{array}$$

which shows that the former process gives the result true to 10 places of figures, of which we shall add another example.

Multiply 34567812 by $(1001)^8$, so that the result may be true to 12 places of figures.

c	*b*	*a*		
	3456	7812	0000	1
	2	7654	2496	.....8..*a*
		9	6790	...28..*b*
			19	...56..*c*

3459 5475 9305 the required product.

The remaining multipliers, 70, 56, 28, 8, 1, are not necessary in obtaining the first 12 figures of the product of 34567812 by 10001 in the 8th power.

As 28 and 56 are large multipliers, the work may stand thus

c	*b*	*a*			
	3456	7812	0000	 1	
	2	7654	2496	...*a*.. 8	
		6	9136	...*b*..20	28
		2	7654	...*b*.. 8	
			17	...*c*..50	56
			2	...*c*.. 6	

Result, = 345954759305 the same as before.

Perhaps this product might be obtained with greater ease by first multiplying 34567812 by $(10001)^5$, and the product by $(10001)^3$; the operation will stand thus:

345678120000...... 1
172839060...... 5
34568......10
3......10

345850093631 = 34567812 × $(10001)^5$.
103755298...... 3
10376...... 3

345954759305 = twelve places of the product of 34567812 by $(10001)^5 \times (10001)^3 = (34567812) \times (10001)^8$.

Although these methods are extremely simple, yet cases will occur, when one of them will have the preference.

Our next object is to determine the logarithms 1·1; 1·01; 1·001; 1·0001; 1·00001; &c.

It is well known that

$$\log. (1 + n) = \text{M} (n - \tfrac{1}{2}n^2 + \tfrac{1}{3}n^3 - \tfrac{1}{4}n^4 + \tfrac{1}{5}n^5 - \tfrac{1}{6}n^6 + \&c.)$$

M being the modulus, = ·432944819032618276511289, &c.

It is evident that when n is $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, &c., the calculation becomes very simple.

$$\begin{aligned}
M &= \cdot 4342944819032518 \\
\tfrac{1}{2}M &= \cdot 2171472409516259 \\
\tfrac{1}{3}M &= \cdot 1447648273010839 \\
\tfrac{1}{4}M &= \cdot 1085736204758130 \\
\tfrac{1}{5}M &= \cdot 0868588963806504 \\
\tfrac{1}{6}M &= \cdot 0723824136505420 \\
\tfrac{1}{7}M &= \cdot 0720420788433217 \\
\tfrac{1}{8}M &= \cdot 0542868102379065 \\
\tfrac{1}{9}M &= \cdot 0482549424336946 \\
\tfrac{1}{10}M &= \cdot 0434294481903252
\end{aligned}$$

&c. &c., are constants employed to determine the logarithms of 11, 101, 1001, 100001, &c.

To compute the log. of 1·001. In this case $n = \frac{1}{1000}$.

$$\begin{aligned}
+\ \frac{M}{1000} &= \cdot 0004342944819033 \text{ positive} \\
-\ \frac{\frac{1}{2}M}{(1000)^2} &= \underline{\cdot 0000002171472410} \text{ negative} \\
& \cdot 0004340773346623 \\
+\ \frac{\frac{1}{3}M}{(1000)^3} &= \underline{\cdot 0000000001447648} \text{ positive} \\
& \cdot 0004340774794271 \\
-\ \frac{\frac{1}{4}M}{(1000)^4} &= \underline{\cdot 0000000000001086} \text{ negative} \\
& \cdot 0004340774793185 \\
+\ \frac{\frac{1}{6}M}{(1000)^5} &= \underline{\cdot 0000000000000001} \text{ positive} \\
& \cdot 0004340774793186 = \text{the log. of } 1\cdot 001;
\end{aligned}$$

true to sixteen places.

It is almost unnecessary to remark, that, instead of adding and subtracting alternately, as above, the positive and negative terms may be summed separately, which will render the operation more concise.

Positive Terms.	*Negative Terms.*
·0004342944819033	·0000002171472410
1447648	1086
1	·0000002171473496
+ ·0004342945266682	
− 000000217473496	
·0004340774793186 = log. 1·001.	

In a similar manner the succeeding logarithms may be obtained to almost any degree of accuracy.

Log. 1·1 = ·041392685158225 &c. which we call A
1·01 = ·004321373782643 — — B
1·001 = ·000434077479319 — — C
1·0001 = ·000043427276863 — — D
1·00001 = ·000004342923104 — — E
1·000001 = ·000000434294265 — — F
1·0000001 = ·000000043429447 — — G
1·00000001 = ·000000004342945 — — H
1·000000001 = ·000000000434295 — — I
1·0000000001 = ·000000000043430 — — J
1·00000000001 = ·000000000004343 — — K
1·000000000001 = ·000000000000434 — — L
1·0000000000001 = ·000000000000043 — — M
1·00000000000001 = ·000000000000004 — — N
&c. &c. &c.

Without further formality or paraphernalia, for it is presumed that such is not necessary, we shall commence operating, as the method can be acquired with ease, and put in a clearer point of view by proper examples.

Required the logarithm of 542470, to seven places of decimals.

```
     5 4|2 4|7 0|. .|
       3|2 5|4 8|2 0|
           8|1 3|7 1|
            |1 0|8 5|
                   8|
     ----------------
     5 7 5 8|4 2 8 4 = 6 B = ·02592824
           1|7 2 7 5
                   3
     ---------------
Take 5 7 6 0 1 5 6 2 = 3 D = ·00013028.
From 5 7 6 0 4 5 6 9
     ---------------
576) · · · · 3|0 0 7
             2|8 8 0 = 5 E = ·00002171
             -------
               1|2 7
               1|1 5 = 2 F = ·00000087
               -----
                 1|2
                 1|2 = 2 G = ·00000009
                             ---------
                             ·02608119 Take
                            5·76045693 From
                            ----------
```

Hence we have log. 542470 = 5·73437574, which is correct to seven decimal places.

6 B is written to represent 6 times the log. of 1·01.

The nearest number to 542470, whose log. is composed of the same digits as itself, being 576045·6934, &c., our object was to raise 542470· to 576045·69 by multiplying 542470· by some power or powers of 1·1, 1·01, 1·001, 1·0001, &c.

It is here necessary to remark, that A is not employed, because the given number multiplied by 1·1, would exceed 576045·69; for a like reason C is omitted.

Again, when half the figures coincide, the process may be performed (as above) by common division; the part which coincides becoming the divisor; thus, in finding 5 E, 576 is divided into 3007, it goes 5 times, the E showing that there are five figures in each period at this step. For A, there is but one figure in each period; for B, there are two figures; for C, there are three figures in each period, and so on.

Let it be required to calculate the logarithm of 2785·9, true to seven places of decimals.

It will be found more convenient, in this instance, to bring the given number to 3550·26018, the log. of which is 3·55026908.

```
          2|7|8|5|9|0|0|0
           |5|5|7|1|8|0|0
           | |2|7|8|5|9|0
          ---------------
          3 3|7 0|9 3|9 0 = 2 A = ·08278537
            1|6 8|5 4|7 0
                3|3 7|0 9
                    3|3 7
                     |  2
          ---------------
          3 5 4|2 8 9|0 8 = 5 B = ·02160687
               |7 0 8|5 8
                     |3 5
          ---------------
     Take 3 5 4 9|9 8 0 1 = 2 C = ·00086815
     From 3 5 5 0|2 6 0 2
          ---------------
      355) · · · 2|8 0 1 = 7 E = ·00003040
                 2|4 8 5
                 -------
                   3|1 6 = 8 F = ·00000347
                   2|8 4
                   -----
                     3|2 = 9 G = ·00000039
                     3|2         ----------
                         Take     ·10529465
                         From    3·55026018
                                 ----------
               log. 2785·9 =     3·44496553
```

At the Observatory at Paris, *g* = 9·80896 metres, the second being the unit of time, what is the logarithm of 9·80896?

In this example, we shall bring 9·80896 to 9·99999, &c.

```
      9 8|0 8|9 6|0 0|0 0
         |9 8|0 8|9 6|0 0
      ---------------------
      9 9 0|7 0 4|9 6 0|0 = 1 B = ·0043213738
          8|9 1 6|3 4 4|6
               3 5|6 6 5|4
                     8 3|2
      ---------------------
      9 9 9 6|5 7 0 5|3 2 = 9 C = ·0039066973
            2|9 9 8 9|7 2
                    3|0 0
      ---------------------
      9 9 9 5|6 9 8 0 4 = 3 D = ·0001302818
            3|9 9 9 8 3
                      6
      ---------------------
Take  9 9 9 9 9 6 9 7 9 3 = 4 E = ·0000173717
From 1 0 0 0 0 0 0 0 0 0 0
      ---------------------
      · · · · · 3 0 2 0 7
From which we have.........3 F = ·0000013029
                           2 H = ·0000000087
                           7 J = ·0000000003
                                 -----------
                          Take  ·0083770365
                          From 1·0000000000
                                -----------
                Log. 9·80896 = ·9916229635
```

As before observed, 9 C might have been obtained in the following manner:

```
        8 9 0|7 0 4|9 6 0|0 = 1 B, as above.
            4|9 5 3|5 2 4|8
                  9|9 0 7|0
                        9|9
        ---------------------
5 times 9 9 5|6 6 8|4 0 1|7
            3|9 8 2|6 7 3|6
                  5|9 7 3|9
                        4|0
        ---------------------
4 times 9 9 9 6 5 7 0 5 3 2 = 9 C.
```

A French metre is equal to 3·2808992 English feet, required the log. of 3·2808992.

```
e| d| c| b| a|
 |32|80|89|92|00...once
 | 2|29|66|29|44...  7 times from a
    | 6|88|98|88...21     —       b
       |11|48|31...35     —       c
          |11|48...35     —       d
             |  7...21    —       e
 ---------------------
  35 17 56 80 18 = B 7.
```

The manner in which B 7 is obtained is worthy of remark: the multipliers being 1, 7, 21, 35, 35, 21, 7, 1, when 7 times the first line (commencing with the period marked *a*) is obtained, 21 times the same line (commencing with the period marked *b*) is determined by multiplying the 2d line by 3. If the 2d line be again multiplied by 5, we have the 4th line of the multiplier 35; but to multiply by 5, we have only to take the half the product produced by multiplying by 7, advancing the result one figure to the right. Hence, to find the result for 35 is almost as easy as to find the result for 5.

But the object in this case being to bring the proposed number to 35502601815, the process must be continued.

	c	*b*	*a*		
1	351	756	801	8	= B 7, as above.
9	3	165	811	2	
36		12	663	2	
84			29	6	
	354	935	305	8	= C 9

The 2d (or 9) line is produced by beginning at *a*, but the multiplication may be performed by subtracting 3517568 from 35175680; the 36 line is produced by beginning at *b*, observing to carry from the preceding figure, making the usual allowance when the number is followed by 5, 6, 7, 8, or 9. The 36 line may be produced by multiplying the 9 line by 4, beginning one period more to the left. To multiply by 84 is not apparently so convenient, for 84 × 352 = 29|568; and as only one figure of the period 568 is required, when the proper allowance is made, the result becomes 29|6.

But, since 84 is equal to 36 × $2\frac{1}{3}$, we have only to multiply the 36 line by 2, and add $\frac{1}{3}$ of it; with such management, the work will stand thus:—

351	756	801	8	= B 7, as before	
3	165	811	2	= 9 times	
	12	663	2	= 36 times	
		24	3	= 72 times	= 84 times
		4	2	= 12 times	
354	935	305	8	= C 9	

This amounts to very little more than adding the above numbers together.

Many other contractions will suggest themselves, when the multipliers are large: thus, to multiply any number 57837 by 9, as alluded to above, is easily effected, by the following well-known process:—Subtract the first figure to the right from 10, the second from the first, the third from the second, and so on.

$$\text{Thus, } 57837 \times 9 = \begin{cases} 578370\ldots\text{ten times} \\ \underline{\ \ 57837}\ldots\text{once} \\ 520533\ldots\text{nine times} \end{cases}$$

Such simple observations are to be found in every book on mental arithmetic, and therefore require but little attention here.

The whole work of the previous example will stand thus:—

```
                3 2|8 0|8 9|9 2|0 0
                  2|2 9|6 6|2 9|4 4
                      6|8 8|9 8|8 8
                        1 1|4 8|3 1
                            1 1|4 8 + 7
                -------------------
B 7 =           3 5 1|7 5 6|8 0 1|8 = ·0302496165 = 7 B
                      3|1 6 5|8 1 1|2
                        1 2|6 6 3|2
                              2 9|6
                -------------------
C 9 =           3 5 4 9|3 5 3 0|5 8 = ·0039066973 = 9 C
                       |7 0 9 8|7 1
                               |3 5
                -------------------
D 2 =           3 5 5 0 0|6 2 9 6 4 = ·0000868546 = 2 D
                        1|7 7 5 0 3
                                  4
                -------------------
Take E 5 =      3 5 5 0 2 4 0 4 7 1 = ·0000217146 = 5 E
From            3 5 5 0 2 6 0 1 8 2
                -------------------
   3550). . . . . . . . . 1|9 7 1 1
   F 5                    1|7 7 5 0 = ·0000021715 = 5 F
                            -------
                            1|9 6 1
   G 5                      1|7 7 5 = ·0000002172 = 5 G
                              -----
                              1|8 6
   H 5                        1|7 8 = ·0000000217 = 5 H
                                ---
                                 |8
   I 2                           |7 = ·0000000009 = I 2
                                  -
                                  1|
   J 3                            1| = ·0000000001 = J 3
                                      -----------
                            Take      ·0342672944
                            From     3·5502601816
                                     ------------
                  Log. 3280·8992  =  3·5159928972
                ∴ log. 3·2808992  =  0·5159928972.
```

The constant sidereal year consists of 365·25636516 days; what is the log. of this number?

In this case it is better to bring *the constant* 35502601816 to 36525636516, instead of bringing the given number to the constant, as in the former examples.

3 5 5 0 2 6 0 1 8 1 6
7 1 0 0 5 2 0 3 6
3 5 5 0 2 6 0
B 2 = 3 6 2 1 6 2 0 4 1 1 2 = ·0086427476 = 2 B
2 8 9 7 2 9 6 3 3
1 0 1 4 0 5 4
2 0 2 8
C 8 = 3 6 5 0 6 9 4 9 8 2 7 = ·0034726298 = 8 C
1 8 2 5 3 4 7 5
3 6 5 1
Take D 5 = 3 6 5 2 5 2 0 6 9 5 3 = ·0002171364 = 5 D
From 3 6 5 2 5 6 3 6 5 1 6
36525·2) 4 2 9 5 6 3
E 1 = 3 6 5 2 5 2 = ·0000043429 = 1 E
6 4 3 1 1
F 1 = 3 6 5 2 5 = ·0000004343 = 1 F
2 7 7 8 6
G 7 = 2 5 5 6 8 = ·0000003040 = 7 G
2 2 1 8
H 6 = 2 1 9 1 = ·0000000261 = 6 H
I 0 2 7
J 7 = 2 5 = ·0000000003 = 7 J
·0123376214
Add 3·5502601816

Hence, log. 3652·5636516 = 3·5625978030
∴ log. 365·25636516 = 2·562597803.

M. Regnault determined with the greatest care the density of mercury to be 13·59593 at the temperature 0°, centigrade. It is required to calculate the log. of 13·59593, to eight places of decimals.

In this case it is better to bring the given number to *the constant* 1371288574.

1 3 5 9 5 9 3 0 0
1 0 8 7 6 7 4
3 8 0 7
8
C 8 = 1 3 7 0 5 0 7 8 8 = ·003472630 = 8 C
6 8 5 2 5
1 4
Subtract D 5 = 1 3 7 1 1 9 3 2 8 = ·000217136 = 5 D
From 1 3 7 1 2 8 8 5 7
9 5 2 9 = ·000026058 = E 6
E 6 = 8 2 2 7
1 3 0 2
F 9 = 1 2 3 4 = ·000003909 = F 9
6 8
H 5 = 6 9 = ·000000022 = H 5
·003719755

Take ·003719755
From ·137128857

log. 1·359593 = ·133409102
∴ log. 13·59593 = 1·133409102.

TO DETERMINE THE NUMBER CORRESPONDING TO A GIVEN LOGARITHM.

This problem has been very much neglected—so much so, that none of our elementary books ever allude to a method of computing the number answering to a given logarithm. When an operation is performed by the use of logarithms, it is very seldom that the resulting logarithm can be found in the table; we have, therefore, to find the nearest less logarithm, and the next greater, and correct them by proportion, so that there may be found an intermediate number that will agree with the given logarithm, or nearly so. But although the *proportional parts of the difference* abridge this process, we can only find a number appertaining to any logarithm to seven places of figures when using our best modern tables. As, however, the tabular logarithms extend only to a degree of approximation, fixed generally at seven decimal places, all of which, except those answering to the number 10 and its powers, err, either in excess or defect, the maximum limit of which is ½ in the last decimal, and since both errors may conspire, the 7th figure cannot be depended on as strictly true, unless the proposed logarithm falls between the limits of log. 10000 and log. 22200.

Indubitably we are now speaking of extreme cases, but since it is not an unfrequent occurrence that some calculations require the most rigid accuracy, and many resulting logarithms may be extended beyond the limits of the table, this subject ought to have a place in a work like the present. It is not part of the present design to enter into a strict or formal demonstration of the following mode of finding the number corresponding to a given logarithm, as the operation will be fully explained by suitable examples.

What number corresponds to the logarithm 3·4449555?

The next less constant log. to the one proposed is 2·37581209, or rather, 3·37581209, when the characteristic or index is increased by a unit.

First from 3·44496555
take 3·37581209
·06915346
·04139269 = 1 A
·02776077
·02592824 = 6 B
..183253
173631 = 4 C
....9622
8685 = 2 D
.....937

Secondly.

2|3 7 5 8 1 2 0 9 constant
|2 3 7 5 8 1 2 1 = A 1
2 6 1 3 3 9 3 3|0
1 5 6 8 0 3 6|0
3 9 2 0 0 9
5 2 2 7
3 9
2 7 7|4 1 6 9 6 5 = B 6
1|1 0 9 6 6 8
1|6 6 4
1
2 7 8 5|2 8 2 9|8 = C 4

.....937
869 = 2 E
......68
43 = 1 F
......25
22 = 5 G
.......3
3 = 7 H

2 7 8 5 2 8 2 9 8 = C 4
5 5 7 0 6
3
2 7 8 5 8 4 0 0 7 = D 2
5 5 7 2 = E 2
2 7 9 = F 1
1 3 9 = G 5
1 9 = H 7
2 7 8 5 9 0 0 1 6

∴ 2785·90016 is the number sought.

What number corresponds to the logarithm 5·73437574?

When the index of this log. is reduced by a unit, the nearest next less constant is 4·66924683.

From 4·73437574
Take 4·66924683
·6512891
4139269.........1 A
·2373622
2160687.........5 B
··212035
173631.........4 C
...39304
39085.........9 D
.....219
217.........5 F
.......2.........0 G
2.........4 H

There is neither the equal of this number, nor a less, obtainable from E, ∴ E 0, or E, is omitted.

Then, 4 6 6 9 2 4 6 8 3
4 6 6 9 2 4 6 8.........A 1
5 1 3 6 1 7 1 5 1
2 5 6 8 0 8 5 8
5 1 3 6 1 7
5 1 3 6
2 6
5 3 9 8 1 6 7 8 8.........B 5
2 1 5 9 2 6 7
3 2 3 9
2
5 4 1 9 7 9 2 9 6.........C 4
4 8 7 7 8 1
1 9 5
5 4 2 4 6 7 2 7 2.........D 9
2 7 1 2.........F 5
2 2.........H 4
5 4 2 4 7 0 0 0 6

∴ 542470·006 is the number whose logarithm is 5·73437574.

Had the given logarithm represented a decimal with a positive index, the required number would be 0·000054247, &c.; or if written with a negative index, as $\overline{5}$·73437574, the result would be the same, for the characteristic $\overline{5}$, shows how many places the first significant figure is below unity.

Required the number corresponding to log. 2·3727451.

The constant 100000000 is the one to be employed in this case.

```
1·3727451 the given log. minus 1 in the index.
1·0000000
---------
 ·3727451
 3725342.........9 A
 -------
...2109
   1737.........4 D
   ----
....372
    347.........8 E
    ---
.....25
     22.........5 F
     --
      3
      3.........7 G
      -
      1 0 0 0 0 0 0 0 Constant.
        9 0 0 0 0 0 0
        3 6 0 0 0 0 0
          8 4 0 0 0 0
          1 2 6 0 0 0
            1 2 6 0 0
                8 4 0
                  3 6
                    9
   ----------------
   2 3 5 7 9 4 8 5   A 9
           9 4 3 2
                 1
   ----------------
   2 3 5 8 8 9 1 8   D 4
           1 8 9 7   E 8
             1 1 8   F 5
               1 6   G 7
   ----------------
   2 3 5 9 0 9 4 9
```

∴ 235·90949 is the required number, and the seconds in the diurnal apparent motion of the stars.

$$235·90949'' = 3'\ 55·90949''.$$

Let it be required to find the *hyperbolic* logarithm of any number, as 3·1415926536. The common log. of this number is ·49714987269 (33), and the common log. of this log. is $\overline{1}$·6964873.

The modulus of the common system of logarithms is ·4342944819, &c.

∴ 1 : 4342944819 : : hyperbolic log. N : common log. N.

To distinguish the hyperbolic logarithm of the number N from its common logarithm, it is necessary to write the hyp. log. Log. N, and the common logarithm log. N.

Hence, 4342944819 × Log. N = log. N;
or log. (·4342944819) + log. (log. N) = log. (log. N).

∴ log. (Log. N) = log. (log. N) − $\bar{1}$·6377843; for $\bar{1}$·6377843 = log. ·4342944819.

Now, to work the above example, from $\bar{1}$·6964873
take $\bar{1}$·6377843
·0587030, the number corresponding to this *com. log.* will be the *hyp. log.* of 3·1415927. ·0587030 must be reduced to ·0000000 which is known to be the log. of 1.

·0587030
0413927 1 A
.173103
172855 4 B
....248
217 5 E
.....31
30 7 F
......1 2 G

1 A = 110000000
4400000
66000
440
1
114466441 = B 4
5723 = E 5
801 = F 7
23 = G 2
114472988

∴ 1·14472988 is the hyperbolic log. of 3·1415927, true to the last figure; for the hyp. log. 3·1415926535898 = 1·1447298858494.

The reason of this operation is very clear, because
$1 \times 1{\cdot}1 \times (1{\cdot}01)^4 \times (1{\cdot}00001)^5 \times (1{\cdot}000001)^7 \times (1{\cdot}0000001)^2 =$ 1·14472988.

This example answers the purpose of illustration, but the hyp. log. of 3·1415927 can be more readily found by dividing its com. log. ·49714987269 by the constant ·4342944819, which is termed the modulus of the common system of logarithms.

Suppose it is known that $\bar{1}$·3426139 is the log. of the decimal which a *French litre* is of an English gallon. Required the decimal.

The index, $\bar{1}$, may be changed to any other characteristic, so as to suit any of *the constants*, as the alteration is easily allowed for when the work is completed. In this instance, it is best to put +1 instead of $\bar{1}$.

From 1·3426139
Take 1·0000000
·3426139
3311415 = 8 A
·0114724
..86427 = 2 B
28297
26045 = 6 C
2252

100000000 Constant
80000000
28000000
5600000
700000
56000
2800
80
1
214358881 = A 8

2252
2171 = 5 D
81
43 = 1 E
38
35 = 8 F
3
3 = 7 G

214358881 = A 8
42871 78
21436
218667495 = B 2
1312005
3280
4
219982784 = C 6
109991
22
220092797 = D 5
2201 = E 1
1761 = F 8
754 = G 7
220096913

∴ The French litre = ·2200969 English gallons.

In measuring heights by the barometer, it is necessary to know the ratio of the density of the mercury to that of the air.

At Paris, a *litre* of air at 0° centigrade, under a pressure of 760 millimetres, weighs 1·293187 grammes. At the level of the sea, in latitude 45°, it weighs 1·292697 grammes. A *litre* of water, at its maximum density, weighs 1000 grammes, and a *litre* of mercury, at the temperature of 0° cent., weighs 13595·93 grammes:

$$\therefore \frac{13595{\cdot}93}{1{\cdot}292697} = \text{the ratio at } 45^\circ$$

Now, log. 13595·93 = 4·133409102 (29)
and log. 1·292697 = 0·111496744 (30)
4·021912358 = the log. of the ratio at 45°.

To find the number corresponding to this log., it is necessary to reject the index for the present, and reduce the decimal part to zero. By this means the necessity of using any of the constants is superseded.

·021912358
·021606869 = 5 B
. . . 305489
303991 = 7 D
. 1498
1303 = 3 F
. 195
174 = 4 G
. . . . 21
17 = 4 H
4
4 = 9 I

1000000000
500000000
1000000
10000
50
1051010050 = B 5
73571
22
105174598 = D 7
316 = F 3
42 = G 4
4 = H 4
1 = I 9
105174961

$$\therefore \text{ by logarithms, } \frac{13595{\cdot}93}{1{\cdot}292697} = 10517{\cdot}49, \text{ \&c., which is easily}$$

verified by common division.

M. Regnault found that, at Paris, the litre of atmospheric air weighs 1·293187 grammes; the litre of nitrogen 1·256167 grammes; a litre of oxygen, 1·429802 grammes; of hydrogen, 0·089578 grammes; and of carbonic acid, 1·977414 grammes. But, strictly considered, these numbers are only correct for the locality in which the experiments were made; that is for the latitude of 48° 50′ 14″ and a height about 60 metres above the level of the sea; M. Regnault finds the weight of the litre of air under the parallel of 45° latitude, and at the same distance from the centre of the earth as that which the experiments were tried, to be 12·926697.

Assuming this as the standard, he deduces for any other latitude, any other distance from the centre of the earth, the formula,

$$w = \frac{1{\cdot}292697\ (1{\cdot}00001885)\ (1 - 0{\cdot}002837)\ \cos.\ 2\,\lambda}{1 + \dfrac{2\,h}{\mathrm{R}}}$$

Here, w is the weight of the litre of air, R the mean radius of the earth = 6366198 metres, h the height of the place of observation above the mean radius, and λ the latitude of the place.

At Philadelphia, lat. 39° 56′ 51·5″, suppose the radius of the earth to be 6367653 metres, the weight of the litre of air will be 1·2914892 grammes. The ratio of the density of mercury to that of air at the level of the sea at Philadelphia is 10527·735 to 1; required the number of degrees in an arc whose length is equal to that of the radius.

$$\text{As } 3{\cdot}14159265358 98 : 1 :: \frac{360}{2} : \text{the required degrees.}$$

$$\begin{aligned} \text{Log. } 360 &= 2{\cdot}556302500767 \\ \text{log. } 3{\cdot}14159265359 &= \underline{0{\cdot}497149872694} \\ & \ 2{\cdot}059452623073 \\ \text{log. } 2 &= \underline{0{\cdot}301029995664} \\ & \ 1{\cdot}758122632409 = \text{the log. of the} \end{aligned}$$

number required.

When the index of this log. is changed into 4, the nearest next less constant is 4·669246832878.

From	4·758122632409	4 6 6 9 2 4 6 8 3 2 8 7 8	= Constant
Take	4·669246832878	9 3 3 8 4 9 3 6 6 5 7 6	
	·088875799531	4 6 6 9 2 4 6 8 3 2 9	
2 A =	·82785370316	5 6 4 9 7 8 8 6 6 7 7 8 3	= A 2
	..6090429215	5 6 4 9 7 8 8 6 6 7 8	
1 B =	4321373783	5 7 0 6 2 8 6 5 5 4 4 6 1	= B 1
	..1769055432	2 2 8 2 5 1 4 6 2 1 8	
4 C =	1736309917	3 4 2 3 7 7 1 9	
	32745515	2 2 8 2 5	
7 E =	30400462	6	
	2345053	5 7 2 9 1 4 5 9 6 1 2 2 9	= C 4

	2345053	5729145961229 = C 4
5 F =	2171471	401040217
	173582	12031
3 G =	130288	5729547013477 = E 7
	43294	28647735
9 H =	39087	57
	4207	5729575661269 = F 5
9 I =	3909	1718873 = G 3
	298	515662 = H 9
6 J =	261	51566 = I 9
	37	3438 = J 6
8 K =	35	458 = K 8
		29 = L 5
	2	5729577951295 = the number required.
5 L =	2	

But the original index is 1; ∴ 57·29577951295° are the number of degrees in an arc the length of which is equal to that of the radius.

The above result may be easily verified by common division, a method, no doubt, which would be preferred by many, for logarithms are seldom used when the ordinary rules of arithmetic can be applied with any reasonable facility. However, this example, like many others, is introduced to show with what ease and correctness the number corresponding to a given log. can be obtained. The extent, also, by far exceeds that obtainable by any tables extant.

Other computations give,

$$r^\circ = 57{\cdot}2957795130^\circ = 57^\circ\ 17'\ 44''\ {\cdot}80624$$

the degrees in an arc = radius.

$$r' = 3437{\cdot}7467707849' = 3437'\ 44''\ {\cdot}80624$$

the minutes in an arc = radius.

$$r'' = 206264{\cdot}8062470963$$

the number of seconds in an arc = radius.

The relative mean motion of the moon from the sun in a Julian or fictitious year, of 365¼ days, is 12 cir. 4 signs, 12° 40′ 15·977315′ = 16029615·977315″.

∴ 16029615·977315″ : 1 circumference (= 129600″)
: : 365·25 days
: 29·5305889216 days = the mean synodic month.

This proportion may, for the sake of example, be found by logarithms.

Log. 365·25.........	2·56259022460634
log. 1296000.........	6·11260500153457
	8·67519522614091
log. 16029615·977315 =	7·20492311805406
	1·47027210808685

If the index of this log. be made 2 instead of 1, the nearest next less constant will be 2·375812087593221.

From 2·47027210808685
Take 2·37581208759322
·09446002049363
2 A = 08278537031645
.1167465017718
2 B = 864274756529
..303190261189
6 C = 260446487591
...42743773598
9 D = 39084549177
....3659224421
8 E = 3474338483
.....184885938
4 F = 173717706
......11168232
2 G = 8685889
.......2482343
5 H = 2171473
........310870
7 I = 304006
..........6863
1 J = 4343
..........2520
5 K = 2172
...........348
8 L = 347
2 N = 1

2375812087593222 Const.
475162417518644
23758120875933
2874732625987779 = 2 A
5749465251976
287473262600
2932514751770115 = 2 B
1759508851062
43987721288
586502929
439933
2
2950153886696335 = C 6
2655138498033
10620555400
247811
4
2952810087497633 = D 9
2362248070033
8267877
17
2953046320572677 = E 8
118121855288
17722
2953058132775677 = F 4
590611633
3
2953058723387333 = G 2
14765294 = H 5
2067141 = I 7
29531 = J 1
14765 = K 5
2362 = L 8
6 = N 2
2953058892178332

.·. 29·5305889218 is the number required.

To perform, by logarithms, the ordinary operations of multiplication, division, proportion, or even the extraction of the square root, except in the way of illustration, is not the design of these pages; for such an application of logarithms, in a particular manner only, diminish the labour of the operator. It is not necessary, however, to examine minutely here the instances in which common arithmetic is preferable to artificial numbers; besides, much will depend on the skill and facility of the operator.

TRIGONOMETRY.

ANGULAR MAGNITUDES.—TRIGONOMETRY.—HEIGHT AND DISTANCES.—SPHERICAL TRIGONOMETRY.—THE APPLICATION OF LOGARITHMS TO ANGULAR MAGNITUDES.

PLANE TRIGONOMETRY treats of the relations and calculations of the sides and angles of plane triangles.

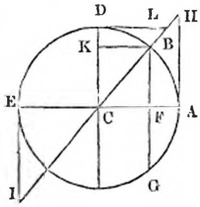
The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; also each degree into 60 minutes, each minute into 60 seconds, and so on.

Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

The measure of any angle is an arc of any circle contained between the two lines which form that angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc.

Hence, a right angle being measured by a quadrant, or quarter of the circle, is an angle of 90 degrees; and the sum of the three angles of every triangle, or two right angles, is equal to 180 degrees. Therefore, in a right-angled triangle, taking one of the acute angles from 90 degrees, leaves the other acute angle; and the sum of two angles, in any triangle, taken from 180 degrees, leaves the third angle; or one angle being taken from 180 degrees, leaves the sum of the other two angles.

Degrees are marked at the top of the figure with a small °, minutes with ′, seconds with ″, and so on. Thus, $57° 30' 12''$ denote 57 degrees 30 minutes and 12 seconds.



The complement of an arc, is what it wants of a quadrant or 90°. Thus, if AD be a quadrant, then BD is the complement of the arc AB; and, reciprocally, AB is the complement of BD. So that, if AB be an arc of 50°, then its complement BD will be 40°.

The supplement of an arc, is what it wants of a semicircle, or 180°. Thus, if ADE be a semicircle, then BDE is the supplement of the arc AB; and, reciprocally, AB is the supplement of the arc BDE. So that, if AB be an arc of 50°, then its supplement BDE will be 130°.

The sine, or right sine, of an arc, is the line drawn from one extremity of the arc, perpendicular to the diameter passing through the other extremity. Thus, BF is the sine of the arc AB, or of the arc BDE.

Hence the sine (BF) is half the chord (BG) of the double arc (BAG).

The versed sine of an arc, is the part of the diameter intercepted between the arc and its sine. So, AF is the versed sine of the arc AB, and EF the versed sine of the arc EDB.

The tangent of an arc is a line touching the circle in one extremity of that arc, continued from thence to meet a line drawn from the centre through the other extremity: which last line is called the secant of the same arc. Thus, AH is the tangent, and CH the secant, of the arc AB. Also, EI is the tangent, and CI the secant, of the supplemental arc BDE. And this latter tangent and secant are equal to the former, but are accounted negative, as being drawn in an opposite or contrary direction to the former.

The cosine, cotangent, and cosecant, of an arc, are the sine, tangent, and secant of the complement of that arc, the co being only a contraction of the word complement. Thus, the arcs AB, BD being the complements of each other, the sine, tangent or secant of the one of these, is the cosine, cotangent or cosecant of the other. So, BF, the sine of AB, is the cosine of BD; and BK, the sine of BD, is the cosine of AB: in like manner, AH, the tangent of AB, is the cotangent of BD; and DL, the tangent of DB, is the cotangent of AB: also, CH, the secant of AB, is the cosecant of BD; and CL, the secant of BD, is the cosecant of AB.

Hence several remarkable properties easily follow from these definitions; as,

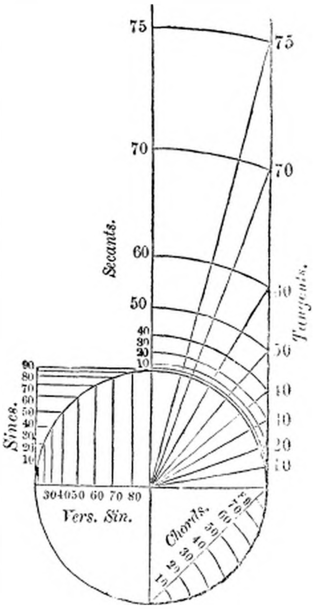
That an arc and its supplement have the same sine, tangent, and secant; but the two latter, the tangent and secant, are accounted negative when the arc is greater than a quadrant or 90 degrees.

When the arc is 0, or nothing, the sine and tangent are nothing, but the secant is then the radius CA. But when the arc is a quadrant AD, then the sine is the greatest it can be, being the radius CD of the circle; and both the tangent and secant are infinite.

Of any arc AB, the versed sine AF, and cosine BK, or CF, together make up the radius CA of the circle. The radius CA, tangent AH, and secant CH, form a right-angled triangle CAH. So also do the radius, sine, and cosine, form another right-angled triangle CBF or CBK. As also the radius, cotangent, and cosecant, another right-angled triangle CDL. And all these right-angled triangles are similar to each other.

The sine, tangent, or secant of an angle, is the sine, tangent, or secant of the arc by which the angle is measured, or of the degrees, &c. in the same arc or angle.

The method of constructing the scales of chords, sines, tangents, and secants, usually engraven on instruments, for practice, is exhibited in the annexed figure.



A trigonometrical canon, is a table exhibiting the length of the sine, tangent, and secant, to every degree and minute of the quadrant, with respect to the radius, which is expressed by unity, or 1, and conceived to be divided into 10000000 or more decimal parts. And further, the logarithms of these sines, tangents, and secants are also ranged in the tables; which are most commonly used, as they perform the calculations by only addition and subtraction, instead of the multiplication and division by the natural sines, &c., according to the nature of logarithms.

Upon this table depends the numeral solution of the several cases in trigonometry. It will therefore be proper to begin with the mode of constructing it, which may be done in the following manner:—

To find the sine and cosine of a given arc.

This problem is resolved after various ways. One of these is as follows, viz. by means of the ratio between the diameter and circumference of a circle, together with the known series for the sine and cosine, hereafter demonstrated. Thus, the semi-circumference of the circle, whose radius is 1, being 3·14159265358979З, &c., the proportion will therefore be,

As the number of degrees or minutes in the semicircle,
Is to the degrees or minutes in the proposed arc,
So is 3·14159265, &c., to the length of the said arc.

This length of the arc being denoted by the letter a; also its sine and cosine by s and c; then will these two be expressed by the two following series, viz.:—

$$s = a - \frac{a^3}{2.3} + \frac{a^5}{2.3.4.5} - \frac{a^7}{2.3.4.5.6.7} + \&c.$$

$$= a - \frac{a^3}{6} + \frac{a^5}{120} - \frac{a^7}{5040} + \&c.$$

$$c = 1 - \frac{a^2}{2} + \frac{a^4}{2.3.4} - \frac{a^6}{2.3.4.5.6} + \&c.$$

$$= 1 - \frac{a^2}{2} + \frac{a^4}{24} - \frac{a^6}{720} + \&c.$$

If it be required to find the sine and cosine of one minute. Then, the number of minutes in 180° being 10800, it will be first, as 10800 : 1 :: 3·14159265, &c. : ·000290888208665 = the length of an arc of one minute. Therefore, in this case,

$a =$ ·0002908882
and $\frac{1}{6}a^3 =$ ·000000000004, &c.
the difference is $s =$ ·0002908882 the sine of 1 minute.
Also, from 1·
take $\frac{1}{2}a^2 =$ 0·0000000423079, &c.
leaves $c =$ ·9999999577 the cosine of 1 minute.

For the sine and cosine of 5 degrees.

Here, as 180° : 5° : : 3·14159265, &c., : ·08726646 = a the length of 5 degrees.

$$\begin{aligned} \text{Hence,} \quad a &= \cdot 08726646 \\ -\tfrac{1}{6}a^3 &= -\cdot 00011076 \\ +\tfrac{1}{120}a^5 &= \cdot 00000004 \\ \text{these collected give } s &= \cdot 08715574 \text{ the sine of } 5^\circ. \end{aligned}$$

$$\begin{aligned} \text{And, for the cosine,} \quad 1 &= 1\cdot \\ -\tfrac{1}{2}a^2 &= -\cdot 00380771 \\ +\tfrac{1}{24}a^4 &= \cdot 00000241 \\ \text{these collected, give } c &= \cdot 99619470 \text{ the consine of } 5^\circ. \end{aligned}$$

After the same manner, the sine and cosine of any other arc may be computed. But the greater the arc is, the slower the series will converge, in which case a greater number of terms must be taken to bring out the conclusion to the same degree of exactness.

Or, having found the sine, the cosine will be found from it, by the property of the right-angled triangle CBF, viz. the cosine CF = $\sqrt{CB^2 - BF^2}$, or $c = \sqrt{1 - s^2}$.

There are also other methods of constructing the canon of sines and cosines, which, for brevity's sake, are here omitted.

To compute the tangents and secants.

The sines and cosines being known, or found, by the foregoing problem; the tangents and secants will be easily found, from the principle of similar triangles, in the following manner:—

In the first figure, where, of the arc AB, BF is the sine, CF or BK the cosine, AH the tangent, CH the secant, DL the cotangent, and CL the cosecant, the radius being CA, or CB, or CD; the three similar triangles CFB, CAH, CDL, give the following proportions:

1. CF : FB : : CA : AH; whence the tangent is known, being a fourth proportional to the cosine, sine, and radius.

2. CF : CB : : CA : CH; whence the secant is known, being a third proportional to the cosine and radius.

3. BF : FC : : CD : DL; whence the cotangent is known, being a fourth proportional to the sine, cosine, and radius.

4. BF : BC : : CD : CL; whence the cosecant is known, being a third proportional to the sine and radius.

Having given an idea of the calculations of sines, tangents, and secants, we may now proceed to resolve the several cases of trigonometry; previous to which, however, it may be proper to add a few preparatory notes and observations, as below.

There are usually three methods of resolving triangles, or the cases of trigonometry—namely, geometrical construction, arithmetical computation, and instrumental operation.

In the first method.—The triangle is constructed by making the parts of the given magnitudes, namely, the sides from a scale of equal parts, and the angles from a scale of chords, or by some other instrument. Then, measuring the unknown parts by the same scales or instruments, the solution will be obtained near the truth.

In the second method.—Having stated the terms of the proportion according to the proper rule or theorem, resolve it like any other proportion, in which a fourth term is to be found from three given terms, by multiplying the second and third together, and dividing the product by the first, in working with the natural numbers; or, in working with the logarithms, add the logs. of the second and third terms together, and from the sum take the log. of the first term; then the natural number answering to the remainder is the fourth term sought.

In the third method.—Or instrumentally, as suppose by the log. lines on one side of the common two-foot scales; extend the compasses from the first term to the second or third, which happens to be of the same kind with it; then that extent will reach from the other term to the fourth term, as required, taking both extents towards the same end of the scale.

In every triangle, or case in trigonometry, there must be given three parts, to find the other three. And, of the three parts that are given, one of them at least must be a side; because the same angles are common to an infinite number of triangles.

All the cases in trigonometry may be comprised in three varieties only; viz.

1. When a side and its opposite angle are given.
2. When two sides and the contained angle are given.
3. When the three sides are given.

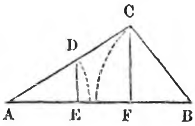
For there cannot possibly be more than these three varieties of cases; for each of which it will therefore be proper to give a separate theorem, as follows:

When a side and its opposite angle are two of the given parts.

Then the sides of the triangle have the same proportion to each other, as the sines of their opposite angles have.

That is,

As any one side,
Is to the sine of its opposite angle;
So is any other side,
To the sine of its opposite angle.



For, let ABC be the proposed triangle, having AB the greatest side, and BC the least. Take AD = BC, considering it as a radius; and let fall the perpendiculars DE, CF, which will evidently be the sines of the angles A and B, to the radius AD or BC. But the triangles ADE, ACF, are equiangular, and therefore AC : CF : : AD or BC : DE; that is, AC is to the sine of its opposite angle B, as BC to the sine of its opposite angle A.

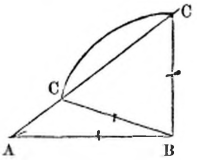
In practice, to find an angle, begin the proportion with a side opposite a given angle. And to find a side, begin with an angle opposite a given side.

An angle found by this rule is ambiguous, or uncertain whether it be acute or obtuse, unless it be a right angle, or unless its magnitude be such as to prevent the ambiguity; because the sine answers to two angles, which are supplements to each other; and accordingly the geometrical construction forms two triangles with the same parts that are given, as in the example below; and when there is no restriction or limitation included in the question, either of them may be taken. The degrees in the table, answering to the sine, are the acute angle; but if the angle be obtuse, subtract those degrees from 180°, and the remainder will be the obtuse angle. When a given angle is obtuse, or a right one, there can be no ambiguity; for then neither of the other angles can be obtuse, and the geometrical construction will form only one triangle.

In the plane triangle ABC,

Given, AB 345 yards
BC 232 yards
angle A 37° 20′

Required the other parts.



Geometrically.—Draw an indefinite line, upon which set off AB = 345, from some convenient scale of equal parts. Make the angle A = 37⅓°. With a radius of 232, taken from the same scale of equal parts, and centre B, cross AC in the two points C, C. Lastly, join BC, BC, and the figure is constructed, which gives two triangles, showing that the case is ambiguous.

Then, the sides AC measured by the scale of equal parts, and the angles B and C measured by the line of chords, or other instrument, will be found to be nearly as below; viz.

AC 174	angle B 27°	angle C 115½°
or 374½	or 78¼	or 64½

Arithmetically.—First, to find the angles at C:

As side	BC 232	log.	2·3654880
To sin. opp. angle A	37° 20′		9·7827958
So side	AB 345		2·5378191
To sin. opp. angle C	115° 36′ or 64° 24		9·9551269

Add angle A	37	20		37	20
The sum	152	56	or	101	44
Taken from	180	00		180	00
Leaves angle B	27	04	or	78	16

Then, to find the side AC:

As sine angle A	37° 20′	log.	9·7827958
To opposite side BC	232		2·365488
So sine angle B	27° 04′		9·6580371
	78 16		9·9908291
To opposite side AC	174·07		2·2407293
or,	374·56		2·5735213

In the plane triangle ABC,

Given, { AB 365 poles
angle A 57° 12′
angle B 24 45 }
Required the other parts.

Ans. { angle C 98° 3′
AC 154·33
BC 309·86 }

In the plane triangle ABC,

Given, { AC 120 feet
BC 112 feet
angle A 57° 27′ }
Required the other parts.

Ans. { angle B 64° 34′ 21″
or, 115 25 39
angle C 57 58 39
or, 7 7 21
AB 112·65 feet
or, 16·47 feet }

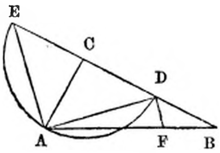
When two sides and their contained angle are given.

Then it will be,

As the sum of those two sides,
Is to the difference of the same sides;
So is the tang. of half the sum of their opposite angles,
To the tang. of half the difference of the same angles.

Hence, because it is known that the half sum of any two quantities increased by their half difference, gives the greater, and diminished by it gives the less, if the half difference of the angles, so found, be added to their half sum, it will give the greater angle, and subtracting it will leave the less angle.

Then, all the angles being now known, the unknown side will be found by the former theorem.



Let ABC be the proposed triangle, having the two given sides AC, BC, including the given angle C. With the centre C, and radius CA, the less of these two sides, describe a semicircle, meeting the other side BC produced in D and E. Join AE, AD, and draw DF parallel to AE.

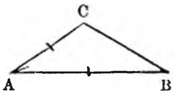
Then, BE is the sum, and BD the difference of the two given sides CB, CA. Also, the sum of the two angles CAB, CBA, is equal to the sum of the two CAD, CDA, these sums being each the supplement of the vertical angle C to two right angles: but the two latter CAD, CDA, are equal to each other, being opposite to the two equal sides CA, CD: hence, either of them, as CDA, is equal to half the sum of the two unknown angles CAB, CBA. Again, the exterior angle CDA is equal to the two interior angles B and DAB; therefore, the angle DAB is equal to the difference between CDA and B, or between CAD and B; consequently, the same angle DAB is equal to half the difference of the unknown angles B and CAB; of which it has been shown that CDA is the half sum.

Now the angle DAE, in a semicircle, is a right angle, or AE is perpendicular to AD; and DF, parallel to AE, is also perpendicular to AD: consequently, AE is the tangent of CDA the half sum and DF the tangent of DAB the half difference of the angles, to the same radius AD, by the definition of a tangent. But, the tangents AE, DF, being parallel, it will be as BE : BD :: AE : DF; that is, as the sum of the sides is to the difference of the sides, so is the tangent of half the sum of the opposite angles, to the tangent of half their difference.

The sum of the unknown angles is found, by taking the given angle from 180°.

In the plane triangle ABC,

Given, {
AB 345 yards
AC 174·07 yards
angle A 37° 20′



Required the other parts.

Geometrically.—Draw AB = 345 from a scale of equal parts. Make the angle A = 37° 20′. Set off AC = 174 by the scale of equal parts. Join BC, and it is done.

Then the other parts being measured, they are found to be nearly as follows, viz. the side BC 232 yards, the angle B 27°, and the angle C 115½°.

Arithmetically.

As sum of sides AB, AC..................	519·07 log.	2·7152259
To difference of sides AB, AC............	170·93	2·2328183
So tangent half sum angles C and B.....	71° 20′	10·4712979
To tangent half difference angles C and B	44 16	9·9888903
Their sum gives angle C	115 36	
Their diff. gives angle B	27 4	

Then, by the former theorem,

As sine angle C 115° 36′, or 64° 24′......log.	9·0551259
To its opposite side AB 345..................	2·5378191
So sine angle A 37° 20′......................	9·7827958
To its opposite side BC 232.................	2·3654890

In the plane triangle ABC,

Given, {
AB 365 poles
AC 154·33
angle A 57° 12′

Required the other parts.

{
BC 309·86
angle B 24° 45′
angle C 98° 3′

In the plane triangle ABC,

Given, {
AC 120 yards
BC 112 yards
angle C 57° 58′ 39″

Required the other parts.

{
AB 112·65
angle A 57° 27′ 0″
angle B 64 34 21

When the three sides of the triangle are given.

Then, having let fall a perpendicular from the greatest angle upon the opposite side, or base, dividing it into two segments, and the whole triangle into two right-angled triangles; it will be,

As the base, or sum of the segments,
Is to the sum of the other two sides;
So is the difference of those sides,
To the difference of the segments of the base.

Then, half the difference of the segments being added to the half sum, or the half base, gives the greater segment; and the same subtracted gives the less segment.

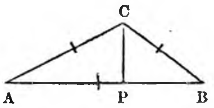
Hence, in each of the two right-angled triangles, there will be known two sides, and the angle opposite to one of them; consequently, the other angles will be found by the first problem.

The rectangle under the sum and difference of the two sides, is equal to the rectangle under the sum and difference of the two segments. Therefore, by forming the sides of these rectangles into a proportion, it will appear that the sums and differences are proportional, as in this theorem.

In the plane triangle ABC,

Given, the sides $\begin{cases} \text{AB } 345 \text{ yards} \\ \text{AC } 232 \\ \text{BC } 174{\cdot}07 \end{cases}$

To find the angles.



Geometrically.—Draw the base AB = 345 by a scale of equal parts. With radius 232, and centre A, describe an arc; and with radius 174, and centre B, describe another arc, cutting the former in C. Join AC, BC, and it is done.

Then, by measuring the angles, they will be found to be nearly as follows, viz. angle A 27°, angle B 37⅓°, and angle C 115½°.

Arithmetically.—Having let fall the perpendicular CP, it will be,

As the base AB : AC + BC :: AC − BC : AP − BP
that is, as 345 : 406·07 :: 57·93 : 68·18 = AP − BP
its half is.......................... 34·09
the half base is..................172·50
the sum of these is...............206·59 = AP
and their difference..............138·41 = BP

Then, in the triangle APC, right-angled at P,

As the side AC.....................	232	........log. 2·3654880
To sine opposite angle...........	90°	.... 10·0000000
So is side AP.......................	206·59	.... 2·3151093
To sine opposite angle ACP.....	62° 56′	.... 9·9496213
Which taken from...........	90 00	
Leaves the angle A.........	27 04	

Again, in the triangle BPC, right-angled at P,

As the side of BC..........	174·07	log. 2·2407239
To sine opposite angle P...	90°	10·0000000
So is side BP................	138·41	2·1411675
To sin. opposite angle BCP	52° 40′.........	9·9004436
Which taken from.....	90 00	
Leaves the angle B...	37 20	
Also, the angle ACP...	62° 56′	
Added to angle BCP...	52 40	
Gives the whole angle ACB...	115 36	

So that all the three angles are as follow, viz. the angle A 27° 4′; the angle B 37° 20′; the angle C 115° 36′.

In the plane triangle ABC,

Given the sides, AB 365 poles, AC 154·33, BC 309·86

To find the angles. angle A 57° 12′, angle B 24 45, angle C 98 3

In the plane triangle ABC,

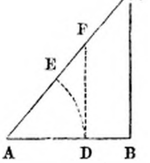
Given the sides, AB 120, AC 112·65, BC 112

To find the angles. angle A 57° 27′ 00″, angle B 57 58 39, angle C 64 34 21

The three foregoing theorems include all the cases of plane triangles, both right-angled and oblique; besides which, there are other theorems suited to some particular forms of triangles, which are sometimes more expeditious in their use than the general ones; one of which, as the case for which it serves so frequently occurs, may be here taken, as follows:—

When, in a right-angled triangle, there are given one leg and the angles; to find the other leg or the hypothenuse; it will be,

As radius, *i. e.* sine of 90° or tangent of 45°
Is to the given leg,
So is the tangent of its adjacent angle
To the other leg;
And so is the secant of the same angle
To the hypothenuse.



AB being the given leg, in the right-angled triangle ABC; with the centre A, and any assumed radius, AD, describe an arc DE, and draw DF perpendicular to AB, or parallel to BC. Now it is evident, from the definitions, that DF is the tangent, and AF the secant, of the arc DE, or of the angle A which is measured by that arc, to the radius AD. Then, because of the parallels BC, DF, it will be as AD : AB :: DF : BC :: AF : AC, which is the same as the theorem is in words.

In the right-angled triangle ABC,

$$\text{Given}\left\{\begin{matrix}\text{the leg AB } 162\\ \text{angle A } 53^\circ\ 7'\ 48''\end{matrix}\right\}\text{ to find AC and BC.}$$

Geometrically.—Make AB = 162 equal parts, and the angle A = 53° 7′ 48″; then raise the perpendicular BC, meeting AC in C. So shall AC measure 270, and BC 216.

Arithmetically.

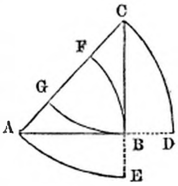
As radius	tang. 45°	log.	10·0000000
To leg AB	162		2·2095150
So tang. angle A	53° 7′ 48″		10·1249371
To leg BC	216		2·3344521
So secant angle A	53° 7′ 48″		10·2218477
To hyp. AC	270		2·4313627

In the right-angled triangle ABC,

$$\text{Given}\left\{\begin{matrix}\text{the leg AB } 180\\ \text{the angle A } 62^\circ\ 40'\end{matrix}\right.$$

To find the other two sides. $\left\{\begin{matrix}\text{AC } 392{\cdot}0147\\ \text{BC } 348{\cdot}2464\end{matrix}\right.$

There is sometimes given another method for right-angled triangles, which is this:



ABC being such a triangle, make one leg AB radius, that is, with centre A, and distance AB, describe an arc BF. Then it is evident that the other leg BC represents the tangent, and the hypothenuse AC the secant, of the arc BF, or of the angle A.

In like manner, if the leg BC be made radius; then the other leg AB will represent the tangent, and the hypothenuse AC the secant, of the arc BG or angle C.

But if the hypothenuse be made radius; then each leg will represent the sine of its opposite angle; namely, the leg AB the sine of the arc AE or angle C, and the leg BC the sine of the arc CD or angle A.

And then the general rule for all these cases is this, namely, that the sides of the triangle bear to each other the same proportion as the parts which they represent.

And this is called, Making every side radius.

OF HEIGHTS AND DISTANCES.

By the mensuration and protraction of lines and angles, are determined the lengths, heights, depths, and distances of bodies or objects.

Accessible lines are measured by applying to them some certain measure a number of times, as an inch, or foot, or yard. But inaccessible lines must be measured by taking angles, or by some such method, drawn from the principles of geometry.

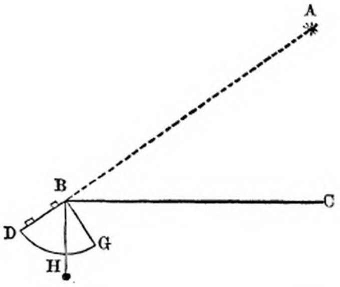
When instruments are used for taking the magnitude of the angles in degrees, the lines are then calculated by trigonometry: in the other methods, the lines are calculated from the principle of similar triangles, without regard to the measure of the angles.

Angles of elevation, or of depression, are usually taken either with a theodolite, or with a quadrant, divided into degrees and minutes, and furnished with a plummet suspended from the centre, and two sides fixed on one of the radii, or else with telescopic sights.

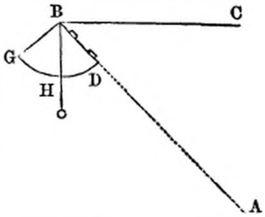
To take an angle of altitude and depression with the quadrant.

Let A be any object, as the sun, moon, or a star, or the top of a tower, or hill, or other eminence; and let it be required to find the measure of the angle ABC, which a line drawn from the object makes with the horizontal line BC.

Fix the centre of the quadrant in the angular point, and move it round there as a centre, till with one eye at D, the other being shut, you perceive the object A through the sights: then will the arc GH of the quadrant, cut off by the plumb line BH, be the measure of the angle ABC, as required.



The angle ABC of depression of any object A, is taken in the same manner; except that here the eye is applied to the centre, and the measure of the angle is the arc GH, on the other side of the plumb line.



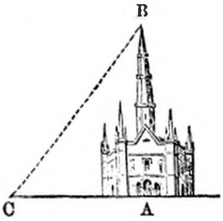
The following examples are to be constructed and calculated by the foregoing methods, treated of in trigonometry.

Having measured a distance of 200 feet, in a direct horizontal line, from the bottom of a steeple, the angle of elevation of its top, taken at that distance, was found to be 47° 30′: from hence it is required to find the height of the steeple.

Construction.—Draw an indefinite line, upon which set off AC = 200 equal parts, for the measured distance. Erect the indefinite perpendicular AB; and draw CB so as to make the angle C = 47° 30′, the angle of elevation; and it is done. Then AB, measured on the scale of equal parts, is nearly 218¼.

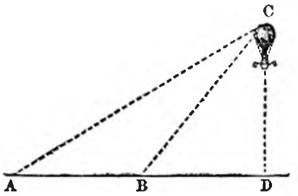
Calculation.

As radius	10·0000000
To AC 200	2·3010300
So tang. angle C 47° 30′	10·0379475
To AB 218·26 required	2·3389775



What was the perpendicular height of a cloud, or of a balloon, when its angles of elevation were 35° and 64°, as taken by two observers, at the same time, both on the same side of it, and in the same vertical plane; their distance, as under, being half a mile, or 880 yards. And what was its distance from the said two observers?

Construction.—Draw an indefinite ground line, upon which set off the given distance AB = 880; then A and B are the places of the observers. Make the angle A = 35°, and the angle B = 64°; and the intersection of the lines at C will be the place of the balloon; from whence the perpendicular CD, being let fall, will be its perpendicular height. Then, by measurement, are found the distances and height nearly, as follows, viz. AC 1631, BC 1041, DC 936.



Calculation.

First, from angle B 64°
Take angle A 35
Leaves angle ACB 29

Then, in the triangle ABC,

As sine angle ACB	29°	9·6855712
To opposite side AB	880	2·9444827
So sine angle A	35°	9·7585913
To opposite side BC	1041·125	3·0175028

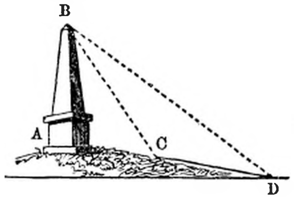
As sine angle ACB	29°	9·6855712
To opposite side AB	880	2·9444827
So sine angle B	116° or 64°	9·9536602
To opposite side AC	1631·442	3·2125717

And, in the triangle BCD,

As sine angle D	90°	10·0000000
To opposite side BC	1041·125	3·0175028
So sine angle B	64°	9·9536602
To opposite side CD	935·757	2·9711630

Having to find the height of an obelisk standing on the top of a declivity, I first measured from its bottom, a distance of 40 feet, and there found the angle, formed by the oblique plane and a line imagined to go to top of the obelisk 41°; but, after measuring on in the same direction 60 feet farther, the like angle was only 23° 45′. What then was the height of the obelisk?

Construction.—Draw an indefinite line for the sloping plane or declivity, in which assume any point A for the bottom of the obelisk, from whence set off the distance AC = 40, and again CD = 60 equal parts. Then make the angle C = 41°, and the angle D = 23° 45′; and the point B, where the two lines meet, will be the top of the obelisk. Therefore AB, joined, will be its height.



Calculation.

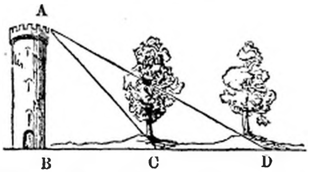
From the angle C 41° 00′
Take the angle D 23 45
Leaves the angle DBC 17 15

Then, in the triangle DBC,
As sine angle DBC 17° 15′.................. 9·4720856
To opposite side DC 60 .... 1·7781513
So sine angle D 24 45.................. 9·6050320
To opposite side CB 81·488.................. 1·9110977

And, in the triangle ABC,
As sum of sides CB, CA 121·488 ..... 2·0845333
To difference of sides CB, CA 41·488 ..... 1·6179225
So tang. half sum angles A, B 69° 30′ .....10·4272623
To tang. half diff. angles A, B 42 24½..... 9·9606516
The diff. of these is angle CBA 27 5½
Lastly, as sine angle CBA 27° 5½′............... 9·6582842
To opposite side CA 40 .... 1·6020600
So sine angle C 41° 0′ .... 9·8169429
To opposite side AB 57·623 .... 1·7607187

Wanting to know the distance between two inaccessible trees, or other objects, from the top of a tower, 120 feet high, which lay in the same right line with the two objects, I took the angles formed by the perpendicular wall and lines conceived to be drawn from the top of the tower to the bottom of each tree, and found them to be 33° and 64½°. What then may be the distance between the two objects?

Construction.—Draw the indefinite ground line BD, and perpendicular to it BA = 120 equal parts. Then draw the two lines AC, AD, making the two angles BAC, BAD, equal to the given angles 33° and 64½°. So shall C and D be the places of the two objects.



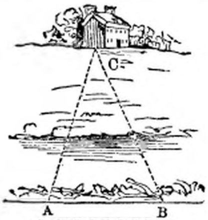
Calculation.—First, In the right-angled triangle ABC,
As radius..10·0000000
To AB....................120 .... 2·0791812
So tang. angle BAC..... 33°.................... 9·8125174
To BC.................77·929 .... 1·8916986

And, in the right-angled triangle ABD,
As radius..10·0000000
To AB....................120 .... 2·0791812
So tang. angle BAD.... 64½°....................10·3215039
To BD...............251·585 .... 2·4006851

From which take BC 77·929
Leaves the dist. CD 173·656 as required.

Being on the side of a river, and wanting to know the distance to a house which was seen on the other side, I measured 200 yards in a straight line by the side of the river; and then at each end of this line of distance, took the horizontal angle formed between the house and the other end of the line; which angles were, the one of them 68° 2′, and the other 73° 15′. What then were the distances from each end to the house?

Construction.—Draw the line AB = 200 equal parts. Then draw AC so as to make the angle A = 68° 2′, and BC to make the angle B = 73° 15′. So shall the point C be the place of the house required.



Calculation.

To the given angle A	68°	2′
Add the given angle B	73	15
Then their sum	141	17
Being taken from	180	0
Leaves the third angle C	38	43

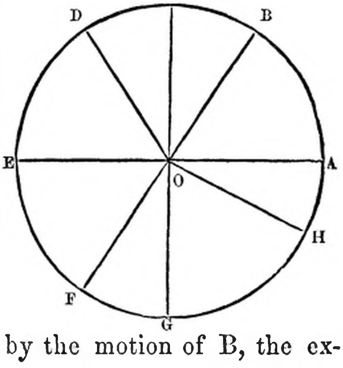
Hence, As sin. angle C 38° 43′.................9·7962062
To op. side AB 200 ....2·3010300
So sin. angle A 68° 2′.................9·9672679
To op. side BC 296·54 ....2·4720917

And, As sin. angle C 38° 43′.................9·7962062
To op. side AB 200 ....2·3010300
So sin. angle B 73° 15′.................9·9811711
To op. side AC 306·19 ....2·4859949

SPHERICAL TRIGONOMETRY.

This Article is taken from a short Practical Treatise on Spherical Trigonometry, by Oliver Byrne, the author of the present work. Published by J. A. Valpy. London, 1835.

As the sides and angles of spherical triangles are measured by circular arcs, and as these arcs are often greater than 90°, it may be necessary to mention one or two particulars respecting them.



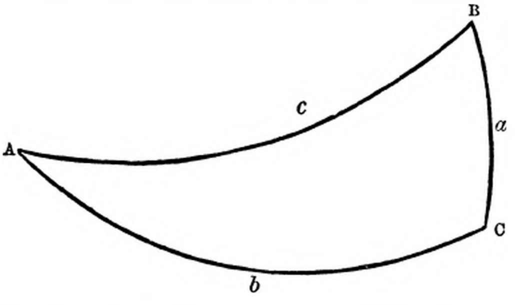
The arc CB, which when added to AB makes up a quadrant or 90°, is called the complement of the arc AB; every arc will have a complement, even those which are themselves greater than 90°, provided we consider the arcs measured in the direction ABCD, &c., as positive, and consequently those measured in the opposite direction as negative. The complement BC of the arc AB commences at B, where AB terminates, and may be considered as generated by the motion of B, the extremity of the radius OB, in the direction BC. But the complement of the arc AD or DC, commencing in like manner at the extremity D, must be generated by the motion of D in the opposite direction, and the angular magnitude AOD will here be diminished by the motion of OD, in generating the complement; therefore the complement of AOD or of AD may with propriety be considered negative.

Calling the arc AB or AD, θ, the complement will be $90° - \theta$; the complement of 36° 44′ 33″ is 53° 15′ 27″; and the complement of 136° 27′ 39″ is negative 46° 27′ 39″.

The arc BE, which must be added to AB to make up a semicircle or 180°, is called the *supplement* of the arc AB. If the arc is greater than 180°, as the arc ADF its supplement, FE measured in the reverse direction is negative. The expression for the supplement of any arc θ is therefore $180° - \theta$; thus the supplement of 112° 29′ 35″ is 67° 30′ 25″, and the supplement of 205° 42′ is negative 25° 42′.

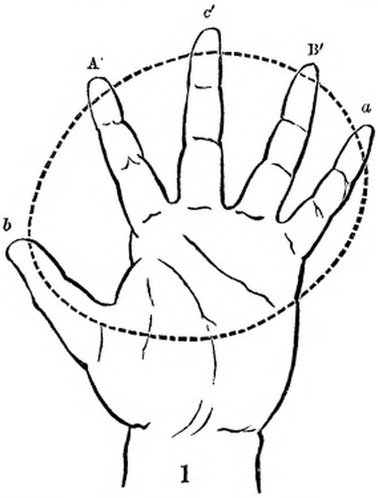
In the same manner as the complementary and supplementary arcs are considered as positive or negative, according to the direction in which they are measured, so are the arcs themselves positive or negative; thus, still taking A for the commencement, or *origin*, of the arcs, as AB is positive, AH will be negative. *In the doctrine of triangles*, we consider only positive angles or arcs, and the magnitudes of these are comprised between $\theta = 0$ and $\theta = 180°$; but in the general theory of angular quantity, we consider both positive and negative angles, according as they are situated above or below the fixed line AO, from which they are measured, that is, according as the arcs by which they are estimated are positive or negative. Thus the angle BOA is positive, and the angle AOH negative. Moreover, in this more extended theory of angular magnitude, an angle may consist of any number of degrees whatever; thus, if the revolving line OB set out from the fixed line OA, and make n revolutions and a part, the angular magnitude generated is measured by n times 360°, plus the degrees in the additional part.

In a right-angled spherical triangle we are to recognise but five parts, namely, the three sides a, b, c, and the two angles A, B; so that the right angle C is omitted.



Let A', c', B', be the complements of A, c, B, respectively, and suppose b, a, B', c', A', to be placed on the hand, as in the annexed figure, and that the fingers stand in a circular order, the parts represented by the fingers thus placed are called circular parts.

If we take any one of these as a middle part, the two which lie next to it, one on each side, will be *adjacent* parts. The two parts immediately beyond the adjacent parts, one on each side, are called the opposite parts.



Thus, taking A' for a middle part, b and c' will be *adjacent* parts, and a and B' are opposite parts.

If we take c' as a middle part, A' and B' are adjacent parts, and b, a, opposite parts.

When B' is a middle part, c', a, become adjacent parts, and A', b, opposite parts.

Again, if we take a as a middle part, then B', b, will be adjacent parts, and c', A', opposite parts.

Lastly, taking b as a middle part, A', a, are adjacent parts, and c', B', opposite parts.

This being understood, Napier's two rules may be expressed as follows:—

I. Rad. × sin. middle part = product of tan. adjacent parts.

II. Rad. × sin. middle part = product of cos. opposite parts.

Both these rules may be comprehended in a single expression, thus,

Rad. sin. mid. = prod. tan. adja. = prod. cos. opp.;

and to retain this in the memory we have only to remember, that the vowels in the contractions *sin.*, *tan.*, *cos.*, are the same as those in the contractions *mid.*, *adja.*, *opp.*, to which they are joined.

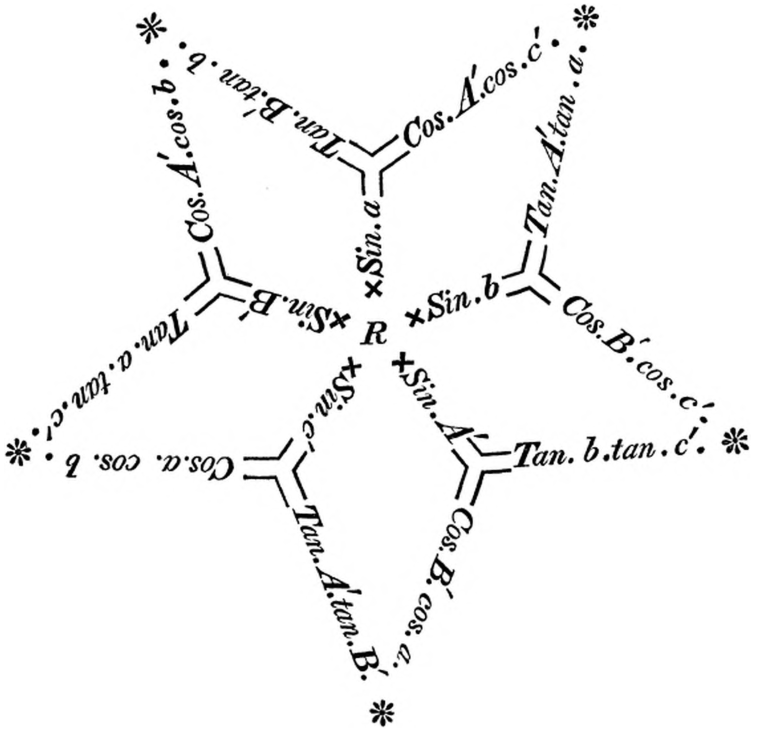
These rules comprehend all the succeeding equations, reading from the centre, R = radius.

In the solution of right-angled spherical triangles, two parts are given to find a third, therefore it is necessary, in the application of this formula, to choose for the middle part that which causes the other two to become either adjacent parts or opposite parts.

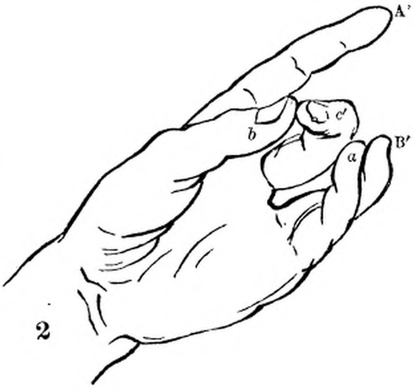
In a right-angled spherical triangle, the hypothenuse

$c = 61° \; 4' \; 56''$; and the angle
$A = 61° \; 50' \; 29''$. Required the adjacent leg?

$$\begin{array}{rrrr|rrrr}
 & 90° & 0' & 00'' & & 90° & 0' & 00'' \\
= & 61 & 4 & 56 & A = & 61 & 50 & 29 \\
\hline
 & 28 & 55 & 04 = c'. & & 28 & 9 & 31 = A.
\end{array}$$


Cos.A′.cos.c′
Tan.A′.tan.a
×Sin.b
Cos.B′.cos.c′
Tan.b.tan.c′
×Sin.A′
Cos.B′.cos.a
Tan.A′.tan.B′
×Sin.c′
Cos.a.cos.b
Tan.a.tan.c′
×Sin.B′
Cos.A′.cos.b
Tan.B′.tan.b
×Sin.a
R

In this example, A′ is selected for the middle part, because then *b* and *c*′ become adjacent parts, as in the annexed figure.



$$\text{Rad.} \times \sin. A' = \tan. b \times \tan. c'.$$

$$\therefore \tan. b = \frac{\text{rad.} \times \sin. A'}{\tan c'}.$$

By Logarithms.

Rad. —..........—10·0000000
Sin. A′—28°9′21″— 9·6738628
19·6738628
Tan. *c*′—28°55′4″— 9·7422808
Tan.*b*′—40°30′16″—9·9315820

The side adjacent to the *given* angle is acute or obtuse, according as the hypothenuse is of the same, or of different species with the *given* angle.

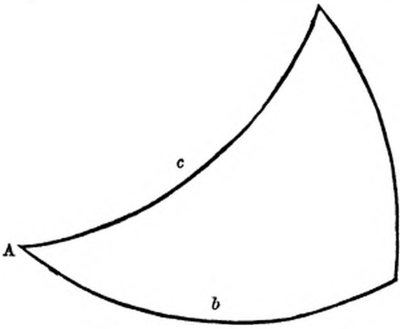
∴ the leg *b* = 40° 30′ 16″, acute.

Supposing the hypothenuse *c* = 113° 55′, and the angle A = 31° 51′, then the adjacent leg *b* would be 117° 34′, obtuse.

In the right-angled spherical triangle ABC, are given the hypothenuse $c = 113° 55'$, and the angle $A = 104° 08'$; to find the opposite leg a.

$$\begin{array}{r}c = 113°\ 55' \\ 90\quad 0 \\ \hline 23\quad 55 = c'.\end{array}$$

$$\begin{array}{r}A = 104°\ 08' \\ 90\quad 0 \\ \hline 14\quad 08 = A'.\end{array}$$



In this example, a is taken for the middle part, then A' and c' are opposite parts. (See the subjoined figure.)

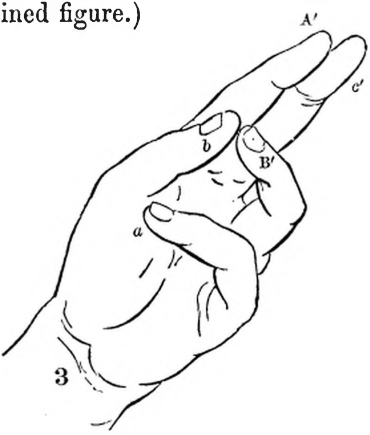
From the general formula, we have,

$$\text{Rad.} \times \text{sin. } a = \text{cos. } A' \times \text{cos. } c'.$$

$$\therefore \text{sin. } a = \frac{\text{cos. } A' \times \text{cos. } c'}{\text{Rad.}}.$$

By Logarithms.

$$\begin{array}{lr}\text{cos. } A' - 14°\ 08' \ldots\ldots & 9\cdot9860509 \\ \text{cos. } c' - 23\quad 55 \ldots\ldots & 9\cdot9610108 \\ \hline & 19\cdot9476617 \\ \text{Radius} \ldots\ldots & 10\cdot0000000 \\ \hline \text{sin. } a \left\{\begin{array}{l}117°\ 34' \\ \ \ 62\quad 26\end{array}\right\} \ldots & 9\cdot9476617\end{array}$$



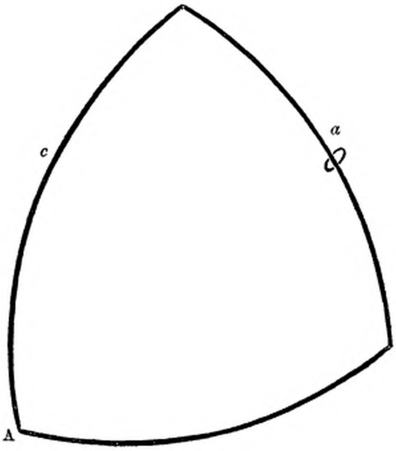
The obtuse side 117° 34′ is the leg required, for the side opposite to the given angle is always of the same species with the given angle.

If in a right-angled spherical triangle, the hypothenuse were 78° 20′, and the angle $A = 37° 25'$, then the opposite leg $a = 36° 31'$, and not 143° 29′, because the given angle is acute.

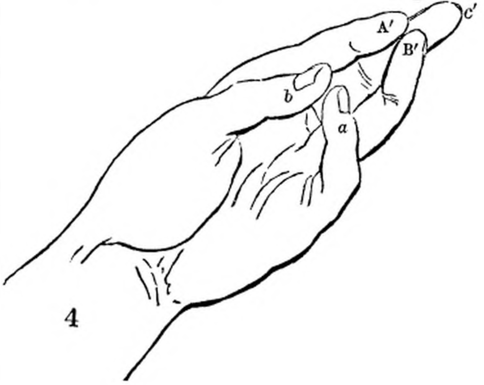
In a right-angled spherical triangle, are given $c = 78° 20'$, and $A = 37° 25'$, to find the angle B.

$$\begin{array}{r}90°\quad 0' \\ c = 78\quad 20 \\ \hline 11\quad 40 = c'.\end{array}$$

$$\begin{array}{r}90°\quad 0' \\ A = 37\quad 25 \\ \hline 52\quad 35 = A'\end{array}$$



Here the complement of the hypothenuse (c') is the *middle part;* and the complement of the angle opposite the perpendicular (A'), and the complement of the angle opposite the base (B') are the *adjacent* parts. This will readily be perceived by reference to the usual figure in the margin.



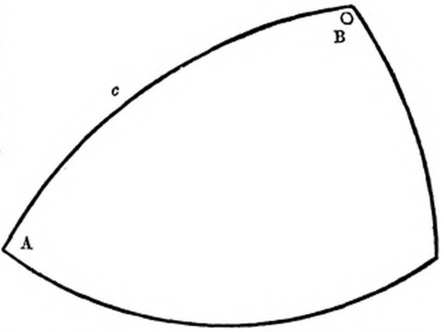
4

$\text{Rad.} \times \sin. c' = \tan. A' \times \tan. B'$;

$$\therefore \tan. B' = \frac{\text{Rad.} \times \sin. c'}{\tan. A'}.$$

By Logarithms.

Rad.................10·0000000
sin. c' — 11° 40′. 9·3058189
19·3058189
tan. A' — 52° 35′ 10·1163279
∴ tan. B' — 8° 48′ 9·1894910
But 90 — B = B′
hence 90 — B′ = B.
90° 0′
8 48
B = 81° 12′.



When the hypothenuse and an angle are given, the other angle is acute or obtuse, according as the given parts are of the same or of different species.

In the above example, both the given parts are acute, therefore the required angle is *acute;* but if one be acute and the other obtuse, then the angle found would be obtuse:—Thus, if the hypothenuse be 113° 55′, and the angle A = 31° 51′; then will B′ = 14° 08′, and the angle B = 104° 08′.

Given the hypothenuse c = 61° 04′ 56″, and the side or leg, a = 40° 30′ 20″, to find the angle adjacent to a.

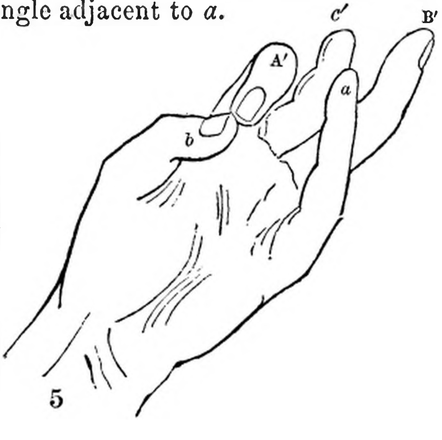
90° 0′ 0″
c = 61 04 56
28 55 04″ = c''.

The three parts are here connected; therefore the complement of B is the *middle part,* a and the complement of c are the adjacent parts.

Hence we have,

$\text{Rad.} \times \sin. B' = \tan. a \times \tan. c'$.

$$\therefore \sin. B' = \frac{\tan. a \times \tan. c'}{\text{Rad.}}$$



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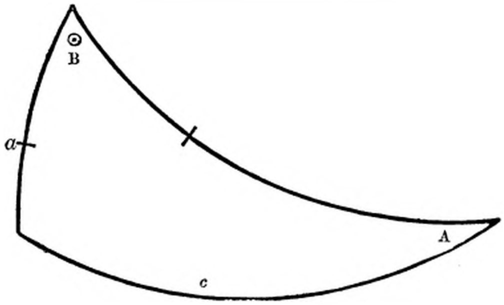


By Logarithms.

tan. a — 40° 30′ 20″ = 9·9315841
tan. c' — 28 55 04 = 9·7422801

19·6738642
Rad..........................10·0000000

sin. B′....28° 09′ 31″...... 9·6738642

$$\begin{array}{r} 90^\circ\ \ 0'\ \ 0'' \\ B' = \underline{28\ \ 09\ \ 31} \\ 61\ \ 50\ \ 29 = B. \end{array}$$

The angle adjacent to the given side is acute or obtuse according as the hypothenuse is of the same or of different species with the given side.

Before working the above example, it was easy to foresee that the angle B would be acute; but suppose the hypothenuse = 70° 20′, and the side a = 117° 34′, then the angle B would be obtuse, because a and c are of different species.

Rule V.—In a spherical triangle, right-angled at c, are given c = 78° 20′ and b = 117° 34′, to find the angle B; opposite the given leg, (see the next diagram.)

In this example, b becomes the middle part, and c' and B′ opposite parts; and therefore, by the rule,

Rad. × sin. b = cos. B′ × cos. c'; that is,

$$\cos. B' = \frac{\text{Rad.} \times \sin. b}{\cos. c'}.$$

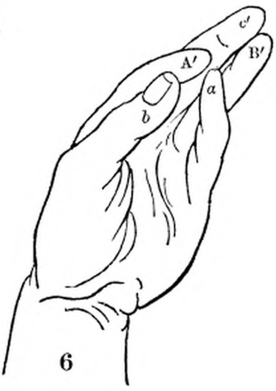
$$90^\circ - 78^\circ\ 20' = 11^\circ\ 40' = c'.$$

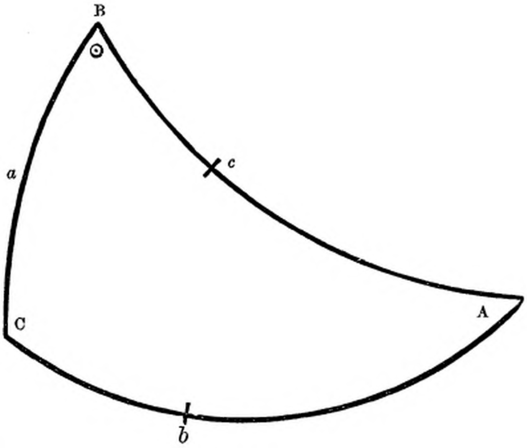
Hence, by Logarithms.

Rad.........................10·0000000
sin. b = sin. 117° 34′ or sin. 62 26 } 9·9476655

19·9476655
cos. c' 11° 40′............ 9·9909338

cos. B′ 25° 09′........... 9·9567317





But since the angle opposite the given side is of the same species with the given side, 90° must be added to B′, to produce B:—viz. $90° + 25° 09' = 115° 09'$.

Given $c = 61° 04' 56''$, and $b = 40° 30' 20''$, to find the other side a.

Here c' is the middle part, a and b the opposite parts; hence by position 4, $a = 50° 30' 30''$.

Given the side $b = 48° 24' 16''$, and the adjacent angle $A = 66° 20' 40''$, to find the side a.

In this instance, b is the middle part, the complement of A and a are adjacent parts. Consequently, $a = 59° 38' 27''$.

In the right-angled spherical triangle ABC,

$$\text{Given} \left\{ \begin{array}{l} \text{The side } a = 59° 38' 27'' \\ \text{Its adjacent angle } B = 52° 32' 55'' \end{array} \right\} \text{to find the angle A.}$$

Answer, 66° 20′ 40″.

The required angle is of the same species as the *given* side, and *vice versa.*

Given the side $b = 49° 17'$, and its adjacent angle $A = 23° 28'$, to find the hypothenuse.

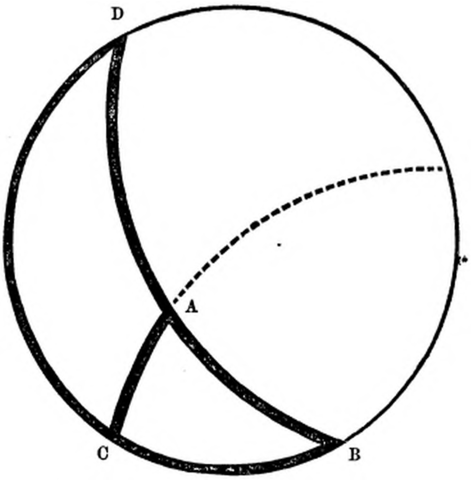
Making A′ the middle part, the others will be adjacent parts, and, therefore, by the first rule we have $c = 51° 42' 37''$.

In a spherical triangle, right-angled at C, are given $b = 29° 12' 50''$, and $B = 37° 26' 21''$, to find the side a.

Taking a for the middle part, the other two will be adjacent parts; hence by the rule,

$$\begin{aligned} \text{Rad.} \times \sin. a &= \tan. b \times \tan. B' \\ \text{that is, rad.} \times \sin. a &= \tan. b \times \cot. B \\ \therefore \sin. a &= \frac{\tan. b \times \cot. B}{\text{rad.}} \end{aligned}$$

In this case, there are two solutions, i. e. a and the supplement of a, because both of them have the same sine. As sin. a is necessarily positive, b and B must necessarily be always of the same species, so that, as observed before, the sides including the right angle are always of the same species as the opposite angles.



In working this example, we find the log. sin. a = 9·8635411, which corresponds to 46° 55′ 02″,

or, 133° 04′ 58″.

It appears, therefore, that a is *ambiguous*, for there exist two right-angled triangles, having an oblique angle, and the opposite side in the one equal to an oblique angle and an opposite side in the other, but the remaining oblique angle in the one the supplement of the remaining oblique angle in the other. These triangles are situated with respect to each other, on the sphere, as the triangles ABC, ADC, in the annexed diagram, in which, with the exception of the common side AC, and the equal angles B, D, the parts of the one triangle are supplements of the corresponding parts of the other.

In a right-angled spherical triangle are

$$\text{Given}\left\{\begin{array}{l}\text{the side } a \ldots\ldots\ldots\ldots = 42^\circ\ 12', \\ \text{its opposite angle A} = \ 48^\circ\end{array}\right\}\text{to find the adjacent angle B.}$$

The complement of the given angle is the middle part; and neither a nor B′ being joined to A′, they are consequently opposite parts; hence, the angle B = 64° 35′, or 115° 25′; this case, like the last, being ambiguous, or doubtful.

Given a = 11° 30′, and A = 23° 30′, to find the hypothenuse c.

c = 30°, or 150°, being ambiguous.

In a right-angled triangle, there are given the two perpendicular sides, viz. a = 48° 24′ 16″, b = 59° 38′ 27″, to find the angle A.

A = 66° 20′ 40″.

Given a = 142° 31′, b = 54° 22′, to find c.

c = 117° 33′.

$$\text{Given}\left\{\begin{array}{l}\text{A} = 37^\circ\ 25' \\ \text{B} = \ 81\ 12\end{array}\right\}\text{Required the side } a.$$

a = 36° 31′.

$$\text{Given}\left\{\begin{array}{l}\text{A} = 66^\circ\ 20'\ 40'' \\ \text{B} = 52\ \ 32\ \ 55\end{array}\right\}\text{to find the hypothenuse } c.$$

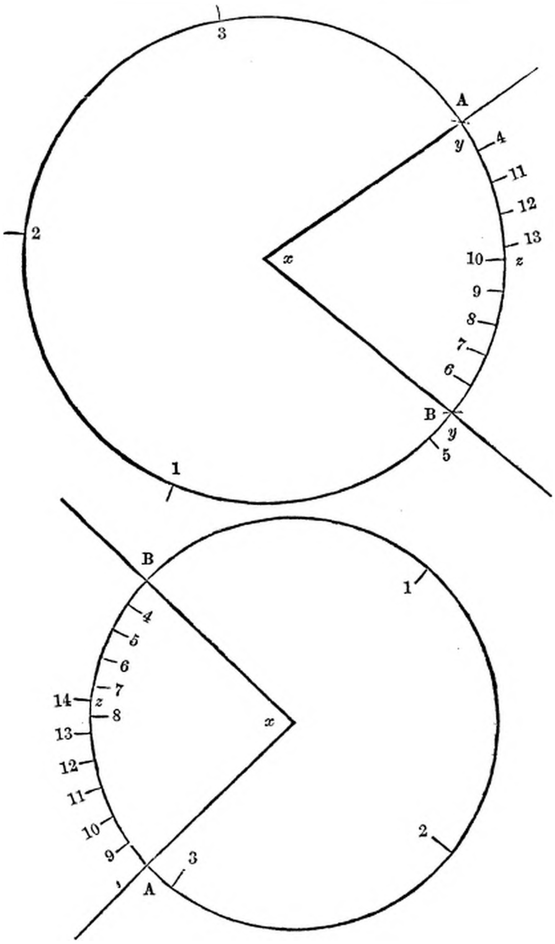
c = 70° 23′ 42″.

MEASUREMENT OF ANGLES.

From the "Civil Engineer and Architect's Journal," for Oct. and Nov. 1847.

A NEW METHOD OF MEASURING THE DEGREES, MINUTES, ETC., IN ANY RECTILINEAR ANGLE BY COMPASSES ONLY, WITHOUT USING SCALE OR PROTRACTOR.

Apply AB = x, from B to 1; from 1 to 2; from 2 to 3; from 3 to 4; from 4 to 5. Then take B 5, in the compasses, and apply it from B to 6; from 6 to 7; from 7 to 8; from 8 to 9; and from 9 to 10, near the middle of the arc AB. With the same opening,



B 5 or A 4, or y, which we shall term it, lay off 4,11, 11,12, and 12,13. Then the arc between 13 and 10 is found to be contained 23 times in the arc AB.

Hence, we have,

$$5x - y = 360°;$$
$$9y + z = x;$$
$$23z = x; \text{ or, } z = \frac{x}{23}.$$
$$\therefore 9y + \frac{x}{23} = x, \qquad \therefore y = \frac{22x}{207}.$$

By substituting this value in the first equation, we obtain,

$$5x - \frac{22x}{207} = 360.$$

$$\frac{1013x}{207} = 360, \text{ and } x = \frac{360 \times 207}{1013} = 73° \ 33'{\cdot}82.$$

Apply AB = x, from B to 1; from 1 to 2; from 2 to 3; from 3 to 4. Then take B 4, in the compasses, and apply it on the arc, from B to 4; from 4 to 5; from 5 to 6; from 6 to 7; and from 7 to 8, near the middle of the arc AB. With the same opening, B 4 = y, lay off A 9, 9,10, 10,11, 11,12, 12,13, and 13,14. The arc between 14 and 8 is found to be contained nearly 24 times in the arc AB. Therefore, we have,

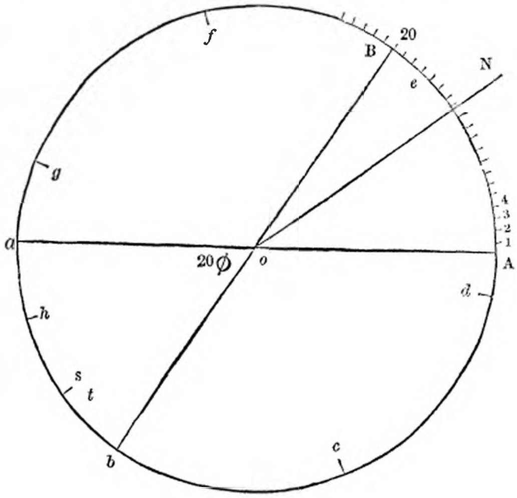
$$4x + y = 360;$$
$$11y - z = x;$$
$$24z = x; \text{ or, } z = \frac{x}{24}.$$
$$\therefore 11y - \frac{x}{24} = x; \qquad \therefore y = \frac{25x}{264}.$$

Substituting this value of y in the first equation,

$$4x + \frac{25x}{264} = 360;$$

$$x = \frac{360 \times 264}{1071} = 88° \ 44'{\cdot}333.$$

How to lay off an angle of any number of degrees, minutes, &c., with compasses only, without the use of scale or protractor.



Let it be required to lay off an angle of 36° 40′ = β. Take any small opening of the compasses less than one-tenth of the radius, and lay off any number of equal small arcs, from A to 1; from 1 to 2; from 2 to 3, &c., until we have laid off an arc, AB, greater than the one required. Draw B b through the centre o, then will the arc $a\,b$ = arc AB, which we shall put $= 20\phi$ in this example, and proceed to measure ab as in the first example. Lay off $a\,b$ from b to c; from c to d; from d to e; from e to f; from f to g. Putting $g\,a = \triangle_1$, then,

$$6 \times 20\phi + \triangle_1 = 360° = \frac{108}{11}\beta\text{; because,}$$

$$\frac{360°}{36°\ 40'} = \frac{21600}{2200} = \frac{108}{11}.$$

Lay off, as before directed, $g\,a, = \triangle_1$, from a to h, from h to s, and b to t; then calling $s\,t$, $\triangle_2$, we have

$$3\triangle_1 + \triangle_2 = 20\phi;$$

and we find that $s\,t$ is contained 28 times in the arc $a\,b$;

$$\therefore 120\phi + \triangle_1 = \frac{108}{11}\beta;\ 3\triangle_1 + \triangle_2 = 20\phi;\ \text{and}\ 28\triangle_2 = 20\phi.$$

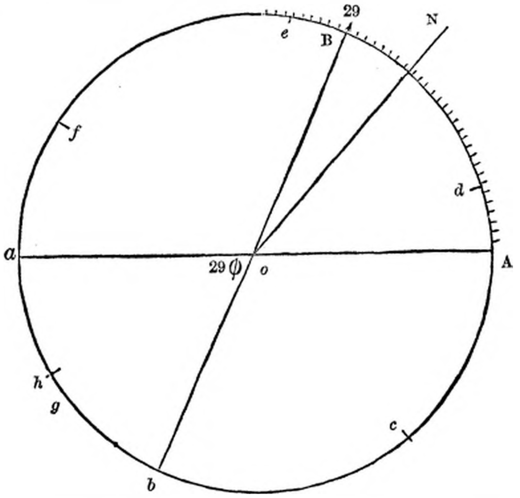
Eliminating $\triangle_1$ and $\triangle_2$, we find

$$\beta = \frac{29205}{2268}\phi = 12{\cdot}9 \text{ times } \phi \text{ nearly;}$$

$\therefore 36°\ 40' = \angle\, \mathrm{A}\,o\,\mathrm{N}$ is laid off with as much ease and certainty as by a protractor.

As a second example, let it be required to lay off an angle of 132° 27′. From 180° 0′ take 132° 27′ = 47° 33′, which put $= \beta$.

$$\frac{360°}{47°\ 33'} = \frac{2400}{317}\ \text{when put} = \frac{v}{\delta},\ \text{then}\ \frac{v}{\delta}\beta = 360° = \pi.$$



We have laid off 29 small arcs from A to B; $29 = \varepsilon$. $\mathrm{AB} = a\,b = b\,c = c\,d = d\,e = c\,f$. And $a\,g = b\,h = a\,f = \triangle_1$; $h\,g = \triangle_2$.

$$\therefore 5 \times 29\phi + \triangle_1 = 360° = \frac{v}{\delta}\beta = m\,e\,\phi \pm \triangle_1 \qquad (1)$$

$$2\triangle_1 - \triangle_2 = 29\phi,\ \text{or}\ n\triangle_1 \pm \triangle_2 = \varepsilon\,\phi \qquad (2)$$

$$13\triangle_2 = 29\phi,\quad \text{or}\quad q\triangle_2 = \varepsilon\,\phi \qquad (3)$$

Eliminating $\triangle_1$ and $\triangle_2$, we have $\beta = \frac{\{mnq \pm (q \mp 1)\}\epsilon\delta}{\nu n q}\phi = \frac{\{5\cdot2\cdot13 + (13 + 1)\}29\cdot317}{2400\cdot2\cdot13}\phi = \frac{1323729}{62400}\phi = 21\frac{1}{4}$ times ϕ very nearly. Hence the line o N determines the angle $a\,o$ N = 132° 27′.

In the expression

$$\beta = \frac{\{mnq \pm (q \mp 1)\}\epsilon\delta}{\nu n q}\phi \quad \text{(R)}$$

substituting the numerals of the first example, then

$$\beta = \frac{\{6\cdot3\cdot28 + (28 - 1)\}20\cdot11}{108\cdot3\cdot28}\phi = \frac{29205}{2268}\phi = 12\cdot9 \text{ times } \phi \text{ nearly,}$$

the result before obtained.

The ambiguous signs of (R) cannot be mistaken or lead to error, if the manner in which it is deduced from (1), (2), (3), be attended to. From (3)

$$\triangle_2 = \frac{\epsilon\phi}{q}; \text{ substituting this value of } \triangle_2, \text{ in (2),}$$

$$n\,\triangle_1 = \epsilon\phi \mp \triangle_2 = \epsilon\phi \mp \frac{\epsilon\phi}{q}; \text{ which, when substituted for } \triangle_1$$

in (1), gives

$$\frac{\nu}{\delta}\beta = m\,\epsilon\phi \pm \frac{1}{n}\left(\epsilon\phi \mp \frac{\epsilon\phi}{q}\right); \text{ from which (R) is found.}$$

This method of measuring angles is more exact than it may appear; for if, in the first example, we take

$$5x - y = 360;\ 9y + z = x;\ \text{and } 20z = x,$$

$$\text{then } x = \frac{64800}{881} = 73°\ 33'\ 85.$$

The first equations gave 73° 33′ 82 when $23z = x$, so it does not matter much whether 20, 21, 22, 23, 24, or 25 times z make x. This fact is particularly worth attention.

Given the three angles to find the three sides.

The following formulas give any side a of any spherical triangle.

$$\sin.\ \tfrac{1}{2}a = \sqrt{\frac{-\cos.\ \tfrac{1}{2}\mathrm{S}\ \cos.\ (\tfrac{1}{2}\mathrm{S} - \mathrm{A})}{\sin.\ \mathrm{B}\ \sin.\ \mathrm{C}}}, \text{ and}$$

$$\cos.\ \tfrac{1}{2}a = \sqrt{\frac{\cos.\ (\tfrac{1}{2}\mathrm{S} - \mathrm{B})\ \cos.\ (\tfrac{1}{2}\mathrm{S} - \mathrm{C})}{\sin.\ \mathrm{B}\ \sin.\ \mathrm{C}.}}$$

Given the three sides to find the three angles.

$$\sin.\ \tfrac{1}{2}\mathrm{A} = \sqrt{\frac{\sin.\ (\tfrac{1}{2}\mathrm{S} - b)\ \sin.\ (\tfrac{1}{2}\mathrm{S} - c)}{\sin.\ b\ \sin.\ c.}}$$

$$\cos.\ \tfrac{1}{2}\mathrm{A} = \sqrt{\frac{\sin.\ \tfrac{1}{2}\mathrm{S}\ \sin.\ (\tfrac{1}{2}\mathrm{S} - a)}{\sin.\ b\ \sin.\ c.}}$$

GRAVITY—WEIGHT—MASS.

SPECIFIC GRAVITY, CENTRE OF GRAVITY, AND OTHER CENTRES OF BODIES. —WEIGHTS OF ENGINEERING AND MECHANICAL MATERIALS.—BRASS, COPPER, STEEL, IRON, WATER, STONE, LEAD, TIN, ROUND, SQUARE, FLAT, ANGULAR, ETC.

1. IN a second, the acceleration of a body falling freely in vacuo is 32·2 feet; what velocity has it acquired at the end of 5 seconds?

$$32{\cdot}2 \times 5 = 161 \text{ feet, the velocity.}$$

2. A cylinder rolling down an inclined plane with an initial velocity of 24 feet a second, and suppose it to acquire each second 5 additional feet velocity; what is its velocity at the end of 3·7 seconds?

$$24 + 3{\cdot}7 \times 5 = 42{\cdot}5 \text{ feet.}$$

3. Suppose a locomotive, moving at the rate of 30 feet a second, (as it is usually termed, with a 30 feet velocity,) and suppose it to lose 5 feet velocity every second; what is its velocity at the end of 3·33 seconds?

The acceleration is — 3·33, negative.

$$\therefore 30 - 5 \times 3{\cdot}33 = 13{\cdot}35 \text{ feet.}$$

4. If a body has acquired a velocity of 36 feet in 11 seconds, by uniformly accelerated motion; what is the space described?

$$\frac{36 \times 11}{2} = 198 \text{ feet.}$$

5. A carriage at rest moves with an accelerated motion over a space of 200 feet in 45 seconds; at what velocity does it proceed at the beginning of the 46th second?

$$\frac{200 \times 2}{45} = 8{\cdot}8889 \text{ feet, the velocity at the end of the 45th second.}$$

The four fundamental formulas of uniformly accelerated motion are

$$v = p\,t; \quad s = \frac{v\,t}{2}; \quad s = \frac{p\,t^2}{2}; \quad s = \frac{v^3}{2p}.$$

v the velocity, p the acceleration, t the time, and s the space.

6. What space will a body describe that moves with an acceleration of 11·5 feet for 10 seconds.

$$\frac{11{\cdot}5 \times (10)^2}{2} = 575 \text{ feet.}$$

7. A body commences to move with an acceleration of 5·5 feet, and moves on until it is moving at the rate of 100 feet a second; what space has it described?

$$\frac{(100)^2}{2 \times 5{\cdot}5} = 909{\cdot}09 \text{ feet.}$$

8. A body is propelled with an initial velocity of 3 feet, and with an acceleration of 8 feet a second; what space is described in 13 seconds?

$$3 \times 13 + \frac{8 \times (13)^2}{2} = 715 \text{ feet.}$$

9. What distance will a body perform in 35 seconds, commencing with a velocity of 10 feet, and being accelerated to move with a velocity of 40 feet at the beginning of the 36th second?

$$\frac{10 + 40}{2} \times 35 = 875 \text{ feet, the distance.}$$

The formulas for a uniformly accelerated motion, commencing with a velocity c, are as follow:—

$$v = c + pt; \quad s = ct + \frac{pt^2}{2}; \quad s = \frac{c + v}{2}t; \quad s = \frac{v^2 - c^2}{2p}.$$

The succeeding formulas are applicable for a uniformly retarded motion with an initial velocity c.

$$v = c - pt; \quad s = ct - \frac{pt^2}{2}; \quad s = \frac{c + v}{2}t; \quad s = \frac{c^2 - v^2}{2p}.$$

10. A body rolls up an inclined plane, with an initial velocity of 50 feet, and suffers a retardation of 10 feet the second; to what height will it ascend?

$$\frac{50}{10} = 5 \text{ seconds, the time.}$$

$$\frac{(50)^2}{2 \times 10} = 125 \text{ feet, the height required.}$$

The free vertical descent of bodies in vacuo offers an important example of uniformly accelerated motion. The acceleration in the previous examples was designated by p, but in the particular motion, brought about by the force of gravity, the acceleration is designated by the letter g, and has the mean value of 32·2 feet.

If this value of g be substituted for p, in the preceding formula, we have,

$$v = 32{\cdot}2 \times t; \; v = 8{\cdot}024964 \times \sqrt{s}; \; s = 16{\cdot}1 \times t^2; \; s = {\cdot}015528 \times v^2;$$
$$t = {\cdot}031056 \times v; \text{ and } t = {\cdot}2492224 \times \sqrt{s}.$$

11. What velocity will a body acquire at the end of 5 seconds, in its free descent?

$$32{\cdot}2 \times 5 = 161 \text{ feet.}$$

12. What velocity will a body acquire, after a free descent through a space of 400 feet?

$$8{\cdot}024964 \times \sqrt{400} = 160{\cdot}49928 \text{ feet.}$$

13. What space will a body pass over in its free descent during 10 seconds?

$$16{\cdot}1 \times (10)^2 = 1610 \text{ feet.}$$

14. A body falling freely in vacuo, has in its free descent acquired a velocity of 112 feet; what space is passed over?

$$\cdot015528 \times (112)^2 = 194\cdot783232 \text{ feet.}$$

15. In what time will a body falling freely acquire the velocity of 30 feet?

$$\cdot031056 \times 30 = \cdot93168 \text{ seconds.}$$

16. In what time will a body pass over a space of 16 feet, falling freely in vacuo?

$$\cdot2492224 \times \sqrt{16} = \cdot9968896 \text{ seconds.}$$

If the free descent of bodies go on, with an initial velocity, which we may call c, the formulas are,

$$v = c + g\,t;\ v = c + 32\cdot2 \times t;\ v = \sqrt{c^2 + 2\,g\,s};\ v = \sqrt{c^2 + 64\cdot4 \times s};$$

$$s = c\,t + g\frac{t^2}{2} = c\,t + 16\cdot1 \times t^2;\ s = \frac{v^2 - c^2}{2\,g} = \cdot015528\,(v^2 - c^2).$$

If a body be projected vertically to height, with a velocity which we shall term c, then the formulas become,

$$v = c - 32\cdot2 \times t;\ v = \sqrt{c^2 - 64\cdot4 \times s};\ s = c\,t - g\,\frac{t^2}{2} =$$

$$c\,t - 16\cdot1 \times t^2;\ s = \frac{c^2 - v^2}{2\,g} = \cdot015528\,(c^2 - v^2).$$

17. What space is described by a body passing from 18 feet velocity to 30 feet velocity during its free descent in vacuo.

From the annexed table, we find that the height due to 30 feet velocity..= 13·97516

The height due to 18................................= 5·03106

Space described.. 8·94410

Since this problem and table are often required in practical mechanics, we shall enter into more particulars respecting it.

$$\text{As } s = \frac{v^2 - c^2}{2\,g} = \frac{v^2}{2\,g} - \frac{c^2}{2\,g},$$

if we put h = height due to the initial velocity c; that is, $h = \frac{c^2}{2\,g}$; and h_1 = the height due to the terminal velocity v; that is, $h_1 = \frac{v^2}{2\,g}$; then,

$s = h_1 - h$, for falling bodies, as in the last example; and

$s = h - h_1$, for ascending bodies.

Although these formulas are only strictly true for a free descent in vacuo, they may be used in air, when the velocity is not great. The table will be found useful in hydraulics, and for other heights and velocities besides those set down, for by inspection it is seen that the height ·201242 answers to the velocity 3·6; and the height 20·12423 to 36; and the height 2012·423 to 360; and so on.

TABLE *of the Heights corresponding to different Velocities, in feet the second.*

Velocity in Feet.	CORRESPONDING HEIGHT IN FEET.									
	0	1	2	3	4	5	6	7	8	9
0	·000000	·000155	·000621	·001398	·002484	·003882	·005590	·007609	·009938	·012578
1	·015528	·018789	·020652	·026242	·0304348	·0349379	·039752	·044876	·050311	·056056
2	·062112	·068478	·075155	·082143	·089441	·097050	·104969	·113199	·121739	·130590
3	·139752	·149224	·159006	·169099	·187888	·190217	·201242	·212577	·224224	·236180
4	·248447	·261025	·273913	·285714	·300621	·314441	·328572	·343013	·357764	·372826
5	·388199	·403882	·419877	·436180	·452795	·469720	·486956	·504503	·522360	·550578
6	·559006	·577795	·596894	·616304	·636025	·656060	·676397	·697050	·718013	·739286
7	·760870	·782764	·804970	·827484	·850310	·873447	·896895	·920652	·944721	·969099
8	·993789	1·018790	1·044100	1·069720	1·095652	1·121895	1·148421	1·175311	1·201482	1·229971
9	1·257764	1·285869	1·314285	1·343012	1·372050	1·401400	1·431055	1·461025	1·491304	1·521894

The following extension is obtained from the foregoing table, by mere inspection, and moving the decimal point as before directed.

Velocity in Feet.	Corresponding Height in Feet.	Velocity in Feet.	Corresponding Height in Feet.	Velocity in Feet.	Corresponding Height in Feet.	Velocity in Feet.	Corresponding Height in Feet.
10	1·552795	19	5·60559	28	12·17392	37	21·25777
11	1·878882	20	6·21118	29	13·05901	38	22·42236
12	2·065218	21	6·84783	30	13·97516	39	23·61802
13	2·624224	22	7·51553	31	14·92237	40	24·84472
14	3·043478	23	8·21429	32	15·90062	41	26·10249
15	3·49379	24	8·94410	33	16·90994	42	27·39131
16	3·97516	25	9·70497	34	18·78883	43	28·57143
17	4·48758	26	10·49690	35	19·02174	44	30·06212
18	5·03106	27	11·31988	36	20·12423	45	31·4441

18. What *mass* does a body weighing 30268 lbs. contain?

$$\frac{30268}{32{\cdot}2} = \frac{302680}{322} = 940 \text{ lbs.}$$

For the mass is equal to the weight divided by g. And g is taken equal to 32·2; but the acceleration of gravity is somewhat variable; it becomes greater the nearer we approach the poles of the earth. It is greatest at the poles and least at the equator, and also diminishes the more a body is above or below the level of the sea. The *mass*, so long as nothing is added to or taken from it, is invariable, whether at the centre of the earth or at any distance from it. If M be the *mass* and W the weight of a body,

$$\text{Then } M = \frac{W}{g} = \frac{W}{32{\cdot}2} = {\cdot}0310559 \text{ W}.$$

19. What is the *mass* of a body whose weight is 200 lbs?

$${\cdot}031055 \times 200 = 6{\cdot}21118 \text{ lbs.}$$

The weight of a body whose *mass* is 200 lbs. is 32·2 × 200 = 6440·0 lbs. It may be remarked, that one and the same steel spring is differently bent by one and the same weight at different places.

The force which accelerates the motion of a heavy body on an inclined plane, is to the force of gravity as the sine of the inclination of the plane to the radius, or as the height of the plane to its length.

The velocity acquired by a body in falling from rest through a given height, is the same, whether it fall freely, or descend on a plane at whatever inclination.

The space through which a body will descend on an inclined plane, is to the space through which it would fall freely in the same time, as the sine of the inclination of the plane to the radius.

The velocities which bodies acquire by descending along chords of the same circle, are as the lengths of those chords.

If the body descend in a curve, it suffers no loss of velocity.

The centre of gravity of a body is a point about which all its parts are in equilibrio.

Hence, if a body be suspended or supported by this point, the body will rest in any position into which it is put. We may, therefore, consider the whole weight of a body as centred in this point.

The common centre of gravity of two or more bodies, is the point about which they would equiponderate or rest in any position. If the centres of gravity of two bodies be connected by a right line, the distances from the common centre of gravity are reciprocally as the weights of the bodies.

If a line be drawn from the centre of gravity of a body, perpendicular to the horizon, it is called *the line of direction*, being the line that the centre of gravity would describe if the body fell freely.

The centre of gyration is that part of a body revolving about an axis, into which if the whole quantity of matter were collected, the same moving force would generate the same angular velocity.

To find the centre of Gyration.—Multiply the weight of the several particles by the squares of their distances from the centre of motion, and divide the sum of the products by the weight of the whole mass; the square root of the quotient will be the distance of the centre of gyration from the centre of motion.

The distances of the centre of gyration from the centre of motion, in different revolving bodies, are as follow:—

In a straight rod revolving about one end, the length × ·5773.

In a circular plate, revolving on its centre, the radius × ·7071.

In a circular plate, revolving about one diameter, the radius × ·5.

In a thin circular ring, revolving about one diameter, radius × ·7071.

In a solid sphere, revolving about one diameter, the radius × ·6325.

In a thin hollow sphere, revolving about one diameter, radius × ·8164.

In a cone, revolving about its axis, the radius of the base × ·5477.

In a right-angled cone, revolving about its vertex, the height × ·866.

In a paraboloid, revolving about its axis, the radius of the base × ·5773.

The centre of percussion is that point in a body revolving about a fixed axis, into which the whole of the force or motion is collected.

It is, therefore, that point of a revolving body which would strike any obstacle with the greatest effect; and, from this property, it has received the name of the centre of percussion.

The centres of oscillation and percussion are in the same point.

If a heavy straight bar, of uniform density, be suspended at one extremity, the distance of its centre of percussion is two-thirds of its length.

In a long slender rod of a cylindrical or prismatic shape, the centre of percussion is nearly two-thirds of the length from the axis of suspension.

In an isosceles triangle, suspended by its apex, the distance of the centre of percussion is three-fourths of its altitude. In a line or rod whose density varies as the distance from the point of suspension, also in a fly-wheel, and in wheels in general, the centre of percussion is distant from the centre of suspension three-fourths of the length.

In a very slender cone or pyramid, vibrating about its apex, the distance of its centre of percussion is nearly four-fifths of its length.

Pendulums of the same length vibrate slower, the nearer they are brought to the equator. A pendulum, therefore, to vibrate seconds at the equator, must be somewhat shorter than at the poles.

When we consider a simple pendulum as a ball, which is suspended by a rod or line, supposed to be inflexible, and without weight, we suppose the whole weight to be collected in the centre of gravity of the ball. But when a pendulum consists of a ball, or any other figure, suspended by a metallic or wooden rod, the length of the pendulum is the distance from the point of suspension to a point in the pendulum, called the *centre of oscillation*, which does not exactly coincide with the centre of gravity of the ball.

If a rod of iron were suspended, and made to vibrate, that point in which all its force would be collected is called its centre of oscillation, and is situated at two-thirds the length of the rod from the point of suspension.

SPECIFIC GRAVITY.

THE comparative density of various substances, expressed by the term *specific gravity*, affords the means of readily determining the bulk from the known weight, or the weight from the known bulk; and this will be found more especially useful, in cases where the substance is too large to admit of being weighed, or too irregular in shape to allow of correct measurement. The standard with which all solids and liquids are thus compared, is that of distilled water, one cubic foot of which weighs 1000 ounces avoirdupois;

and the specific gravity of a *solid* body is determined by the difference between its weight in the air, and in water. Thus,

If the body be *heavier* than water, it will displace a quantity of fluid equal to it in *bulk*, and will lose as much weight on immersion as that of an equal bulk of the fluid. Let it be weighed first, therefore, in the air, and then in water, and its weight in the air be divided by the difference between the two weights, and the quotient will be its specific gravity, that of water being unity.

A piece of copper ore weighs $56\frac{1}{4}$ ounces in the air, and $43\frac{3}{4}$ ounces in water; required its specific gravity.

$56{\cdot}25 - 43{\cdot}75 = 12{\cdot}5$ and $56{\cdot}25 \div 12{\cdot}5 = 4{\cdot}5$, the specific gravity.

If the body be *lighter* than water, it will float, and displace a quantity of fluid equal to it in *weight*, the bulk of which will be equal to that only of the part immersed. A heavier substance must, therefore, be attached to it, so that the two may sink in the fluid. Then, the weight of the lighter substance in the air, must be added to that of the heavier substance in water, and the weight of both united, in water, be subtracted from the sum; the weight of the lighter body in the air must then be divided by the difference, and the quotient will be the specific gravity of the lighter substance required.

A piece of fir weighs 40 ounces in the air, and, being immersed in water attached to a piece of iron weighing 30 ounces, the *two* together are found to weigh 3·3 ounces in water, and the iron alone, 25·8 ounces in the water; required the specific gravity of the wood.

$40 + 25{\cdot}8 = 65{\cdot}8 - 3{\cdot}3 = 62{\cdot}5$; and $40 \div 62{\cdot}5 = 0{\cdot}64$, the specific gravity of the fir.

The specific gravity of a *fluid* may be determined by taking a solid body, heavy enough to sink in the fluid, and of known specific gravity, and weighing it both in the air and in the fluid. The difference between the two weights must be multiplied by the specific gravity of the solid body, and the product divided by the weight of the solid in the air: the quotient will be the specific gravity of the fluid, that of water being unity.

Required the specific gravity of a given mixture of muriatic acid and water; a piece of glass, the specific gravity of which is 3, weighing $3\frac{3}{4}$ ounces when immersed in it, and 6 ounces in the air.

$6 - 3{\cdot}75 = 2{\cdot}25 \times 3 = 6{\cdot}75 \div 6 = 1{\cdot}125$, the specific gravity.

Since the weight of a cubic foot of distilled water, at the temperature of 60 degrees, (Fahrenheit,) has been ascertained to be 1000 avoirdupois ounces, it follows that the specific gravities of all bodies compared with it, may be made to express the weight, in ounces, of a cubic foot of each, by multiplying these specific gravities (compared with that of water as unity) by 1000. Thus, that of water being 1, and that of silver, as compared with it, being 10·474, the multiplication of each by 1000 will give 1000 ounces for the cubic foot of water, and 10474 ounces for the cubic foot of silver.

In the following tables of *specific gravities*, the numbers in the *first* column, if taken as whole numbers, represent the weight of a cubic foot in ounces; but if the last *three* figures are taken as decimals, they indicate the specific gravity of the body, water being considered as unity, or 1.

To ascertain the number of cubic feet in a substance, from its weight, the whole weight in *pounds* avoirdupois must be divided by the figures against the name, in the *second* column of the table, taken as whole numbers and decimals, and the quotient will be the contents in cubic feet.

Required the cubic content of a mass of cast-iron, weighing 7 cwt. 1 qr. = 812 lbs.

812 lbs. ÷ 450·5 (the tabular weight) = 1·803 cubic feet.

To find the weight from the measurement or cubic content of a substance, this operation must be reversed, and the number of cubic feet, found by the rules given under "*Mensuration of Solids*," *multiplied* by the figures in the *second* column, to obtain the weight in pounds avoirdupois.

Required the weight of a log of oak, 3 feet by 2 feet 6 inches, and 9 feet long.

$$9 \times 3 \times 2{\cdot}5 = 67{\cdot}5 \text{ cubic feet.}$$

And 67·5 × 58·2 (the tabular weight) = 3928·5 lbs., or 35 cwt. 0 qr. 8½ lbs.

The velocity g, which is the measure of the force of gravity, varies with the latitude of the place, and with its altitude above the level of the sea.

The force of gravity at the latitude of 45° = 32·1803 feet; at any other latitude L, $g = 32{\cdot}1803$ feet $- 0{\cdot}0821 \cos. 2\,L$. If g' represents the force of gravity at the height h above the sea, and r the radius of the earth, the force of gravity at the level of the sea will be $g = g' \left(1 + \frac{5\,h}{4\,r}\right)$.

In the latitude of London, at the level of the sea, $g = 32{\cdot}191$ feet.
Do. Washington, do. do., $g = 32{\cdot}155$ feet.

The length of a pendulum vibrating seconds is in a constant ratio to the force of gravity.

$$\frac{g}{l} = 9{\cdot}8696044.$$

Length of a pendulum vibrating seconds at the level of the sea, in various latitudes.

At the Equator		39·0152 inches.
Washington,	lat. 38° 53′ 23″	39·0958 —
New York,	lat. 40° 42′ 40″	39·1017 —
London,	lat. 51° 31′	39·1393 —
	lat. 45°	39·1270 —
	lat. L	39·1270 in.—0·09982 cos. 2 L.

Specific Gravity of various Substances.

	Weight of a cubic foot in ounces.	Weight of a cubic foot in pounds.
METALS.		
Antimony, fused	6,624	414·0
Bismuth, cast	9,823	614·0
Brass, common, cast	7,824	489·0
cast	8,396	524·8
wire-drawn	8,544	534·0
Copper, cast	8,788	549·2
wire-drawn	8,878	554·9
Gold, pure, cast	19,258	1203·6
22 carats, stand	17,486	1093·0
20 carats, trinket	15,709	982·0
Iron, cast	7,207	450·5
bars	7,788	486·8
Lead, cast	11,352	709·5
litharge	6,300	393·8
Manganese	7,000	437·5
Mercury, solid, 40° below 0°	15,632	977·0
at 32 deg. Fahr.	13,619	851·2
at 60 deg.	13,580	848·8
at 212 deg.	13,375	836·8
Nickel, cast	7,807	488·0
Platina, crude, grains	15,602	975·1
purified	19,500	1218·8
hammered	20,337	1271·1
rolled	22,069	1379·4
wire-drawn	21,042	1315·1
Silver, cast, pure	10,474	654·6
Parisian standard	10,175	636·0
French coin	10,048	628·0
shilling, Geo. III.	10,534	658·4
Steel, soft	7,833	489·6
hardened	7,840	490·0
tempered	7,816	488·5
tempered and hard	7,818	488·6
Tin, pure Cornish	7,291	455·6
Tungsten	6,066	379·1
Uranium	6,440	402·5
Wolfram	7,119	445·0
Zinc, usual state	6,862	429·0
pure	7,191	449·5
WOODS.		
Ash	845	52·9
Beech	852	53·2
Box, Dutch	912	57·0
French	1,328	83·0
Brazilian	1,031	64·5
Cedar, American	561	35·1
Indian	1,315	82·2
Cherry-tree	715	44·8
Cocoa	1,040	65·0
Cork	240	15·0
Ebony, Indian	1,209	75·6
American	1,331	83·2
Elm	671	42·0
Fir, yellow	657	41·1
white	569	35·6
Lignum-vitæ	1,333	83·4
Lime-tree	604	37·8
Logwood	913	57·1
Mahogany	1,063	66·5
Maple	750	47·0
Oak, heart of, old	1,170	73·1
dry	932	58·2
Vine	1,327	83·0
Walnut	671	42·0
Willow	585	36·6
Yew	807	50·5
STONES, EARTHS, ETC.		
Alabaster, yellow	2,699	168·8
white	2,730	170·6
Borax	1,714	107·1
Brick earth	2,000	125·0
Chalk	2,784	174·0
Coal, Cannel	1,270	79·4
Newcastle	1,270	79·4
Staffordshire	1,240	77·5
Scotch	1,300	81·2
Emery	4,000	250·0
Flint, black	2,582	162·0
Glass, flint	2,933	170·9
white	2,892	168·2
Granite, Aberd. blue	2,625	164·1
Cornish	2,662	166·4
Egyptian, red	2,654	165·9
" gray	2,728	170·5
STONES.—*Continual.*		
Grindstone	2,143	134·0
Gypsum, opaque	2,168	135·5
semi-transparent	2,306	144·1
Jet, bituminous	1,259	78·8
Lime-stone	3,182	199·0
Marble	2,700	168·8
Mill-stone	2,484	155·2
Porcelain, China	2,385	149·1
Portland-stone	2,570	160·6
Pumice stone	915	57·2
Paving-stone	2,416	151·0
Purbeck-stone	2,601	162·6
Rotten-stone	1,981	124·0
Slate, common	2,672	167·0
new	2,854	178·4
Stone, common	2,520	157·5
rag	2,470	154·4
Sulphur, native	2,033	127·1
melted	1,991	124·5
LIQUIDS.		
Acetic acid	1,007	63·0
Acetous acid	1,025	64·1
Alcohol, commercial	837	52·3
highly rectified	829	51·8
Ammonia, liquid	897	56·1
Beer	1,023	68·0
Ether, sulphuric	739	46·2
Milk of cows	1,032	64·5
Muriatic acid	1,194	74·6
Nitric acid	1,271	79·5
highly concentrated	1,583	99·0
Oil of almonds, sweet	917	57·4
hemp-seed	926	58·0
linseed	940	58·8
olives	915	57·3
poppies	924	57·8
rape-seed	919	57·5
turpentine, essence	870	54·4
whales	923	57·8
Spirits of wine, commercial	837	52·4
highly rectified	829	51·9
Sulphuric acid	1,841	115·1
highly concentrated	2,125	133·0
Turpentine, liquid	991	62·0
Vinegar, distilled	1,010	63·1
Water, rain, or distilled	1,000	62·5
sea	1,026	64·1
MISCELLANEOUS SUBSTANCES.		
Beeswax	965	60·4
Butter	942	59·0
Camphor	989	62·0
Fat, beef or mutton	923	57·8
hogs'	937	58·6
Honey	1,450	90·6
Indigo	769	48·1
Ivory	1,826	114·1
Lard	948	59·2
Opium	1,336	83·5
Spermaceti	943	59·0
Sugar, white	1,606	100·4
Tallow	942	59·0
GASES.		
Atmospheric air being estimated as 1.		
Atmospheric, or common air		1·000
Ammoniacal gas		·590
Azote		·969
Carbonic acid		1·520
Carbonic oxide		·960
Carburetted hydrogen		·491
Chlorine		·470
Hydrogen		·074
Muriatic acid gas		1·278
Nitrous gas		1·094
Nitrous acid gas		2·427
Oxygen		1·104
Steam		·690
Sulphuretted hydrogen		1·777
Sulphurous acid		2·193

Table *of the Weight of a Foot in length of Flat and Rolled Iron.*

Thickness in inches and parts.	BREADTH IN INCHES AND PARTS OF AN INCH.															
	4	3¾	3½	3¼	3	2¾	2½	2¼	2	1¾	1½	1⅜	1¼	1	¾	½
⅛	1·68	1·57	1·47	1·36	1·26	1·15	1·05	0·94	0·84	0·73	0·63	0·57	0·52	0·42	0·31	0·21
3/16	2·52	2·36	2·20	2·04	1·89	1·73	1·57	1·41	1·26	1·10	0·94	0·86	0·78	0·63	0·47	0·31
¼	3·36	3·15	2·94	2·73	2·52	2·31	2·10	1·89	1·68	1·47	1·26	1·18	1·05	0·84	0·63	0·42
⅜	5·04	4·72	4·41	4·09	3·78	3·46	3·15	2·83	2·52	2·20	1·89	1·73	1·57	1·26	0·94	0·63
½	6·72	6·30	5·88	5·46	5·04	4·62	4·20	3·78	3·36	2·94	2·52	2·31	2·10	1·68	1·26	
⅝	8·40	7·87	7·35	6·82	6·30	5·77	5·25	4·72	4·20	3·67	3·15	2·88	2·62	2·10	1·57	
¾	10·08	9·45	8·82	8·19	7·56	6·93	6·30	5·66	5·04	4·41	3·78	3·46	3·15	2·52		
⅞	11·76	11·02	10·29	9·45	8·82	8·08	7·35	6·61	5·88	5·14	4·41	4·04	3·67	2·94		
1	13·44	12·60	11·76	10·92	10·08	9·24	8·40	7·56	6·72	5·87	5·04	4·62	4·20			
1⅛	15·12	14·16	13·20	12·28	11·34	10·39	9·45	8·50	7·56	6·60	5·67	5·19	4·72			
1¼	16·80	15·75	14·70	13·65	12·60	11·55	10·50	9·45	8·40	7·35	6·30	5·77				
1⅜	18·48	17·32	16·16	15·01	13·86	12·70	11·55	10·39	9·24	8·07						
1½	20·18	18·90	17·64	16·38	15·12	13·86	12·60	11·34	10·08	8·80						
1¾	23·54	22·05	20·58	19·11	17·64	16·17	14·70	13·22								
2	26·88	25·20	23·52	21·84	20·16	18·48	16·80	15·12								
2½	33·65	31·50	29·40	27·39	25·20	23·10										
3	40·32	37·80	35·28	32·76												
3½	47·04															

Table *of the Weight of Cast-iron Pipes, in lengths.*

Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.		
Inch.	Inch.	Feet.	C.	qr.	lb.	Inch.	Inch.	Feet.	C.	qr.	lb.	Inch.	Inch.	Feet.	C.	qr.	lb.
1	¼	3½			12	6½	⅜	9	2	0	16	11½	½	9	5	0	7
	⅜	3½			21		½	9	2	3	20		⅝	9	6	1	12
1½	¼	4½			21		⅝	9	3	2	21		¾	9	7	2	8
	⅜	4½		1	4		¾	9	4	1	21		1	9	10	1	2
2	¼	6		1	8		1	9	6	0	14	12	½	9	5	0	24
	⅜	6		2	0	7	½	9	3	0	7		⅝	9	6	2	8
2½	¼	6		1	16		⅝	9	3	3	20		¾	9	7	3	20
	⅜	6		2	10		¾	9	4	3	5		1	9	10	3	0
	½	6		3	10		1	9	6	2	4	12½	½	9	5	1	16
3	¼	9		2	20	7½	½	9	3	1	6		⅝	9	6	3	9
	⅜	9	1	0	6		⅝	9	4	0	22		¾	9	8	1	0
	½	9	1	1	12		¾	9	5	0	10		1	9	11	0	21
	⅝	9	1	3	6		1	9	7	0	0	13	½	9	5	2	20
	¾	9	2	1	0	8	½	9	3	2	4		⅝	9	7	0	14
3½	¼	9		3	0		⅝	9	4	1	25		¾	9	8	2	7
	⅜	9	1	0	21		¾	9	5	1	18		1	9	11	2	12
	½	9	1	2	14		1	9	7	1	16	13½	½	9	5	3	7
	⅝	9	2	0	8	8½	½	9	3	3	2		⅝	9	7	1	12
	¾	9	2	2	0		⅝	9	4	2	26		¾	9	8	3	16
4	⅜	9	1	1	10		¾	9	5	2	22		1	9	11	3	24
	½	9	1	3	12		1	9	7	3	8	14	½	9	6	0	4
	⅝	9	2	1	12	9	½	9	4	0	0		⅝	9	7	2	16
	¾	9	2	2	21		⅝	9	5	0	4		¾	9	9	1	0
4½	⅜	9	1	2	2		¾	9	6	0	2		1	9	12	1	14
	½	9	2	0	4		1	9	8	0	26	14½	½	9	6	0	24
	⅝	9	2	2	14	9½	½	9	4	0	18		⅝	9	7	3	14
	¾	9	3	0	21		⅝	9	5	1	0		¾	9	9	2	2
5	⅜	9	1	2	22		¾	9	6	1	6		1	9	12	3	6
	½	9	2	1	10		1	9	8	2	20	15	½	9	6	1	21
	⅝	9	2	3	17	10	½	9	4	1	10		¾	9	9	3	7
	¾	9	3	1	24		⅝	9	5	1	26		1	9	13	0	26
5½	⅜	9	1	3	10		¾	9	4	2	14		1¼	9	16	3	5
	½	9	2	2	0		1	9	9	0	8	15½	½	9	6	2	14
	⅝	9	3	0	18	10½	½	9	4	2	14		¾	9	10	9	10
	¾	9	3	3	7		⅝	9	5	3	7		1	9	13	2	17
	1	9	5	0	12		¾	9	7	0	0		1¼	9	17	1	6
6	⅜	9	2	0	0		1	9	9	2	0	16	½	9	7	0	22
	½	9	2	2	21	11	½	9	4	3	14		¾	9	10	1	20
	⅝	9	3	1	17		⅝	9	6	0	11		1	9	14	0	8
	¾	9	4	0	16		¾	9	7	1	7		1¼	9	17	3	14
	1	9	5	2	20		1	9	9	3	20		1½	9	21	3	4

TABLE *of the Weight of one Foot Length of Malleable Iron.*

SQUARE IRON.		ROUND IRON.			
Scantling.	Weight.	Diameter.	Weight.	Circumference.	Weight.
Inches.	Pounds.	Inches.	Pounds.	Inches.	Pounds.
¼	0·21	¼	0·16	1	0·26
⅜	0·47	⅜	0·37	1¼	0·41
½	0·84	½	0·66	1½	0·59
⅝	1·34	⅝	1·03	1¾	0·82
¾	1·89	¾	1·48	2	1·05
⅞	2·57	⅞	2·02	2¼	1·34
1	3·36	1	2·63	2½	1·65
1⅛	4·25	1⅛	3·33	2¾	2·01
1¼	5·25	1¼	4·12	3	2·37
1⅜	6·35	1⅜	4·98	3¼	2·79
1½	7·56	1½	5·93	3½	3·24
1⅝	8·87	1⅝	6·96	3¾	3·69
1¾	10·29	1¾	8·08	4	4·23
1⅞	11·81	1⅞	9·27	4½	5·35
2	13·44	2	10·55	5	6·61
2¼	17·01	2¼	13·35	5½	7·99
2½	21·00	2½	16·48	6	9·51
2¾	25·41	2¾	19·95	6½	11·18
3	30·24	3	23·73	7	12·96
3½	41·16	3¼	27·85	7½	14·78
4	53·76	3½	32·32	8	16·92
4½	68·04	3¾	37·09	8½	19·21
5	84·00	4	42·21	9	21·53
6	120·96	4½	53·41	10	26·43
7	164·64	5	65·93	12	31·99

The following tables are rendered of great utility by means of this table:—

The weight of Water being 1·
———— Copper = 8·8
———— Brass = 8·4
———— Iron, cast = 7·2
———— Lead = 11·3
———— Zinc = 7·2
———— Gun-metal = 8·7
———— Sand = 1·5
———— Coal = 1·25
———— Brick = 2·0
———— Stone = 2·5
———— Timber, average = 0·85

Suppose it be required to ascertain the weight of a cast iron pipe 26¼ inches outside and 23¾ inside, the length being 6½ feet.

Opposite 26¼ in the table is

$$234{\cdot}8576 \times 7{\cdot}2 \times 6{\cdot}5 = 10991{\cdot}135.$$

And opposite 23¾ in the table is

$$\begin{array}{r} 192{\cdot}2856 \times 7{\cdot}2 \times 6{\cdot}5 = \underline{8998{\cdot}966} \text{ subtract} \\ 1992{\cdot}169 \text{ lbs. avr.} \end{array}$$

The succeeding table contains the surface and solidity of spheres, together with the edge or dimensions of equal cubes, the length of equal cylinders, and the weight of water in avoirdupois pounds:—

Surface and Solidity of Spheres.

Diameter.	Surface.	Solidity.	Cube.	Cylinder.	Water in lbs.
1 in.	3·1416	·5236	·8060	·6666	·0190
$\frac{1}{16}$	3·5465	·6280	·8563	·7082	·0227
$\frac{1}{8}$	3·9760	·7455	·9067	·7500	·0270
$\frac{3}{16}$	4·4301	·8767	·9571	·7917	·0317
$\frac{1}{4}$	4·9087	1·0226	1·0075	·8333	·0370
$\frac{5}{16}$	5·4117	1·1838	1·0578	·8750	·0428
$\frac{3}{8}$	5·9395	1·3611	1·1082	·9166	·0500
$\frac{7}{16}$	6·4918	1·5553	1·1586	·9583	·0563
$\frac{1}{2}$	7·0686	1·7671	1·2090	1·0000	·0640
$\frac{9}{16}$	7·6699	2·0000	1·2593	1·0416	·0723
$\frac{5}{8}$	8·2957	2·2467	1·3097	1·0833	·0813
$\frac{11}{16}$	8·9461	2·5161	1·3601	1·1349	·0910
$\frac{3}{4}$	9·6211	2·8061	1·4105	1·1666	·1015
$\frac{13}{16}$	10·3206	3·1176	1·4608	1·2083	·1128
$\frac{7}{8}$	11·0446	3·4514	1·5112	1·2500	·1250
$\frac{15}{16}$	11·7932	3·8081	1·5616	1·2916	·1377
2 in.	12·5664	4·1888	1·6020	1·3333	·1516
$\frac{1}{16}$	13·3640	4·5938	1·6633	1·3750	·1662
$\frac{1}{8}$	14·1862	5·0243	1·7127	1·4166	·1818
$\frac{3}{16}$	15·0330	5·4807	1·7631	1·4582	·1982
$\frac{1}{4}$	15·9043	6·9640	1·8135	1·5000	·2160
$\frac{5}{16}$	16·8000	6·4749	1·8638	1·5516	·2342
$\frac{3}{8}$	17·7205	7·0143	1·9142	1·5832	·2540
$\frac{7}{16}$	18·6655	7·5828	1·9646	1·6250	·2743
$\frac{1}{2}$	19·6350	8·1812	2·0150	1·6666	·2960
$\frac{9}{16}$	20·6290	8·8103	2·0653	1·7082	·3187
$\frac{5}{8}$	21·6475	9·4708	2·1157	1·7500	·3426
$\frac{11}{16}$	22·6907	10·1634	2·1661	1·7915	·3676
$\frac{3}{4}$	23·7583	10·8892	2·2165	1·8332	·3939
$\frac{13}{16}$	24·8505	11·6485	2·2668	1·8750	·4213
$\frac{7}{8}$	25·9672	12·4426	2·3172	1·9165	·4501
$\frac{15}{16}$	27·1084	13·2718	2·3676	1·9582	·4800
3 in.	28·2744	14·1372	2·4180	2·0000	·5114
$\frac{1}{16}$	29·4647	15·0392	2·4683	2·0415	·5440
$\frac{1}{8}$	30·6796	15·9790	2·5187	2·0832	·5780
$\frac{3}{16}$	31·9191	16·9570	2·5691	2·1250	·6133
$\frac{1}{4}$	33·1831	17·9742	2·6195	2·1665	·6401
$\frac{5}{16}$	35·3715	19·0311	2·6698	2·2082	·6884
$\frac{3}{8}$	35·7847	20·1289	2·7202	2·2500	·7281
$\frac{7}{16}$	37·1224	21·2680	2·7706	2·2915	·7693
$\frac{1}{2}$	38·4846	22·4493	2·8210	2·3332	·8120
$\frac{9}{16}$	39·8713	23·6735	2·8713	2·3750	·8561
$\frac{5}{8}$	41·2825	24·9415	2·9217	2·4166	·9021
$\frac{11}{16}$	42·7183	26·2539	2·9712	2·4582	·9496
$\frac{3}{4}$	44·1787	27·6117	3·0225	2·5000	·9987
$\frac{13}{16}$	45·6636	29·0102	3·0728	2·5415	1·0493
$\frac{7}{8}$	47·1730	30·4659	3·1232	2·5832	1·1020
$\frac{15}{16}$	48·7070	31·9640	3·1730	2·6250	1·1561
4 in.	50·2656	33·5104	3·2240	2·6665	1·1974
$\frac{1}{16}$	51·8486	35·1058	3·2743	2·7082	1·2698
$\frac{1}{8}$	53·4562	36·7511	3·3247	2·7500	1·3293
$\frac{3}{16}$	55·0884	38·4471	3·3751	2·7915	1·3906
$\frac{1}{4}$	56·7451	40·1944	3·4255	2·8332	1·4538
$\frac{5}{16}$	58·4262	42·0461	3·4758	2·8750	1·5208
$\frac{3}{8}$	60·1321	43·8463	3·5262	2·9165	1·5860
$\frac{7}{16}$	61·8625	45·7524	3·5766	2·9582	1·6550

Diameter.	Surface.	Solidity.	Cube.	Cylinder.	Water in lbs.
$\frac{1}{2}$	63·6174	47·7127	3·6270	3·0000	1·7258
$\frac{9}{16}$	65·3968	49·7290	3·6773	3·0415	1·7987
$\frac{5}{8}$	67·2007	51·8006	3·7277	3·0832	1·8736
$\frac{11}{16}$	69·0352	53·9290	3·7781	3·1250	1·9506
$\frac{3}{4}$	70·8823	56·1151	3·8285	3·1665	2·0297
$\frac{13}{16}$	72·7599	58·3595	3·8788	3·2080	2·1109
$\frac{7}{8}$	74·6620	60·6629	3·9292	3·2500	2·1942
$\frac{15}{16}$	76·5887	62·9261	3·9796	3·2913	2·2760
5 in.	78·5400	65·4500	4·0300	3·3332	2·3673
$\frac{1}{16}$	80·5157	67·9351	4·0803	3·3750	2·4572
$\frac{1}{8}$	82·5160	70·4824	4·1307	3·4155	2·5453
$\frac{3}{16}$	84·5409	73·0926	4·1811	3·4582	2·6438
$\frac{1}{4}$	86·5903	75·7664	4·2315	3·5000	2·7605
$\frac{5}{16}$	88·6641	78·5077	4·2818	3·5414	2·8396
$\frac{3}{8}$	90·7627	81·3083	4·3322	3·5832	2·9407
$\frac{7}{16}$	92·8858	84·1777	4·3820	3·6250	3·0447
$\frac{1}{2}$	95·0334	87·1139	4·4330	3·6665	3·1509
$\frac{9}{16}$	97·2053	90·1175	4·4633	3·7080	3·2595
$\frac{5}{8}$	99·4021	93·1875	4·5337	3·7500	3·3706
$\frac{11}{16}$	101·6233	96·3304	4·5841	3·7913	3·4843
$\frac{3}{4}$	103·8691	99·5412	4·6345	3·8330	3·6004
$\frac{13}{16}$	106·1394	102·8225	4·6848	3·8750	3·7191
$\frac{7}{8}$	108·4342	106·1754	4·7352	3·9163	3·8404
$\frac{15}{16}$	110·7536	109·5973	4·7856	3·9580	3·9641
6 in.	113·0976	113·0976	4·8360	4·0000	4·0907
$\frac{1}{16}$	115·4660	116·6688	4·8863	4·0417	4·2200
$\frac{1}{8}$	117·8590	120·3139	4·9367	4·0833	4·3517
$\frac{3}{16}$	120·2771	124·0374	4·9871	4·1250	4·4874
$\frac{1}{4}$	122·7187	127·8320	5·0375	4·1666	4·6236
$\frac{5}{16}$	125·1852	131·7053	5·0878	4·2083	4·7638
$\frac{3}{8}$	127·6765	135·6563	5·1382	4·2500	4·9067
$\frac{7}{16}$	130·1923	139·6854	5·1886	4·2917	5·0524
$\frac{1}{2}$	132·7326	143·7936	5·2390	4·3332	5·2010
$\frac{9}{16}$	135·2974	147·9815	5·2893	4·3750	5·3525
$\frac{5}{8}$	137·8867	152·2499	5·3377	4·4165	5·5069
$\frac{11}{16}$	140·5006	156·5997	5·3901	4·4583	5·6786
$\frac{3}{4}$	143·1391	161·0315	5·4405	4·5000	5·8245
$\frac{13}{16}$	145·8021	167·5461	5·4908	4·5416	6·0601
$\frac{7}{8}$	148·4896	170·1682	5·5412	4·5832	6·1550
$\frac{15}{16}$	151·2017	174·8270	5·5916	4·6250	6·3235
7 in.	153·9384	179·5948	5·6420	4·6665	6·4960
$\frac{1}{16}$	156·6995	184·4484	5·6923	4·7082	6·6725
$\frac{1}{8}$	159·4852	189·3882	5·7427	4·7500	6·8502
$\frac{3}{16}$	162·2955	194·1165	5·7931	4·7915	7·0212
$\frac{1}{4}$	165·1303	199·5325	5·8435	4·8332	7·2171
$\frac{5}{16}$	167·9895	204·7371	5·8938	4·8750	7·4053
$\frac{3}{8}$	170·8735	210·0331	5·9442	4·9166	7·5970
$\frac{7}{16}$	173·7520	215·4172	5·9946	4·9582	7·7916
$\frac{1}{2}$	176·7150	220·8937	6·0450	5·0000	7·9897
$\frac{9}{16}$	179·6725	226·7240	6·0953	5·0415	8·2006
$\frac{5}{8}$	182·6545	232·1235	6·1457	5·0832	8·3960
$\frac{11}{16}$	185·6611	237·8883	6·1961	5·1250	8·6044
$\frac{3}{4}$	188·6923	243·7276	6·2465	5·1665	8·8157
$\frac{13}{16}$	191·7480	249·4720	6·2968	5·2082	9·0234
$\frac{7}{8}$	194·8282	255·7121	6·3472	5·2500	9·2491
$\frac{15}{16}$	197·9330	261·9673	6·3976	5·2913	9·4753
8 in.	201·0624	268·0832	6·4480	5·3330	9·6965
$\frac{1}{16}$	204·2162	274·4156	6·4983	5·3750	9·9260

Diameter.	Surface.	Solidity.	Cube.	Cylinder.	Water in lbs.
1/8	207·3946	280·8469	6·5487	5·4164	10·1583
3/16	210·5976	287·3780	6·5991	5·4581	10·3944
1/4	213·8251	294·0095	6·6495	5·5000	10·6343
5/16	217·0770	300·7422	6·6998	5·5414	10·8778
3/8	220·3537	307·5771	6·7502	5·5831	11·1250
7/16	223·6549	314·5147	6·8006	5·6250	11·3760
1/2	226·9806	321·5553	6·8510	5·6664	11·6306
9/16	230·3308	328·7012	6·9013	5·7080	11·8891
5/8	233·7055	335·9517	6·9517	5·7500	12·1514
11/16	237·1048	343·3079	7·0021	5·7913	12·4170
3/4	240·5287	350·7710	7·0525	5·8330	12·6874
13/16	243·9771	358·3412	7·1028	5·8750	12·9612
7/8	247·4500	366·0199	7·1532	5·9163	13·2390
15/16	250·9475	373·8073	7·2036	5·9580	13·5206
9 in.	254·4696	381·7017	7·2540	6·0000	13·8062
1/16	258·0261	389·7118	7·3043	6·0417	14·0959
1/8	261·5872	397·8306	7·3547	6·0833	14·3895
3/16	265·1829	406·0613	7·4051	6·1250	14·6872
1/4	268·8031	414·4048	7·4555	6·1667	14·9890
5/16	272·4477	421·2907	7·5058	6·2083	15·2381
3/8	276·1171	431·4361	7·5562	6·2500	15·6050
7/16	279·8110	440·1294	7·6066	6·2916	15·9195
1/2	283·5294	448·9215	7·6570	6·3333	16·2375
9/16	287·2723	457·8500	7·7073	6·3750	16·5604
5/8	291·0397	466·8763	7·7557	6·4166	16·6869
11/16	294·8310	476·0304	7·8081	6·4582	17·2180
3/4	298·4483	485·3035	7·8585	6·5000	17·5534
13/16	302·4894	494·6952	7·9088	6·5415	17·8931
7/8	306·3550	504·2094	7·9592	6·5832	18·2373
15/16	310·9452	513·8436	8·0096	6·6250	18·5857
10 in.	314·1600	523·6000	8·0600	6·6666	18·6786
1/16	318·0992	533·4789	8·1103	6·7083	19·2960
1/8	322·0630	543·4814	8·1607	6·7500	19·6577
3/16	326·0514	553·6081	8·2111	6·7916	20·0240
1/4	330·0643	563·8603	8·2615	6·8333	20·3948
5/16	334·1016	574·2371	8·3118	6·8750	20·6682
3/8	338·1637	584·7415	8·3622	6·9166	21·1501
7/16	342·2503	595·3677	8·4126	6·9582	21·5344
1/2	346·3614	606·1318	8·4630	7·0000	21·9238
9/16	350·4970	617·0207	8·5133	7·0416	22·3176
5/8	354·6571	628·0387	8·5637	7·0833	22·7162
11/16	358·8418	639·1871	8·6141	7·1250	23·1194
3/4	363·0511	650·4666	8·6645	7·1666	23·5274
13/16	367·2849	661·8580	8·7148	7·2082	23·9394
7/8	371·5432	673·4222	8·7652	7·2500	24·3577
15/16	375·8261	685·0997	8·8156	7·2915	24·7801
11 in.	380·1336	696·9116	8·8660	7·3330	25·2073
1/16	384·4655	708·9106	8·9163	7·3750	25·6414
1/8	388·8220	720·9409	8·9667	7·4165	26·0764
3/16	393·2031	733·1599	9·0171	7·4582	26·5184
1/4	397·6087	745·5004	9·0675	7·5000	26·5657
5/16	402·0387	758·0104	9·1178	7·5414	27·4162
3/8	406·4935	770·6440	9·1682	7·5832	27·8742
7/16	410·7728	783·5787	9·2186	7·6250	28·3420
1/2	415·4766	796·3301	9·2690	7·6664	28·8033
9/16	420·0049	809·3844	9·3193	7·7080	29·2754
5/8	424·5576	822·5807	9·3697	7·7500	29·7527
11/16	429·1351	835·9695	9·4201	7·7913	30·2370

Diameter.	Surface.	Solidity.	Cube.	Cylinder.	Water in lbs.
¾	433·7371	849·4035	9·4705	7·8330	30·7229
13/16	438·3636	863·0283	9·5208	7·8750	31·2157
⅞	443·0146	876·7999	9·5772	7·9163	31·3883
15/16	447·6902	890·7070	9·6216	7·9580	32·2169
12 in.	452·3904	904·7808	9·6720	8·0000	32·7259
¼	471·4363	962·5158	9·8735	8·1666	34·8142
½	490·8750	1022·656	10·0750	8·3332	36·9886
¾	506·7064	1085·251	10·2765	8·5000	39·2535
13 in.	530·9304	1150·337	10·4780	8·6666	41·6077
¼	551·5471	1218·000	10·6790	8·8332	44·0551
½	572·5566	1288·252	10·8810	9·0000	46·5961
¾	593·9587	1361·346	11·0825	9·1665	49·2399
14 in.	615·7536	1436·758	11·2840	9·3332	51·9675
¼	637·9411	1515·106	11·4855	9·5000	54·8014
½	660·5214	1596·260	11·6870	9·6665	57·7367
¾	683·4943	1680·265	11·8885	9·8332	60·7751
15 in.	706·8600	1767·150	12·0900	10·0000	64·0178
¼	730·6183	1856·988	12·2915	10·1666	67·1672
½	754·7694	1949·821	12·4930	10·3332	70·5250
¾	779·3131	2045·697	12·6940	10·5000	73·9929
16 in.	804·2496	2144·665	12·8960	10·6666	77·5725

TABLE *containing the Weight of Flat Bar Iron,* 1 *foot in length, of various breadths and thicknesses.*

Breadth in Inches.	THICKNESS IN PARTS OF AN INCH.									
	¼	5/16	⅜	7/16	½	9/16	⅝	¾	⅞	1 inch.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1 in.	0·83	1·04	1·25	1·45	1·66	1·87	2·08	2·50	2·91	3·33
1⅛	0·93	1·17	1·40	1·64	1·87	2·00	2·34	2·81	3·28	3·75
1¼	1·04	1·30	1·56	1·82	2·08	2·34	2·60	3·12	3·74	4·16
1⅜	1·14	1·43	1·71	2·00	2·29	2·57	2·86	3·43	4·01	4·58
1½	1·25	1·56	1·87	2·18	2·50	2·81	3·12	3·75	4·37	5·00
1⅝	1·35	1·69	2·03	2·36	2·70	3·04	3·38	4·06	4·73	5·41
1¾	1·45	1·82	2·18	2·55	2·91	3·28	3·64	4·37	5·10	5·83
1⅞	1·56	1·95	2·34	2·73	3·12	3·51	3·90	4·68	5·46	6·25
2 in.	1·66	2·08	2·50	2·91	3·33	3·75	4·16	5·00	5·83	6·66
2⅛	1·77	2·21	2·65	3·09	3·54	3·98	4·42	5·31	6·19	7·08
2¼	1·87	2·34	2·81	3·28	3·75	4·21	4·68	5·62	6·56	7·50
2⅜	1·97	2·47	2·96	3·46	3·95	4·45	4·94	5·93	6·92	7·91
2½	2·08	2·60	3·12	3·64	4·16	4·68	5·20	6·25	7·29	8·33
2⅝	2·18	2·73	3·28	3·82	4·37	4·92	5·46	6·56	7·65	8·75
2¾	2·29	2·86	3·43	4·01	4·58	5·15	5·72	6·87	8·02	9·16
2⅞	2·39	2·99	3·59	4·19	4·79	5·39	5·98	7·18	8·38	9·58
3 in.	2·50	3·12	3·75	4·37	5·00	5·62	6·25	7·50	8·75	10·00
3¼	2·70	3·38	4·06	4·73	5·41	6·09	6·77	8·12	9·47	10·83
3½	2·91	3·64	4·37	5·10	5·83	6·56	7·29	8·75	10·20	11·66
3¾	3·12	3·90	4·68	5·46	6·25	7·03	7·81	9·37	10·93	12·50
4 in.	3·33	4·16	5·00	5·83	6·66	7·50	8·33	10·00	11·66	13·33
4¼	3·54	4·42	5·31	6·19	7·08	7·96	8·85	10·62	12·39	14·16
4½	3·75	4·68	5·62	6·56	7·50	8·43	9·37	11·25	13·12	15·00
4¾	3·95	4·94	5·93	6·92	7·91	8·90	9·89	11·87	13·85	15·33
5 in.	4·17	5·20	6·25	7·29	8·33	9·37	10·41	12·50	14·58	16·66
5¼	4·37	5·46	6·56	7·65	8·75	9·84	10·93	13·12	15·31	17·50
5½	4·58	5·72	6·87	8·02	9·16	10·31	11·45	13·75	16·04	18·33
5¾	4·79	5·98	7·18	8·38	9·58	10·78	11·97	14·37	16·77	19·16
6 in.	5·00	6·26	7·50	8·75	10·00	11·25	12·50	15·00	17·50	20·00

TABLE *combining the Specific Gravities and other Properties of Bodies. Water the standard of comparison, or* 1000.

Names.	Specific gravity.	METALS.							
		Melting points in degrees of Fahrenheit.	Contraction in parts of an inch per lineal foot from the fluid to the average temperature in solid state.	Ultimate cohesive strength of an inch sq. prism in tons.	Scale of wire-drawing ductility.	Scale of laminable ductility.	Ratio of hardness.	Scale as conductors of electricity.	Ratio of power in the conduction of heat.
Platinum . .	19500	3280	. .	. .	3	5	. .	. .	3·8
Pure Gold .	19258	2016	. .	. .	1	1	1·8	3	10·0
Mercury . .	13500	. .	. .	. .	. .	. .	. .	. .	. .
Lead	11352	612	·319	·81	8	7	1·0	6	1·8
Pure Silver .	10474	1873	. .	. .	2	2	2·4	2	9·7
Bismuth . .	8923	476	·156	1·45	. .	. .	2·0	. .	. .
Copper, cast .	8788	1996	·193	8·51	. .	. .	. .	. .	. .
" wrought	8910	. .	. .	15·08	5	3	2·8	1	8·9
Brass, cast .	7824	1900	·210	8·01	. .	. .	to any degree	. .	. .
" sheet .	8396	. .	. .	12·23	6	6	. .	. .	8·6
Iron, cast .	7264	2786	·125	7·87	. .	. .	to any degree	. .	. .
" bar . .	7700	. .	·137	25·00	4	8	4·7	4	3·7
Steel, soft .	7833	. .	·133	58·91	. .	. .	. .	. .	. .
" hard .	7816	. .	. .	. .	. .	. .	to any degree	. .	. .
Tin, cast . .	7291	442	·278	2·11	8	4	1·2	5	3·0
Zinc, cast . .	7190	773	·329	5·06	7	8	1·6	7	3·6

Names.	STONES, EARTHS, ETC.			
	Specific gravity.	Weight of a cubic foot in lbs.	Cubic feet in a ton.	Tons required to crush 1½-in. cubes.
Marble, average	2730	170·00	13	9·25
Granite, ditto .	2651	165·68	13½	6·2
Purbeck stone .	2601	162·56	13¾	9·0
Portland ditto .	2570	160·62	14	4·5
Bristol ditto . .	2554	159·62	14	. .
Millstone	2484	155·25	14½	. .
Paving stone . .	2415	150·93	14¾	5·7
Craigleith ditto	2362	147·62	15	5·0
Grindstone . . .	2143	133·93	16¾	6·6
Chalk, Brit. . .	2781	173·81	12¾	0·5
Brick	2000	125·00	17	0·8
Coal, Scotch . .	1300	81·15	27½	. .
" Newcastle	1270	79·37	27¼	. .
" Staffordsh.	1240	77·50	29	. .
" Cannel . .	1238	77·37	29	. .

TABLE *containing the Weight of Columns of Water, each one foot in length, and of Various Diameters, in lbs. avoirdupois.*

Diam	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.
3 in.	3·0672	9 in.	27·6120	15 in.	76·7004	21 in.	150·2376	27 in.	248·5116	33 in.	371·2344
⅛	3·3288	⅛	28·3848	⅛	77·9844	⅛	152·1288	⅛	250·8180	⅛	374·0520
¼	3.6000	¼	29·1672	¼	79·2792	¼	153·9348	¼	253·1352	¼	376·8004
⅜	3·8820	⅜	29·9604	⅜	80·5836	⅜	155·7396	⅜	255·4632	⅜	379·4592
½	4·1748	½	30·7657	½	81·9000	½	157·5780	½	257·8008	½	382·5684
⅝	4·4784	⅝	31·6524	⅝	83·2260	⅝	159·4152	⅝	260·1504	⅝	385·4292
¾	4·7928	¾	32·4060	¾	84·5628	¾	161·2644	¾	262·5096	¾	388·2996
⅞	5·1180	⅞	33·2424	⅞	85·9104	⅞	163·1220	⅞	264·8796	⅞	391·1820
4 in.	5·4540	10 in.	34·0884	16 in.	87·2688	22 in.	164·9928	28 in.	267·2616	34 in.	394·0740
⅛	5·7996	⅛	34·9464	⅛	88·6368	⅛	166·8732	⅛	269·6532	⅛	396·9768
¼	6·1572	¼	35·8152	¼	90·0168	¼	168·7632	¼	272·0544	¼	399·8928
⅜	6·5244	⅜	36·6936	⅜	91·4176	⅜	170·6652	⅜	275·6672	⅜	402·8088
½	6·9024	½	37·5828	½	92·8080	½	172·5780	½	276·8916	½	405·7500
⅝	7·2912	⅝	38·4828	⅝	94·2192	⅝	174·5004	⅝	279·3252	⅝	408·6948
¾	7·6908	¾	39·3936	¾	95·6412	¾	176·4336	¾	281·7708	¾	411·4116
⅞	8·1012	⅞	40·3152	⅞	97·0740	⅞	178·3776	⅞	284·2260	⅞	414·6180
5 in.	8·5212	11 in.	41·2476	17 in.	98·5176	23 in.	180·3324	29 in.	286·6920	35 in.	417·5952
⅛	8·9532	⅛	42·1908	⅛	99·9720	⅛	182·2980	⅛	289·1688	⅛	420·5844
¼	9·3948	¼	43·1436	¼	101·4372	¼	184·2744	¼	291·6564	¼	423·5832
⅜	9·8484	⅜	44·1084	⅜	102·9120	⅜	186·2616	⅜	294·1548	⅜	426·5928
½	10·3126	½	45·0828	½	104·3988	½	188·2584	½	296·5548	½	429·6120
⅝	10·7856	⅝	46·0680	⅝	105·8952	⅝	190·2672	⅝	299·1828	⅝	432·6432
¾	11·2704	¾	47·0640	¾	107·4024	¾	192·2856	¾	301·7124	¾	435·6840
⅞	11·7660	⅞	48·0708	⅞	108·9204	⅞	194·3184	⅞	304·2540	⅞	438·7368
6 in.	12·2712	12 in.	49·0384	18 in.	110·4492	24 in.	196·3548	30 in.	306·8052	36 in.	441·7992
⅛	12·7884	⅛	50·1168	⅛	111·9888	⅛	198·4056	⅛	309·3672	¼	447·9573
¼	13·3152	¼	51·1548	¼	113·5392	¼	200·4672	¼	311·9400	½	454·1678
⅜	13·8540	⅜	52·2048	⅜	115·0992	⅜	203·5384	⅜	314·5224	¾	460·4105
½	14·4024	½	53·2644	½	116·6712	½	204·6216	½	317·1168	37 in.	466·6960
⅝	14·9616	⅝	54·3348	⅝	118·2528	⅝	206·7144	⅝	319·7220	¼	473·0240
¾	15·5316	¾	55·4760	¾	119·8452	¾	208·8192	¾	322·3368	½	479·3946
⅞	16·1124	⅞	56·4804	⅞	121·4484	⅞	210·9336	⅞	324·9624	¾	485·8078
7 in.	16·7028	13 in.	57·6108	19 in.	123·0624	25 in.	213·0588	31 in.	327·6000	38 in.	492·2637
⅛	17·3052	⅛	58·7244	⅛	124·6872	⅛	215·1948	⅛	330·2472	¼	498·7621
¼	17·9172	¼	59·8476	¼	126·3228	¼	217·3416	¼	332·9052	½	505·3032
⅜	18·5412	⅜	60·9828	⅜	127·9680	⅜	219·4980	⅜	335·5728	¾	511·9979
½	19·1748	½	62·1276	½	129·6252	½	221·6664	½	338·2524	39 in.	518·4132
⅝	19·8192	⅝	63·2832	⅝	131·5320	⅝	223·8444	⅝	340·9428	¼	525·1821
¾	20·4744	¾	64·4496	¾	132·9696	¾	226·0344	¾	343·6428	½	531·8936
⅞	21·1404	⅞	65·6268	⅞	134·6580	⅞	228·2340	⅞	346·3536	¾	538·6478
8 in.	21·8172	14 in.	66·8148	20 in.	136·3562	26 in.	230·4444	32 in.	349·0764	40 in.	545·4445
⅛	22·5036	⅛	68·0136	⅛	138·0572	⅛	232·6644	⅛	351·8088	¼	552·2839
¼	23·2020	¼	69·2220	¼	139·7880	¼	234·8576	¼	354·5520	½	559·1659
⅜	23·9100	⅜	70·4424	⅜	141·5184	⅜	237·1404	⅜	357·3048	¾	566·0904
½	24·5288	½	71·6724	½	143·2608	½	239·3928	½	360·0696	41 in.	573·0577
⅝	25·3524	⅝	72·9120	⅝	145·0128	⅝	241·6572	⅝	362·8452	½	587·1199
¾	26·0988	¾	74·1648	¾	146·7756	¾	243·9312	¾	365·6304	42 in.	601·3526
⅞	26·8500	⅞	75·4272	⅞	148·5492	⅞	246·2160	⅞	368·4276	60 in.	799·2426

TABLE *containing the Weight of Square Bar Iron, from* 1 *to* 10 *feet in length, and from* ¼ *of an inch to* 6 *inches square.*

Inches square.	LENGTH OF THE BARS IN FEET.									
	1 foot.	2 feet.	3 feet.	4 feet.	5 feet.	6 feet.	7 feet.	8 feet.	9 feet.	10 feet.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
¼	0·2	0·4	0·6	0·8	1·1	1·3	1·5	1·7	1·9	2·1
⅜	0·5	1·0	1·4	1·9	2·4	2·9	3·3	3·8	4·3	4·8
½	0·8	1·7	2·5	3·4	4·2	5·1	5·9	6·8	7·6	8·5
⅝	1·3	2·6	4·0	5·3	6·6	7·9	9·2	10·6	11·0	13·2
¾	1·9	3·8	5·7	7·6	9·5	11·4	13·3	15·2	17·1	19·0
⅞	2·6	5·2	7·8	10·4	12·9	15·5	18·1	20·7	23·3	25·9
1 in.	3·4	6·8	10·1	13·5	16·9	20·3	23·7	27·0	30·4	33·8
1⅛	4·3	8·6	12·8	17·1	21·4	25·7	29·9	34·2	38·5	42·8
1¼	5·3	10·6	15·8	21·1	26·4	31·7	37·0	42·2	47·5	52·8
1⅜	6·4	12·8	19·2	25·6	32·0	38·3	44·7	51·1	57·5	63·9
1½	7·6	15·2	22·8	30·4	38·0	45·6	53·2	60·8	68·4	76·0
1⅝	8·9	17·9	26·8	35·7	44·6	53·6	62·5	71·4	80·3	89·3
1¾	10·4	20·7	31·1	41·4	51·8	62·1	72·5	82·8	93·2	103·5
1⅞	11·9	23·8	35·6	47·5	59·4	71·3	83·2	95·1	106·9	118·8
2 in.	13·5	27·0	40·6	54·1	67·6	81·1	94·6	108·2	121·7	135·2
2⅛	15·3	30·5	45·8	61·1	76·3	91·6	106·8	122·1	137·4	152·6
2¼	17·1	34·2	51·3	68·4	85·6	102·7	119·8	136·9	154·0	171·1
2⅜	19·1	38·1	57·2	76·3	95·3	114·4	133·5	152·5	171·6	190·7
2½	21·1	42·8	63·4	84·5	105·6	126·7	147·8	169·0	190·1	211·2
2⅝	23·3	46·6	69·9	93·2	116·5	139·8	163·0	186·3	209·6	232·9
2¾	25·6	51·1	76·7	102·2	127·8	153·4	178·9	204·5	230·0	255·6
2⅞	27·9	55·9	83·8	111·8	139·7	167·6	195·7	223·5	251·5	279·4
3 in.	30·4	60·8	91·2	121·7	152·1	182·5	212·9	243·3	273·7	304·2
3⅛	33·0	66·0	99·0	132·0	165·1	198·1	231·1	264·1	297·1	330·1
3¼	35·7	71·4	107·1	142·8	178·5	214·2	249·9	285·6	321·3	357·0
3⅜	38·5	77·0	115·5	154·0	192·5	231·0	269·5	308·0	346·5	385·0
3½	41·4	82·8	124·2	165·6	207·0	248·4	289·8	331·3	372·7	414·1
3⅝	44·4	88·8	133·3	177·7	222·1	266·5	310·9	355·3	399·8	444·2
3¾	47·5	95·1	142·6	190·1	237·7	285·2	332·7	380·3	427·8	475·3
3⅞	50·8	101·5	152·3	203·0	253·8	304·5	355·3	406·0	456·8	507·6
4 in.	54·1	108·2	162·3	216·3	270·4	324·5	378·6	432·7	486·8	540·8
4⅛	57·5	115·0	172·6	230·1	287·6	345·1	402·6	460·1	517·7	575·2
4¼	61·1	122·1	183·2	244·2	305·3	366·3	427·4	488·4	549·5	610·6
4⅜	64·7	129·4	194·1	258·8	323·5	388·2	452·9	517·6	582·3	647·0
4½	68·4	136·9	205·3	273·8	342·2	410·7	479·1	547·6	616·0	684·5
4⅝	72·3	144·6	216·9	289·2	361·5	433·8	506·1	578·4	650·7	723·1
4¾	76·3	152·5	228·8	305·1	381·3	457·6	533·8	610·1	686·4	762·6
4⅞	80·3	160·7	241·0	321·3	401·7	482·0	562·3	642·7	723·0	803·3
5 in.	84·5	169·0	253·4	337·9	422·4	506·9	591·4	675·8	760·3	844·8
5¼	93·2	186·3	279·5	372·7	465·8	559·0	652·2	745·3	838·5	931·7
5½	102·2	204·5	306·7	409·0	511·2	613·4	715·7	817·9	920·2	1022·4
5¾	111·8	223·5	335·3	447·0	558·8	670·5	782·3	894·0	1005·8	1117·6
6 in.	121·7	243·3	365·0	486·7	608·3	730·0	841·6	973·3	1009·5	1216·6

TABLE *of the Weight of a Square Foot of Sheet Iron in lbs. avoirdupois, the thickness being the number on the wire-gauge. No.* 1 *is* $\frac{5}{16}$ *of an inch; No.* 4, ¼; *No.* 11, ⅛, *&c.*

No. on wire-gauge	1	2	3	4	5	6	7	8	9	10	11
Pounds avoir.......	12·5	12	11	10	9	8	7·5	7	6	5·68	5
No. on wire-gauge	12	13	14	15	16	17	18	19	20	21	22
Pounds avoir.......	4·62	4·31	4	3·95	3	2·5	2·18	1·93	1·62	1·5	1·37

TABLE *of the Weight of a Square Foot of Boiler Plate Iron, from ⅛ to* 1 *inch thick, in lbs. avoirdupois.*

⅛	3/16	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾	13/16	⅞	15/16	1 in.
5	7·5	10	12·5	15	17·5	20	22·5	25	27·5	30	32·5	35	37·5	40

TABLE *containing the Weight of Round Bar Iron, from* 1 *to* 10 *feet in length, and from ¼ of an inch to* 6 *inches diameter.*

Inches diameter.	LENGTH OF THE BARS IN FEET.									
	1 foot.	2 feet.	3 feet.	4 feet.	5 feet.	6 feet.	7 feet.	8 feet.	9 feet.	10 feet.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
¼	0·2	0·3	0·5	0·7	0·8	1·0	1·2	1·3	1·5	1·7
⅜	0·4	0·7	1·1	1·5	1·9	2·2	2·6	3·0	3·4	3·7
½	0·7	1·3	2·0	2·7	3·3	4·0	4·6	5·3	6·0	6·6
⅝	1·0	2·1	3·1	4·2	5·2	6·3	7·3	8·3	9·4	10·4
¾	1·5	3·0	4·5	6·0	7·5	9·0	10·5	11·9	13·4	14·9
⅞	2·0	4·1	6·1	8·1	10·2	12·2	14·2	16·3	18·3	20·3
1 in.	2·7	5·3	8·0	10·6	13·3	15·9	18·6	21·2	23·9	26·5
1⅛	3·4	6·7	10·1	13·4	16·8	20·2	23·5	26·9	30·2	33·6
1¼	4·2	8·3	12·5	16·7	20·9	25·0	29·2	33·4	37·5	41·7
1⅜	5·0	10·0	15·1	20·1	25·1	30·1	35·1	40·2	45·2	50·2
1½	6·0	11·9	17·9	23·9	29·9	35·8	41·8	47·8	53·7	59·7
1⅝	7·0	14·0	21·0	28·0	35·1	42·1	49·1	56·1	63·1	70·1
1¾	8·1	16·3	24·4	32·5	40·6	48·8	56·9	65·0	73·2	81·3
1⅞	9·3	18·7	28·0	37·3	46·7	56·0	65·3	74·7	84·0	93·3
2 in.	10·6	21·2	31·8	42·5	53·1	63·7	74·3	84·9	95·5	106·2
2⅛	12·0	24·0	36·0	48·0	59·9	71·9	83·9	95·9	107·9	119·9
2¼	13·4	26·9	40·3	53·8	67·2	80·6	94·1	107·5	121·0	134·4
2⅜	15·0	30·0	44·9	60·0	74·9	89·9	104·8	119·8	134·8	149·8
2½	16·7	33·4	50·1	66·8	83·5	100·1	116·8	133·6	150·2	166·9
2⅝	18·3	36·6	54·9	73·2	91·5	109·8	128·1	146·3	164·6	182·9
2¾	20·1	40·2	60·2	80·3	100·4	120·5	140·5	160·6	180·7	200·8
2⅞	21·9	43·9	65·8	87·8	109·7	131·7	153·6	175·6	197·5	219·4
3 in.	23·9	47·8	71·7	95·6	119·4	143·3	167·2	191·1	215·0	238·9
3⅛	25·9	51·9	77·8	103·7	129·6	155·6	181·5	207·4	233·3	259·3
3¼	28·0	56·1	84·1	112·2	140·2	168·2	196·3	224·3	253·4	280·4
3⅜	30·2	60·5	90·7	121·0	151·2	181·4	211·7	241·9	272·2	302·4
3½	32·5	65·0	97·5	130·0	162·6	195·1	227·6	260·1	292·6	325·1
3⅝	34·9	69·8	104·7	139·5	174·4	209·3	244·2	279·1	314·0	348·9
3¾	37·3	74·7	112·0	149·3	186·7	224·0	261·3	298·7	336·0	373·3
3⅞	39·9	79·7	119·6	159·5	199·3	239·2	279·0	318·9	358·8	398·6
4 in.	42·5	84·9	127·4	169·9	212·3	254·8	297·2	339·7	382·2	424·6
4⅛	45·2	90·3	135·5	180·7	225·9	271·0	316·2	361·4	406·6	451·7
4¼	48·0	95·9	143·9	191·8	239·8	287·7	335·7	383·6	431·6	479·5
4⅜	50·8	101·6	152·4	203·3	254·1	304·9	355·7	406·5	457·3	508·2
4½	53·8	107·5	161·3	215·0	268·8	322·6	376·3	430·1	483·8	537·6
4⅝	56·8	113·6	170·4	227·2	283·9	340·7	397·5	454·3	511·1	567·9
4¾	60·0	119·8	179·7	239·6	299·5	359·4	419·3	479·2	539·1	599·0
4⅞	63·1	126·2	189·3	252·4	315·5	378·6	441·7	504·8	567·8	630·9
5 in.	66·8	133·5	200·3	267·0	333·8	400·5	467·3	534·0	600·8	667·5
5¼	73·2	146·3	219·5	292·7	365·9	439·0	512·2	585·4	658·5	731·7
5½	80·3	160·6	240·9	321·2	401·5	481·8	562·1	642·4	722·7	803·0
5¾	87·8	175·6	263·3	351·1	438·9	526·7	614·4	702·2	790·0	877·8
6 in.	95·6	191·1	286·7	382·2	477·8	573·3	668·9	764·4	860·0	955·5

TABLE *of the Weight of Cast Iron Plates, per Superficial Foot, from one-eighth of an inch to one inch thick.*

⅛ inch.	¼ inch.	⅜ inch.	½ inch.	⅝ inch.	¾ inch.	⅞ inch.	1 inch.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
4 13⅜	9 10⅝	14 8	19 5⅜	24 2¾	29 0	33 13⅜	38 10¾

TABLE *containing the Weight of Cast Iron Pipes,* 1 *foot in length.*

Diameter of Bore in Inches.	THICKNESS IN INCHES.							
	⅜	½	⅝	¾	⅞	1 inch.	1⅛	1¼
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1½	6·9	9·9	......	......	......	......	......	......
2	8·8	12·3	16·1	20·3	......	......	......	......
2½	10·6	14·7	19·2	23·9	......	......	......	......
3	12·4	17·2	22·2	27·6	33·3	39·3	45·6	......
3½	14·2	19·6	25·3	31·3	37·6	44·2	51·1	......
4	16·8	22·1	28·4	35·0	41·9	49·1	56·6	64·4
4½	18·0	24·5	31·4	38·7	46·2	54·0	62·1	70·6
5	19·8	27·0	34·5	42·3	50·5	58·9	67·6	76·7
5½	21·6	29·5	37·6	46·0	54·8	63·8	73·2	82·8
6	23·5	31·9	40·7	49·7	59·1	68·7	78·7	88·8
6½	25·3	34·4	43·7	53·4	63·4	73·4	84·2	95·1
7	27·2	36·8	46·8	56·8	67·7	78·5	89·7	101·2
7½	29·0	39·1	49·9	60·7	72·0	83·5	95·3	107·4
8	30·8	41·7	52·9	64·4	76·2	88·4	100·8	113·5
8½	32·9	44·4	56·2	68·3	80·8	93·5	106·5	119·9
9	34·5	46·6	59·1	71·8	84·8	98·2	111·8	125·8
9½	36·3	49·1	62·1	75·5	89·1	103·1	117·4	131·9
10	38·2	51·5	65·2	79·2	93·4	108·0	122·8	138·1
10½	......	54·0	68·2	82·8	97·7	112·9	128·4	144·2
11	......	56·4	71·3	86·5	102·0	117·8	133·9	150·3
11½	......	58·9	74·3	90·1	106·3	122·7	139·4	156·4
12	......	61·3	77·4	93·6	110·6	127·6	145·0	162·6
13	......	......	82·7	101·2	118·2	137·4	154·1	173·5
14	......	......	89·5	108·2	126·5	146·2	165·3	185·2
15	......	......	95·2	115·7	135·3	156·2	176·2	198·1
16	......	......	......	123·3	143·1	166·1	187·5	211·3
17	......	......	......	130·2	152·5	178·5	198·2	223·4
18	......	......	......	137·0	161·2	185·3	209·1	235·6
19	......	......	......	......	169·2	195·7	222·3	247·1
20	......	......	......	......	178·1	205·2	233·2	259·0
21	......	......	......	......	......	214·1	243·5	273·2
22	......	......	......	......	......	223·0	254·8	285·4
23	......	......	......	......	......	233·4	265·5	298·3
24	......	......	......	......	......	245·2	277·5	310·6

TABLE *containing the Weight of Solid Cylinders of Cast Iron, one foot in length, and from* ¾ *of an inch to* 14 *inches diameter.*

Diameter in Inches.	Weight in Lbs.	Diameter in Inches.	Weight in Lbs.	Diameter in Inches.	Weight in Lbs.	Diameter in Inches.	Weight in Lbs.
¾	1·39	2⅞	20·48	4⅞	58·72	7¾	148·87
⅞	1·88	3 in.	22·35	5 in.	61·96	8 in.	158·63
1 in.	2·47	3⅛	24·20	5⅛	64·66	8¼	168·15
1⅛	3·13	3¼	26·18	5¼	68·31	8½	179·08
1¼	3·87	3⅜	28·23	5⅜	71·00	8¾	189·00
1⅜	4·68	3½	30·36	5½	74·98	9 in.	200·77
1½	5·57	3⅝	32·57	5⅝	78·65	9¼	211·12
1⅝	6·54	3¾	34·85	5¾	81·95	9½	223·70
1¾	7·59	3⅞	37·21	5⅞	85·81	9¾	235·31
1⅞	8·71	4 in.	39·66	6 in.	89·23	10 in.	247·87
2 in.	9·91	4⅛	41·80	6¼	96·82	10½	273·27
2⅛	11·19	4¼	44·77	6½	104·72	11 in.	299·92
2¼	12·54	4⅜	47·00	6¾	112·93	11½	327·81
2⅜	13·98	4½	50·19	7 in.	121·45	12 in.	356·93
2½	15·49	4⅝	52·71	7¼	130·28	13	418·99
2⅝	17·08	4¾	55·92	7½	139·42	14	485·83
2¾	18·74						

TABLE *containing the Weight of a Square Foot of Copper and Lead, in lbs. avoirdupois, from* $\frac{1}{32}$ *to* $\frac{1}{2}$ *an inch in thickness, advancing by* $\frac{1}{32}$.

Thickness.	Copper.	Lead.
$\frac{1}{32}$	1·45	1·85
$\frac{1}{16}$	2·90	3·70
$\frac{3}{32}$	4·35	5·54
$\frac{1}{8}$	5·80	7·39
$\frac{1}{8} + \frac{1}{32}$	7·26	9·24
$\frac{1}{8} + \frac{1}{16}$	8·71	11·08
$\frac{1}{8} + \frac{3}{32}$	10·16	12·93
$\frac{1}{4}$	11·61	14·77
$\frac{1}{4} + \frac{1}{32}$	13·07	16·62
$\frac{1}{4} + \frac{1}{16}$	14·52	18·47
$\frac{1}{4} + \frac{3}{32}$	15·97	20·31
$\frac{3}{8}$	17·41	22·16
$\frac{3}{8} + \frac{1}{32}$	18·87	24·00
$\frac{3}{8} + \frac{1}{16}$	20·32	25·85
$\frac{3}{8} + \frac{3}{32}$	21·77	27·70
$\frac{1}{2}$	23·22	29·55

TABLE *for finding the Weight of Malleable Iron, Copper, and Lead Pipes,* 12 *inches long, of various thicknesses, and any diameter required.*

Thickness.	Malleable Iron.	Copper.	Lead.
. $\frac{1}{32}$ of an inch.	·104	·121	·1539
$\frac{1}{16}$	·208	·2419	·3078
$\frac{3}{32}$	·3108	·3628	·4616
$\frac{1}{8}$	·414	·4838	·6155
$\frac{1}{8} + \frac{1}{32}$	·518	·6047	·7694
$\frac{1}{8} + \frac{1}{16}$	·621	·7258	·9232
$\frac{1}{8} + \frac{3}{32}$	·725	·8466	1·0771
$\frac{1}{4}$	·828	·9678	1·231

RULE.—Multiply the circumference of the pipe in inches by the numbers opposite the thickness required, and by the length in feet; the product will be the weight in avoirdupois lbs. nearly.

Required the weight of a copper pipe 12 feet long, 15 inches in circumference, $\frac{1}{8} + \frac{1}{16}$ of an inch in thickness.

$$·7258 \times 15 = 10·817 \times 12 = 130·644 \text{ lbs. nearly.}$$

TABLE *of the Weight of a Square Foot of Millboard in lbs. avoirdupois.*

Thickness in inches......	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$
Weight in lbs.............	·688	1·032	1·376	1·72	2·064

TABLE *containing the Weight of Wrought Iron Bars* 12 *inches long in lbs. avoirdupois.*

Inch.	Round.	Square.	Inch.	Round.	Square.
$\frac{1}{4}$	·163	·208	$2\frac{1}{2}$	16·32	20·80
$\frac{3}{8}$	·367	·467	$2\frac{5}{8}$	18·00	22·89
$\frac{1}{2}$	·653	·830	$2\frac{3}{4}$	19·76	25·12
$\frac{5}{8}$	1·02	1·30	$2\frac{7}{8}$	21·59	27·46
$\frac{3}{4}$	1·47	1·87	3	23·52	29·92
$\frac{7}{8}$	2·00	2·55	$3\frac{1}{4}$	27·60	35·12
1	2·61	3·32	$3\frac{1}{2}$	32·00	40·80
$1\frac{1}{8}$	3·31	4·21	$3\frac{3}{4}$	36·72	46·72
$1\frac{1}{4}$	4·08	5·20	4	41·76	53·12
$1\frac{3}{8}$	4·94	6·28	$4\frac{1}{4}$	47·25	60·00
$1\frac{1}{2}$	5·88	7·48	$4\frac{1}{2}$	52·93	67·24
$1\frac{5}{8}$	6·90	8·78	$4\frac{3}{4}$	58·92	74·95
$1\frac{3}{4}$	8·00	10·20	5	65·28	83·20
$1\frac{7}{8}$	9·18	11·68	$5\frac{1}{4}$	72·00	91·56
2	10·44	13·28	$5\frac{1}{2}$	79·04	100·48
$2\frac{1}{8}$	11·80	15·00	$5\frac{3}{4}$	86·36	109·82
$2\frac{1}{4}$	13·23	16·81	6	94·08	119·68
$2\frac{3}{8}$	14·73	18·74	7	128·00	163·20

TABLE *of the Proportional Dimensions of* 6*-sided Nuts for Bolts from* $\frac{1}{4}$ *to* $2\frac{1}{2}$ *inches diameter.*

Diameter of bolts........	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
Breadth of nuts..........	$\frac{11}{16}$	$\frac{13}{16}$	1	$1\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{15}{16}$	$2\frac{1}{8}$
Breadth over the angles	$\frac{3}{4}$	$1\frac{5}{6}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{8}{16}$	$1\frac{13}{16}$	2	$2\frac{1}{4}$	$2\frac{7}{16}$
Thickness.................	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{7}{16}$
Diameter of bolts........	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	
Breadth of nuts..........	$2\frac{5}{16}$	$2\frac{1}{2}$	$2\frac{11}{16}$	$2\frac{7}{8}$	$3\frac{1}{16}$	$3\frac{1}{4}$	$3\frac{5}{8}$	4	
Breadth over the angles	$2\frac{11}{16}$	$2\frac{7}{8}$	$3\frac{1}{8}$	$3\frac{5}{16}$	$3\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{3}{16}$	$4\frac{5}{8}$	
Thickness.................	$1\frac{9}{16}$	$1\frac{11}{16}$	$1\frac{13}{16}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	

TABLE *of the Specific Gravity of Water at different temperatures, that at* 62° *being taken as unity.*

70° F.	·99913	52° F.	1·00076
68	·99936	50	1·00087
66	·99958	48	1·00095
64	·99980	46	1·00102
62	1·	44	1·00107
58	1·00035	42	1·00111
56	1·00050	40	1·00113
54	1·00064	38	1·00115

The difference of temperatures between 62° and 39°·2, where water attains its greatest density, will vary the bulk of a gallon rather less than the third of a cubic inch.

TABLE *of the Weight of Cast Iron Balls in pounds avoirdupois, from* 1 *to* 12 *inches diameter, advancing by an eighth.*

Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.
1	·14	4¾	14·76	8½	84·56
1⅛	·20	4⅞	15·95	8⅝	88·34
1¼	·27	5	17·12	8¾	92·24
1⅜	·37	5⅛	18·54	8⅞	96·26
1½	·47	5¼	19·93	9	100·39
1⅝	·59	5⅜	21·39	9⅛	104·62
1¾	·74	5½	22·91	9¼	108·98
1⅞	·91	5⅝	24·51	9⅜	113·46
2	1·10	5¾	26·18	9½	118·06
2⅛	1·32	5⅞	27·91	9⅝	122·77
2¼	1·57	6	29·72	9¾	127·63
2⅜	1·84	6⅛	31·64	9⅞	132·60
2½	2·15	6¼	33·62	10	137·71
2⅝	2·49	6⅜	35·67	10⅛	142·91
2¾	2·86	6½	37·80	10¼	148·28
2⅞	3·27	6⅝	40·10	10⅜	153·78
3	3·72	6¾	42·35	10½	159·40
3⅛	4·20	6⅞	44·74	10⅝	165·16
3¼	4·72	7	47·21	10¾	171·05
3⅜	5·29	7⅛	49·79	10⅞	177·10
3½	5·80	7¼	52·47	11	183·29
3⅝	6·56	7⅜	55·23	11⅛	189·60
3¾	7·26	7½	58·06	11¼	196·10
3⅞	8·01	7⅝	60·04	11⅜	202·67
4	8·81	7¾	64·09	11½	209·43
4⅛	9·67	7⅞	67·25	11⅝	216·32
4¼	10·57	8	70·49	11¾	223·40
4⅜	11·53	8⅛	73·85	11⅞	230·57
4½	12·55	8¼	77·32	12	237·94
4⅝	13·62	8⅜	80·88		

TABLE *of the Weight of Flat Bar Iron,* 12 *inches long, in lbs. avoirdupois.*

Thickness.		⅛	3/16	¼	⅜	½	⅝	¾	⅞	1 inch.
Breadth in inches.	½	·21	·31	·42	·63					
	¾	·31	·47	·63	·94	1·26	1·57			
	1	·42	·63	·84	1·26	1·68	2·10	2·52	2·94	
	1¼	·52	·78	1·05	1·57	2·10	2·62	3·15	3·67	4·20
	1⅜	·57	·86	1·18	1·73	2·31	2·88	3·46	4·04	4·62
	1½	·63	·94	1·26	1·89	2·52	3·15	3·78	4·41	5·04
	1¾	·73	1·10	1·47	2·20	2·94	3·67	4·41	5·14	5·87
	2	·84	1·26	1·68	2·52	3·36	4·20	5·06	5·88	6·72
	2¼	·96	1·41	1·89	2·83	3·78	4·72	5·66	6·61	7·56
	2½	1·05	1·57	2·10	3·15	4·20	5·25	6·30	7·35	8·40
	2¾	1·15	1·73	2·31	3·46	4·62	5·77	6·93	8·08	9·24
	3	1·26	1·89	2·52	3·78	5·04	6·30	7·56	8·82	10·08
	3¼	1·36	2·04	2·73	4·09	5·46	6·82	8·19	9·55	10·92
	3½	1·47	2·20	2·94	4·41	5·88	7·35	8·82	10·29	11·76
	3¾	1·57	2·36	3·15	4·72	6·30	7·87	9·45	11·02	12·60
	4	1·68	2·52	3·36	5·04	6·72	8·40	10·08	11·76	13·44
	4½	1·89	2·83	3·73	5·67	7·56	9·45	11·34	13·23	15·12
	5	2·10	3·15	4·12	6·30	8·40	10·50	12·60	16·70	17·80
	6	2·52	3·78	5·04	7·56	10·08	12·60	15·12	17·64	20·16

Weight of a copper rod 12 inches long and 1 inch diameter = 3·039 lbs.
Weight of a brass rod 12 inches long and 1 inch diameter = 2·86 lbs.

BRASS.—*Weight of a Lineal Foot of Round and Square.*

Diameter.	Weight of round.	Weight of square.	Diameter.	Weight of round.	Weight of square.
Inches.	Lbs.	Lbs.	Inches.	Lbs.	Lbs.
$\frac{1}{4}$	·17	·22	$1\frac{3}{4}$	8·66	11·03
$\frac{3}{8}$	·39	·50	$1\frac{7}{8}$	9·95	12·66
$\frac{1}{2}$	·70	·90	2	11·32	14·41
$\frac{5}{8}$	1·10	1·40	$2\frac{1}{8}$	12·78	16·27
$\frac{3}{4}$	1·59	2·02	$2\frac{1}{4}$	14·32	18·24
$\frac{7}{8}$	2·16	2·75	$2\frac{3}{8}$	15·96	20·32
1	2·83	3·60	$2\frac{1}{2}$	17·68	22·53
$1\frac{1}{8}$	3·58	4·56	$2\frac{5}{8}$	19·50	24·83
$1\frac{1}{4}$	4·42	5·63	$2\frac{3}{4}$	21·40	27·25
$1\frac{3}{8}$	5·35	6·81	$2\frac{7}{8}$	23·39	29·78
$1\frac{1}{2}$	6·36	8·00	3	25·47	32·43
$1\frac{5}{8}$	7·47	9·51			

STEEL.—*Weight of One Foot of Round Steel.*

Diameter in inches and parts.	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2
Weight in lbs. and decimal parts.	·167	·376	·669	1·04	1·5	2·05	2·67	3·38	4·18	5·06	6·02	7·07	8·2	9·41	11·71

TABLES OF THE WEIGHTS OF ROLLED IRON,

Per lineal foot, of various sections, illustrated in the accompanying cuts, viz. Parallel Angle Iron, equal and unequal sides; Taper Angle Iron; Parallel T *Iron, equal and unequal depth and width; Taper* T *Iron; Sash Iron; and Permanent and Temporary Rails.*

TABLE I.—*Parallel Angle Iron, of equal sides.* (Fig. 1.)

Length of sides A B, in inches.	Uniform thickness throughout.	Weight of one lineal foot in lbs.
Inches.	Inches.	
3	$\frac{3}{8}$	8·0
$2\frac{3}{4}$	$\frac{3}{8}$	7·0
$2\frac{1}{2}$	$\frac{3}{8}$	5·75
$2\frac{1}{4}$	5-16ths	4·5
2	$\frac{1}{4}$ full	3·75
$1\frac{3}{4}$	$\frac{1}{4}$	3·0
$1\frac{1}{2}$	$\frac{1}{4}$	2·5
$1\frac{3}{8}$	No. 6 wire-gauge	1·75
$1\frac{1}{4}$	8	1·5
$1\frac{1}{8}$	9	1·25
1	10	1·0
$\frac{7}{8}$	10	·875
$\frac{3}{4}$	11	·625
$\frac{5}{8}$	11	·568
$\frac{1}{2}$	12	·5

Fig. 1.

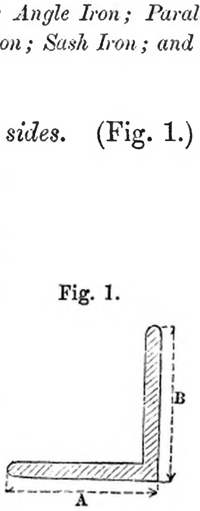


TABLE II.—*Parallel Angle Iron, of unequal sides.* (Fig. 2.)

Length of side A, in inches.	Length of side B, in inches.	Uniform thickness throughout.	Weight of one lineal foot in lbs.
Inches.	Inches.	Inches.	
3½	5	⅜	9·75
3	5	⅜	8·75
3	4	5-16ths	7·5
2¼	4	5-16ths	6·75
2¼	4	¼	5·75
2	4	¼	5·5
2½	3	¼	4·75
2	2½	¼	3·375
1½	2	¼	2·875
1½	2	3-16ths	2·25

Fig. 2.

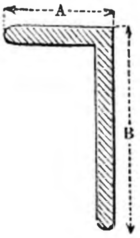


TABLE III.—*Taper Angle Iron, of equal sides.* (Fig. 3.)

Length of sides, A A, in inches.	Thickness of edges at B.	Thickness of root at C.	Weight of one lineal foot in lbs.
Inches.	Inches.	Inches.	
4	½	⅝	14·0
3	½	⅝	10·375
2¾	7-16ths	9-16ths	8·25
2½	⅜	½	6·5
2¼	5-16ths, full	7-16ths	5·0
2	¼ full	5-16ths, full	3·875
1¾	¼	5-16ths	3·25
1½	¼ bare	5-16ths, bare	2·625

Fig. 3.

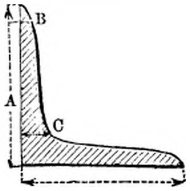


TABLE IV.—*Parallel* T *Iron, of unequal width and depth.* (Fig. 4.)

Width of top table A, in inches.	Total depth B, in inches.	Uniform thickness of top table C.	Uniform thickness of rib D.	Weight of one lineal foot in lbs.
Inches.	Inches.	Inches.	Inches.	
5	6	½	½	15·75
4½	3¼	½	9.16ths	13·25
4	3	⅜	⅜	8·875
3½	3	⅜	⅜	8·25
3½	4	½	½	12·5
2½	3	⅜	⅜	7·0
2¼	2	5-16ths	⅜ full	4·5
2	1½	5-16ths	5-16ths	4·0
1¾	2	¼	¼	3·125
1½	2	¼	¼	2·875
1¼	1½	¼	¼	2·375
1	1¼	3-16ths	3-16ths	1·5
¾	1	3-16ths	3-16ths	1·125

Fig. 4.

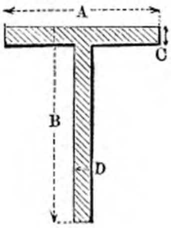


TABLE V.—*Parallel T Iron, of equal depth and width.* (Fig. 5.)

Width of top table, and total depth A A.	Uniform thickness throughout.	Weight of one lineal foot in lbs.
Inches.	Inches.	
6	½	
5	7-16ths	13·75
4	⅜	9·75
3½	⅜	8·5
3	⅜	7·5
2½	5-16ths	4·625
2¼	5-16ths	4·5
2	5-16ths	3·75
1¾	¼	3·0
1½	¼	2·25
1¼	¼	1·75
1	3-16ths	1·0
⅞	⅛	·725
¾	⅛	·625

Fig. 5.

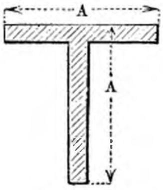
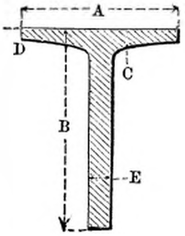


TABLE VI.—*Taper T Iron.* (Fig. 6.)

Fig. 6.



Width of top table A, in inches.	Total depth B, in inches.	Thickness of top table at root C.	Thickness of top table at edges D.	Uniform thickness of rib E.	Weight of one lineal foot in lbs.
Inches.	Inches.	Inches.	Inches.	Inches.	
3	3¼	½	⅜	7-16ths	8·0
3	2⅝	7-16ths	⅜	½	8·0
2½	3	7-16ths	5-16ths	5-16ths	5·25
2	2½	⅝	½	½	6·5
2	1½	⅜ full	5-16ths	⅜	3·5
2	1½	5-16ths	¼	¼	2·875

TABLE VII.—*Sash Iron.* (Fig. 7.)

Total depth A.	Depth of rebate B.	Width at edge C.	Greatest width D.	Weight of one lineal foot in lbs.
Inches.	Inches.		Inches.	
2	1	No. 9 wire-gauge	5-8ths	1·75
1¾	¾	7	9-16ths	1·625
1½	¾	6	9-16ths	1·25
1⅜	⅝	10	9-16ths	1·125
1¼	⅝	10	9-16ths	1·0
1	½	⅛	½	·75

Fig. 7.

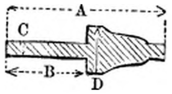


Fig. 8.

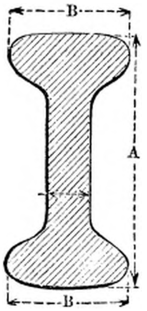


TABLE VIII.—*Rails equal top and bottom Tables.* (Fig. 8.)

Depth A, in inches.	Width across top and bottom B B, in inches.	Thickness of rib C.	Weight of one lineal foot in lbs.
Inches.	Inches.	Inches.	
5	2⅝	¾	25·0
4½	2½	¾	23·33
4½	2½	⅝	21·66

TABLE IX.—*Temporary Rails.* (Fig. 9.)

Top width A, in inches.	Rib width B, in inches.	Bed width C, in inches.	Total depth D, in inches.	Thickness of bed E.	Weight of one lineal foot in lbs.
Inches.	Inches.	Inches.	Inches.	Inches.	
1½	⅝	3	2	7-16ths	9·0
1¾	⅝	3	2½	½	12·0
1⅞	⅝	4	3	½	16·0
2	⅝	4	3	½	17·33

Fig. 9.

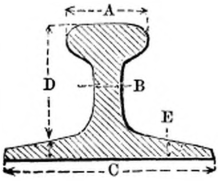


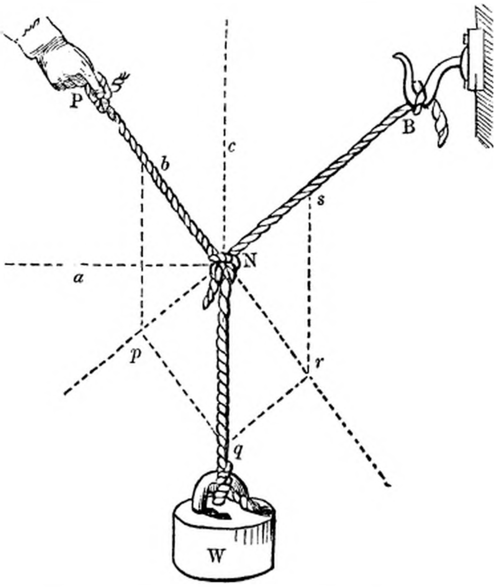
TABLE *of Natural Sines, Co-sines, Tangents, Co-tangents, Secants, and Co-secants, to every degree of the Quadrant.*

Deg.	Sines.	Co-sines.	Tangents.	Co-tangents.	Secants.	Co-secants.	Degree.
0	·00000	1·00000	·00000	Infinite.	1·00000	Infinite.	90
1	·01745	·99985	·01746	57·2900	1·00015	57·2987	89
2	·03490	·99939	·03492	28·6363	1·00061	28·6537	88
3	·05234	·99863	·05241	19·0811	1·00137	19·1073	87
4	·06976	·99756	·06993	14·3007	1·00244	14·3356	86
5	·08716	·99619	·08749	11·4301	1·00382	11·4737	85
6	·10453	·99452	·10510	9·51236	1·00551	9·56677	84
7	·12187	·99255	·12278	8·14435	1·00751	8·20551	83
8	·13917	·99027	·14054	7·11537	1·00983	7·18530	82
9	·15643	·98769	·15838	6·31375	1·01246	6·39245	81
10	·17365	·98481	·17633	5·67128	1·01543	5·75877	80
11	·19081	·98163	·19438	5·14455	1·01872	5·24084	79
12	·20791	·97815	·21256	4·70463	1·02234	4·80973	78
13	·22495	·97437	·23087	4·33148	1·02630	4·44541	77
14	·24192	·97030	·24933	4·01078	1·03061	4·13356	76
15	·25882	·96593	·26795	3·73205	1·03528	3·86370	75
16	·27564	·96126	·28675	3·48741	1·04030	3·62796	74
17	·29237	·95630	·30573	3·27085	1·04569	3·42030	73
18	·30902	·95106	·32492	3·07768	1·05146	3·23607	72
19	·32557	·94552	·34433	2·90421	1·05762	3·07155	71
20	·34202	·93969	·36397	2·74748	1·06418	2·92380	70
21	·35837	·93358	·38386	2·60509	1·07114	2·79043	69
22	·37461	·92718	·40403	2·47509	1·07853	2·66947	68
23	·39073	·92050	·42447	2·35585	1·08636	2·55930	67
24	·40674	·91355	·44523	2·24004	1·09464	2·45859	66
25	·42262	·90631	·46631	2·14451	1·10338	2·36620	65
26	·43837	·89879	·48773	2·05030	1·11260	2·28117	64
27	·45399	·89101	·50952	1·96261	1·12233	2·20869	63
28	·46947	·88295	·53171	1·88073	1·13257	2·13005	62
29	·48481	·87462	·55431	1·80405	1·14335	2·06266	61
30	·50000	·86603	·57735	1·73205	1·15470	2·00000	60
31	·51504	·85717	·60086	1·66428	1·16663	1·94160	59
32	·52992	·84805	·62487	1·60033	1·17918	1·88708	58
33	·54464	·83867	·64941	1·53986	1·19236	1·83608	57
34	·55919	·82904	·67451	1·48256	1·20622	1·78829	56
35	·57358	·81915	·70021	1·42815	1·22077	1·74345	55
36	·58778	·80902	·72654	1·37638	1·23607	1·70130	54
37	·60181	·79863	·75355	1·32704	1·25214	1·66164	53
38	·61566	·78801	·78129	1·27994	1·26902	1·62427	52
39	·62932	·77715	·80978	1·23490	1·28676	1·58902	51
40	·64279	·76604	·83910	1·19175	1·30541	1·55572	50
41	·65606	·75471	·86929	1·15037	1·32511	1·52425	49
42	·66913	·74314	·90040	1·11061	1·34561	1·49448	48
43	·68200	·73135	·93251	1·07237	1·36706	1·46628	47
44	·69466	·71934	·96569	1·03553	1·39012	1·43956	46
45	·70711	·70711	1·00000	1·00000	1·41421	1·41421	45
Deg.	Co-sines.	Sines.	Co-tangents.	Tangents.	Co-secants.	Secants.	Degree.

MOMENT OF INERTIA.

CORDS, KNOTS, NODES, CHAIN-BRIDGE.—ANGULAR VELOCITY.—RADIUS OF GYRATION.

1. If the cord qNB, be fixed at the extremity B, and stretched by a weight of 500 lbs. at the extremity q, and the middle knot or node N, by a force of 255 lbs. pulling upwards, under an angle aNb of 54°; what is the tension and position of NB.



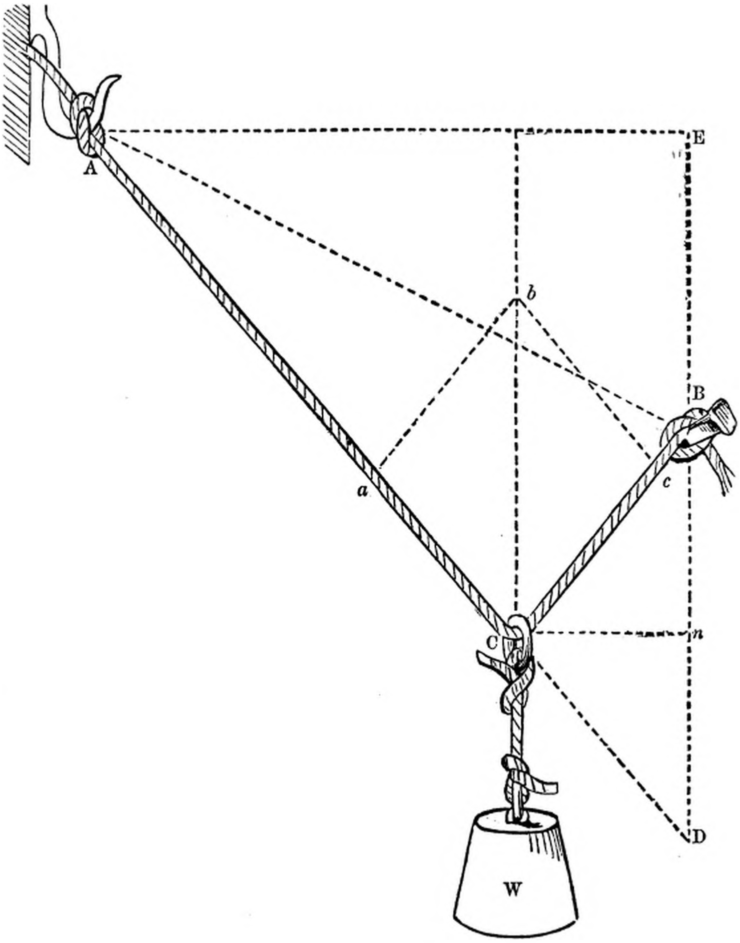
Angle qNr = 180° − angle qNP; and 90° − aNb = bNc = qNr = 36°; cos. 36° = ·80902.

$\sqrt{500^2 + 255^2 - 2 \times 255 \times 500 \times \text{cos. } 36°}$ = 329·7 lbs., the magnitude of the tension.

$\frac{500 \text{ sin. } 36°}{329·7}$ = ·891386 = sine of angles bNs, or angle BNr = 63° 2′.

2. Between the points A and B, a cord 10 feet in length is stretched by a weight W of 500 lbs. suspended to it by a ring; the horizontal distance AE = 6·6 feet, and the vertical distance BE = 3·2 feet; required the position of the ring C, the tensions, and directions of the rope.

The tensions of the cords AC, CB are equal, and angle ACb = angle bCB.



$$AD = AC + CB = 10 \text{ feet.}$$

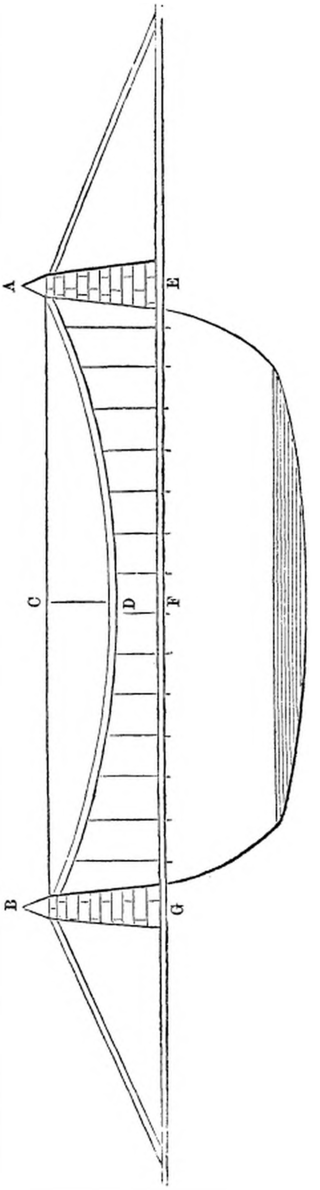
$$\sqrt{(10^2 - 6{\cdot}6^2)} = 7{\cdot}5126 = ED;\ BD = 7{\cdot}5126 - 3{\cdot}2 = 4{\cdot}3126$$

$Dn = \frac{4{\cdot}3126}{2} = 2{\cdot}1563$; $7{\cdot}5126 : 2{\cdot}1563 :: 10 : \frac{21{\cdot}563}{7{\cdot}5126} = 2{\cdot}87 = CD = CB$; and $CA = 10 - 2{\cdot}87 = 7{\cdot}13$.

$$\frac{Bn}{Bc} = \text{cosine } bCB = \frac{2{\cdot}1563}{2{\cdot}87} = {\cdot}75132.$$

$\therefore \angle bCB = 41° 18'$; $\frac{W}{2 \cos. 41° 18'} = \frac{500}{1{\cdot}50264} = 332{\cdot}7$ lbs., the tension on the cord CB, which is equal to the tension on AC.

3. Let 500,000 lbs. be the whole weight on a chain-bridge whose span AB = 400 feet, and height of the arc CD = 40 feet; required the tensions and other circumstances respecting the chains.



The tangent of the angles of inclination of the ends of the chain is equal $\frac{40 \times 2}{200} = \cdot 40000$, the angle answering to this natural tangent is 21° 48′.

The vertical tension at each point of suspension is = half the weight = 250000; the horizontal tension at the points of suspension = 250000 × cot. 21° 48′ = $\frac{250000}{\cdot 4}$ = 625000 lbs.

The whole tension at one end will be

$$\sqrt{625000^2 + 250000^2} = 673146 \text{ lbs.}$$

4. Suppose the piston of a steam engine, with its rod, weighs 1000 lbs.; it has no velocity at its highest and lowest positions, but in the middle the velocity is a maximum and equal 10 ft.; what effect will it accumulate by virtue of its inertia in the first half of its path, and give out again in the second half; and what is the mean force which would be requisite to accelerate the motion of the piston in the first half of its path, which is the same as that which it would exert in the second half by its retardation, the length of stroke being 8 feet.

According to the principle of *vis viva*, the effect which the piston will accumulate by virtue of its inertia in the first half of its path, and give out again in the second half = $\frac{10^2}{2 \times 32\cdot 2} \times 1000 = 1552\cdot 794$ units of work. Half the path of the piston = 4 feet; hence,

$$\frac{1552\cdot 794}{4} = 388\cdot 1985 \text{ lbs., the mean force.}$$

Moment of Inertia, or the Moment of Rotation, or the Moment of the Mass, is the sum of the products of the particles of the mass and the squares of their distances from the axis of rotation.

5. If a body at rest, but capable of turning round a fixed axis A, possesses a moment of inertia of 121 units of work, the measures taken in feet and pounds, made to turn by means of a cord and weight of 36 lbs., lying over a pulley in a path of 10 feet; what are the circumstances of the motion.

$$\sqrt{\frac{2 \times 36 \times 10}{121}} = 2{\cdot}439347 \text{ feet, the angular velocity of the}$$

body, which call v; so that each point at the distance of one foot from the axis of revolution will describe, after the accumulation of 121 units of work, 2·44 feet in a second.

$$6{\cdot}2832 = \text{circumference of a circle 2 feet in diameter,}$$

$$\frac{6{\cdot}2832}{2{\cdot}44} = 2{\cdot}6 \text{ seconds, the time of one revolution.}$$

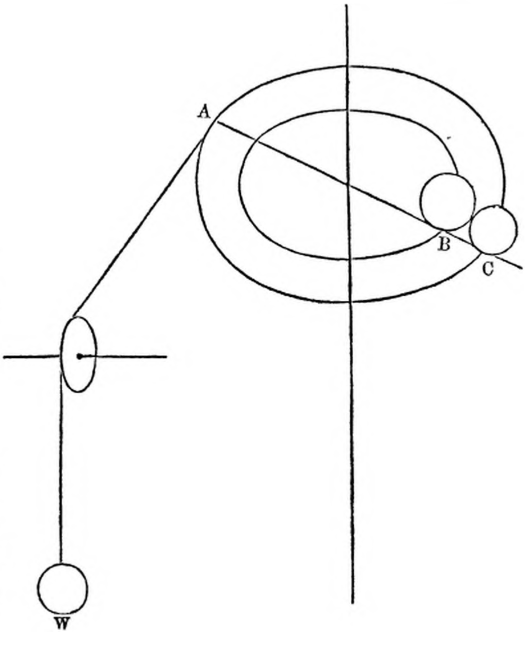
6. If an angular velocity of 3 feet passes into a velocity of 7 feet; what mechanical effect will a *mass* produce so moving, supposing the *moment of inertia* to be 200, the measures taken in feet and pounds.

According to the principles of *vis viva*,

$$(7^2 - 3^2)\frac{200}{2} = 4000 \text{ units of work, which may be 40 lbs.}$$

raised 100 feet, 80 lbs. raised 50 feet, 400 lbs. raised 10 feet; and so on.

7. The weight of a rotating mass B is 500 lbs., its distance OB from the axis of rotation 3 feet, the weight W, constituting the moving force, 90 lbs., its arm AO = OC = 4 feet; required the circumstances of the motion that ensues.



$$\left\{90 + \frac{3^2}{4^3}500\right\} \div 32{\cdot}2 = 11{\cdot}53 \text{ lbs.,}$$

the inert *mass* accelerated by the force of W. And it is well known that the force divided by the *mass* gives the acceleration.

$\therefore \frac{90}{11{\cdot}53} = 7{\cdot}806$, the acceleration of the motion of W. The angular acceleration in a circle 1 foot from the axis $= \frac{7{\cdot}806}{4} = 1{\cdot}9515$.

After 10 seconds the acquired angular velocity will be

$$1{\cdot}9515 \times 10 = 19{\cdot}515.$$

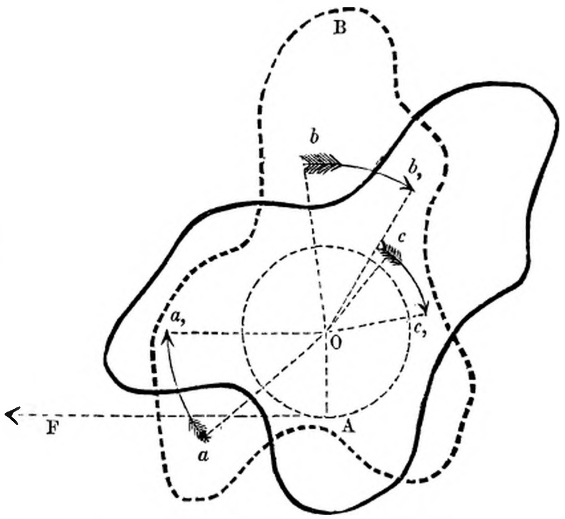
And the corresponding distance $= \frac{1{\cdot}9515 \times 10^2}{2} = 97{\cdot}575$ feet, measured on a circle one foot from O.

The space described by the weight W is $\frac{7{\cdot}806 \times 10^2}{2} = 390{\cdot}3$ feet, which is the same as the space described by C. The circumference of a circle one foot from C $= 3{\cdot}1416$.

$$\therefore \frac{97{\cdot}575}{3{\cdot}1416} = 31{\cdot}059 \text{ revolutions.}$$

In the rotation of a body AB about a fixed axis O, all its points describe equal angles in equal times. If the body rotate in a certain time through the angle θ°, or arc $\phi = \frac{\theta^\circ}{180^\circ}\pi$, radius $= 1$; and hence, $\pi = 3{\cdot}141592$, &c.; the elements of the body, a, b, c, &c., at the distances $oa = x_1$, $ob = x_2$, &c. from the axis, will describe the arcs or spaces $aa_1 = \phi x_1$, $bb_1 = \phi x_2$, $cc_1 = \phi x_3$, &c. If the angular velocity, that is, the velocity of those points of the body which are distant a unit of length, a foot, from the axis of revolution, be put $= z$, then the simultaneous velocities of the elements of the mass at the distances x_1, x_2, x_3, &c., will be,

$$z x_1,\ z x_2,\ z x_3,\ \text{\&c.}$$



And if a be the *mass* of the element at a; b the *mass* of the element at b; c the *mass* of the element at c, &c., their *vis viva* will be,

$$(z x_1)^2 a,\ (z x_2)^2 b,\ (z x_3)^2 c,\ \text{\&c.}$$

And the sum of the *vis viva* of the whole body $=$

$$z^2 (x_1^2 a + x_2^2 b + x_3^2 c, \text{\&c.})$$

According to our definition, $x_1^2 a + x_2^2 b + x_3^2 c$, &c. is the *moment of inertia*, which may be represented by R; then $z^2 R$ is the *vis viva* of a body revolving with the angular velocity z. Therefore, to communicate to a body in a state of rest an angular velocity z, a *mechanical effect* $F s$, or force × space = $\frac{1}{2}$ the *vis viva*, must be expended; that is, $F s = \frac{1}{2} z^2 R$, or, which is the same thing, a body performing the units of work $F s$, passes from the angular velocity z to a state of rest. In general, if the initial angular velocity $= v$, and the terminal angular velocity $= z$, the units of work will be,

$$F s = \frac{z^2 - v^2}{2} \times R.$$

The moment of inertia of a body about an axis not passing through the centre of gravity is equivalent to its moment of inertia about an axis running parallel to it through the centre of gravity, increased by the product of the mass of the body and the square of the distance of the two centres.

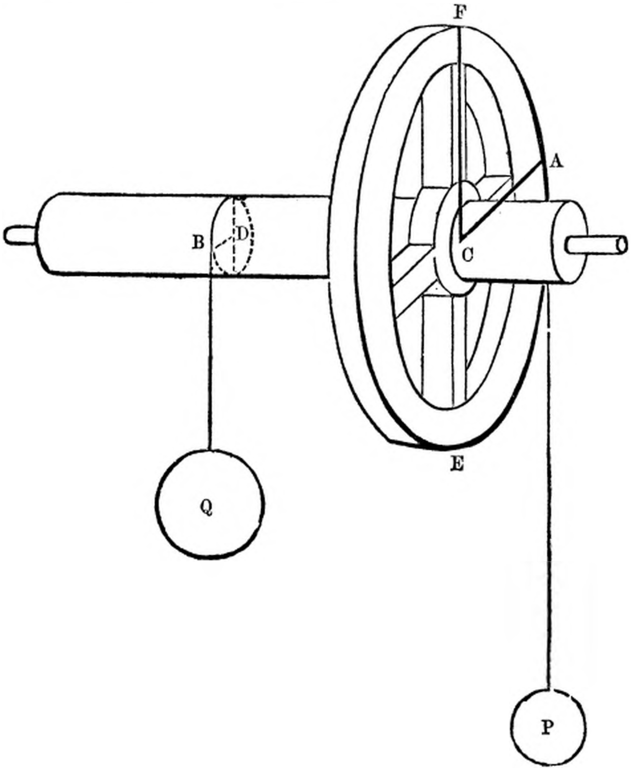
It is necessary to know the moments of inertia of the principal geometrical bodies, because they very often come into application in mechanical investigations. If these bodies be homogeneous, as in the following we will always suppose to be the case, the particles of the mass M_1, M_2, &c. are proportional to the corresponding particles of the volume V_1, V_2, &c.; and hence the measure of the moment of inertia may be replaced by the sum of the particles of the volume, and the squares of their distances from the axis of revolution. In this sense, the moments of inertia of lines and surfaces may also be found.

If the whole mass of a body be supposed to be collected into one point, its distance from the axis may be determined on the supposition that the mass so concentrated possesses the same moment of inertia as if distributed over its space. This distance is called the *radius of gyration*, or *of inertia*. If R be the moment of inertia, M the mass, and r the radius of gyration, we then have $M r^2 = R$, and hence $r = \sqrt{\frac{R}{M}}$. We must bear in mind that this radius by no means gives a determinate point, but a circle only, within whose circumference the mass may be considered as arbitrarily distributed.

If into the formula $R_1 = R + M e^2$, expressed in the words above printed in italics, we introduce $R = M r^2$ and $R_1 = M r_1^2$, we obtain $r_1^2 = r^2 + e^2$; that is, the square of the radius of gyration referred to a given axis = the square of the radius of gyration referred to a parallel line of gravity, plus the square of the distance between the two axes.

Wheel and axle.—The theory of the moment of inertia finds its most frequent application in machines and instruments, because in these rotary motions about a fixed axis are those which generally present themselves.

If two weights, P and Q, act on a wheel and axle ACDB, with the arms $CA = a$ and $DB = b$ through the medium of perfectly flexible strings, and if the radius of the gudgeons be so small that their friction may be neglected, it will remain in equilibrium if the statical moments P . CA and Q . DB are equal, and therefore $Pa = Qb$. But if the moment of the weight P is greater than that of Q, therefore $Pa > Qb$, P will descend and Q ascend; if $Pa < Qb$, P will ascend and Q descend. Let us now examine the conditions of motion in the case that $Pa > Qb$. The force corresponding to the weight Q and acting at the arm b generates at the arm a a force $\frac{Qb}{a}$, which acts opposite to the force corresponding to the weight P, and hence there is a residuary moving force $P - \frac{Qb}{a}$ acting at A. The mass $\frac{Q}{g}$ is reduced by its transference from the distance b to that of a to $\frac{Qb^2}{ga^2}$; hence the mass moved by $P - \frac{Qb}{a}$ is $M = \left(P + \frac{Qb^2}{a^2}\right) \div g$, or, if the moment of inertia of



the wheel and axle without the weights P and Q $= \frac{G y^2}{g}$, and, therefore, its inert mass reduced to A $= \frac{G y^2}{g a^2}$, we have, more exactly,

$$M = \left(P + \frac{Q b^2}{a^2} + \frac{G y^2}{a^2}\right) \div g = (P a^2 + Q b^2 + G y^2) \div g a^2.$$

From thence it follows that the accelerated motion of the weight P, together with that of the circumference of the wheel, namely,

$$p = \frac{\text{moving force}}{\text{mass}} = \frac{P - \frac{Q b}{a}}{P a^2 + Q b^2 + G y^2} g a^2 = \frac{P a - Q b}{P a^2 + Q b^2 + G y^2} g a;$$

on the other hand, the accelerated motion of the ascending weight Q, or of the circumference of the axle, is,

$$q = \frac{b}{a} p = \frac{P a - Q b}{P a^2 + Q b^2 + G y^2} g b.$$

The tension of the string by P is $S = P - \frac{P p}{g} = P\left(1 - \frac{p}{g}\right)$, that of the string by Q is $T = Q + \frac{Q q}{g} = Q\left(1 + \frac{q}{g}\right)$; hence the pressure on the gudgeon is,

$$S + T = P + Q - \frac{P p}{g} + \frac{Q q}{g} = P + Q - \frac{(P a - Q b)^2}{P a^2 + Q b^2 + G y^2};$$

the pressure, therefore, on the gudgeons for a revolving wheel and axle is less than for one in a state of equilibrium. Lastly, from the accelerating forces p and q, the rest of the relations of motion may be found; after t seconds, the velocity of P is $v = p t$, of Q is $v_1 = q t$, and the space described by P is $s = \frac{1}{2} p t^2$, by Q is $s_1 = \frac{1}{2} q t^2$.

Let the weight P at the wheel be = 60 lbs., that at the axle Q = 160 lbs., the arm of the first CA $= a =$ 20 inches, that of the second DB $= b =$ 6 inches; further, let the axle consist of a solid cylinder of 10 lbs. weight, and the wheel of two iron rings and four arms, the rings of 40 and 12 lbs., the arms together of 15 lbs. weight; lastly, let the radii of the greater ring AE = 20 and 19 inches, that of the less FG = 8 and 6 inches; required the conditions of motion of this machine. The moving force at the circumference of the wheel is,

$$P - \frac{b}{a} Q = 60 - \frac{6}{20} 160 = 60 - 48 = 12 \text{ lbs.},$$

the moment of inertia of the machine, neglecting the masses of the gudgeons and the strings, is equivalent to the moment of inertia of the axle $= \frac{W b^2}{2} = \frac{10 \,.\, 6^2}{2} = 180$, plus the moment of the smaller ring $= \frac{R_1 (r_1^2 + r_2^2)}{2} = \frac{12\,(8^2 + 6^2)}{2} = 600$, plus the moment of the larger ring $= \frac{40\,(20^2 + 19^2)}{2} = 15220$, plus the moment of the arms, approximately $= \frac{A\,(\rho_1^{\,3} - \rho_2^{\,3})}{3\,(\rho_1 - \rho_2)} = \frac{A\;\rho_1^{\,2} + \rho_1\rho_2 + \rho_2^{\,2})}{3} = \frac{15\,(19^2 + 19 \times 8 + 8^2)}{3} = 2885$; hence, collectively, $G\,y^2 = 180 + 600 + 15220 + 2885 = 18885$, or for foot measure $= \frac{18885}{144} = 131{\cdot}14$. The collective mass, reduced to the circumference of the wheel is,

$$= \left(P + \frac{Q\,b^2 + G\,y^2}{a^2}\right) \div g = \left[60 + 160\left(\frac{6}{20}\right)^2 + \frac{18885}{20^2}\right] \div g = \left(60 + 160 \times 0{\cdot}09 + \frac{18885}{400}\right) 0{\cdot}031 = 121{\cdot}61 \times 0{\cdot}031 = 337 \text{ lbs.}$$

Accordingly, the accelerated motion of the weight P, together with that of the circumference of the wheel, is,

$$p = \frac{P - \frac{b}{a}\,Q}{\frac{P + Q\,b^2 + G\,y^2}{a^2}}\,g = \frac{12}{3{\cdot}77} = 3{\cdot}183 \text{ feet; on the other}$$

hand, that of Q is $q = \frac{b}{a}\,p = \frac{6}{20}\,3{\cdot}183 = 0{\cdot}954$ feet; further, the tension of the string by P is $= \left(1 - \frac{p}{g}\right) P = \left(1 - \frac{3{\cdot}133}{32{\cdot}2}\right) 60 = 54{\cdot}07$ lbs.; that by Q, on the other hand, $Q = \left(1 + \frac{q}{g}\right) Q = (1 + 0{\cdot}925 \times 0{\cdot}032)\,160 = 1{\cdot}030\;160 = 164{\cdot}8$ lbs.; and consequently the pressure on the gudgeons $S + T = 54{\cdot}06 + 164{\cdot}80 = 218{\cdot}86$ lbs., or inclusive of the weight of the machine $= 218{\cdot}86 + 77 = 295{\cdot}86$ lbs. After 10 seconds, P has acquired the velocity $p\,t = 3{\cdot}084 \times 10 = 30{\cdot}84$ feet, and described the space $s = \frac{v\,t}{2} = 30{\cdot}84 \times 5 = 154{\cdot}2$ feet, and Q has ascended a height $\frac{b}{a}\,s = 0{\cdot}3 \times 154{\cdot}2 = 46{\cdot}26$ feet.

The weight P which communicates to the weight Q the accelerated motion $q = \frac{P\,a\,b - Q\,b^2}{P\,a^2 + Q\,b^2 + G\,y^2}\,g$, may also be replaced by another weight P_1, without changing the acceleration of the motion Q, if it act at the arm a_1, for which,

$$\frac{P_1\,a_1 - Q\,b}{P_1\,a_1^{\,2} + Q\,b^2 + G\,y^2} = \frac{P\,a - Q\,b}{P\,a^2 + Q\,b + G\,y^2}.$$

The magnitude $\frac{P\,a^2 + Q\,b^2 + G\,y^2}{P\,a - Q\,b}$, represented by k, and we obtain $a_1^{\,2}, - k\,a_1 = - \frac{Q\,b\,(b + k) + G\,y^2}{P_1}$, and the arm in question,

$$a_1 = \tfrac{1}{2}\,k \pm \sqrt{\left(\frac{k}{2}\right)^2 - \frac{Q\,b\,(b + k) + G\,y^2}{P_1}}.$$

We may also find by help of the differential calculus, that the motion of Q is most accelerated by the weight P, when the arm of the latter corresponds to the equation $P\,a^2 - 2\,Q\,a\,b = Q\,b^2 + G\,y^2$, therefore,

$$a = \frac{b\,Q}{P} + \sqrt{\left(\frac{b\,Q}{P}\right)^2 + \frac{Q\,b^2 + G\,y^2}{P}}.$$

The formula found above assumes a complicated form if the friction of the gudgeons and the rigidity of the cord are taken into account. If we represent the statical moments of both resistances by $F\,r$, we must then substitute for the moving force $P - \frac{b}{a}\,Q$, the value $P - \frac{Q\,b + F\,r}{a}$, whence the acceleration of Q comes out,

$$q = \frac{(P\,a - F\,r)\,b - Q\,b^2}{P\,a^2 + Q\,b^2 + G\,y^2}\,g \text{ and } a = \frac{Q\,b + F\,r}{P} + \sqrt{\left(\frac{Q\,b + F\,r}{P}\right)^2 + \frac{Q\,b^2 + G\,y^2}{P}}.$$

The weights P = 30 lbs. Q = 80 lbs. act at the arms $a = 2$ feet. and $b = \frac{1}{2}$ foot of a wheel and axle, and their moments of inertia $G\,y^2$ amount to 60 lbs.; then the accelerated motion of the ascending weight Q is,

$$q = \frac{30 \times 2 \times \frac{1}{2} - 80 \times (\frac{1}{2})^2}{30 \times 2^2 + 80 \times (\frac{1}{2})^2 + 60}\,g = \frac{30 - 20}{120 + 20 + 60}\,32{\cdot}2 = \frac{322}{200} =$$

1·61 feet. But if a weight $P_1 = 45$ lbs. generates the same acceleration in the motion of Q, the arm of P_1 is then,

$$a_1 = \frac{k}{2} \pm \sqrt{\left(\frac{k}{2}\right)^2 - \frac{80 \times \frac{1}{2}\,(\frac{1}{2} + k) + 60}{45}}, \text{ or as } k = \frac{200}{60 - 40} =$$

10, a_1 is $= 5 \pm \sqrt{25 - \frac{32}{3}} = 5 \pm \frac{1}{3}\,11{\cdot}358 = 5 \pm 3{\cdot}786 = 8{\cdot}786$ feet, or 1·214 feet.

The accelerated motion of Q comes out greatest if the arm of the force or radius of the wheel amount to,

$$a = \frac{\frac{1}{2} \times 80}{30} + \sqrt{\left(\frac{40}{30}\right)^2 + \frac{20 + 60}{30}} = \frac{4}{3} + \sqrt{\frac{16}{9} + \frac{24}{9}} = \frac{4 + \sqrt{40}}{3} =$$

3·4415 feet, and q is $= \left(\frac{30 \times 1{\cdot}7207 - 20}{30 \times (3{\cdot}4415)^2 + 80}\right) g = \frac{31{\cdot}621}{435{\cdot}32}\,g =$ 2·339 feet.

The statical moment of the friction, together with the rigidity of the string, is $F\,r = 8$; then, instead of $Q\,b$, we must put $Q\,b + F\,r = 40 + 8 = 48$; whence it follows that,

$$a = \frac{48}{30} + \sqrt{\left(\frac{40}{30}\right)^2 + \frac{8}{3}} = 1{\cdot}6 + \sqrt{5{\cdot}227} = 3{\cdot}886,$$ and the correspondent maximum accelerating force

$$q = \frac{30 \times 1{\cdot}943 - 8 \times \frac{1}{2} - 20}{30 \times (3{\cdot}886)^2 + 80}\,g = \frac{34{\cdot}29}{533} \times 32{\cdot}2 = 2{\cdot}071 \text{ feet.}$$

WEIGHT, ACCELERATION, AND MASS.

PARALLELOGRAM OF FORCES.—THE PRINCIPLE OF VIRTUAL VELOCITIES.—MECHANICAL POWERS: CONTINUOUS CIRCULAR MOTION, GEARING, TEETH OF WHEELS, DRUMS, PULLEYS, PUMPING ENGINES, ETC.

1. If a weight of 10 lbs., moved by the hand, ascends with a 3 feet acceleration, what is the pressure on the hand?

$$10\,(1 + \frac{3}{32\cdot2}) = 10\cdot93168 \text{ lbs.}$$

If a weight of 10 lbs., moved by the hand, descends with a 3 feet acceleration, the pressure on the hand will be 9·06832 lbs., for then

$$10\,(1 - \frac{3}{32\cdot2}) = 9\cdot06832.$$

If w be the weight of the *mass* acted upon by the force of the hand, and also by the force of gravity, as $g = 32\cdot2$, the *mass* moved by the sum or difference of these forces will be $= \frac{w}{g}$. If P be the pressure on the hand, and p its acceleration, the body falls with the force $\frac{w}{g}p$; it also falls with the force $w - \text{P}$; hence,

$$w - \text{P} = \frac{w}{g}p \quad \therefore \text{P} = (1 - \frac{p}{g})\,w.$$

When the body is ascending, then p is negative,

$$\text{and } w + \text{P} = \frac{w}{g}(-p) \quad \therefore \text{P} = (1 + \frac{p}{g})\,w.$$

2. If a body of 200 lbs. be moved on a smooth horizontal track, by the joint action of two forces, and describes a space of 10 feet in the first second, what is the amount of each of these forces; the first makes an angle of 35° with the track upon which the body moves, and the other an angle of 50°?

In solving this question, the natural sines of the angles 35°, 50°, and of their sum 85°, will be required. We shall first take these from the table:

sin. 35° = ·57358
sin. 50° = ·76604
sin. 85° = ·99619.

The acceleration is = 20 feet, that is, twice the space passed over in the first second,

$\frac{200}{32\cdot2}$ = the *mass*, and $\frac{200}{32\cdot2} \times 20 = 124\cdot224$ lbs., the force of the resultant, in the direction of the track upon which the body moves.

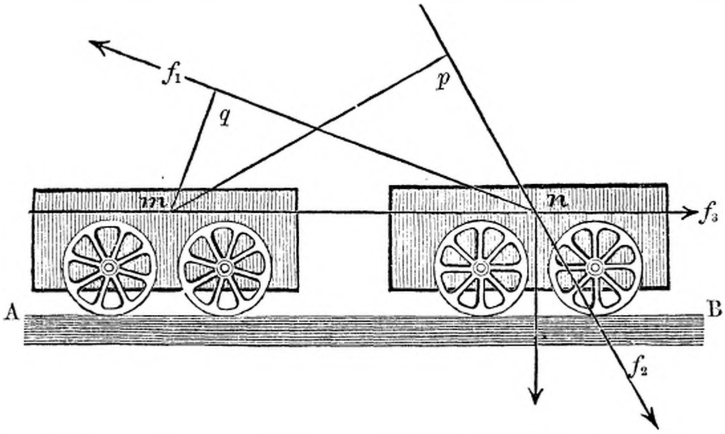
$$\text{One of the components} = \frac{124{\cdot}224 \text{ sin. } 35^\circ}{\text{sin. } (35^\circ + 50^\circ)} = 71{\cdot}52 \text{ lbs.}$$

$$\text{The other component} = \frac{124{\cdot}224 \text{ sin. } 50^\circ}{\text{sin. } (35^\circ + 50^\circ)} = 95{\cdot}52 \text{ lbs.}$$

These, and the like results, may be obtained with greater ease by logarithms.

$$\begin{array}{lcr}
\text{Log. } 124{\cdot}224 & = & 2{\cdot}0942055 \\
\text{Log. sin. } 35^\circ & = & \underline{9{\cdot}7585913} \\
 & & 11{\cdot}8527968 \\
\text{Log. sin. } 85^\circ & = & \underline{9{\cdot}9983442} \\
\text{Log. of } 71{\cdot}52413 & = & 1{\cdot}8544526 \\
\text{Log. } 124{\cdot}224 & = & 2{\cdot}0942055 \\
\text{Log. sin. } 50^\circ & = & \underline{9{\cdot}8842540} \\
 & & 11{\cdot}9784595 \\
\text{Log. sin. } (85^\circ) & = & \underline{9{\cdot}9983442} \\
\text{Log. of } 95{\cdot}5247 & = & 1{\cdot}9801153
\end{array}$$

3. A carriage weighing 8000 lbs. is moved forward by a force f_1 of 500 lbs. upon a horizontal surface AB; during the motion, two resistances have to be overcome, one horizontal of 100 lbs., the amount of friction, represented in the figure by f_3, the other f_2 of 200 lbs. acting downwards; the angles $f_3 n f_2$ and $f_1 n m$, which the directions of these forces make with the horizon, are 61° and 21° respectively: it is required to know what work the force f_1 will perform by converting a 5 feet initial velocity of the carriage into a 20 feet velocity.



If we put $x = nm$, the distance the carriage moves in passing from a 5 to a 20 feet velocity,

The work of the force $f_1 = f_1 \times nq = 500 \times \cos. 21^\circ \times x$.
The work of the force $f_3 = (-f_3) \times nm = -100 \times x$.
The work of the force $f_2 = (-f_2) \times np = -200 \times \cos. 61^\circ \times x$.

Consequently, the work of the effective force will be $269 \cdot 828 \times x = \{500 \times \cdot 94358 - 100 - 200 \times \cdot 48481\} x$, since the natural cosine of $21° = \cdot 93358$, and the natural cosine of $61° = \cdot 48481$.

But according to the principle of *vis viva*, the work done is equal to

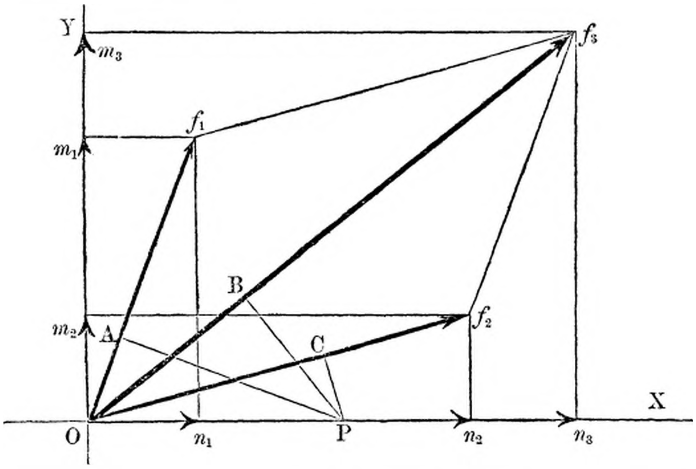
$$\frac{20^2 - 5^2}{64 \cdot 4} \times 8000 = 46589 \cdot 82.$$

$$\therefore 269 \cdot 828 \times x = 46589 \cdot 82 \text{ and } x = \frac{46589 \cdot 82}{269 \cdot 828} =$$

772·665 feet, the space passed over by the carriage.

This question is solved on the PRINCIPLE OF VIRTUAL VELOCITIES, which we shall explain, as it is of essential service in practical mechanics.

This explanation depends on what is technically termed the "*Parallelogram of Forces.*"



When a material point O, is acted upon by two forces f_1, f_2, whose directions Of_1, Of_2, make with each other an angle, if Of_1, Of_2 represent the magnitudes and directions of the forces, the diagonal of the parallelogram $O\,f_1\,f_3\,f_2$ represents the resultant in magnitude and direction; that is, the diagonal represents a single force equal to the combined actions of the forces represented by the sides. And if the sides of the parallelogram represent the accelerations of the forces, the diagonal represents the resultant acceleration. Draw through O, two axes OX and OY, at right angles to each other, and resolve the forces f_1 and f_2, as well as their resultant f_3, into components in the directions of these axes; namely, f_1 into n_1 and m_1; f_2 into n_2 and m_2; and f_3 into n_3 and m_3. The forces in one axis are n_1, n_2, and n_3; and those in the other m_1, m_2, and m_3. And by the parallelogram of forces it is well known that

$$n_3 = n_1 + n_2 \text{ and } m_3 = m_1 + m_2. \quad \text{(E)}.$$

Now if we take in the axis OX any point P, and let fall from it the perpendiculars PA, PB, PC, on the directions of the forces f_1, f_3, f_2, we obtain the following similar right-angled triangles, namely,

OAP and O $n_1 f_1$ are similar;
OBP and O $n_3 f_3$ ———— ;
OCP and O $n_2 f_2$ ———— ;

$\therefore \frac{On_1}{Of_1} = \frac{OA}{OP} = \frac{n_1}{f_1}$ and $n_1 = \frac{AO}{OP} f_1$. It is easily seen also that $n_2 = \frac{CO}{OP} f_2$; and $n_3 = \frac{BO}{OP} f_3$.

If the values be substituted in (E), we obtain

$$BO \times f_3 = CO \times f_2 + AO \times f_1.$$

From the similarity of these triangles, and the remaining equation of (E), we can readily find that

$$PB \times f_3 = PA \times f_1 + PC \times f_2.$$

The equation becomes more compact by putting

OA, OC, OB, respectively equal s_1, s_2 s_3; and
PA, PC, PB, ———————— q_1, q_2, q_3.
Then $f_3 s_3 = f_2 s_2 + f_1 s_1$ and $f_3 q_2 = f_2 q_2 + f_1 q_1$.

The same holds good with any number of forces f_1, f_2, f_3, &c., and their resultant f_n, that is

$$f_n s_n = f_1 s_1 + f_2 s_2 + f_3 s_3 + \text{\&c.}$$
$$\text{and } f_n q_n = f_1 q_1 + f_2 q_2 + f_3 q_3 + \text{\&c.}$$

If the point of application O, move in a straight line to P, then OA $= s_1$ is called the space of the force f_1, and $f_1 s_1$ the work done by the force f_1, in moving the body from O to P. OB is the space of the resultant, and the product $f_3 s_3$, the work done by it. $f_2 s_2$ is the work done by f_2 in moving the material point O from O to P. Hence the work done by the resultant is equal to all the work done by the component forces, as we have shown,

$$f_n s_n = f_1 s_1 + f_2 s_2 + f_3 s_3 + \text{\&c.}$$

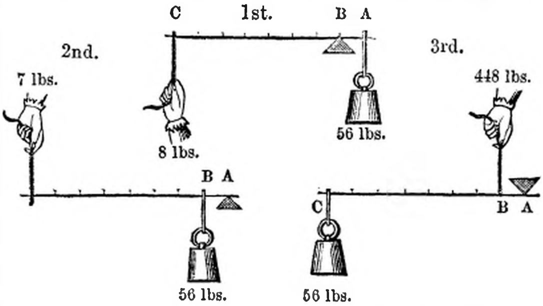
PRINCIPLES AND PRACTICAL APPLICATIONS OF MECHANICAL POWERS.

MECHANICAL Powers, or the Elements of Machinery, are certain simple mechanical arrangements whereby weights may be raised or resistances overcome with the exertion of less power or strength than is necessary without them.

They are usually accounted six in number, viz. the *lever*, the *wheel* and *axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw;* but properly two of these comprise the whole, namely, the *lever* and *inclined plane*,—the wheel and axle being only a lever of the first kind, and the pulley a lever of the second,—the wedge and the screw being also similarly allied to that of the inclined plane: however, although such seems to be the case in these respects, yet they each require, on account of their various modifications, a peculiar rule of estimation adapted expressly to the different circumstances in which they are individually required to act.

THE LEVER.

Levers, according to mode of application, as the following, are distinguished as being of the first, second, or third kind; and although levers of equal lengths produce different effects, the general principles of estimation in all are the same; namely, the power is to the weight or resistance, as the distance of the one end to the fulcrum is to the distance of the other end to the same point.



In the *first kind*, the power is to the resistance, as the distance AB is to the distance BC.

In the *second*, the power is to the resistance, as the distance AB is to that of AC; and,

In the *third*, the resistance is to the power, as the distance AB is to that of AC.

RULE, *first kind.*—Divide the longer by the shorter end of the lever from the fulcrum, and the quotient is the effective force that the power applied is equal to.

Let the handle of a pump equal 65 inches in length, and 10 inches from the shortest end to centre of motion; what is the amount of effective leverage thereby obtained?

$$65 - 10 = 55, \text{ and } \frac{55}{10} = 5\tfrac{1}{2} \text{ to } 1.$$

Required the situation of the fulcrum on which to rest a lever of 15 feet, so that $2\frac{1}{2}$ cwt. placed at one end may equipoise 30 cwt. at the other, the weight of the lever not being taken into account.

$\dfrac{15 \times 2{\cdot}5}{2{\cdot}5 + 30} = 1{\cdot}154$ feet from the end on which the 30 cwt. is to be placed.

It is by the second kind of lever that the greatest effect is obtained from any given amount of power; hence the propriety of the application of this principle to the working of force pumps, and shearing of iron, as by the lever of a punching-press, &c.

RULE, *second kind.*—Divide the whole length of lever, or distance from power to fulcrum, by the distance from fulcrum to weight, and the quotient is the proportion of effect that the power is to the weight or resistance to be overcome.

Required the amount of effect or force produced by a power of

50 lbs. on the ram of a Bramah's pump, the length of the lever being 3 feet, and distance from ram to fulcrum $4\frac{1}{2}$ inches.

3 feet = 36 inches, and $\frac{36}{4\cdot5} = 8$, or the power and resistance are to each other as 8 to 1; hence $50 \times 8 = 400$ lbs. force upon the ram.

The lever on the safety valve of a steam boiler is of the *third kind*, the action of the steam being the power, and the weight or spring-balance attached the resistance; but in such application the action of the lever's weight must also be taken into account.

THE WHEEL AND PINION, OR CRANE.

The mechanical advantage of the wheel and axle, or crane, is as the velocity of the weight to the velocity of the power; and being only a modification of the first kind of lever, it of course partakes of the same principles.

RULE.—*To determine the amount of effective power produced from a given power by means of a crane with known peculiarities.*—Multiply together the diameter of the circle described by the winch, or handle, and the number of revolutions of the pinion to 1 of the wheel; divide the product by the barrel's diameter in equal terms of dimensions, and the quotient is the effective power to 1 of exertive force.

Let there be a crane the winch of which describes a circle of 30 inches in diameter; the pinion makes 8 revolutions for 1 of the wheel, and the barrel is 11 inches in diameter; required the effective power in principle, also the weight that 36 lbs. would raise, friction not being taken into account.

$\frac{30 \times 8}{11} = 21\cdot8$ to 1 of exertive force; and $21\cdot8 \times 36 = 784\cdot8$ lbs.

RULE.—*Given any two parts of a crane, to find the third, that shall produce any required proportion of mechanical effect.*—Multiply the two given parts together, and divide the product by the required proportion of effect; the quotient is the dimensions of the other parts in equal terms of unity.

Suppose that a crane is required, the ratio of power to effect being as 40 to 1, and that a wheel and pinion 11 to 1 is unavoidably compelled to be employed, also the throw of each handle to be 16 inches; what must be the barrel's diameter on which the rope or chain must coil?

$16 \times 2 = 32$ inches diameter described by the handle.

And $\frac{32 \times 11}{40} = 8\cdot8$ inches, the barrel's diameter.

THE PULLEY.

The principle of the pulley, or, more practically, the block and tackle, is the distribution of weight on various points of support; the mechanical advantage derived depending entirely upon the flexibility and tension of the rope, and the number of pulleys or sheives in the lower or rising block: hence, by blocks and tackle of the usual kind, the power is to the weight as the number of cords attached to the lower block; whence the following rules.

Divide the weight to be raised by the number of cords leading *to*, *from*, or *attached* to the lower block; and the quotient is the power required to produce an equilibrium, provided friction did not exist.

Divide the weight to be raised by the power to be applied; the quotient is the number of sheives in, or cords attached to the rising block.

Required the power necessary to raise a weight of 3000 lbs. by a four and five-sheived block and tackle, the four being the movable or rising block.

Necessarily there are nine cords leading to and from the rising block.

$$\text{Consequently } \frac{3000}{9} = 333 \text{ lbs., the power required.}$$

I require to raise a weight of 1 ton 18 cwt., or 4256 lbs.; the amount of my power to effect this object being 500 lbs., what kind of block and tackle must I of necessity employ?

$\frac{4256}{500} = 8{\cdot}51$ cords; of necessity there must be 4 sheives or 9 cords in the rising block.

As the effective power of the crane may, by additional wheels and pinions, be increased to any required extent, so may the pulley and tackle be similarly augmented by purchase upon purchase.

THE INCLINED PLANE.

The *inclined plane* is properly the second elementary power, and may be defined the lifting of a load by regular instalments. In principle it consists of any right line not coinciding with, but lying in a sloping direction to, that of the horizon; the standard of comparison of which commonly consists in referring the rise to so many parts in a certain length or distance, as 1 in 100, 1 in 200, &c.,—the first number representing the perpendicular height, and the latter the horizontal length in attaining such height, both numbers being of the same denomination, unless otherwise expressed; but it may be necessary to remark, that the inclination of a plane, the sine of inclination, the height per mile, or the height for any length, the ratio, &c., are all synonymous terms.

The advantage gained by the inclined plane, when the power acts in a parallel direction to the plane, is as the length to the height or angle of inclination: hence the rule. Divide the weight by the ratio of inclination, and the quotient equal the power that will just support that weight upon the plane. Or, multiply the weight by the height of the plane, and divide by the length,—the quotient is the power.

Required the power or equivalent weight capable of supporting a load of 350 lbs. upon a plane of 1 in 12, or 3 feet in height and 36 feet in length.

$$\frac{350}{12} = 29{\cdot}16 \text{ lbs., or } \frac{350 \times 3}{36} = 29{\cdot}16 \text{ lbs. power, as before.}$$

The weight multiplied by the length of the base, and the product divided by the length of the incline, the quotient equal the pressure or downward weight upon the incline.

TABLE *showing the Resistance opposed to the Motion of Carriages on different Inclinations of Ascending or Descending Planes, whatever part of the insistent weight they are drawn by.*

Tons.	Hundreds.									
		100	200	300	400	500	600	700	800	900
		·01	·005	·00333	·0025	·002	·00167	·00143	·00125	·00111
10	·1	·00909	·00476	·00322	·00244	·00196	·00164	·00141	·00123	·0011
20	·05	·00833	·00454	·00312	·00238	·00192	·00161	·00139	·00122	·00109
30	·0333	·00769	·00435	·00303	·00232	·00189	·00159	·00137	·0012	·00107
40	·025	·00714	·00417	·00294	·00227	·00185	·00156	·00135	·00119	·00106
50	·02	·00667	·004	·00286	·00222	·00182	·00154	·00133	·00118	·00105
60	·0166	·00625	·00385	·00278	·00217	·00178	·00151	·00131	·00116	·00104
70	·0143	·00588	·0037	·0027	·00213	·00175	·00149	·0013	·00115	·00103
80	·0125	·00555	·00357	·00263	·00208	·00172	·00147	·00128	·00114	·00102
90	·0111	·00526	·00345	·00256	·00204	·00169	·00145	·00126	·00112	·00101

Although this table has been calculated particularly for carriages on railway inclines, it may with equal propriety be applied to any other incline, the amount of traction on a level being known.

Application of the preceding Table.

What weight will a tractive power of 150 lbs. draw up an incline of 1 in 340, the resistance on the level being estimated at $\frac{1}{240}$th part of the insistent weight?

In a line with 40 in the left-hand column and under 200 is ·00417
Also in the same line and under 390 is........................ ·00294
Added together = ·00711

$$\text{Then } \frac{150}{{\cdot}00711} = 21097 \text{ lbs. weight drawn up the plane.}$$

What weight would a force of 150 lbs. draw down the same plane, the fraction on the level being the same as before?

Friction on the level = ·00417
Gravity of the plane = ·00294 subtract
= ·00123

$$\text{And } \frac{150}{{\cdot}00123} = 121915 \text{ lbs. weight drawn down the plane.}$$

Example of incline when velocity is taken into account.—A power of 230 lbs., at a velocity of 75 feet per minute, is to be employed for moving weights up an inclined plane 12 feet in height and 163 feet in length, the least velocity of the weight to be 8 feet per minute; required the greatest weight that the power is equal to.

$$\frac{230 \times 75 \times 163}{12 \times 8} = \frac{2811750}{96} = 29288 \text{ lbs., or } 13{\cdot}25 \text{ tons.}$$

TABLE *of Inclined Planes, showing the ascent or descent per yard, and the corresponding ascent or descent per chain, per mile; and also the ratio.*

Per yard.		Per chain.	Per mile.	Ratio.	· Per yard.		Per chain.	Per mile.	Ratio.
In parts of an in.	In dec'ls. of an inch.	Inches.	Feet.	1 inch.	In parts of an in.	In decimals of an inch.	Inches.	Feet.	1 inch.
$\frac{1}{64}$	·0156	·344	2·29	2304	$\frac{7}{16}$	·4375	9·625	64·17	82
$\frac{1}{48}$	·0208	·458	3·06	1728	$\frac{1}{2}$	·5	11	73·33	72
$\frac{1}{32}$	·0312	·687	4·58	1152	$\frac{9}{16}$	·5625	12·375	82·5	64
$\frac{1}{24}$	·0417	·917	6·11	864	$\frac{7}{12}$	·5833	12·833	85·56	62
$\frac{1}{16}$	·0625	1·375	9·17	576	$\frac{3}{5}$	·6	13·2	88	60
$\frac{1}{12}$	·0833	1·833	12·22	432	$\frac{5}{8}$	·625	13·75	91·67	58
$\frac{1}{10}$	·1	2·2	14·67	360	$\frac{2}{3}$	·6667	14·667	97·78	54
$\frac{1}{8}$	·125	2·75	18·33	288	$\frac{11}{16}$	·6875	15·125	100·83	52
$\frac{1}{6}$	·1667	3·667	24·44	216	$\frac{7}{10}$	·7	15·4	102·67	51
$\frac{3}{16}$	·1875	4·125	27·50	192	$\frac{3}{4}$	·75	16·5	110	48
$\frac{1}{5}$	·2	4·4	29·33	180	$\frac{4}{5}$	·8	17·6	117·33	45
$\frac{1}{4}$	·25	5·5	36·67	144	$\frac{13}{16}$	·8125	17·875	119·17	44
$\frac{3}{10}$	·3	6·6	44	120	$\frac{5}{6}$	·8333	18·333	122·22	43
$\frac{5}{16}$	·3125	6·875	45·83	115	$\frac{7}{8}$	·875	19·25	128·33	41
$\frac{1}{3}$	·3333	7·333	48·89	108	$\frac{9}{10}$	·9	19·8	132	40
$\frac{3}{8}$	·375	8·25	55	96	$\frac{11}{12}$	·9167	20·167	134·44	39
$\frac{2}{5}$	·4	8·8	58·67	20	$\frac{15}{16}$	·9375	20·625	137·5	38
$\frac{5}{12}$	·4167	9·167	61·11	86	1	1	22	146·67	36

THE WEDGE.

The wedge is a double inclined plane; consequently its principles are the same: hence, when two bodies are forced asunder by means of the wedge in a direction parallel to its head,—Multiply the resisting power by half the thickness of the head or back of the wedge, and divide the product by the length of one of its inclined sides; the quotient is the force equal to the resistance.

The breadth of the back or head of a wedge being 3 inches, and its inclined sides each 10 inches, required the power necessary to act upon the wedge so as to separate two substances whose resisting force is equal to 150 lbs.

$$\frac{150 \times 1{\cdot}5}{10} = 22{\cdot}5 \text{ lbs.}$$

When only one of the bodies is movable, the whole breadth of the wedge is taken for the multiplier.

THE SCREW.

The screw, in principle, is that of an inclined plane wound around a cylinder, which generates a spiral of uniform inclination, each revolution producing a rise or traverse motion equal to the pitch of the screw, or distance between two consecutive threads,—the pitch being the height or angle of inclination, and the circumference the length of the plane when a lever is not applied; but the lever being a necessary qualification of the screw, the circle which it describes is taken, instead of the screw's circumference, as the length of the plane: hence the mechanical advantage is, as the circumference of the circle described by the lever where the power acts, is to the pitch of the screw, so is the force to the resistance in principle.

Required the effective power obtained by a screw of $\frac{7}{8}$ inch pitch, and moved by a force equal to 50 lbs. at the extremity of a lever 30 inches in length.

$$\frac{30 \times 2 \times 3{\cdot}1416 \times 50}{{\cdot}875} = 10760 \text{ lbs.}$$

Required the power necessary to overcome a resistance equal to 7000 lbs. by a screw of $1\frac{1}{4}$ inch pitch, and moved by a lever 25 inches in length.

$$\frac{7000 \times 1{\cdot}25}{25 \times 2 \times 3{\cdot}1416} = 55{\cdot}73 \text{ lbs. power.}$$

In the case of a screw acting on the periphery of a toothed wheel, the power is to the resistance, as the product of the circle's circumference described by the winch or lever, and radius of the wheel, to the product of the screw's pitch, and radius of the axle, or point whence the power is transmitted; but observe, that if the screw consist of more than one helix or thread, the apparent pitch must be increased so many times as there are threads in the screw. Hence, *to find what weight a given power will equipoise:*

Rule.—Multiply together the radius of the wheel, the length of the lever at which the power acts, the magnitude of the power, and the constant number 6·2832; divide the product by the radius of the axle into the pitch of the screw, and the quotient is the weight that the power is equal to.

What weight will be sustained in equilibrio by a power of 100 lbs. acting at the end of a lever 24 inches in length, the radius of the axle, or point whence the power is transmitted, being 8 inches, the radius of the wheel 14 inches, the screw consisting of a double thread, and the apparent pitch equal $\frac{5}{8}$ of an inch?

$$\frac{14 \times 24 \times 100 \times 6{\cdot}2832}{{\cdot}625 \times 2 \times 8} = 21111{\cdot}55 \text{ lbs., or } 9{\cdot}4 \text{ tons, the}$$

power sustained.

If an endless screw be turned by a handle of 20 inches, the threads of the screw being distant half an inch; the screw turns a toothed wheel, the pinion of which turns another wheel, and the pinion of this another wheel, to the barrel of which a weight W is attached; it is required to find the weight a man will be able to sustain, who acts at the handle with a force of 150 lbs., the diameters of the wheels being 18 inches, and those of the pinions and barrel 2 inches.

$$150 \times 20 \times 3{\cdot}1416 \times 2 \times 18^3 = W \times 2^3 \times \tfrac{1}{2};$$
$$\therefore W = 12269 \text{ tons.}$$

CONTINUOUS CIRCULAR MOTION.

In mechanics, circular motion is transmitted by means of *wheels*, *drums*, or *pulleys;* and accordingly as the driving and driven are of equal or unequal diameters, so are equal or unequal velocities produced: hence the principle on which the following rules are founded.

Rule.—*When time is not taken into account.*—Divide the greater diameter, or number of teeth, by the lesser diameter, or number of teeth, and the quotient is the number of revolutions the lesser will make for 1 of the greater.

How many revolutions will a pinion of 20 teeth make for 1 of a wheel with 125?

$$125 \div 20 = 6{\cdot}25, \text{ or } 6\tfrac{1}{4} \text{ revolutions.}$$

Intermediate wheels, of whatever diameters, so as to connect communication at any required distance apart, cause no variation of velocity more than otherwise would result were the first and last in immediate contact.

Rule.—*To find the number of revolutions of the last, to* 1 *of the first, in a train of wheels and pinions.*—Divide the product of all the teeth in the driving, by the product of all the teeth in the driven, and the quotient equal the ratio of velocity required.

Required the ratio of velocity of the last, to 1 of the first, in the following train of wheels and pinions; viz., *pinions driving*,—the first of which contains 10 teeth, the second 15, and third 18;—*wheels driven*,—first 15 teeth, second 25, and third 32.

$\frac{10 \times 15 \times 18}{15 \times 25 \times 32} = {\cdot}225$ of a revolution the wheel will make to 1 of the pinion.

A wheel of 42 teeth giving motion to one of 12, on which shaft is a pulley of 21 inches diameter, driving one of 6; required the number of revolutions of the last pulley to 1 of the first wheel.

$$\frac{42 \times 21}{12 \times 6} = 12{\cdot}25, \text{ or } 12\tfrac{1}{4} \text{ revolutions.}$$

Where increase or decrease of velocity is required to be communicated by wheel-work, it has been demonstrated that the number of teeth on each pinion should not be less than 1 to 6 of its wheel, unless there be some other important reason for a higher ratio.

Rule.—*When time must be regarded.*—Multiply the diameter, or number of teeth in the driver, by its velocity in any given time, and divide the product by the required velocity of the driven; the quotient equal the number of teeth, or diameter of the driven, to produce the velocity required.

If a wheel containing 84 teeth makes 20 revolutions per minute, how many must another contain to work in contact, and make 60 revolutions in the same time?

$$\frac{84 \times 20}{60} = 28 \text{ teeth.}$$

From a shaft making 45 revolutions per minute, and with a pinion 9 inches diameter at the pitch line, I wish to transmit motion at 15 revolutions per minute; what at the pitch line must be the diameter of the wheel?

$$\frac{45 \times 9}{15} = 27 \text{ inches.}$$

Required the diameter of a pulley to make 16 revolutions in the same time as one of 24 inches making 36.

$$\frac{24 \times 36}{16} = 54 \text{ inches.}$$

RULE.—*The distance between the centres and velocities of two wheels being given, to find their proper diameters.*—Divide the greatest velocity by the least; the quotient is the ratio of diameter the wheels must bear to each other. Hence, divide the distance between the centres by the ratio plus 1; the quotient equal the radius of the smaller wheel; and subtract the radius thus obtained from the distance between the centres; the remainder equal the radius of the other.

The distance of two shafts from centre to centre is 50 inches, and the velocity of the one 25 revolutions per minute, the other is to make 80 in the same time; the proper diameters of the wheels at the pitch lines are required.

$80 \div 25 = 3{\cdot}2$, ratio of velocity, and $\frac{50}{3{\cdot}2 + 1} = 11{\cdot}9$, the radius of the smaller wheel; then $50 - 11{\cdot}9 = 38{\cdot}1$, radius of larger; their diameters are $11{\cdot}9 \times 2 = 23{\cdot}8$, and $38{\cdot}1 \times 2 = 76{\cdot}2$ inches.

To obtain or diminish an accumulated velocity by means of wheels and pinions, or wheels, pinions, and pulleys, it is necessary that a proportional ratio of velocity should exist, and which is simply thus attained:—Multiply the given and required velocities together, and the square root of the product is the mean or proportionate velocity.

Let the given velocity of a wheel containing 54 teeth equal 16 revolutions per minute, and the given diameter of an intermediate pulley equal 25 inches, to obtain a velocity of 81 revolutions in a machine; required the number of teeth in the intermediate wheel, and diameter of the last pulley.

$\sqrt{81 \times 16} = 36$ mean velocity.

$\frac{54 \times 16}{36} = 24$ teeth, and $\frac{25 \times 36}{81} = 11{\cdot}1$ inches, diameter of pulley.

To determine the proportion of wheels for screw cutting by a lathe.—In a lathe properly adapted, screws to any degree of pitch, or number of threads in a given length, may be cut by means of a leading screw of any given pitch, accompanied with change wheels and pinions; course pitches being effected generally by means of one wheel and one pinion with a *carrier*, or *intermediate wheel*, which cause no variation or change of motion to take place: hence the following

RULE.—Divide the number of threads in a given length of the screw which is to be cut, by the number of threads in the same length of the leading screw attached to the lathe; and the quotient is the ratio that the wheel on the end of the screw must bear to that on the end of the lathe spindle.

Let it be required to cut a screw with 5 threads in an inch, the leading screw being of $\frac{1}{2}$ inch pitch, or containing 2 threads in an inch; what must be the ratio of wheels applied?

$5 \div 2 = 2{\cdot}5$, the ratio they must bear to each other.

Then suppose a pinion of 40 teeth be fixed upon for the spindle,—

$40 \times 2{\cdot}5 = 100$ teeth for the wheel on the end of the screw.

But screws of a greater degree of fineness than about 8 threads in an inch are more conveniently cut by an additional wheel and pinion, because of the proper degree of velocity being more effectively attained; and these, on account of revolving upon a stud, are commonly designated the *stud-wheels*, or *stud-wheel* and *pinion;* but the mode of calculation and ratio of screw are the same as in the preceding rule;—hence, all that is further necessary is to fix upon any 3 wheels at pleasure, as those for the spindle and stud-wheels,—then multiply the number of teeth in the spindle-wheel by the ratio of the screw, and by the number of teeth in that wheel or pinion which is in contact with the wheel on the end of the screw; divide the product by the stud-wheel in contact with the spindle-wheel, and the quotient is the number of teeth required in the wheel on the end of the leading screw.

Suppose a screw is required to be cut containing 25 threads in an inch, the leading screw as before having 2 threads in an inch, and that a wheel of 60 teeth is fixed upon for the end of the spindle, 20 for the pinion in contact with the screw-wheel, and 100 for that in contact with the wheel on the end of the spindle;—required the number of teeth in the wheel for the end of the leading screw.

$$25 \div 2 = 12{\cdot}5, \text{ and } \frac{60 \times 12{\cdot}5 \times 20}{100} = 150 \text{ teeth.}$$

Or, suppose the spindle and screw-wheels to be those fixed upon, also any one of the stud-wheels, to find the number of teeth in the other.

$$\frac{60 \times 12{\cdot}5}{150 \times 100} = 20 \text{ teeth, or } \frac{60 \times 12{\cdot}5 \times 20}{150} = 100 \text{ teeth.}$$

TABLE *of Change Wheels for Screw Cutting, the leading screw being of ½ inch pitch, or containing two threads in an inch.*

Number of threads in inch of screw.	Number of teeth in		Number of threads in inch of screw.	Number of teeth in				Number of threads in inch of screw.	Number of teeth in			
	Lathe spindle-wheel.	Leading screw-wheel.		Lathe spindle-wheel.	Wheel in contact with spindle-wheel.	Pinion in contact with screw-wheel.	Leading screw-wheel.		Lathe spindle-wheel.	Wheel in contact with spindle-wheel.	Pinion in contact with screw-wheel.	Leading screw-wheel.
1	80	40	8¼	40	55	20	60	19	50	95	20	100
1¼	80	50	8½	90	85	20	90	19½	80	120	20	130
1½	80	60	8¾	60	70	20	75	20	60	100	20	120
1¾	80	70	9½	90	90	20	95	20¼	40	90	20	90
2	80	90	9¾	40	60	20	65	21	80	120	20	140
2¼	80	90	10	60	75	20	80	22	60	110	20	120
2½	80	100	10½	50	70	20	75	22½	80	120	20	150
2¾	80	110	11	60	55	20	120	22¾	80	130	20	140
3	80	120	12	90	90	20	120	23¾	40	95	20	100
3¼	80	130	12¾	60	85	20	90	24	65	120	20	130
3½	80	140	13	90	90	20	130	25	60	100	20	150
3¾	80	150	13½	60	90	20	90	25½	30	85	20	90
4	40	80	13¾	80	100	20	110	26	70	130	20	140
4¼	40	85	14	90	90	20	140	27	40	90	20	120
4½	40	90	14¼	60	90	20	95	27½	40	100	20	110
4¾	40	95	15	90	90	20	150	28	75	140	20	150
5	40	100	16	60	80	20	120	28½	30	90	20	95
5½	40	110	16¼	80	100	20	130	30	70	140	20	150
6	40	120	16½	80	110	20	120	32	30	80	20	120
6½	40	130	17	45	85	20	90	33	40	110	20	120
7	40	140	17½	80	100	20	140	34	30	85	20	120
7½	40	150	18	40	60	20	120	35	60	140	20	150
8	30	120	18¾	80	100	20	150	36	30	90	20	120

TABLE *by which to determine the Number of Teeth, or Pitch of Small Wheels.*

Diametral pitch.	Circular pitch.	Diametral pitch.	Circular pitch.
3	1·047	9	·349
4	·785	10	·314
5	·628	12	·262
6	·524	14	·224
7	·449	16	·196
8	·393	20	·157

Required the number of teeth that a wheel of 16 inches diameter will contain of a 10 pitch.

$16 \times 10 = 160$ teeth, and the circular pitch = ·314 inch.

What must be the diameter of a wheel for a 9 pitch of 126 teeth?

$\frac{126}{9} = 14$ inches diameter, circular pitch ·349 inch.

The pitch is reckoned on the diameter of the wheel instead of the circumference, and designated wheels of 8 pitch, 12 pitch, &c.

TABLE *of the Diameters of Wheels at their pitch circle, to contain a required number of teeth at a given pitch.*

Number of teeth.	PITCH OF THE TEETH IN INCHES.													
	1 in.	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2 in.	2⅛	2¼	2½	2¾	3 in.
	DIAMETER AT THE PITCH CIRCLE IN FEET AND INCHES.													
10	0 3¼	0 3⅝	0 4	0 4½	0 4⅞	0 5¼	0 5⅝	0 6	0 6½	0 6⅞	0 7¼	0 8	0 8¾	0 9⅝
11	0 3½	0 4	0 4⅜	0 5	0 5⅜	0 5¾	0 6¼	0 6⅝	0 7	6 7½	0 7⅞	0 8¾	0 9¾	0 10⅝
12	0 3⅞	0 4⅜	0 4⅞	0 5⅜	0 5⅞	0 6⅜	0 6¾	0 7⅛	0 7⅝	0 8⅛	0 8⅝	0 9⅝	0 10⅝	0 11½
13	0 4⅛	0 4¾	0 5¼	0 5¾	0 6¼	0 6⅞	0 7⅜	0 7⅞	0 8⅜	0 8⅞	0 9⅜	0 10⅜	0 11½	1 0½
14	0 4½	0 5	0 5⅝	0 6¼	0 6¾	0 7⅜	0 7⅞	0 8½	0 9	0 9½	0 10	0 11¼	1 0⅜	1 1½
15	0 4⅞	0 5⅜	0 6	0 6⅝	0 7¼	0 7⅞	0 8½	0 9	0 9⅝	0 10¼	0 10¾	1 0	1 1¼	1 2⅜
16	0 5⅛	0 5¾	0 6⅜	0 7	0 7⅝	0 8⅜	0 9	0 9⅝	0 10¼	0 10⅞	0 11½	1 0¾	1 2	1 3⅜
17	0 5½	0 6⅛	0 6¾	0 7½	0 8¼	0 8⅞	0 9⅝	0 10¼	0 10⅞	0 11½	1 0¼	1 1½	1 2⅞	1 4¼
18	0 5¾	0 6½	0 7⅛	0 8	0 8⅝	0 9⅜	0 10	0 10¾	0 11½	1 0¼	1 0⅞	1 2⅜	1 3¾	1 5¼
19	0 6	0 6⅞	0 7½	0 8⅜	0 9⅛	0 9⅞	0 10⅝	0 11⅜	1 0⅛	1 0⅞	1 1⅝	1 3⅛	1 4⅝	1 6¼
20	0 6⅜	0 7⅛	0 8	0 8⅞	0 9⅝	0 10⅜	0 11¼	1 0	1 0¾	1 1½	1 2⅜	1 4	1 5½	1 7⅛
21	0 6¾	0 7½	0 8⅜	0 9¼	0 10	0 11	0 11¾	1 0½	1 1½	1 2¼	1 3	1 4¾	1 6⅜	1 8⅛
22	0 7	0 7⅞	0 8¾	0 9⅝	0 10⅝	0 11½	1 0⅜	1 1⅛	1 2	1 2⅞	1 3¾	1 5½	1 7¼	1 9
23	0 7⅜	0 8¼	0 9⅛	0 10	0 11	1 0	1 0⅞	1 1¾	1 2⅝	1 3½	1 4½	1 6⅜	1 8	1 10
24	0 7⅝	0 8⅝	0 9½	0 10½	0 11½	1 0½	1 1½	1 2⅜	1 3⅜	1 4¼	1 5¼	1 7⅛	1 9	1 10⅞
25	0 8	0 9	0 10	0 11	1 0	1 1	1 2	1 2⅞	1 3⅞	1 4⅞	1 6	1 8	1 9⅞	1 11⅞
26	0 8¼	0 9¼	0 10⅜	0 11½	1 0½	1 1½	1 2½	1 3½	1 4½	1 5½	1 6⅝	1 8¾	1 10¾	2 0⅞
27	0 8⅝	0 9⅝	0 10¾	0 11⅞	1 1	1 2	1 3	1 4⅛	1 5¼	1 6¼	1 7⅜	1 9½	1 11⅝	2 1¾
28	0 9	0 10	0 11¼	1 0¼	1 1¼	1 2½	1 3⅝	1 4⅝	1 5¾	1 6⅞	1 8	1 10¼	2 0½	2 2¾
29	0 9¼	0 10⅜	0 11⅝	1 0¾	1 1⅞	1 3	1 4⅛	1 5⅜	1 6½	1 7⅝	1 8¾	1 11⅛	2 1⅜	2 3¾
30	0 9½	0 10¾	1 0	1 1⅛	1 2⅜	1 3½	1 4½	1 6	1 7⅛	1 8¼	1 9½	2 0	2 2¼	2 4⅝
31	0 9⅞	0 11⅛	1 0⅜	1 1⅝	1 2⅞	1 4	1 5⅜	1 6½	1 7¾	1 9	1 10¼	2 0¾	2 3⅛	2 5⅝
32	0 10¼	0 11½	1 0¾	1 2	1 3⅜	1 4⅝	1 5⅞	1 7⅛	1 8⅜	1 9⅝	1 11	2 1½	2 4	2 6½
33	0 10½	0 11⅞	1 1⅛	1 2½	1 3¾	1 5⅛	1 6½	1 7¾	1 9	1 10⅜	1 11⅝	2 2¼	2 4⅞	2 7½
34	0 10⅞	1 0⅛	1 1⅝	1 3	1 4¼	1 5⅝	1 7	1 8⅜	1 9⅝	1 11	2 0⅜	2 3	2 5¾	2 8½
35	0 11⅛	1 0⅝	1 2	1 3⅜	1 4¾	1 6⅛	1 7½	1 9	1 10⅛	1 11¾	2 1	2 3⅞	2 6⅝	2 9½
36	0 11½	1 1	1 2⅜	1 3¾	1 5¼	1 6⅝	1 8	1 9½	1 10⅞	2 0⅜	2 2	2 4⅝	2 7½	2 10⅜
37	0 11¾	1 1¼	1 2¾	1 4¼	1 5⅝	1 7¼	1 8⅝	1 10	1 11½	2 1	2 2½	2 5½	2 8⅜	2 11⅜
38	1 0⅛	1 1⅝	1 3¼	1 4⅝	1 6¼	1 7⅝	1 9¼	1 10¾	2 0¼	2 1¾	2 3¼	2 6¼	2 9¼	3 0¼
39	1 0⅜	1 2	1 3½	1 5	1 6⅝	1 8¼	1 9¾	1 11⅜	2 0⅞	2 2⅜	2 4	2 7	2 10⅛	3 1¼
40	1 0¾	1 2⅜	1 4	1 5½	1 7⅛	1 8¾	1 10⅜	1 11⅞	2 1½	2 3	2 4⅝	2 7⅞	2 10⅞	3 2¼
41	1 1	1 2¾	1 4⅜	1 6	1 7⅝	1 9¼	1 10⅞	2 0½	2 2⅛	2 3¾	2 5⅜	2 8⅝	2 11⅞	3 3⅛
42	1 1⅜	1 3	1 4¾	1 6⅜	1 8	1 9¾	1 11½	2 1	2 2¾	2 4½	2 6	2 9⅜	3 0¾	3 4⅛
43	1 1⅝	1 3½	1 5⅛	1 6⅞	1 8⅝	1 10¼	2 0	2 1⅝	2 3⅜	2 5	2 6¾	2 10¼	3 1⅝	3 5
44	1 2	1 3¾	1 5½	1 7¼	1 9	1 10¾	2 0½	2 2¼	2 4	2 5¾	2 7½	2 11	3 2½	3 6
45	1 2⅜	1 4⅛	1 6	1 7¾	1 9½	1 11⅜	2 1	2 2⅞	2 4⅝	2 6½	2 8¼	2 11¾	3 3⅜	3 7
46	1 2⅝	1 4½	1 6⅜	1 8⅛	1 10	1 11⅞	2 1⅝	2 3½	2 5¼	2 7⅛	2 9	3 0⅝	3 4¼	3 7⅞
47	1 2⅞	1 4⅞	1 6¾	1 8⅝	1 10½	2 0⅜	2 1⅞	2 4	2 6	2 7⅞	2 9⅝	3 1½	3 5⅛	3 8⅞
48	1 3¼	1 5¼	1 7⅛	1 9	1 11	2 0⅞	2 2¾	2 4⅝	2 6½	2 8½	2 10⅜	3 2¼	3 6	3 9⅞
49	1 3½	1 5⅝	1 7½	1 9½	1 11½	2 1⅜	2 3⅜	2 5¼	2 7¼	2 9⅛	2 11	3 3	3 6⅞	3 10⅞
50	1 3⅞	1 6	1 8	1 9⅞	2 0	2 1⅞	2 3⅞	2 5⅞	2 7⅞	2 9⅞	2 11¾	3 3¾	3 7¾	3 11¾
51	1 4¼	1 6¼	1 8⅜	1 10⅜	2 0⅜	2 2⅜	2 4½	2 6½	2 8½	2 10½	3 0½	3 4½	3 8⅝	4 0¾
52	1 4½	1 6⅝	1 8¾	1 10¾	2 0⅞	2 2⅞	2 4⅞	2 7⅛	2 9⅛	2 11⅛	3 1¼	3 5⅜	3 9½	4 1⅝
53	1 4⅞	1 6⅞	1 9⅛	1 11¼	2 1¼	2 3⅜	2 5½	2 7⅝	2 9¾	2 11⅞	3 2	3 6¼	3 10½	4 2⅝
54	1 5¼	1 7⅜	1 9½	1 11⅝	2 1¾	2 3⅞	2 6	2 8¼	2 10⅜	3 0½	3 2⅝	3 7	3 11	4 3½
55	1 5½	1 7¾	1 9⅞	2 0	2 2¼	2 4½	2 6⅝	2 8⅞	2 11	3 1¼	3 3½	3 7¾	4 0⅛	4 4½
56	1 5⅞	1 8⅛	1 10¼	2 0½	2 2¾	2 4⅞	2 7⅛	2 9⅜	2 11⅝	3 1⅞	3 4⅛	3 8½	4 1	4 5½
57	1 6⅛	1 8½	1 10⅝	2 0⅞	2 3¼	2 5¼	2 7¾	2 10	3 0¼	3 2½	3 4⅞	3 9⅜	4 1⅞	4 6⅜
58	1 6½	1 8¾	1 11	2 1⅜	2 3⅝	2 6	2 8¼	2 10⅝	3 0⅞	3 3¼	3 5½	3 10⅛	4 2¾	4 7⅜
59	1 6¾	1 9⅛	1 11½	2 1⅞	2 4⅛	2 6½	2 8⅞	2 11¼	3 1½	3 4	3 6¼	3 11⅛	4 3⅝	4 8⅜
60	1 7⅛	1 9⅜	1 11⅞	2 2¼	2 4⅝	2 7	2 9⅜	2 11¾	3 2¼	3 4⅝	3 7	3 11¾	4 4½	4 9¼
61	1 7½	1 9⅞	2 0¼	2 2¾	2 5¼	2 7½	2 10	3 0⅜	3 2⅞	3 5¼	3 7¾	4 0½	4 5⅜	4 10¼
62	1 7¾	1 10¼	2 0⅝	2 3⅛	2 5⅝	2 8	2 10½	3 1	3 3½	3 6	3 8½	4 1⅜	4 6¼	4 11¼
63	1 8	1 10½	2 1	2 3½	2 6	2 8⅝	2 11	3 1⅝	3 4⅛	3 6⅝	3 9⅛	4 2⅛	4 7⅛	5 0⅛
64	1 8⅜	1 10⅞	2 1½	2 4	2 6½	2 9⅛	2 11⅝	3 2¼	3 4¾	3 7¼	3 9⅞	4 3	4 8	5 1⅛
65	1 8⅝	1 11¼	2 1⅞	2 4½	2 7	2 9⅝	3 0¼	3 2⅞	3 5⅜	3 8	3 10½	4 3¾	4 8⅞	5 2
66	1 9	1 11⅝	2 2¼	2 4⅞	2 7½	2 10⅛	3 0⅞	3 3⅜	3 6	3 8⅝	3 11¼	4 4½	4 9¾	5 3
67	1 9⅜	2 0	2 2⅝	2 5⅜	2 8	2 10⅝	3 1⅜	3 4	3 6½	3 9⅜	4 0	4 5⅜	4 10⅝	5 4
68	1 9⅝	2 0⅜	2 3	2 5¾	2 8½	2 11⅛	3 1⅞	3 4⅝	3 7¼	3 10	4 0¾	4 6⅛	4 11½	5 5
69	1 9⅞	2 0¾	2 3½	2 6¼	2 8⅞	2 11¾	3 2½	3 5⅛	3 7⅞	3 10⅝	4 1½	4 7	5 0⅜	5 6
70	1 10¼	2 1	2 3⅞	2 6⅝	2 9⅜	3 0¼	3 3	3 5¾	3 8½	3 11⅜	4 2⅛	4 7¾	5 1¼	5 6⅞

Number of teeth.	PITCH OF THE TEETH IN INCHES.													
	1 in.	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2 in.	2⅛	2¼	2½	2¾	3 in.
	DIAMETER AT THE PITCH CIRCLE IN FEET AND INCHES.													
71	1 10⅝	2 1½	2 4¼	2 7	2 9⅞	3 0¾	3 3½	3 6⅜	3 9¼	4 0	4 2⅞	4 8½	5 2⅛	5 7¾
72	1 10⅞	2 1¾	2 4⅝	2 7½	2 10⅜	3 1¼	3 4⅛	3 6¾	3 9⅞	4 0¾	4 3½	4 9¼	5 3	5 8¾
73	1 11¼	2 2⅛	2 5	2 8	2 10⅞	3 1¾	3 4⅝	3 7½	3 10½	4 1⅜	4 4¼	4 10	5 3⅞	5 9¾
74	1 11½	2 2½	2 5½	2 8⅜	2 11⅜	3 2¼	3 5¼	3 7⅞	3 11⅛	4 2	4 5	4 10⅞	5 4¾	5 10⅝
75	1 11⅞	2 2⅞	2 5⅞	2 8⅞	2 11⅞	3 2¾	3 5¾	3 8¾	3 11¾	4 2¾	4 5¾	4 11¾	5 5⅝	5 11⅝
76	2 0¼	2 3¼	2 6¼	2 9¼	3 0¼	3 3¼	3 6⅜	3 9⅜	4 0⅜	4 3½	4 6½	5 0½	5 6½	6 0½
77	2 0½	2 3½	2 6⅝	2 9¾	3 0¾	3 3⅞	3 6⅞	3 9⅞	4 1	4 4	4 7⅛	5 1¼	5 7⅜	6 1½
78	2 0⅞	2 3⅞	2 7	2 10⅛	3 1½	3 4⅜	3 7½	3 10½	4 1⅝	4 4¾	4 7⅞	5 2	5 8¼	6 2½
79	2 1⅛	2 4¼	2 7⅜	2 10½	3 1¾	3 4⅞	3 8	3 11⅛	4 2¼	4 5½	4 8½	5 2⅞	5 9⅛	6 3½
80	2 1½	2 4⅝	2 7⅞	2 11	3 2¼	3 5⅜	3 8½	3 11¾	4 3	4 6⅛	4 9¼	5 3⅝	5 10	6 4⅜
81	2 1¾	2 5	2 8¼	2 11½	3 2⅝	3 5⅞	3 9⅛	4 0⅜	4 3½	4 6¾	4 10	5 4½	5 10⅞	6 5⅜
82	2 2⅛	2 5⅜	2 8⅝	2 11⅞	3 3⅛	3 6⅜	3 9⅝	4 0⅞	4 4¼	4 7½	4 10¾	5 5¼	5 11¾	6 6⅜
83	2 2½	2 5¾	2 9	3 0⅜	3 3⅝	3 6⅞	3 10¼	4 1½	4 4⅞	4 8⅛	4 11½	5 6	6 0⅝	6 7¼
84	2 2¾	2 6	2 9⅜	3 0¾	3 4	3 7½	3 10¾	4 2⅛	4 5½	4 8⅞	5 0⅛	5 6⅞	6 1½	6 8¼
85	2 3	2 6⅜	2 9¾	3 1¼	3 4½	3 7⅞	3 11¼	4 2¾	4 6⅛	4 9½	5 0⅞	5 7⅝	6 2⅜	6 9⅛
86	2 3⅜	2 6¾	2 10¼	3 1⅝	3 5⅛	3 8½	3 11⅞	4 3¼	4 6¾	4 10⅛	5 1⅝	5 8½	6 3¼	6 10⅛
87	2 3¾	2 7⅛	2 10⅝	3 2	3 5½	3 9	4 0½	4 3⅞	4 7⅜	4 10⅞	5 2¼	5 9¼	6 4⅛	6 11
88	2 4	2 7½	2 11	3 2½	3 6	3 9½	4 1	4 4½	4 8	4 11½	5 3	5 10	6 5	7 0
89	2 4⅜	2 7⅞	2 11⅜	3 2⅞	3 6½	3 10	4 1½	4 5⅛	4 8⅝	5 0¼	5 3¾	5 10¾	6 5⅞	7 1
90	2 4⅝	2 8¼	2 11¾	3 3¼	3 7	3 10½	4 2⅛	4 5¾	4 9¼	5 0⅞	5 4½	5 11⅝	6 6¾	7 2
91	2 4⅞	2 8½	3 0¼	3 3⅞	3 7½	3 11	4 2¾	4 6¼	4 9⅞	5 1½	5 5⅛	6 0⅜	6 7⅝	7 2⅞
92	2 5¼	2 8⅞	3 0⅝	3 4¼	3 7⅞	3 11⅝	4 3¼	4 7	4 10½	5 2¼	5 5⅞	6 1	6 8½	7 3⅞
93	2 5⅝	2 9¼	3 1	3 4¾	3 8⅜	4 0⅛	4 3⅞	4 7½	4 11¼	5 2⅞	5 6⅝	6 2	6 9⅜	7 4⅞
94	2 5⅞	2 9⅝	3 1⅜	3 5⅛	3 8⅞	4 0⅝	4 4⅜	4 8⅛	4 11¾	5 3½	5 7⅜	6 2¾	6 10¼	7 5¾
95	2 6¼	2 10	3 1¾	3 5½	3 9⅜	4 1⅛	4 4⅞	4 8¾	5 0½	5 4¼	5 8	6 3½	6 11⅛	7 6¾
96	2 6½	2 10⅜	3 2¼	3 6	3 9⅞	4 1⅝	4 5½	4 9⅜	5 1⅛	5 5	5 8¾	6 4⅜	7 0	7 7⅝
97	2 6⅞	2 10¾	3 2⅝	3 6½	3 10¼	4 2⅛	4 6	4 10	5 1¾	5 5⅝	5 9½	6 5¼	7 0⅞	7 8⅝
98	2 7¼	2 11	3 3	3 6⅞	3 10¾	4 2¾	4 6½	4 10½	5 2⅜	5 6¼	5 10⅛	6 6	7 1¾	7 9½
99	2 7½	2 11⅜	3 3⅜	3 7⅜	3 11¼	4 3¼	4 7⅛	4 11	5 3	5 7	5 11	6 6¾	7 2⅝	7 10½
100	2 7⅞	2 11⅞	3 3¾	3 7¾	3 11¾	4 3¾	4 7¾	4 11⅝	5 3⅝	5 7⅝	5 11⅝	6 7½	7 3½	7 11½
101	2 8¼	3 0⅛	3 4⅛	3 8¼	4 0¼	4 4¼	4 8¼	5 0¼	5 4¼	5 8¼	6 0¼	6 8⅜	7 4⅜	8 0½
102	2 8½	3 0½	3 4½	3 8⅝	4 0⅝	4 4¾	4 8⅞	5 1	5 5	5 9	6 1	6 9⅛	7 5¼	8 1⅜

TABLE *of the Strength of the Teeth of Cast Iron Wheels at a given velocity.*

Pitch of teeth in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth in horse power, at			
			3 feet per second.	4 feet per second.	6 feet per second.	8 feet per second.
3·99	1·9	7·6	20·57	27·43	41·14	54·85
3·78	1·8	7·2	17·49	23·32	34·98	46·64
3·57	1·7	6·8	14·73	19·65	29·46	39·28
3·36	1·6	6·4	12·28	16·38	24·56	32·74
3·15	1·5	6	10·12	13·50	20·24	26·98
2·94	1·4	5·6	8·22	10·97	16·44	21·92
2·73	1·3	5·2	6·58	8·78	13·16	17·54
2·52	1·2	4·8	5·18	6·91	10·36	13·81
2·31	1·1	4·4	3·99	5·32	7·98	10·64
2·1	1·0	4	3·00	4·00	6·00	8·00
1·89	·9	3·6	2·18	2·91	4·36	5·81
1·68	·8	3·2	1·53	2·04	3·06	3·08
1·47	·7	2·8	1·027	1·37	2·04	2·72
1·26	·6	2·4	·64	·86	1·38	1·84
1·05	·5	2	·375	·50	·75	1·00

ADDITIONAL EXAMPLES ON THE VELOCITY OF WHEELS, DRUMS, PULLEYS, ETC.

If a wheel that contains 75 teeth makes 16 revolutions per minute, required the number of teeth in another to work in it, and make 24 revolutions in the same time.

$$\frac{75 \times 16}{24} = 50 \text{ teeth.}$$

A wheel, 64 inches diameter, and making 42 revolutions per minute, is to give motion to a shaft at the rate of 77 revolutions in the same time: required the diameter of a wheel suitable for that purpose.

$$\frac{64 \times 42}{77} = 34{\cdot}9 \text{ inches.}$$

Required the number of revolutions per minute made by a wheel or pulley 20 inches diameter, when driven by another of 4 feet diameter, and making 46 revolutions per minute.

$$\frac{48 \times 46}{20} = 110{\cdot}4 \text{ revolutions.}$$

A shaft, at the rate of 22 revolutions per minute, is to give motion, by a pair of wheels, to another shaft at the rate of 15½; the distance of the shafts from centre to centre is 45½ inches; the diameters of the wheels at the pitch lines are required.

$$\frac{45{\cdot}5 \times 15{\cdot}5}{22 + 15{\cdot}5} = 18{\cdot}81 \text{ radius of the driving wheel.}$$

$$\text{And } \frac{45{\cdot}5 \times 22}{22 + 15{\cdot}5} = 26{\cdot}69 \text{ radius of the driven wheel.}$$

Suppose a drum to make 20 revolutions per minute, required the diameter of another to make 58 revolutions in the same time.

58 ÷ 20 = 2·9, that is, their diameters must be as 2·9 to 1; thus, if the one making 20 revolutions be called 30 inches, the other will be 30 ÷ 2·9 = 10·345 inches diameter.

Required the diameter of a pulley, to make 12½ revolutions in the same time as one of 32 inches making 26.

$$\frac{32 \times 26}{12{\cdot}5} = 66{\cdot}56 \text{ inches diameter.}$$

A shaft, at the rate of 16 revolutions per minute, is to give motion to a piece of machinery at the rate of 81 revolutions in the same time; the motion is to be communicated by means of two wheels and two pulleys with an intermediate shaft; the driving wheel contains 54 feet, and the driving pulley is 25 inches diameter; required the number of teeth in the other wheel, and the diameter of the other pulley.

$\sqrt{81 \times 16} = 36$, the mean velocity between 16 and 81; then, $\frac{16 \times 54}{36} = 24$ teeth; and $\frac{36 \times 25}{81} = 11{\cdot}11$ inches, diameter of pulley.

Suppose in the last example the revolutions of one of the wheels to be given, the number of teeth in both, and likewise the diameter of each pulley, to find the revolutions of the last pulley.

$$\frac{16 \times 54}{24} = 36, \text{ velocity of the intermediate shaft;}$$

$$\text{and } \frac{36 \times 25}{11{\cdot}11} = 81, \text{ the velocity of the machine.}$$

Table *for finding the radius of a wheel when the pitch is given, or the pitch of a wheel when the radius is given, that shall contain from* 10 *to* 150 *teeth, and any pitch required.*

Number of Teeth.	Radius.	Number of Teeth.	Radius.	Number of Teeth.	Radius.	Number of Teeth.	Radius.
10	1·618	46	7·327	81	12·895	116	18·464
11	1·774	47	7·486	82	13·054	117	18·623
12	1·932	48	7·645	83	13·213	118	18·782
13	2·089	49	7·804	84	13·370	119	18·941
14	2·247	50	7·963	85	13·531	120	19·101
15	2·405	51	8·122	86	13·690	121	19·260
16	2·563	52	8·281	87	13·849	122	19·419
17	2·721	53	8·440	88	14·008	123	19·578
18	2·879	54	8·599	89	14·168	124	19·737
19	3·038	55	8·758	90	14·327	125	19·896
20	3·196	56	8·917	91	14·486	126	20·055
21	3·355	57	9·076	92	14·645	127	20·214
22	3·513	58	9·235	93	14·804	128	20·374
23	3·672	59	9·394	94	14·963	129	20·533
24	3·830	60	9·553	95	15·122	130	20·692
25	3·989	61	9·712	96	15·281	131	20·851
26	4·148	62	9·872	97	15·440	132	21·010
27	4·307	63	10·031	98	15·600	133	21·169
28	4·465	64	10·190	99	15·759	134	21·328
29	4·624	65	10·349	100	15·918	135	21·488
30	4·788	66	10·508	101	16·077	136	21·647
31	4·942	67	10·667	102	16·236	137	21·806
32	5·101	68	10·826	103	16·395	138	21·965
33	5·260	69	10·985	104	16·554	139	22·124
34	5·419	70	11·144	105	16·713	140	22·283
35	5·578	71	11·303	106	16·873	141	22·442
36	5·737	72	11·463	107	17·032	142	22·602
37	5·896	73	11·622	108	17·191	143	22·761
38	6·055	74	11·781	109	17·350	144	22·920
39	6·214	75	11·940	110	17·509	145	23·079
40	6·373	76	12·099	111	17·668	146	23·238
41	6·532	77	12·258	112	17·827	147	23·397
42	6·691	78	12·417	113	17·987	148	23·556
43	6·850	79	12·576	114	18·146	149	23·716
44	7·009	80	12·735	115	18·305	150	23·875
45	7·168						

Rule.—Multiply the radius in the table by the pitch given, and the product will be the radius of the wheel required.

Or, divide the radius of the wheel by the radius in the table, and the quotient will be the pitch of the wheel required.

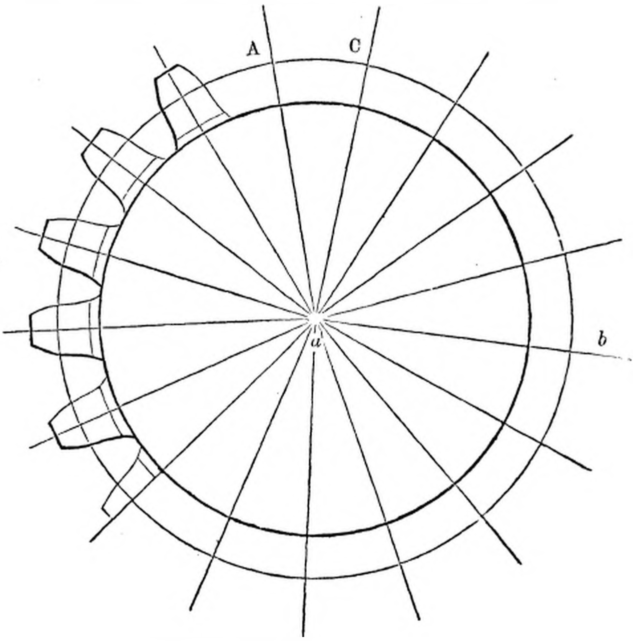
Required the radius of a wheel to contain 64 teeth, of 3 inch pitch.

$$10{\cdot}19 \times 3 = 30{\cdot}57 \text{ inches.}$$

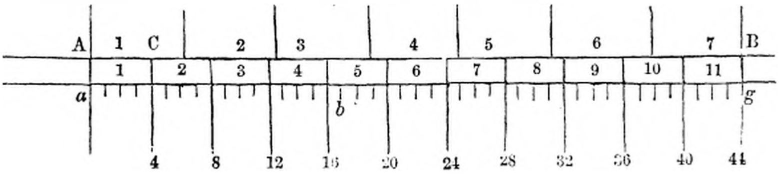
What is the pitch of a wheel to contain 80 teeth, when the radius is 25·47 inches?

$$25{\cdot}47 \div 12{\cdot}735 = 2 \text{ inch pitch.}$$

Or, set off upon a straight line AB seven times the pitch AC given; divide that, or another exactly the same length, into eleven equal parts; call each of those divisions four, or each of those divisions will be equal to four teeth upon the radius. If a circle be made with any number (20) of these equal parts as radius, AC the pitch will go that number (20) of times round the circle.



Were it required to find the diameter of a wheel to contain 17 teeth, the construction would be as follows:—



Thus, 4 divisions and $\frac{1}{4}$ of another equal the radius of the wheel, that is $a_1\, b_1 = a\, b$, and $A_1\, C_1 = AC$.

Regular approved proportions for wheels with flat arms in the middle of the ring, and ribs or feathers on each side.—The length of the teeth = $\frac{6}{9}$ the pitch, besides clearance, or $\frac{5}{7}$ the pitch, clearance included.

Thickness of the teeth	$\frac{4}{9}$	the pitch.
Breadth on the face................................	$2\frac{1}{2}$	—
Edge of the rim....................................	$\frac{4}{9}$	—
Rib projecting inside the rim...................	$\frac{4}{9}$	—
Thickness of the flat arms.......................	$\frac{4}{9}$	—

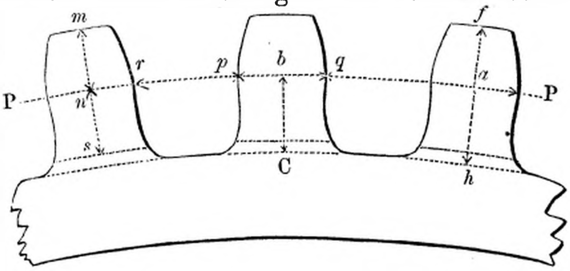
Breadth of the arms at the points = 2 teeth and $\frac{1}{4}$ the pitch, getting broader towards the centre of the wheel in the proportion of $\frac{1}{2}$ inch to every foot in length.

Thickness of the ribs, or feathers, $\frac{1}{4}$ the pitch.

Thickness of metal round the eye, or centre, $\frac{7}{9}$ the pitch.

Wheels made with plain arms, the teeth are in the same proportion as above; the ring and the arms are each equal to one cog or tooth in thickness, and the metal round the eye same as above, in feathered wheels.

These proportions differ, though slightly, in different works and in different localities; but they are the most commonly employed, and are besides the most consistent with good and accurate workmanship. For the sake of more easy reference, we collect them into a table, which the annexed diagram will serve fully to explain. They stand thus:—



ab = Pitch of teeth = 1 pitch.
mn = Depth to pitch line, PP, = $\frac{3}{10}$ —.
$ns + nm$ = Working depth of tooth, = $\frac{6}{10}$ —.
$Cb - ns$ = Bottom clearance, = $\frac{1}{10}$ —.
fh = Whole depth to root, = $\frac{7}{10}$ —.
pq = Thickness of tooth, = $\frac{5}{11}$ —.
rp = Width of space, = $\frac{6}{11}$ —.

The use of the following table is very evident, and the manner of applying it may be rendered still more obvious by the following examples:—

$$\pi = 3{\cdot}1416.$$

1. Given a wheel of 88 teeth, $2\frac{1}{2}$ inch pitch, to find the diameter of the pitch circle. Here the tabular number in the second column answering to the given pitch is ·7958, which multiplied by 88 gives 70·03 for the diameter required.

2. Given a wheel of 5 feet (60 inches) diameter, $2\frac{3}{4}$ inch pitch, to find the number of teeth. Here the factor in the third column

corresponding to the given pitch is 1·1333, which multiplied by 60 gives 68 for the number of teeth.

It may, however, so happen that the answer found in this manner contains a fraction—which being inadmissible by the nature of the question, it becomes necessary to alter slightly the diameter of the pitch circle. This is readily accomplished by taking the nearest whole number to the answer found, and finding the modified diameter by means of the second column. The following case will fully explain what is meant:

3. Given a wheel 33 inches diameter, 1¾ inch pitch, to find the number of teeth. The corresponding factor is 1·7952, which multiplied by 33 gives 59·242 for the number of teeth, that is, 59¼ teeth nearly. Now, 59 would here be the nearest whole number; but as a wheel of 60 teeth may be preferred for convenience of calculation of speeds, we may adopt that number and find the diameter corresponding. The factor in the second column answering to 1¾ pitch is ·557, and this multiplied by 60 gives 33·4 inches as the diameter which the wheel ought to have.

Pitch in inches and parts of an inch.	$D = \frac{P}{\pi} \times N$ Rule.—To find the diameter in inches, multiply the number of teeth by the tabular number answering to the given pitch.	$N = \frac{\pi}{P} \times D$ Rule.—To find the number of teeth, multiply the given diameter in inches by the tabular number answering to the given pitch.
Values of P	Values of $\frac{P}{\pi}$	Values of $\frac{\pi}{P}$
6	1·9095	·5236
5	1·5915	·6283
4½	1·4270	·6981
4	1·2732	·7854
3½	1·1141	·8976
3	·9547	1·0472
2¾	·8754	1·1333
2½	·7958	1·2566
2¼	·7135	1·3963
2	·6366	1·5708
1⅞	·5937	1·6755
1¾	·5570	1·7952
1⅝	·5141	1·9264
1½	·4774	2·0944
1⅜	·4377	2·2848
1¼	·3979	2·5132
1⅛	·3568	2·7926
1	·3183	3·1416
⅞	·2785	3·5904
¾	·2387	4·1888
⅝	·1989	5·0266
½	·1592	6·2832
⅜	·1194	8·3776
¼	·0796	12·5664

Rule.—*To find the power that a cast iron wheel is capable of transmitting at any given velocity.*—Multiply the breadth of the teeth, or face of the wheel, in inches, by the square of the thickness of one tooth, and divide the product by the length of the teeth, the quotient is the strength in horse power at a velocity of 136 feet per minute.

Required the power that a wheel of the following dimensions ought to transmit with safety, namely,

Breadth of teeth...............7½ inches,
Thickness.......................1·4
And length.....................2

$$1{\cdot}4^2 = 1{\cdot}96, \text{ and } \frac{7{\cdot}5 \times 1{\cdot}96}{2} = 7{\cdot}35 \text{ horse power.}$$

The strength at any other velocity is found by multiplying the power so obtained by any other required velocity, and by ·0044, the quotient is the power at that velocity.

Suppose the wheel as above, at a velocity of 320 feet per minute.

$$7{\cdot}35 \times 320 \times {\cdot}0044 = 10{\cdot}3488 \text{ horse power.}$$

ON THE MAXIMUM VELOCITY AND POWER OF WATER WHEELS.

OF UNDERSHOT WHEELS.

THE term "undershot" is applied to a wheel when the water strikes at, or below, the centre; and the greatest effect is produced when the periphery of the wheels moves with a velocity of ·57 that of the water; hence, to find the velocity of the water, multiply the square root or the perpendicular height of the fall in feet by 8, and the product is the velocity in feet per second.

Required the maximum velocity of an undershot wheel, when propelled by a fall of water 6 feet in height.

$\sqrt{6} = 2{\cdot}45 \times 8 = 19{\cdot}6$ feet, velocity of water.

And $19{\cdot}6 \times {\cdot}57 = 11{\cdot}17$ feet per second for the wheel.

OF BREAST AND OVERSHOT WHEELS.

Wheels that have the water applied between the centre and the vertex are styled breast wheels, and overshot when the water is brought over the wheel and laid on the opposite side; however, in either case the maximum velocity is $\frac{2}{3}$ that of the water; hence, to find the head of water proper for a wheel at any velocity, say:

As the square of 16·083, or 258·67, is to 4, so is the square of the velocity of the wheel in feet per second to the head of water required. By *head* is understood the distance between the aperture of the sluice and where the water strikes upon the wheel.

Required the head of water necessary for a wheel of 24 feet diameter, moving with a velocity of 5 feet per second.

$$\frac{5 \times 3}{2} = 7{\cdot}5 \text{ feet, velocity of the water.}$$

And $258{\cdot}67 : 4 :: 7{\cdot}5^2 : {\cdot}87$ feet, head of water required.

But one-tenth of a foot of head must be added for every foot of increase in the diameter of the wheel, from 15 to 20 feet, and ·05 more for every foot of increase from 20 to 30 feet, commencing with five-tenths for a 15 feet wheel.

This additional head is intended to compensate for the friction of water in the aperture of the sluice to keep the velocity as 3 to 2 of the wheel; thus, in place of ·87 feet head for a 24 feet wheel, it will be ${\cdot}87 + 1{\cdot}2 = 2{\cdot}07$ feet head of water.

If the water flow from under the sluice, multiply the square root of the depth in feet by 5·4, and by the area of the orifice also in feet, and the product is the quantity discharged in cubic feet per second.

Again, if the water flow over the sluice, multiply the square root of the depth in feet by 5·4, and $\frac{2}{3}$ of the product multiplied by the length and depth, also in feet, gives the number of cubic feet discharged per second nearly.

Required the number of cubic feet per second that will issue from the orifice of a sluice 5 feet long, 9 inches wide, and 4 feet from the surface of the water.

$$\sqrt{4} = 2 \times 5{\cdot}4 = 10{\cdot}8 \text{ feet velocity.}$$

$$\text{And } 5 \times {\cdot}75 \times 10{\cdot}8 = 40{\cdot}5 \text{ cubic feet per second.}$$

What quantity of water per second will be expended over a wear, dam, or sluice, whose length is 10 feet, and depth 6 inches?

$$\sqrt{{\cdot}5} = {\cdot}2236 \times 5{\cdot}4 = \frac{1{\cdot}20744 \times 2}{3} = {\cdot}80496 \text{ feet velocity.}$$

Then $10 \times {\cdot}5 = 5$ feet, and ${\cdot}80496 \times 5 = 4{\cdot}0248$ cubic feet per second nearly.

In estimating the power of water wheels, half the head must be added to the whole fall, because 1 foot of fall is equal to 2 feet of head; call this the effective perpendicular descent; multiply the weight of the water per second by the effective perpendicular descent and by 60; divide the product by 33,000, and the quotient is the effect expressed in horse power.

Given 16 cubic feet of water per second, to be applied to an undershot wheel, the head being 12 feet; required the power produced.

$$12 \div 2 = 6 \text{ and } \frac{6 \times 16 \times 62{\cdot}5 \times 60}{33000} = 10{\cdot}9 \text{ horse power nearly.}$$

Given 16 cubic feet of water per second, to be applied to a high breast or an overshot wheel, with 2 feet head and 10 feet fall; required the power.

$$2 \div 2 = 1 \text{ and } \frac{\overline{1 + 10} \times 16 \times 62{\cdot}5 \times 60}{33000} = 20 \text{ horse power.}$$

Only about two-thirds of the above results can be taken as real communicative power to machinery.

OF THE CIRCLE OF GYRATION IN WATER WHEELS.

The centre or circle of gyration is that point in a revolving body into which, if the whole quantity of matter were collected, the same moving force would generate the same angular velocity, which renders it of the utmost importance in the erection of water wheels, and the motion ought always to be communicated from that point when it is possible.

Rule.—*To find the circle of gyration.*—Add into one sum twice the weight of the shrouding, buckets, &c., multiplied by the square of the radius, ⅔ of the weight of the arms, multiplied by the square of the radius, and the weight of the water multiplied by the square of the radius also; divide the sum by twice the weight of the shrouding, arms, &c., added to the weight of the water, and the square root of the quotient is the distance of the circle of gyration from the centre of suspension nearly.

Required the distance of the centre of gyration from the centre of suspension in a water wheel 22 feet diameter, shrouding, buckets, &c. = 18 tons, arms = 12 tons, and water = 10 tons.

$$\begin{array}{rl} 22 \div 2 = 11 \text{ and } 11^2 = & 121 \\ \text{Then, } 18 \times 2 = 36 \times 121 = & 4356 \\ \tfrac{2}{3} \text{ of } 12 = 8 \times 121 = & 968 \\ \text{water} = 10 \times 121 = & 1210 \\ \hline & 6534 \end{array}$$

And $\overline{18 + 12} \times 2 = 60 + 10 = 70$; hence,

$$\sqrt{\frac{6534}{70}} = 9\cdot6 \text{ feet from the centre of suspension nearly.}$$

TABLE *of Angles for Windmill Sails.*

Number.	Angle with the Plane of Motion.	
1	18°	24°
2	19	21
3	18	18
4	16	14
5	12½	9
6	7	3 extremity.

The radius is supposed to be divided into six equal parts, and $\frac{1}{6}$ from the centre is called 1, the extremity being denoted by 6.

The first column contains the angles according to an old custom; but experience has taught us that the angles in the second column are preferable.

THE VELOCITY OF THRESHING MACHINES, MILLSTONES, BORING IRON, ETC.

The drum or beaters of a threshing machine ought to move with a velocity of about 3000 feet per minute; hence, divide 11460 by the diameter of the drum in inches; or 955 by the diameter of the drum in feet; and the quotient is the number of revolutions required per minute. And the feeding rollers must make half the revolutions of the drum, when their diameters are about 3½ inches.

If the machine is driven by horses, their velocity ought to be from 2½ to 3 times round a 24 feet ring per minute.

Divide 500 by the diameter of a millstone, in feet, or 6000 by the diameter in inches, and the quotient is the number of revolutions required per minute.

In boring cast iron the cutters ought to have a velocity of about 108 inches per minute, or divide 36 by the diameter in inches, the quotient is the number of revolutions of the boring head per minute. And divide 100 by the diameter in inches, the quotient is the number of revolutions per minute, for turning wrought iron in general, and about half that velocity for cast iron.

OF PUMPS AND PUMPING ENGINES.

PUMPS are chiefly designated by the names of lifting and force pumps; lifting pumps are applied to wells, &c., where the height of the bucket, from the surface of the water, must not exceed 33 feet; this being nearly equal to the pressure of the atmosphere, or the height to which water would be forced up into a vacuum by the pressure of the atmosphere. Force pumps are applicable on all other occasions, as raising water to any required height, supplying boilers against the force of the steam, hydrostatic presses, &c.

The power required to raise water to any height is as the weight and velocity of the water with an addition of about $\frac{1}{5}$ of the whole power for friction; hence the

RULE.—Multiply the perpendicular height of the water, in feet, by the velocity, also in feet, and by the square of the pump's diameter in inches, and again by ·341; (this being the weight of a column of water 1 inch diameter, and 12 inches high, in lbs. avoirdupois;) divide the product by 33,000, and $\frac{1}{5}$ of the quotient added to the whole quotient will be the number of horse power required.

Required the power necessary to overcome the resistance and friction of a column of water 4 inches diameter, 60 feet high, and flowing with a velocity of 130 feet per minute.

$$\frac{60 \times 130 \times 4^2 \times \cdot341}{33000} = \frac{1\cdot3}{5} = \cdot26 + 1\cdot3 = 156 \text{ horse power nearly.}$$

Hot liquor pumps, or pumps to be employed in raising any fluid where steam is generated, require to be placed in the fluid, or as low as the bottom of it, on account of the steam filling the pipes, and acting as a counterpoise to the atmosphere; and the diameter of the pipes to and from a pump ought not to be less than $\frac{2}{3}$ of the pump's diameter.

RULE.—*The diameter of a pump and velocity of the water given, to find the quantity discharged in gallons, or cubic feet, in any given time.*—Multiply the velocity of the water, in feet per minute, by the square of the pump's diameter in inches, and by ·041 for gallons, or ·005454 for cubic feet, and the product will be the number of gallons, or cubic feet, discharged in the given time nearly.

What is the number of gallons of water discharged per hour by a pump 4 inches diameter, the water flowing at the rate of 130 feet per minute?

$$130 \times 60 = 7800 \text{ feet per hour.}$$
$$\text{And, } 7800 \times 4^2 \times \cdot041 = 5116\cdot8 \text{ gallons.}$$

RULE 1.—*The length of stroke and number of strokes given, to find the diameter of a pump, and number of horse power that will discharge a given quantity of water in a given time.*—Multiply the number of cubic feet by 2201, and divide the product by the velocity of the water, in inches, and the square root of the quotient will be the pump's diameter, in inches.

2. Multiply the number of cubic feet by 62·5, and by the perpendicular height of the water in feet, divide the product by 33,000, then will $\frac{1}{5}$ of the quotient, added to the whole quotient, be the number of horse power required.

Required the diameter of a pump, and number of horse power, capable of filling a cistern 20 feet long, 12 feet wide, and $6\frac{1}{2}$ feet deep, in 45 minutes, whose perpendicular height is 53 feet; the pump to have an effective stroke of 26 inches, and make 30 strokes per minute.

$$20 \times 12 \times 6{\cdot}5 = 1560 \text{ cubic feet, and}$$

$$\frac{1560}{45} = 34{\cdot}66 \text{ cubic feet per minute.}$$

$$\text{Then, } \frac{34{\cdot}66 \times 2201}{\sqrt{26 \times 30}} = 9{\cdot}89 \text{ inches diameter of pump.}$$

$$\text{And } \frac{34{\cdot}66 \times 62{\cdot}5 \times 53}{33000} = \frac{3{\cdot}48}{5} = {\cdot}69 + 3{\cdot}48 = 4{\cdot}17 \text{ horse}$$

power.

RULE.—*To find the time a cistern will take in filling, when a known quantity of water is going in, and a known portion of that water is going out, in a given time.*—Divide the content of the cistern, in gallons, by the difference of the quantity going in, and the quantity going out, and the quotient is the time in hours and parts that the cistern will take in filling.

If 30 gallons per hour run in and $22\frac{1}{2}$ gallons per hour run out of a cistern capable of containing 200 gallons, in what time will the cistern be filled?

$30 - 22{\cdot}5 = 7{\cdot}5$, and $200 \div 7{\cdot}5 = 26{\cdot}666$, or 26 hours and 40 minutes.

To find the time a vessel will take in emptying itself of water.—Mr. O'Neill ascertained, from very accurate experiments, that a vessel, 3·166 feet long and 2·705 inches diameter, would empty itself in 3 minutes and 16 seconds, through an orifice in the bottom, whose area is ·0141 inches; and another 6·458 feet long, the diameter and orifice, as before, would do the same in 4 minutes and 40 seconds; hence, from these experiments, a rule is obtained, namely,

Multiply the square root of the depth in feet by the area of the falling surface in inches, divide the product by the area of the orifice, multiplied by 3·7, and the quotient is the time required in seconds, nearly.

How long will it require to empty a vessel of water, 9 feet high, and 20 inches diameter, through a hole $\frac{3}{4}$ inch in diameter?

$\sqrt{9} = 3$, the square root of the depth,
314·16 inches, area of the falling surface,
·4417 inches, area of the orifice;

Then, $\frac{314 \cdot 16 \times 3}{\cdot 4417 \times 3 \cdot 7} = 576 \cdot 7$ seconds, or 9 minutes and 36 seconds.

On the pressure of fluids.—The side of any vessel containing a fluid sustains a pressure equal to the area of the side, multiplied by half the depth; thus,

Suppose each side of a vessel to be 12 feet long and 5 feet deep, when filled with water, what pressure is upon each side?

12 × 5 = 60 feet, the area of the side,
2·5 feet = half the depth, and
62·5 lbs. = the weight of a cubic foot of water.

Then, 60 × 2·5 × 62·5 = 9375 lbs.

RULE.—*To find the weight that a given power can raise by a hydrostatic press.*—Multiply the square of the diameter of the ram in inches by the power applied in lbs., and by the effective leverage of the pump-handle; divide the product by the square of the pump's diameter, also in inches, and the quotient is the weight that the power is equal to.

What weight will a power of 50 lbs. raise by means of a hydrostatic press, whose ram is 7 inches diameter, pump $\frac{7}{8}$, and the effective leverage of the pump-handle being as 6 to 1?

$$\frac{7^2 \times 50 \times 6}{\cdot 875^2} = 19200 \text{ lbs., or 8 tons 11 cwt.}$$

In the following rules for pumping engines the boiler is supposed to be loaded with about $2\frac{1}{2}$ lbs. per square inch, and the barometer attached to the condenser indicating 26 inches on an average, or 13 lbs., = $15\frac{1}{2}$ lbs., from which deduct $\frac{1}{3}$ for friction, leaves a pressure of 10 lbs. nearly upon each square inch of the piston.

RULE.—*To find the diameter of a cylinder to work a pump of a given diameter for a given depth.*—Multiply the square of the pump's diameter in inches by $\frac{1}{3}$ of the depth of the pit in fathoms, and the square root of the product will be the cylinder's diameter in inches.

Required the diameter of a cylinder to work a pump 12 inches diameter and 27 fathoms deep.

$$\sqrt{(12^2 \times 9)} = 36 \text{ inches diameter.}$$

RULE.—*To find the diameter of a pump, that a cylinder of a given diameter can work at a given depth.*—Divide three times the square of the cylinder's diameter in inches by the depth of the pit in fathoms, and the square root of the quotient will be the pump's diameter in inches.

What diameter of a pump will a 36-inch cylinder be capable of working 27 fathoms deep?

$$\sqrt{\frac{36^2 \times 3}{27}} = 12 \text{ inches diameter.}$$

RULE.—*To find the depth from which a pump of a given diameter will work by means of a cylinder of a given diameter.*—Divide three times the square of the cylinder's diameter in inches by the square of the pump's diameter also in inches, and the quotient will be the depth of the pit in fathoms.

Required the depth that a cylinder of 36 inches diameter will work a pump of 12 inches diameter.

$$\sqrt{\frac{36^2 \times 3}{144}} = 27 \text{ fathoms.}$$

An inelastic body of 30 lbs. weight, moves with a 3 feet velocity, and is struck by another inelastic body having a 7 feet velocity, the two will then proceed, after the blow, with the velocity

$$v = \frac{50 \times 7 + 30 \times 3}{50 + 30} = \frac{350 + 90}{80} = \frac{44}{8} = \frac{11}{2} = 5\tfrac{1}{2} \text{ feet.}$$

To cause a body of 120 lbs. weight to pass from a velocity $c_2 = 1\frac{1}{2}$ feet into a 2 feet velocity v, it is struck by a heavy body of 50 lbs., what velocity will the body acquire? Here

$$c_1 = v + \frac{(v - c_2)\, M_2}{M_1} = 2 + \frac{(2 - 1\cdot5) \times 120}{50} = 2 + \frac{6}{5} = 3\cdot2$$

feet.

Two perfectly elastic spheres, the one of 10 lbs. the other of 16 lbs. weight, impinge with the velocities 12 and 6 feet against each other, what will be their velocities after impact? Here $M_1 = 10$ and $c_1 = 12$ feet, but $M_2 = 16$ and $c_2 = -6$ feet, hence the loss of velocity of the first body will be

$$c_1 - v_1 = \frac{2 \times 16\,(12 + 6)}{10 + 16} = \frac{2 \times 16 \times 18}{26} = 22\cdot154 \text{ feet; and}$$

the gain in velocity of the other, $v_2 - c_2 = \frac{2 \times 10 \times 18}{26} = 13\cdot846$ feet. From this the first body after impact will recoil with the velocity $v_1 = 12 - 22\cdot154 = -10\cdot154$ feet; and the other with that of $-6 + 13\cdot846 = 7{,}846$ feet. Moreover, the measure of *vis viva* of the two bodies after impact $= M_1 v_1^2 + M_2 v_2^2 = 10 \times 10\cdot154^2 + 16 \times 7\cdot846^2 = 1031 + 985 = 2016$, as likewise of that before impact, namely: $M_1 c_1^2 + M_2 c_2^2 = 10 \times 12^2 + 16 \times 6^2 = 1440 + 576 = 2016$. Were these bodies inelastic, the first would only lose in velocity $\frac{c_1 - v_1}{2} = 11\cdot077$ feet, and the other gain $\frac{v_2 - c_2}{2} = 6\cdot923$ feet; the first would still retain, after impact, the velocity $12 - 11\cdot077 = 0\cdot923$ feet, and the second take up the velocity $-6 + 6\cdot923 = 0\cdot923$, and the loss of mechanical effect would be $(2016 - (10 + 16)\, 0\cdot923^2) \div 2g = (2016 - 2\cdot22) \times 0\cdot0155 = 29\cdot35$ ft. lbs.

CENTRIPETAL AND CENTRIFUGAL FORCE.

1. What is the centrifugal force of a body weighing 20 lbs. that describes a circle of 10 feet radius 200 times in a minute?

$\cdot000331 \times 200^2 \times 20 \times 10 = 2648$ lbs., the centrifugal force. ·00331 is a constant number.

It is a well established fact that the centrifugal force is to the weight of the body as double the height due to the velocity is to the radius of revolution. Hence, this question may be thus solved:

$20 \times 3\cdot1416 = 62\cdot832$, the circumference of the circle of 10 feet radius.

$62\cdot832 \times 200 = 12566\cdot4$ feet, the space passed over by the weight in one minute.

$\frac{12566\cdot4}{60} = 209\cdot44$ feet, the space described in a second, which is called the velocity.

$\frac{(209\cdot44)^2}{64\cdot4} = 681\cdot136$ feet, the height due to the velocity.

If F be the centrifugal force—

$$F : 20 :: 1362\cdot272 : 10.$$

$\therefore F = \frac{1362\cdot272 \times 20}{10} = 2724\cdot544$ lbs. The former rule gives 2648 lbs.

2. What is the centrifugal force at the equator on a body weighing 300 lbs., supposing the radius of the earth = 21000000 feet, and the time of rotation = 86400″ = 24 hours?

$F = 1\cdot224 \times \frac{21000000 \times 300}{86400^2} = 1\cdot03298$ lbs., or one pound very nearly. 1·224 is a constant multiplier.

$3\cdot1416 \times 21000000 = 65973600$ feet, $\frac{1}{2}$ the circumference of the earth at the equator.

$\frac{2 \times 65973600}{86400} = 1527\cdot16$ feet, the velocity of the weight each second.

$\frac{(1527\cdot16)^2}{64\cdot4} = 36214\cdot56$, the height due to the velocity.

$$F : 300 :: 72429\cdot12 : 21000000.$$

$F = \frac{72429\cdot12 \times 300}{21000000} = 1\cdot0347$ nearly, as by the former approximate method.

3. If a body weighing 100 lbs. describe a circle of 10 feet radius 300 times a minute, what is the diameter of a cast iron cylindrical

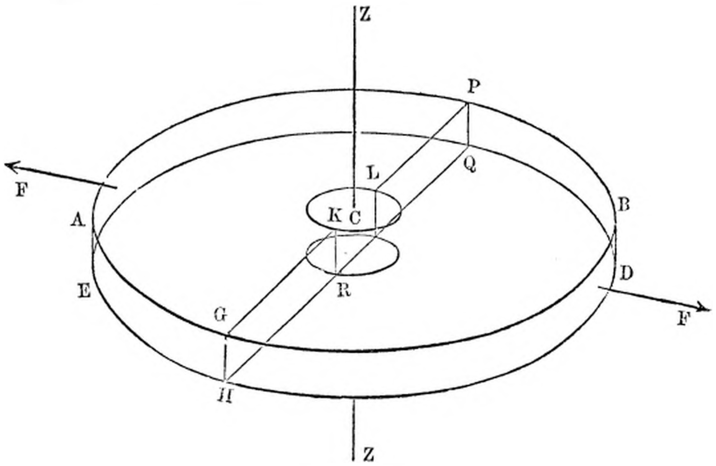
rod, connecting the body with the axis, that will safely support this weight? The centrifugal force will be,

$$\cdot 000331 \times 300^2 \times 100 \times 10 = 29790 \text{ lbs.}$$

From the strength of materials, page 281, we find that the ultimate cohesive strength for each circular inch of cross sectional area is 14652 lbs.; but one-third of this weight, or 4884 lbs., can only be applied with safety.

$$\therefore \sqrt{\frac{29790}{4884}} = 2\cdot 46982 \text{ inches, the diameter of the cylindrical rod.}$$

4. The dimensions, the density, and strength of a millstone ABDE are given; it is required to find the angular velocity v, in consequence of which rupture will take place on account of the centrifugal force.



If we put the radius of the millstone $= r_1 = 24$ inches $=$ CG; the radius $=$ CK of its eye $= r_2 = 4$ inches; the height PQ $=$ GH $= l = 12$ inches; the density $= t = 2500 =$ specific gravity of the millstone; and the modulus of strength $=$ K $=$ 750 lbs. $=$ the ultimate cohesive strength of each square inch of cross sectional area in the section PH, supposing the centrifugal forces $-$ F and $+$ F to cause the separation in this section.

$$(r_1 - r_2)\, l = \text{area of parallelogram GR.}$$

Hence, the force in lbs. required to cause rupture will be,

$2\,(r_1 - r_2)\, l \times \mathrm{K}$; the weight of the stone $\mathrm{G} = \pi\,(r_1^2 - r_2^2)\, l\gamma$, and the radius of gyration of each half of the stone, i. e. the distance of its centre of gravity from the axis of rotation $r = \frac{4}{3\pi} \times \frac{r_1^3 - r_2^3}{r_1^2 - r_1^2}$. At the moment of rupture, the centrifugal force of half the stone is equivalent to the strength; we hence obtain the equation of condition $\omega \times \frac{1}{2} \frac{G r}{g} = 2 (r_1 - r_2) l K$, i. e. $\omega^2 \times \frac{2}{3} (r_1^3 - r_2^3) \frac{l \gamma}{g} = 2 (r_1 - r_2) l K$; or leaving out $2 l$ on both sides, it follows that

$$\omega = \sqrt{\frac{3 g (r_1 - r_2) K}{(r_1^3 - r_2^3) \gamma}} = \sqrt{\frac{3 g K}{(r_1^2 + r_1 r_2 + r_2^2) \gamma}}.$$

If $r_1 = 2$ feet $= 24$ inches, $r_2 = 4$ inches, $K = 750$ lbs., and the specific gravity of the millstone $= 2·5$; therefore the weight of a cubic inch of its mass $= \frac{62·5 \times 2·5}{1728} = 0·0903$ lbs.; it follows that the angular velocity at the moment of rupture is,

$$\omega = \sqrt{\frac{3 \times 12 \times 32·2 \times 750}{688 \times 0·9903}} = \sqrt{\frac{869400}{62·1264}} = 112·1 \text{ inches.}$$

If the number of rotations per minute $= n$, we have then $\omega = \frac{2 \pi n}{60}$; hence, inversely, $n = \frac{30 \omega}{\pi}$, but here $= \frac{30 \times 112·1}{\pi} = 1070$. The average number of rotations of such a millstone is only 120, therefore 9 times less.

With what velocity must a body of 8 lbs. impinge against another at rest of 25 lbs., in order that the last may have a velocity of 2 feet? Were the bodies inelastic, we should then have to put: $v = \frac{M_1 c_1}{M_1 + M_2}$, i. e. $2 = \frac{8 \times c_1}{8 + 25}$, hence $c_1 = \frac{33}{4} = 8\frac{1}{4}$ feet, the required velocity; but were they elastic, we should have $v_2 = \frac{2 M_1 c_1}{M_1 + M_2}$; hence, $c_1 = \frac{33}{8} = 4\frac{1}{8}$ feet.

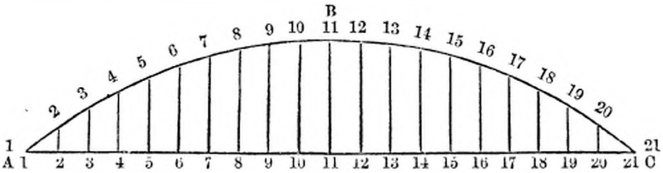
If in a machine, 16 blows per minute take place between two inelastic bodies $M_1 = \frac{1000}{g}$ lbs. and $M_2 = \frac{1200}{g}$ lbs., with the velocities $c_1 = 5$ feet, and $c_2 = 2$ feet, then the loss in mechanical effect from these blows will be: $L = \frac{16}{60} \times \frac{(5 - 2)^2}{2g} \times \frac{1000·1200}{2200} = \frac{4}{15} \times 9 \times \frac{1}{64·4} \times \frac{6000}{11} = 0·576 \times \frac{400}{11} = 20·94$ units of work per second.

If two trains upon a railroad of 120000 lbs. and 160000 lbs. weight, come into collision with the velocities $c_1 = 20$, and $c_2 = 15$ feet, there will ensue a loss of mechanical effect expended upon the destruction of the locomotives and carriages, which in the case of perfect inelasticity of the impinging parts, will amount to $= \frac{(20 + 15)^2}{2g} \times \frac{120000 \times 160000}{280000} = 35^2 \times \frac{1}{64·4} \times \frac{1920000}{28} =$ 1344000 ft. lbs., or units of work.

SHIP-BUILDING AND NAVAL ARCHITECTURE.

Two rules, by which the principal calculations in the art of ship-building are made, may be employed to measure the area or superficial space enclosed by a curve, and a straight line taken as a base.

Rule I.—If the area bounded by the curve line ABC and the straight line AC is required to be estimated, by the rule, the base AC is divided into an even number of equal parts, to give an odd number of points of division.



Where the base AC is divided into twenty equal parts, giving twenty-one points of division, and the lines 1·1, 2·2, 3·3, &c., are drawn from these points at right angles or square to AC, to meet the curve ABC, these lines, 1·1, 2·2, 3·3, &c., are denominated ordinates, and the linear measurement of them, on a scale of parts, is taken and used in the following general expression of the rule.

$$\text{Area} = \{A + 4\,P + 2\,Q\}\,\frac{r}{3}.$$

Where A = sum of the first and last ordinates, or 1·1 and 21·21.

4 P = sum of the even ordinates multiplied by 4.

Or, {2d + 4th + 6th + 8th + 10th + 12th + 14th + 16th + 18th + 20th} × 4.

2 Q = sum of the remaining ordinates; or,

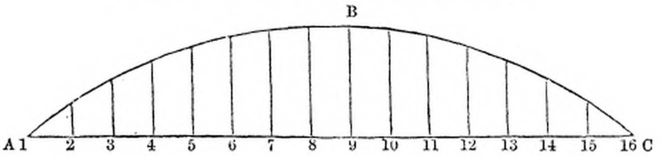
{3d + 5th + 7th + 9th + 11th + 13th + 15th + 17th + 19th} × 2.

And r is equal to the linear measurement of the common interval between the ordinates, or one of the equal divisions of the base AC. This rule, for determining the area contained under the curve and the base, may be put under another form; for as the

$\text{Area} = \{A + 4\,P + 2\,Q\} \times \frac{r}{3}$; it may be transferred into

$$\text{Area} = \left\{\frac{A}{2} + 2\,P + Q\right\} \times \frac{2\,r}{3}.$$

The practical advantages to be derived from this modification of the general rule will appear when the methods of calculation are further developed.



RULE II.—If the base AC be so divided that the equal intervals are in number a multiple of the numeral 3, then the total number of the points of division, and consequently the ordinates to the curve, will be a multiple of the numeral 3 with one added, and the area under the curve ABC, and the base AC, can be determined by the following general expression:

$$\text{Area} = \{A + 2\,P + 3\,Q\} \times \frac{3r}{8}.$$

Where A = sum of the first and last ordinates, or 1 and 16.

2 P = sum of the 4th, 7th, 10th, 13th, multiplied by 2, or ordinates bearing the distinction of being in position as multiples of the numeral 3, with one added.

3 Q, the sum of the remaining ordinates, multiplied by 3, or of the 2d, 3d, 5th, 6th, 7th, 8th, 9th, 11th, 12th, 14th, and 15th, multiplied by 3.

The number of equal divisions for this rule must be either 3, 6, 9, 12, or 15, &c., being multiples of the numeral 3, whence the ordinates will be in number under such divisions, multiples of the numeral 3, with one added.

This rule admits also of a modification in form, to make it more convenient of application.

$$\text{For area} = \{A + 2\,P + 3\,Q\} \times \frac{3}{8}\,r.$$

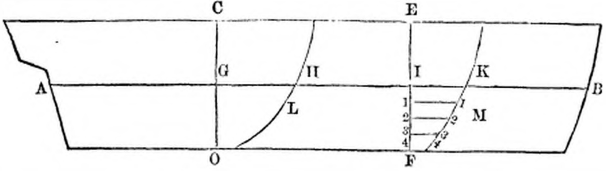
As before advanced for the change adopted in the general expression for the first rule, the utility of this modification of the second rule will be observable when the calculations on the immersed body are proceeded with.

The rules are formed under the supposition that in the first rule the curve ABC, which passes through the extremities of the ordinates, is a portion of a common parabola, while in the second rule the curve is assumed to be a cubic parabola; the results to be obtained from an indiscriminate use of either of these rules, differ from each other in so trifling a degree, (considered practically and not mathematically,) as not to sensibly affect the deductions derived by them.

William O'Neill, or, as English writers term him, William Neal, was the first to rectify a curve of any sort; this curve was the semi-cubical parabola; these rules, of such use in the art of shipbuilding, were first given by him, but as is usual, claimed by English pretenders.

The foregoing rules, when applied to the measurement of the immersed portion of a floating body, as the displacement of a ship, are used as follows.

The ship is considered as being divided longitudinally by equidistant athwartship or transverse vertical planes, the boundaries of which planes give the external form of the vessel at the respective stations, and therefore the comparative forms of any intermediate portion of it.



If the ship be immersed to the line AB, considered as the line of the proposed deepest immersion or lading, the curves HLO and KMF would give the external form of the ship at the positions G and I in that line; and the areas GHLO, IKMF contained under the curves HLO, KMF, the right lines GH, IK, (the half-breadths of the plane of proposed flotation AB at the points G and I,) and the right lines GO, IF, the immersed depths of the body at those points are the areas to be measured; and if the areas obtained be represented by linear measurements, and are set off on lines drawn at right angles to the line AB at their respective stations, a curve bounding the representative areas would be formed, and the measurement by the rules of the area contained under this curve, and the right line, AB, or length of the ship on the load-water line, would give the sum of the areas thus represented, and thence the solid contents of the immersed portion of the ship in cubic feet of space. In accordance with this application of those rules to measure the displacement of the ship, the usual practice is to divide the ship into equidistant vertical and longitudinal planes, the longitudinal planes being parallel to the load-water section or horizontal section formed by the proposed deepest immersion.

To measure the areas of these planes after they have been delineated by the draughtsman, the constructor divides the depth of each of the vertical sections, or the length of each horizontal section, into such a number of equal divisions as will make either one or the other of the rules 1 or 2 applicable. If the first rule be preferred, the equal divisions must be of an even number, so that there may be an odd number of ordinates; while the use of the second rule, to measure the area, will require the equal divisions of the base to be in number a multiple of the numeral 3, which will make the ordinates to be in number a multiple of the numeral 3, with one added. From the points of equal divisions in the respective sections thus determined, perpendicular ordinates are drawn to meet the curve, or the external form of the transverse planes of the body; and a table for the ordinates thus obtained, having been made, as shown page 467, the measures by scale of the respective ordinates are therein inserted.

For the area IKMF, the linear measurements of IK, 1·1, 2·2, 3·3, 4·4, are taken by a scale of parts, and inserted in the column marked 5, page 467, the whole length AB of the load-water line being divided into 10 equal divisions, and the area IKMF being supposed as the fifth from B, the fore extreme of the load-water line. To apply the first rule to the measurement of the area of No. 5 section, the ordinates are extracted from the table, page 467, and operated upon as directed by the rule; viz.

$$\text{Area} = \{A + 4\,P + 2\,Q\} \times \frac{r}{3}.$$

IK, or first,	1·1, or 2d,	2·2 or 3d,
4·4, or last,	3·3, or 4th,	× 2.
————	added together	or 2 Q.
added together = A.	and × 4 = 4 P.	

$$\text{By rule, area} = \{A + 4\,P + 2\,Q\} \times \frac{r}{3}.$$

$$\text{Whence area} = \{(IK + 4{\cdot}4) + (1{\cdot}1 + 3{\cdot}3)\,4 + 2{\cdot}2 \times 2\} \times \frac{r}{3} =$$

area IKMF; and, in a similar manner, may the several areas of the other transverse sections be determined.

When these areas have all been thus measured, they are to be summed by the same rules; the areas themselves being considered as lines, and the result will give the solid for displacement in cubic feet. To shorten this tedious application of the formula, the arrangement of having double-columned tables of ordinates was introduced, as shown on page 484, and for the more ready use of this enlarged table, the modifications in the formula 467, before alluded to, were adopted, that of

$$\text{Area} = \left\{ A + 4\,P + 2\,Q \right\} \times \frac{r}{3} = \left\{ \frac{A}{2} + 2\,P + Q \right\} \times \frac{2r}{3},$$

and that of

$$\text{Area} = \left\{ A + 2P + 3\,Q \right\} \times \frac{3r}{8} = \left\{ \frac{A}{2} + P + 1{\cdot}5\,Q \right\} \times \tfrac{3}{4}\,r,$$

as rendering the required number of figures much less, whereby accuracy of calculation is insured and time is saved.

In using a table of ordinates constructed for this method of calculation, the linear measurement of the several ordinates of vertical section 5 and the corresponding ones of all the others would be inserted in the double columns prepared for them, in the following order:—

In the first and last lines of the enlarged table for the ordinates, distinguishable by $\frac{A}{2}$, in the left-hand column of each pair, the measurements of the first and last ordinates of the respective areas are placed, and in the right-hand column of each pair one-half of such measurements, as being one-half of the first and last ordinates of each vertical section or area. In the lines distinguished by 2 P, in the left-hand column, the measurements of the even ordinates of each respective area are placed, which having been multiplied by two, the result is placed in the respective right-hand columns prepared for each vertical section; while in those lines of the table distinguished by Q, the measurements of the ordinates themselves are placed in the right-hand columns, as not requiring by the modification of the rules any operation to be used on them, before being taken into the sum forming the sub-multiple of the respective areas.

It may here with propriety be suggested, that in practice the insertion of the linear measurements of the ordinates in the table in red ink will be found useful, and that after such has been done, by the upper line of figures in the table of ordinates thus arranged, being divided by two, the second line of figures being multipled by two, and so on with the others as shown by the table, and the results thus obtained being inserted in their respective right-hand columns as before described, great facility and despatch of calculation are afforded to the constructor.

That this method will yield a correct measurement of the areas will be evident by an inspection of the terms of the general expression of area $= \left\{\frac{A}{2} + 2P + Q\right\} \times \frac{2r}{3}$, which are placed against the several lines of the table of ordinates. And it will be equally apparent, that the sum total of the figures inserted in the right-hand columns appropriated to each section is a sub-multiple of the area of each section, and that these results arising from the use of the form for area of $\left\{\frac{A}{2} + 2P + Q\right\}$ will be one-half of those that would be obtained by abstracting the ordinates from the table, page 467, and using them in the expression $A + 4P + 2Q$; and therefore to complete the calculation for the areas by the rule, the first results for the areas must be multiplied by $\frac{2r}{3}$, and the last by $\frac{r}{3}$, where r is equal to the common interval or equal division of the base in linear feet; or the part of the expression for areas of $\left\{\frac{A}{2} + 2P + Q\right\}$ must be multiplied by $\frac{2r}{3}$, to make it equivalent to $\{A + 4P + 2Q\} \times \frac{r}{3}$.

The sub-multiples of the areas of the vertical sections thus determined, require to be summed together for the solid of displacement, and by considering the sub-multiples of the areas to be, as before stated, represented by lines or proportionate ordinates, O'Neill's rules, by the same table of ordinates with an additional column, may be made available to the development of the solid of displacement. For the sectional areas being represented by lines, by the first rule, one-half the first and last areas, added to the sum of the products arising from multiplying the even ordinates or representative areas by two, together with the odd ordinates or the areas as given by the tables, and these being placed in the additional column of the table prepared for them, the sub-multiple of the solid of displacement will be given.

The operation will stand thus: Sub-multiple of each of the areas $= \left\{ \frac{A}{2} + 2P + Q \right\}$, or each area will be $\frac{2r}{3}$ less than the full result, and the representative lines for the areas will be diminished in that proportion; and having used these sub-multiples of the areas thus diminished in the second operation for obtaining the sub-multiple of the solid of displacement under the same rule, the results will again be $\frac{2r'}{3}$ less than the true result; therefore the sum thus determined will have to be multiplied by the quantity $\frac{2r}{3} \times \frac{2r'}{3}$, to give the solid required. In this expression, of $\frac{2r}{3} \times \frac{2r'}{3}$, $r =$ the equal distances taken in the vertical planes to obtain the respective vertical areas; $r' =$ the equal distances at which the vertical areas are apart on the longitudinal plane of the ship.

The displacement being thus determined, by an arrangement of an enlarged table of ordinates, the functions arising from the sub-multiples of the areas of the vertical sections being placed in O'Neill's rules to ascertain the displacement, may be used in the table of ordinates to find the distance of the centre of gravity of the immersed body from any assumed vertical plane; and also the distance that the same point—"the centre of gravity of displacement" —is in depth from the load-water or line of deepest immersion, and that from the considerations which follow:—

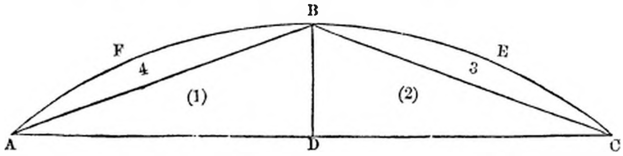
In a system of bodies, the centre of gravity of it is found by multiplying the magnitude or density of each body by its respective distance from the beginning of the system, and dividing the sum of such products by the sum of the magnitudes or densities. The displacement of a ship may be considered as made up of a succession of vertical immersed areas; and if it be assumed that the moments arising from multiplying the area of each section by its relative distance from an initial plane may be represented by successive lineal measurements, the general rules will furnish the summation of such moments; and the displacement or sum of the areas has been obtained by a similar process, from whence, by the rule for finding the centre of gravity of a system as before given, the distance of the common centre of gravity from the assumed initial plane would be ascertained, by dividing the sum of the moments of the areas by the sum of the areas or the solid of displacement.

To extend this reasoning to the enlarged table of ordinates used for the second method of calculation: The sub-multiples of the respective areas, when put into the formulas to obtain the proportionate solid of displacement, are relatively changed in value to give that solid, and consequently the moments of such functions of the vertical areas will be to each other in the same ratio; and the sum of these proportionate moments, if considered as lines, can be ascertained by multiplying the functions of the areas by their relative distances from the assumed initial plane, or by the number of the equal intervals of division they are respectively from it, and afterwards, by the rules, summing these results, forming the sum of the moments of the sub-multiples of the functions of the vertical areas: and the proportionate sub-multiple for the displacement is shown on the table; the division therefore of the former, or the sum of the proportional moments of the functions of the areas, by the proportionate sub-multiple for the displacement, will give the distance (in intervals of equal division) that the centre of gravity of the displacement is from the initial plane, which being multiplied by the value in feet of the equal intervals between the areas, will give the distance in feet from the assumed initial plane, or from the extremity of the base line of the proportional sectional areas for displacement. This reasoning will apply equally to finding the position of the centre of gravity of the body immersed, both as respects length and depth, and on the enlarged tables for construction given, (pages 484 and 485,) the constructor, by adopting this arrangement, will at once have under his observation the calculations *on*, and the results *of*, the most important elements of a naval construction.

The foregoing tabular system, for the application of O'Neill's rules to the calculations required on the immersed volume of a ship's bottom, led to a lineal delineation of the numerical results of the tables, and thence the development of a curve of sectional areas, on a base equivalent to the length of the immersed portion of the body, or of the length at the load-water line. To effect this, the sub-multiples of the sectional areas, taken from the tables for calculation, are severally divided by such a constant number as to make their delineation convenient; then these thus further reduced sub-multiples of the areas, being set off at their respective positions on the base, formed by the length of the load-water line, a curve passed through the extreme points of these measurements, will bound an area, that to the depth used for the common divisor would form a zone, representative of the solid of displacement. The accuracy of such a representation will be easily admitted, if the former reasoning is referred to.

To obtain the solid of displacement from this representative area, the load-water line or plane of deepest immersion is considered as being divided lengthwise into two equal parts, which assumption divides the base of the curve of sectional areas also into two equal portions: the line of representative area to that medial point is then drawn to the curve, and triangles are formed on each side of it by joining the point where it meets the curve with the extremities of the base line; this arrangement divides the representative area into four parts, two triangles which are equal, viz. 1 and 2, and two other areas which are contained under the hypothenuse of these triangles and the curves of sections, or 3 and 4 of the annexed diagram.

Diagram of a Curve of Sectional Areas.



ABCDA equal sectional area, representative of the half displacement as a zone of a given common depth.

AC equal the length of the load-water section from the fore-side of the rabbet of the stem to the aft-side of the rabbet of the post, and D the point of equal division.

BD, the representative area of half the immersed vertical section at the medial point D, joining B with the points A and C, will complete the division of the representative area ABCDA.

ABD and CBD, under such considerations, are equal triangles.

BECB, BFAB, areas, bounded respectively by the hypothenuse AB or BC of the triangles and the curve of sectional areas; and, supposing the curves AFB and BEC to be portions of common parabolas, the solid of displacement will be in the following terms:

The area of each of the triangles is equal to $\frac{1}{4}$ of AC × BD; hence the sum of the two $= \frac{1}{2}$ of AC × BD: the hypothenuse AB or BC $= \sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]}$, and the area of BECB if considered as approximating to a common parabola $= \sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]} \times \frac{2}{3}$ of the greatest perpendicular on the hypothenuse BC.

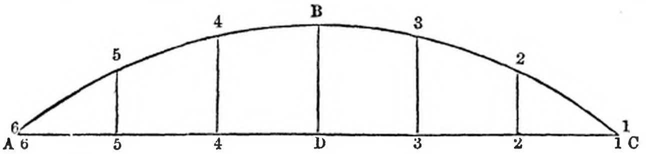
Area of BFAB under the same assumption $= \sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]} \times \frac{2}{3}$ of the greatest perpendicular on the hypothenuse AB; whence the whole displacement will be expressed by $\frac{1}{2}$ AC × BD × $\sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]} \times \frac{2}{3}$ of the greatest perpendicular on the hypothenuse B C + $\sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]} \times \frac{2}{3}$ of the greatest perpendicular on the hypothenuse AB.

By a similar method, from the light draught of water, or the depth of immersion on launching the ship, the light displacement, or the weight of the hull or fabric, may be delineated and estimated; and the representative curve for it being placed relatively on the same base as that used for the representative curve for the load displacement, the area contained between the curve bounding the representative area for the load displacement, and the curve bounding the representative area for the light displacement, will be a representative area of the sum of the weights to be received on board, and point out their position to bring the ship from the light line of flotation, or the line of immersion due to the weight of the hull when completed in every respect, to that of the deepest immersion, or the proposed load-water line of the constructor—a representation that would enable the constructor to apportion the weights to be placed on board to the upward pressure of the water, and thence approximate to the stowage that would insure the easiest movements of a ship in a sea.

By an inspection of the diagram of the curve of sectional areas, it will clearly be seen that the representative area for displacement under the division of it, into the triangles 1 and 2, and parabolic portions of the area 3 and 4, will point out the relative capacities of the displacement, under the fore and after half-lengths of the base or load-water line; for, by construction, the triangles ABD and CBD are equal, and therefore the comparative values of the areas BECB and BFAB, or of $\sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]} \times \frac{2}{3}$ of the greatest perpendicular on the hypothenuse BC, compared with $\sqrt{\left[\left(\frac{AC}{2}\right)^2 + BD^2\right]} \times \frac{2}{3}$ of the greatest perpendicular on the hypothenuse AB, or of the relative values of the greatest perpendiculars on the hypothenuses BC and AB, will give the relative capacities of the fore and after portions of the immersed body or the displacement.

The representative area ABCDA admits also of a measurement by the second rule.

Let BD, as before, be the representative area at the middle point.



Divide AD or DC into three equal portions, then the equal divisions being a multiple of 3, the second rule is applicable to measure the areas ABDA or BCDB; for the area ABDA =

$$\left\{6{,}6 + BD + 2 \times 0 + 3\ \{\overline{4{,}4 + 5{,}5}\}\right\}\frac{3r}{8};\ 6{,}6 = 0;$$

$$= \left\{BD + 3\ \{4{,}4 + 5{,}5\}\right\}\frac{DC}{8};\ \text{and area BCDB} =$$

$$\left\{\overline{1{,}1 + BD} + 2 \times 0 + 3 \times \{2{,}2 + 3{,}3\}\right\}\frac{3r}{8},\ \text{where } 1{,}1 = 0$$

$$\left\{BD + 3 \times \{\overline{2{,}2 + 3{,}3}\}\right\}\frac{AD}{8} = BCDB,$$ and the displacement $= \left\{BD + 3 \times \{4{,}4 + 5{,}5\}\right\}\frac{DC}{8} + \left\{BD + 3 \times \{2{,}2 + 3{,}3\}\right\} \times \frac{AD}{8}$ × by the constant divisor of the areas, or the depth of the zone in feet.

The rules given by O'Neill for the measurement of the immersed portion of the body of a ship, having been theoretically stated, the practical application of them will be given on the construction.

The immersed part of a ship, being a portion of the parallelopipedon formed by the three dimensions;—length on the load-water line, from the foreside of the rabbet of the stem to the aftside of the rabbet of the stern-post; extreme breadth in midships of the load-water section; and depth of immersion in midships from the lower edge of the rabbet of the keel;—it would seem that the first step towards the reduction of the parallelopipedon, or oblong, into the required form, would be to find what portion of it would be of the same contents as the proposed displacement of the ship—a knowledge of which would enable the constructor, by a comparison of the result with a similar element of an approved ship, to determine whether the principal dimensions assumed would (under the form intended) give an immersed body equal to carrying the proposed weights or lading.

The relative capacities of the immersed bodies contained under the fore and after lengths of the load-water line must next be fixed, and the constructor in this very important element of a construction will find little to guide him from the results of past experience and practice. From deductions on approved ships of rival constructors it will be developed, that in this essential element, "the relative difference between the two bodies," they vary from 1 to 13 per cent. on the whole displacement.

The relative capacities of the fore and after bodies of immersion under the proposed load-water line would seem at the first glance of the subject to be a fixed and determinate quantity, as being a conclusion easily arrived at from a knowledge of the proportions due to the superincumbent weights—under such a consideration, the weight of the anchors, bowsprit, and foremast would necessarily be supposed to require an excess in the body immersed under the fore half-length of the load-water line over that immersed under the after half-length of the same element.

In a ship, the necessary arrangement of the weights, to preserve the proposed relative immersion of the extremes or the intended draught of water, would be pointed out by a delineated curve of sectional areas, described as before directed; but a want of that system, or of some other, has often caused an error in the actual draught of water, and that under a great relative excess of the volumes of displacement in the fore and after portions of the immersed body.

The men-of-war brigs built to a construction-draught of water 12 ft. 9 in. forward, 14 ft. 4 in. abaft, giving 1 ft. 7 in. difference, had under such a construction a difference of displacement between the immersed bodies under the fore and after half-lengths of the load-water line that was equivalent to 10·4 tons for every 100 tons of the vessel's total displacement or weight; but these ships, when stowed and equipped for sea, came to the load-draught of water of 14 ft. 2 in. forward, 14 ft. 3 in. aft,—difference 1 inch, or an immersion of the fore extreme of 18 inches more than was intended by the constructor. The reason of this practical departure from the proposed line of flotation of the constructor was, that the internal space or hold of the ship necessarily followed the external form, giving a hold proportionate to the displacement contained under the several portions of the body; but an injudicious disposal of the stores (in placing the weights too far forward) made them more than equivalent to the upward pressure of the water at the respective portions of the proposed immersion of the body, and thence arose the error or excess in the fore immersion by giving a greater draught of water than was designed. The stowage of a ship's hold, under a reference to the representative area for the displacement, contained between the curves of sectional areas developed for the light and load displacements, would prevent similar errors under any extent to which the relative capacity of the two bodies might be carried. This relative capacity of the two bodies will affect the form of the vessel's extremes, giving a short or long bow, a clear or full run to the rudder; for the whole displacement *being a fixed quantity, if the portion of it under the fore half-length of the load-water line be increased*, it must be followed *by a proportionate diminution of the portion of the displacement under the after half-length of the load-water line, so that the total volume of the displacement may remain the same*, which arrangement will give a proportionately full bow and clean run, and *vice versâ*.

The curve of sectional areas under the foregoing considerations is also applicable to a comparison of the relative qualities of ships of the same rate, by showing at one view the distribution of the volume of displacement in each ship, under the draught of water which has been found on trial to give the greatest velocity; based on which, deductions may be made from the relative capacities of the bodies pointed out by the sectional curves, that will serve to guide the naval constructor in future constructions.

The curve of sectional areas is also available for forming a scale to measure the amount of displacement of a ship to any assumed or given draught of water. To effect this, on the sheer draught or longitudinal plan of the ship between the load-water line, or that of deepest immersion, and the line denoting the upper edge of the rabbet of the keel, draw intermediate lines parallel to the load-water line as denoting lines of intermediate immersion between the keel and load-water line; these lines may be placed equidistant from each other, but they are not necessarily required to be so. Find the curve of sectional areas, due to each immersion of the ship denoted by these lines, and measure the areas bounded respectively by these curves, in the manner as before directed for the load displacement: these results will give the magnitudes of the immersed portions of the body in cubic feet, which being divided by 35, the mean of the number of cubic feet of salt or fresh water that are equivalent to a ton in weight, will give their respective weights in tons.

Assume a line of scale for depth, or mean draught of water, the lower part of which is to be considered the underside of the false keel of the ship, and set off on this line, by means of a scale of parts, the depths of the immersions at the middle section of the longitudinal plan; draw lines (at the points thus obtained) perpendicular to this assumed line for depth or draught of water, and having determined a scale to denote the tons, set off on each line by this scale the tons ascertained by the curves of sectional areas to be due to the respective immersions of the body; then a curve passed through these points will be one on which the weights in tons due to the intermediate immersions of the body may be ascertained; or, the displacement of a ship to the mean of a given draught may be found by setting up the mean depth on the scale, showing the draught of water—transferring that depth to the curve for tonnage, and then carrying the point thus obtained on the curve for tonnage to the scale of tons, which will give the number of tons of displacement to that depth of immersion or draught of water.

Description of the several plans to be delineated by the draughtsman, previous to the commencement of the calculations.

Sheer Plan.—A projection of the form of the vessel on a longitudinal and vertical plane, assumed to pass through the middle of the ship, and on which the position of any point in her may be fixed with respect to height and length.

Half-breadth Plan.—The form of the vessel projected on to a longitudinal and horizontal plane, assumed to pass through the extreme length of ship, and on which the position of any point in the ship may be fixed for length and breadth.

Body Plan.—The forms of the vertical and athwartship sections of the ship, projected on to a vertical and athwartship plane, assumed to pass through the largest athwartship and vertical section of her, and on which plan the position of any point in the ship may be fixed for height and breadth.

These plans conjointly will determine every possible point required; for, by inspection, it will be found—

That the sheer and half-breadth plans have one dimension common to both, viz.:..........Length.
Half-breadth and body plane......................Breadth.
Sheer and body plane...............................Height.

For sheer plan gives length and height...... } of the same point.
Half-breadth plan gives length and breadth } of the same point.
Body plan gives breadth and height......... } of the same point.

Which dimensions form the co-ordinates for any point in the solid, and must determine the position of it.

The point C in the load-water section AB, has for its co-ordinates to fix its position,

The length, 1·5 of the half-breadth plan.
Height, 5·C of the sheer plan,
And the breadth, 1·C of the body plan of section.

And the same for any other point of the solid or of the ship.

In the sheer plan, AB represents the line of deepest immersion, *a a*, *b b*, *c c*, *d d*, lines drawn parallel to that line at a distance of ·9 feet, making with AB an odd number of ordinates for the use of the first general rule for the area, where area $= \{A + 4P + 2Q\} \times \frac{r}{3}$, and A = the sum of the first and last ordinates.

P = the sum of the even ordinates, as 2, 4.
Q = the sum of the odd ordinates, as 3, &c.

The line AB, or length of the load-water line, is bisected at C, and AC, CB are thence equal; C being the middle point of the load-water line, the spaces BC, AC are again divided into four equal divisions, giving five ordinates for each space, at a distance apart of 5·5 feet.

This arrangement will give the immersed body of the vessel, as being divided into two parts under an equal division of the load-water line, and an odd number of ordinates in each section of the body for the application of the first general rule given for finding the areas of the vertical sections and thence the displacement.

The half-breadth plan delineates the form of the body immersed for length and breadth, the line AB of the sheer plan being represented in the half-breadth plan by the line marked AB, and *a a*, *b b*, *c c*, *d d*, of the sheer plan by the lines similarly distinguished in the half-breadth plan.

The body plan gives the form of the body in the depth, the lines distinguished 5·5 in the sheer and half-breadth plans being in the body plan developed by the curve 5·5·5, giving the external form of the ship at the section 5·5; the same reasoning applies to the other divisions of the load-water line AB.

A pile of 400 lbs. weight is driven by the last round of 20 blows of a 500 lbs. heavy ram, falling from a height of 5 feet; 6 inches deeper, what resistance will the ground offer, or what load will the pile sustain without penetrating deeper?

Here $G = 400$, $G_1 = 700$ lbs., $H = 5$, and $s = \frac{0·5}{20} = 0·025$ feet, whereby it is supposed that the pile penetrates equally far for each blow.

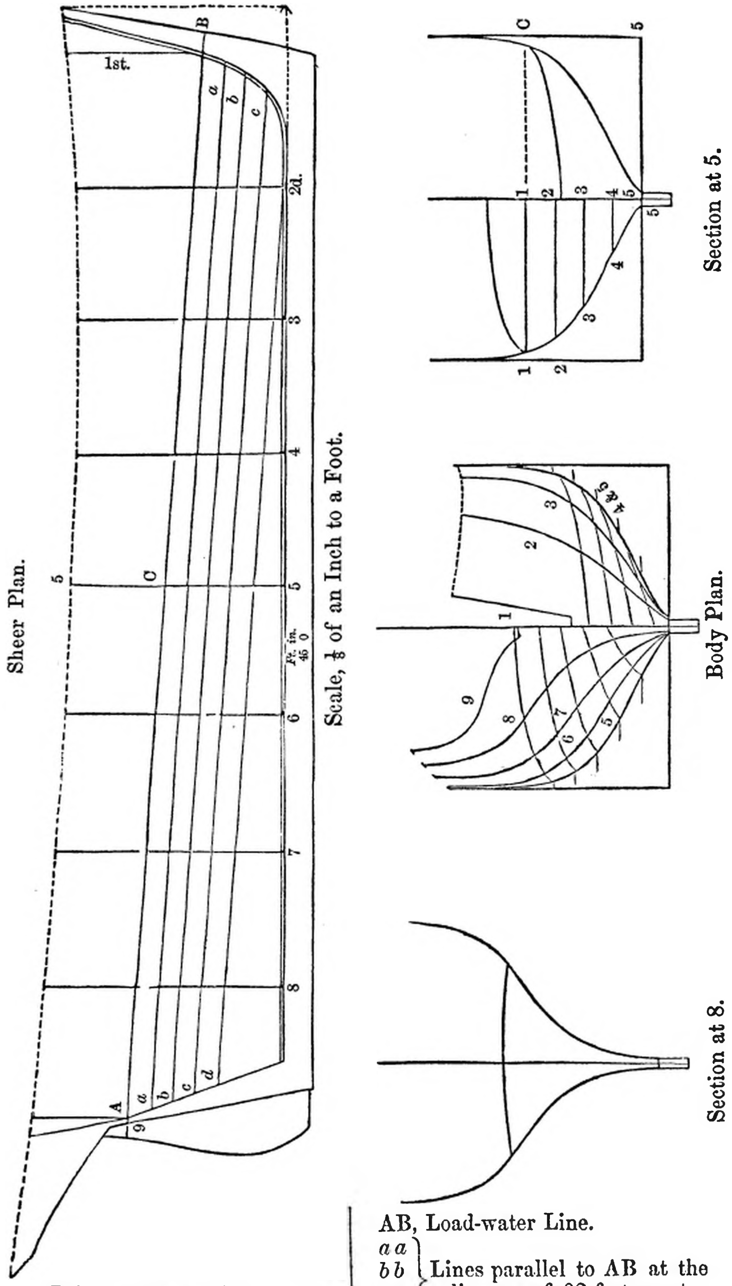
$$P = \left(\frac{700}{700 + 400}\right)^2 \frac{400 \times 5}{0·025} = \left(\frac{7}{11}\right)^2 \times 80000 = 32400 \text{ lbs.},$$

the ram, not during penetration, remaining upon the pile.

$$P = \frac{700^2 \times 5}{1100 \times 0·025} = \frac{4900}{11} \times 200 = 89100 \text{ lbs.},$$ the ram remaining upon the pile during penetration.

For duration, with security, such piles are only loaded from $\frac{1}{100}$ to $\frac{1}{10}$ of their strength.

CONSTRUCTION DRAUGHT OF A YACHT OF 36 TONS MEASUREMENT BY OLD RULE FOR TONNAGE.


Sheer Plan.
Scale, $\frac{1}{8}$ of an Inch to a Foot.
Ft. in. 45 0
1st.
2d.
A
B
C
a
b
c
d
3
4
5
6
7
8
9
Section at 5.
Body Plan.
Section at 8.

Principal Dimensions.

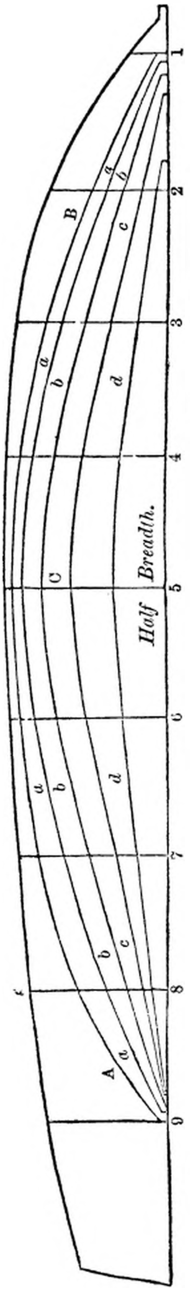
	Ft.	In.
Length for Tonnage..........	45	0
Keel for Tonnage.............	36	$10\frac{3}{4}$
Breadth for do.	13	6
Burthen in Tons...............	35	$\frac{72}{94}$

AB, Load-water Line.

a a, *b b*, *c c*, *d d* } Lines parallel to AB at the distance of ·92 feet apart.

Draught of Water.

	Ft.	In.
Afore.........................	4	6
Abaft.........................	7	6



Calculations required for the construction drawing of a yacht of 36 *tons.*—1*st. Usual mode of calculating the displacement by vertical and horizontal sections.*

Table *of Ordinates for Yacht of* 36 *Tons.*

Distinguishing No. of the sections.	1	2	3	4	(5)	6	7	8	9	
1′ A	·4	3·0	5·0	6·0	6·3	6·1	5·4	3·7	·4	r = the distance between the ordinates used for the vertical section = ·92 feet.
2′ P	·35	2·4	4·2	5·6	5·6	5·5	4·4	2·6	·35	
3′ Q	·3	1·7	3·2	4·4	5·0	4·6	3·4	1·7	·3	r' = the distance between the ordinates used for the horizontal sections = 5·5 feet.
4′ P	·25	1·0	2·2	3·2	3·8	3·4	2·4	1·1	·25	
5′ A	·2	·4	1·3	2·0	2·4	2·0	1·4	·6	·2	

From this Table the following application of O'Neill's rule, No. 1, is usually made to obtain the volume of displacement to the draught of water shown on the drawing as the load-water line, or line of proposed deepest immersion, designated by AB.

General terms of the rule:—

$$\text{Area} = \left\{ A + 4P + 2Q \right\} \times \frac{r}{3}.$$

To find $\frac{1}{2}$ the area of vertical section 1, fore body:—

$$\left.\begin{array}{l}\text{A = sum of}\\ \text{the first}\\ \text{and last}\end{array}\right\} \begin{array}{r}\cdot4\\ \\ \underline{\cdot2}\\ \cdot6 = \text{A}\end{array} \quad \left.\begin{array}{l}\text{4 P = four times the sum}\\ \text{of the even ordinates,}\\ \text{or of (2) and (4)......}\end{array}\right\} \begin{array}{r}\cdot35\\ \\ \underline{\cdot25}\\ \cdot60 = \text{P}\\ \underline{4\cdot}\\ 2\cdot4 = 4\text{ P}\end{array}$$

$$\left.\begin{array}{r}2\text{ Q} = \text{twice the sum of the odd}\\ \text{ordinates, or of (3)}\end{array}\right\} \times \begin{array}{r}\cdot3 = \text{Q}\\ \underline{2}\\ \cdot60 = 2\text{ Q}\end{array}$$

Whence the area, which is equal to

$$\left\{ A + 4P + 2Q \right\} \times \frac{r}{3} = \left\{ \cdot6 + 2\cdot4 + \cdot6 \right\} \times \frac{\cdot92}{3}.$$

$$3\cdot6 \times \frac{\cdot92}{3} = 1\cdot2 \times \cdot92 = 1\cdot104 = \tfrac{1}{2} \text{ area of}$$

section 1.

Which sum is half the area of the section 1, and is kept in that form of the half-measurement for the convenience of calculation.

Fore Body.

Vertical Section 2.

3·0	2·4	1·7
·4	1·0	2
3·4 = A	3·4 = P	3·4 = 2 Q

4
13·6 = 4 P
3·4 = A
3·4 = 2 Q
20·4 = A + 4 P + 2 Q
·92 = r
408
1836
3) 18·768
6·256 = $\frac{1}{2}$ area of Section 2.

Vertical Section 3.

5·0	4·2	3·2
1·3	2·2	2
6·3 = A	6·4 = P	6·4 = 2 Q

4
25·6 = 4 P
6·3 = A
6·4 = 2 Q
38·3 = A + 4 P + 2 Q
·92 = r
766
3447
3) 35·236
11·745 = $\overline{A + 4P + 2Q} \times \frac{r}{3} = \frac{1}{2}$ area of Section 3.

Vertical Section 4.

6·0	5·6	4·4
2·0	3·2	2
8·0 = A	8·8 = P	8·8 = 2 Q

4
35·2 = 4 P
8·0 = A
8·8 = 2 Q
52·0 = A + 4 P + 2 Q
·92 = r
1040
4680
3) 47·840
15·946 = $\overline{A + 4P + 2Q} \times \frac{r}{3} = \frac{1}{2}$ area of Section 4.

Vertical Section 5.

6·3
2·4
8·7 = A

5·6
3·8
9·4 = P
4
37·6 = 4 P
8·7 = A
10·0 = 2 Q
56·3 = A + 4 P + 2 Q
·92 = r
1126
5067
3) 51·796
$17{\cdot}265 = \overline{A + 4P + 2Q} \times \frac{r}{3} = \frac{1}{2}$ area of Section 5.

5·0
2
10·0 = 2 Q

Half areas of Vertical Sections 1, 2, 3, 4, and 5.

No.	feet
No. 1	1·104 feet.
2	6·256
3	11·745
4	15·946
5	17·265

Displacement of the body under the fore half-length of the load-water line by the vertical sections, or the summation of the vertical areas 1, 2, 3, 4, and 5, by the formula for the solid, as being equal to

$\left\{ A' + 4P' + 2Q' \right\} \times \frac{r'}{3}$ where A' = sum of 1st and 5th areas.
P' = " 2d and 4th areas.
Q' = " 3d area.

And r' = distance between the vertical sections, or 5·5 feet.

1... 1·104
5...17·265
18·369 = A′

2... 6·256
4...15·946
22·202 = P′
4
88·808 = 4 P′
18·369 = A′
23·490 = 2 Q′
130·667 = A′ + 4 P′ + 2 Q′
5·5 = r'
653335
653335
3) 718·6685
$239{\cdot}556 = \overline{A' + 4P' + 2Q'} \times \frac{r'}{3}$ = cubic ft. of space in ½ fore-body.
2
479·112 = cubic feet of space in fore-body.

3...11·745 = Q′
2
23·490 = 2 Q′

Displacement of the body immersed under the after half-length of the load-water line by the vertical areas 5, 6, 7, 8, and 9 of the Table of ordinates.

Vertical Section 6.

5, as fore body.	6·1	5·5	4·6 = Q
17·265	2·0	3·4	2
	8·1 = A	8·9 = P	9·2 = 2 Q

4
35·6 = 4 P
8·1 = A
9·2 = 2 Q
52·9 = A + 4 P + 2 Q
·92 = r
1058
4761
3) 48·668

$$16{\cdot}222 = \overline{A+4P+2Q}\times\frac{r}{3} = \left\{\begin{array}{l}\tfrac{1}{2}\text{ area of}\\ \text{Section 6.}\end{array}\right.$$

Vertical Section 7.

5·4	4·4	3·4 = Q
1·4	2·4	2
6·8 = A	6·8 = P	6·8 = 2 Q

4
27·2 = 4 P
6·8 = 2 Q
6·8 = A
40·8 = A + 4 P + 2 Q
·92 = r
816
3672
3) 37·536

$$12{\cdot}512 = \overline{A+4P+2Q}\times\frac{r}{3} = \left\{\begin{array}{l}\tfrac{1}{2}\text{ area of}\\ \text{Section 7.}\end{array}\right.$$

Vertical Section 8.

3·7	2·6	1·7 = Q
·6	1·1	·2
4·3 = A	3·7 = P	3·4 = 2 Q

4
14·8 = 4 P
4·3 = A
3·4 = 2 Q
22·5 = A + 4 P + 2 Q
·92 = r
450
2025
3) 20·700

$$6{\cdot}9 = \overline{A+4P+2Q}\times\frac{r}{3} = \left\{\begin{array}{l}\tfrac{1}{2}\text{ area of}\\ \text{Section 8.}\end{array}\right.$$

Vertical Section 9.

$$\begin{array}{lll} {\cdot}4 & {\cdot}35 & {\cdot}3 = Q \\ {\cdot}2 & {\cdot}25 & 2 \\ \hline {\cdot}6 = A & {\cdot}60 = P & {\cdot}6 = 2\,Q \end{array}$$

$$\begin{array}{rl} {\cdot}60 & = P \\ 4 & \\ \hline 2{\cdot}4 & = 4\,P \\ {\cdot}6 & = A \\ {\cdot}6 & = 2\,Q \\ \hline 3{\cdot}6 & = A + 4\,P + 2\,Q \\ {\cdot}92 & = r \\ \hline 72 & \\ 324 & \\ \hline 3\,)\,3{\cdot}312 & \\ \hline 1{\cdot}104 & = \overline{A + 4\,P + 2\,Q} \times \frac{r}{3} = \frac{1}{2} \text{ area of Section 9.} \end{array}$$

Half areas of the vertical sections 5, 6, 7, 8, and 9.

Sections.	Areas.
5	17·265
6	16·22
7	12·512
8	6·9
9	1·104

Displacement of the after-body under the after half-length of the load-water line by the vertical sections, or the summation of the immersed areas of the vertical sections 5, 6, 7, 8, and 9 by the formula for the solid as being equal to

$$\overline{A' + 4\,P' + 2\,Q'} \times \frac{r'}{3}$$

where A' = sum of the 5th and 9th areas.
P' = " 6th and 8th areas.
Q' = " 7th area.

and r' = the distance between the vertical sections, or 5·5 ft.

$$\begin{array}{lll} 5\ldots 17{\cdot}265 & 6\ldots 16{\cdot}22 & 7\ldots 12{\cdot}512 = Q' \\ 9\ldots\ 1{\cdot}104 & 8\ldots\ 6{\cdot}900 & 2 \\ \hline 18{\cdot}369 = A' & 23{\cdot}120 = P' & 25{\cdot}024 = 2\,Q' \end{array}$$

$$\begin{array}{rl} 23{\cdot}120 & = P' \\ 4 & \\ \hline 92{\cdot}480 & = 4\,P' \\ 25{\cdot}024 & = 2\,Q' \\ 18{\cdot}369 & = A' \\ \hline 135{\cdot}873 & = A' + 4\,P' + 2\,Q' \\ 5{\cdot}5 & = r \\ \hline 679{\cdot}365 & \\ 67{\cdot}936 & \\ \hline 3\,)\,747{\cdot}3015 & \\ \hline 249{\cdot}1005 & = \overline{A' + 4\,P' + 2\,Q'} \times \frac{r'}{3} = \text{cubic ft.} \\ 2 & \text{in } \frac{1}{2} \text{ after-body.} \\ \hline 498{\cdot}2010 & \text{After-body in cubic ft. of space.} \end{array}$$

Displacement of Fore-body by Horizontal Sections.

Horizontal Section 1′.

0·4
6·3
6·7 = A′

6·0
3·0
9·0 = P
4
36·00 = 4 P
10·00 = 2 Q
6·70 = A
52·70 = A + 4 P + 2 Q
5·5 = r
2635
2635
3) 289·85

$96{\cdot}61 = \overline{A + 4P + 2Q} \times \frac{r}{3} = \frac{1}{2}$ area of Section 1′.

5·0 = Q
2
10·0 = Q

Horizontal Section 2′.

·35
5·60
5·95 = A

5·7
2·4
8·1 = P
4
32·4 = 4 P
8·4 = 2 Q
5·95 = A
46·75 = A + 4 P + 2 Q
5·5 = r
23375
23375
3) 257·125

$85{\cdot}708 = \overline{A + 4P + 2Q} \times \frac{r}{3} = \frac{1}{2}$ area of Section 2′.

4·2 = Q
2
8·4 = 2 Q

Horizontal Section 3′.

·3
5·0
5·3 = A

4·4
1·7
6·1 = P
4
24·4 = 4 P
5·3 = A
6·4 = 2 Q
36·1 = A + 4 P + 2 Q
5·5 = r
1805
1805
3) 198·55

$66{\cdot}18 = \overline{A + 4P + 2Q} \times \frac{r}{3} = \frac{1}{2}$ area of Section 3′.

3·2 = Q
2
6·4 = 2 Q

Horizontal Section 4′.

·25
3·8
4·05 = A

3·2
1·0
4·2 = P
4
16·8 = 4 P
4·05 = A
4·40 = 2 Q
25·25 = A + 4 P + 2 Q
5·5 = r
12625
12625
3) 138·875

$$46{\cdot}291 = \overline{A + 4P + 2Q} \times \frac{r}{3} = \left\{ \begin{array}{l} \frac{1}{2} \text{ area of} \\ \text{Section } 4'. \end{array} \right.$$

2·2 = Q
2
4·4 = 2 Q

Horizontal Section 5′.

·2
2·4
2·6 = A

2·0
·4
2·4 = P
4
9·6 = 4 P
2·6 = A
2·6 = 2 Q
14·8 = A + 4 P + 2 Q
5·5 = r
740
740
3) 81·40

$$27{\cdot}13 = \overline{A + 4P + 2Q} \times \frac{r}{3} = \left\{ \begin{array}{l} \frac{1}{2} \text{ area of} \\ \text{Section } 5'. \end{array} \right.$$

1·3 = Q
2
2·6 = 2 Q

Displacement of the fore-body under the fore half-length of the load-water line by horizontal sections, or the summation of the horizontal sections of the fore-body 1′, 2′, 3′, 4′, and 5′, by the formula for the solid, as being equal to

$$\overline{A' + 4P' + 2Q'} \times \frac{r}{3};$$

where A' = sum of the 1′st and 5′th areas;
P' = " 2′d and 4′th areas;
Q' = " 3′d area;
and r = the distance between the horizontal sections, or ·92 feet.

Half areas of the Horizontal Sections 1′, 2′, 3′, 4′, and 5′.

1′ = 96·61.
2′ = 85·708.
3′ = 66·18.
4′ = 46·29.
5′ = 27·13.

Areas.	Areas.	Areas.
1′...96·61	2′...85·708	3′...66·18 = Q'
5′...27·13	4′...46·290	2
$123{\cdot}74 = A'$	$131{\cdot}998 = P'$	$132{\cdot}36 = 2\,Q'$
	4	

$527{\cdot}992 = 4\,P'$
$123{\cdot}740 = A'$
$132{\cdot}360 = 2\,Q'$

$784{\cdot}092 = A' + 4\,P' + 2\,Q'$
$\cdot 92 = r$

1568184
7056828

3) 721·36464

$240{\cdot}45 = \overline{A'+4P'+2Q'} \times \frac{r}{3} = \left\{ \begin{array}{l} \text{cubic ft. in} \\ \tfrac{1}{2}\text{fore-body.} \end{array} \right.$
2

480·90 = fore-body by horizontal sections in cubic feet of space.

Displacement, by horizontal sections of the body immersed under the after half-length of the load-water line, or by the horizontal areas 1′, 2′, 3′, 4′, and 5′, of the table of ordinates.

Calculated areas of 1′, 2,′ 3′, 4′, and 5′.

Section 1′ *After-body.*

6·3	6·1	5·4 = Q
·4	3·7	2
$6{\cdot}7 = A$	$9{\cdot}8 = P$	$10{\cdot}8 = 2\,Q$
	4	

$39{\cdot}2 = 4\,P$
$10{\cdot}8 = 2\,Q$
$6{\cdot}7 = A$

$56{\cdot}7 = A + 4\,P + 2\,Q$
$5{\cdot}5 = r$

2835
2835

3) 311·85

$103{\cdot}95 = \overline{A + 4\,P + 2\,Q} \times \frac{r'}{3} = \left\{ \begin{array}{l} \tfrac{1}{2}\text{ area of} \\ \text{Section } 1'. \end{array} \right.$

Section 2′ After-body.

$$\begin{array}{rlrlrl}
5{\cdot}6 & & 5{\cdot}5 & & 4{\cdot}4 & \\
{\cdot}35 & & 2{\cdot}6 & & 2 & \\
\cline{1-1}\cline{3-3}\cline{5-5}
5{\cdot}95 & = \mathrm{A} & 8{\cdot}1 & = \mathrm{P} & 8{\cdot}8 & = 2\,\mathrm{Q} \\
 & & 4 & & & \\
\cline{3-3}
 & & 32{\cdot}40 & = 4\,\mathrm{P} & & \\
 & & 5{\cdot}95 & = \mathrm{A} & & \\
 & & 8{\cdot}80 & = 2\,\mathrm{Q} & & \\
\cline{3-3}
 & & 47{\cdot}15 & = \mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q} & & \\
 & & 5{\cdot}5 & = r & & \\
\cline{3-3}
 & & 23575 & & & \\
 & & 23575 & & & \\
 & & 3\,)\,\overline{259{\cdot}325} & & & \\
\cline{3-3}
 & & 86{\cdot}441 & = \overline{\mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q}} \times \frac{r'}{3} = \tfrac{1}{2}\text{ area of Section }2'. & &
\end{array}$$

Section 3′ After-body.

$$\begin{array}{rlrlrl}
5{\cdot}0 & & 4{\cdot}6 & & 3{\cdot}4 & = \mathrm{Q} \\
{\cdot}3 & & 1{\cdot}7 & & 2 & \\
\cline{1-1}\cline{3-3}\cline{5-5}
5{\cdot}3 & = \mathrm{A} & 6{\cdot}3 & = \mathrm{P} & 6{\cdot}8 & = 2\,\mathrm{Q} \\
 & & 4 & & & \\
\cline{3-3}
 & & 25{\cdot}2 & = 4\,\mathrm{P} & & \\
 & & 5{\cdot}3 & = \mathrm{A} & & \\
 & & 6{\cdot}8 & = 2\,\mathrm{Q} & & \\
\cline{3-3}
 & & 37{\cdot}3 & = \mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q} & & \\
 & & 5{\cdot}5 & = r & & \\
\cline{3-3}
 & & 1865 & & & \\
 & & 1865 & & & \\
 & & 3\,)\,\overline{205{\cdot}15} & & & \\
\cline{3-3}
 & & 68{\cdot}38 & = \overline{\mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q}} \times \frac{r'}{3} = \tfrac{1}{2}\text{ area of Section }3'. & &
\end{array}$$

Section 4′ After-body.

$$\begin{array}{rlrlrl}
3{\cdot}8 & & 3{\cdot}4 & & 2{\cdot}4 & = \mathrm{Q} \\
{\cdot}25 & & 1{\cdot}1 & & 2 & \\
\cline{1-1}\cline{3-3}\cline{5-5}
4{\cdot}05 & = \mathrm{A} & 4{\cdot}5 & = \mathrm{P} & 4{\cdot}8 & = 2\,\mathrm{Q} \\
 & & 4 & & & \\
\cline{3-3}
 & & 18{\cdot}00 & = 4\,\mathrm{P} & & \\
 & & 4{\cdot}05 & = \mathrm{A} & & \\
 & & 4{\cdot}80 & = 2\,\mathrm{Q} & & \\
\cline{3-3}
 & & 26{\cdot}85 & = \mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q} & & \\
 & & 5{\cdot}5 & = r' & & \\
\cline{3-3}
 & & 13425 & & & \\
 & & 13425 & & & \\
 & & 3\,)\,\overline{147{\cdot}675} & & & \\
\cline{3-3}
 & & 49{\cdot}225 & = \overline{\mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q}} \times \frac{r'}{3} = \tfrac{1}{2}\text{ area of Section }4'. & &
\end{array}$$

Section 5′ After-body.

2·4
·2
$2{\cdot}6 = A$

2·0
·6
$2{\cdot}6 = P$
4
$10{\cdot}4 = 4P$
$2{\cdot}8 = 2Q$
$2{\cdot}6 = A$
$15{\cdot}8 = A + 4P + 2Q$
$5{\cdot}5 = r'$
790
790
3)86·90
$28{\cdot}96 = \overline{A + 4P + 2Q} \times \frac{r'}{3} = \frac{1}{2}$ area of Section 5′.

$1{\cdot}4 = Q$
2
$2{\cdot}8 = 2Q$

Displacement by horizontal sections of the after-body under the after half-length of the load-water line, or the summation of the horizontal sections of the after-body, 1′, 2′, 3′, 4′, and 5′, by the formula of the solid, as being equal to

$$\overline{A' + 4P' + 2Q'} \times \frac{r'}{3}.$$

Half areas of the After Horizontal Sections, 1′, 2′, 3′, 4′, and 5′.

Sections.	Areas.
1′	103·95.
2′	86·44.
3′	68·38.
4′	49·22.
5′	28·96.

Areas.
1′...103·95
5′... 28·96
$132{\cdot}91 = A'$

Areas.
2′...86·44
4′...49·22
$135{\cdot}66 = P'$
4
$542{\cdot}64 = 4P'$
$132{\cdot}91 = A'$
$136{\cdot}76 = 2Q'$
$812{\cdot}31 = A' + 4P' + 2Q'$
$\cdot 92 = r$
162462
731079
3)747·3252
$249{\cdot}1084 = \overline{A' + 4P' + 2Q'} \times \frac{r}{3}$ = cubic ft. of
2 $\frac{1}{2}$ after-body by horizontal sections.
498·2168 = After-body by horizontal sections in cubic feet of space.

Areas.
$3'\ldots 68{\cdot}38 = Q'$
2
$136{\cdot}76 = 2Q'$

DISPLACEMENT.

By Vertical Sections.	Cubic Feet.	*By Horizontal Sections.*	Cubic Feet.
Fore-body (p. 469)	479·11	Fore-body (p. 474)	480·900
After-body (p. 471)	498·20	After-body (p. 476)	498·216
Sum	977·30	Sum	979·116
Half	488·65	Half	489·558

	Cubic Feet.
By Horizontal Sections	979·116
By Vertical Sections	977·300
Difference	1·816 cubic feet.

Cubic Feet.
979·49 = capacity or displacement in cubic feet of space.

The mean weight of salt and fresh water gives 35 cubic feet of space, when filled with water, to be equivalent to a ton avoirdupois; thence the displacement in cubic feet of space being divided by 35 will give the weight of the volume displaced in tons avoirdupois; or 979·49 being divided by 35 gives

5) 979·49
7) 195·898
27·985 tons, the weight of the calculated immersed body in tons.

AREA OF THE MIDSHIP SECTION, OR OF THE GREATEST TRANSVERSE SECTION.

Section at 5.

1·1...6·3
5·5... ·2
6·5 = A

2·2...6·0
4·4...2·3
8·3 = P
4
33·2 = 4 P

3·3...4·8 = Q
2
9·6 = 2 Q

33·2 = 4 P
6·5 = A
9·6 = 2 Q
49·3 = A + 4 P + 2 Q
1·25 = $\frac{r}{3}$ where r = the depth, from 1 to 5, divided by 4 = 5 ft. by 4 = 1·25 ft.
2465
986
493
3) 61·625
20·541 = $\overline{A + 4P + 2Q} \times \frac{r}{3} = \left\{ \frac{1}{2} \right.$ area of midship section.
2
41·082 = Area of midship section without keel.

LOAD-WATER LINE.

Area of the load-water line, or area of the assumed deepest plane of immersion, delineated on the half-breadth plan, and marked by the curve AB. From the table of ordinates, p. 467, we have—

·4
·4
·8 = A

3·0
6·0
6·1
3·7
18·8 = P
4
75·2 = 4 P
·8 = A
33·4 = 2 Q
109·4 = A + 4 P + 2 Q
5·5 = r'
5470
5470
3)601·70

$$200{\cdot}56 = \overline{A + 4P + 2Q} \times \frac{r'}{3} = \left\{ \begin{array}{l} \tfrac{1}{2} \text{ area of load-} \\ \text{water line.} \end{array} \right.$$

5·0
6·3
5·4
16·7 = Q
2
33·4 = 2 Q

200·56 = ½ area of load-water section in superficial feet.
2
401·12 = area of load-water section, which amount of area being divided by 12, will give the number of cubic feet of space that would be contained in a zone of that area of an inch in depth, and that result being again divided by 35, as the number of cubic feet of water equivalent to a ton in weight, will give the number of tons that will immerse the vessel one inch at that line of immersion.

12)401·12 = area of load-water section in superficial feet.
5)33·42 = cubic feet in zone of one inch in depth.
7)6·684
·955 = tons to the inch of immersion at load-water line.

CENTRE OF GRAVITY OF THE DISPLACEMENT.

Estimated from Section 1, considered as the Initial Plane.

	Distinguishing No. of the Areas.	½ Vertical Areas.		Moments.
From p. 469.	1	1·104	× 0	000·000
	2	6·256	× 1	6·256
	3	11·745	× 2	23·490
	4	16·069	× 3	48·207
	5	17·265	× 4	69·060
From p. 471.	6	16·222	× 5	81·110
	7	12·512	× 6	75·072
	8	6·900	× 7	48·300
	9	1·104	× 8	8·832

Moments placed in the Rule.

$$\text{Sum} = \overline{A + 4\,P + 2\,Q} \times \frac{r'}{3}$$

000·000
8·832
8·832 = A

6·256
48·207
81·110
48·300
183·873 = P
4
735·492 = 4 P
8·832 = A
335·244 = 2 Q
1079·568 = A + 4 P + 2 Q
5·5 = r'
5397840
5397840
3) 5937·6240
1979·208 $= \overline{A + 4\,P + 2\,Q} \times \frac{r'}{3} =$

23·490
69·060
75·072
167·622 = Q
2
335·244 = 2 Q

sum of the moments of half the displacement from section 1, in intervals of space of 5·5 ft.; and the half displacement in cubic feet by vertical sections is 488·650 (p. 477) cubic ft.; whence it is found, by dividing the moment 1979·208 by 488·650, that the distance of the centre of gravity of displacement from the section 1 is as follows:—

488·65) 1979·208 (4·05 intervals from 1.
195460 interval = 5·5 ft.
246080
244325
1755 therefore 4·05 × 5·5 = 22·27 ft. = distance of the centre of gravity of the calculated immersed body from 1.

DEPTH OF THE CENTRE OF GRAVITY OF THE DISPLACEMENT BELOW THE LOAD-WATER SECTION.

Section.	Fore-body. Areas. From p. 473.	After-body. Areas. From p. 476.	Sum of the Areas.	Moments.
1′	96·61	103·95	200·56	× 0 = 000·000
2′	85·708	86·44	172·148	× 1 = 172·148
3′	66·18	68·38	134·56	× 2 = 269·12
4′	46·29	49·22	95·51	× 3 = 286·53
5′	27·13	28·96	56·09	× 4 = 224·36

$$\begin{array}{r} 000{\cdot}00 \\ 224{\cdot}36 \\ \hline 224{\cdot}36 = \mathrm{A} \end{array} \qquad \begin{array}{rl} 172{\cdot}148 & \\ 286{\cdot}530 & \\ \hline 458{\cdot}678 & = \mathrm{P} \\ 4 & \\ \hline 1834{\cdot}712 & = 4\,\mathrm{P} \\ 224{\cdot}360 & = \mathrm{A} \\ 538{\cdot}240 & = 2\,\mathrm{Q} \\ \hline 2597{\cdot}312 & = \mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q} \\ {\cdot}92 & = r \\ \hline 5194624 & \\ 23375808 & \\ \hline 3\,)\,2389{\cdot}52704 & \\ \hline 796{\cdot}509 & = \overline{\mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q}} \times \frac{r}{3} = \end{array} \qquad \begin{array}{rl} 269{\cdot}12 & = \mathrm{Q} \\ 2 & \\ \hline 538{\cdot}24 & = 2\,\mathrm{Q} \end{array}$$

sum of the moments of the half displacement calculated from the load-water line: the half displacement by horizontal sections is 489·588 (p. 477) cubic feet; the sum of the moments of the half displacement 796·509 ft., being divided by that quantity, will give the distance in intervals of ·92 ft.; the centre of gravity of displacement is below the load-water line.

Half solid of displacement. Moments.

489·558) 796·509) 1·62 intervals of ·92 feet; therefore

$$\begin{array}{r} 489558 \\ \hline 3069510 \\ 2937348 \\ \hline 1321620 \\ 979116 \\ \hline 342504 \end{array} \qquad \begin{array}{r} 1{\cdot}62 \\ \times\ {\cdot}92 \\ \hline 324 \\ 1458 \\ \hline 1{\cdot}4904 \end{array}$$

1·4904 ft. = the distance the centre of gravity of the calculated immersed body is below the load-water section.

DISTANCE OF THE CENTRE OF GRAVITY OF THE AREA OF THE LOAD-WATER SECTION FROM SECTION 1.

No. of Section.	Ordinates of Section 1 from the Table, p. 467.	Distances of them in intervals of 5·5 ft. from Section 1.	Moments: being the Product of the Areas by the respective Distances.
1	·4	0	000·00
2	3·0	1	3·0
3	5·0	2	10·0
4	6·0	3	18·0
5	6·3	4	25·2
6	6·1	5	30·5
7	5·4	6	32·4
8	3·7	7	25·9
9	·4	8	3·2

The moments, for summation, put into the rule.

$$\begin{array}{rl} 00{\cdot}0 & \\ 3{\cdot}2 & \\ \hline 3{\cdot}2 & = \mathrm{A} \end{array}$$

$$\begin{array}{rl} 3{\cdot}0 & \\ 18{\cdot}0 & \\ 30{\cdot}5 & \\ 25{\cdot}9 & \\ \hline 77{\cdot}4 & = \mathrm{P} \\ 4 & \\ \hline 309{\cdot}6 & = 4\,\mathrm{P} \\ 3{\cdot}2 & = \mathrm{A} \\ 135{\cdot}2 & = 2\,\mathrm{Q} \\ \hline 448{\cdot}0 & = \mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q} \\ 5{\cdot}5 & = r' \\ \hline 2240 & \\ 2240 & \\ \hline 3\,)\,2464{\cdot}0 & \\ \hline 821{\cdot}3 & = \overline{\mathrm{A} + 4\,\mathrm{P} + 2\,\mathrm{Q}} \times \frac{r'}{3} = \end{array}$$

$$\begin{array}{rl} 10{\cdot}0 & \\ 25{\cdot}2 & \\ 32{\cdot}4 & \\ \hline 67{\cdot}6 & = \mathrm{Q} \\ 2 & \\ \hline 135{\cdot}2 & = 2\,\mathrm{Q} \end{array}$$

sum of the moments of the half area of the load-water section reckoned from 1; the half area of the load-water section is 200·56 feet (p. 478); the distance, therefore, of the centre of gravity of the load-water section from 1 will be found in intervals of space of 5·5 feet, by dividing the sum of these moments by the half area, thus:—

$$\begin{array}{rll} \text{Half Area.} & \text{Moments.} & \text{No.} \\ 200{\cdot}56\,) & 821{\cdot}3333 & (\,4{\cdot}09 \text{ intervals, each} \\ & 80224 & 5{\cdot}5 \text{ ft. in length.} \\ \hline & 190933 & \\ & 180504 & \\ \hline & 10429 & \end{array}$$

and 4·09 × 5·5 = 22·5 ft. gives the distance of the centre of gravity of the load-water section from section 1 of the drawing.

Relative capacities of the bodies immersed under the fore and after lengths of equal division of the load-water line—

By former calculations.

After-body immersed contains........497·79 cubic ft. of space.
Fore-body " " ....481·70 cubic ft. of space.
Difference........ 16·09 =

the excess in cubic feet of space of the body displaced under the after half-length of the load-water line over that under the fore-half of the same line—

Sum of the bodies (by former calculation) or whole displacement in cubic feet of space (p. 477)...... } 979·49

equal to 9·7949 hundreds of cubic feet of space, whence 16·09, or the difference between the two bodies in cubic feet, being divided by 9·7949, or the displacement expressed in terms of the hundreds of cubic feet of space, will give the excess for every hundred cubic feet of the whole displacement.

Displacement in Hundreds of Cubic Feet of Space.	Excess in Cubic Feet of Space.	
9·7949)	16·09000 (1·6	= Ratio of the excess of the after-body of displacement over the fore-body of the same, denoted by a per-centage of the whole displacement.
	97949	
	629510	
	587694	
	·41816	

METACENTRE.

A measure of the comparative stability of a ship, or the height of the metacentre above the centre of gravity of displacement estimated, from the expression $\frac{2}{3} \int \frac{y^3 \, dx}{D}$, in which $\int$ is the sign of integration and signifies sum:—

y = the ordinates of the half-breadth load-water section.
dx = the differential increment of the length of load-water section.
D = displacement of the immersed portion of the body in cubic feet of space.

Ordinates from the table.	Cubes of the Ordinates.
Page 467. ·4	00·064
3·0	27·000
5·0	125·000
6·0	216·000
6·3	250·047
6·1	226·981
5·4	157·464
3·7	50·653
·4	0·064

Cubes placed in O'Neill's rule for summation of

$$\text{Area} = (A + 4P + 2Q) \times \frac{r}{3}$$

00·064
00·064
·128 = A

27·000
216·000
226·981
50·653
520·634 = P
4
2082·536 = 4 P
1065·022 = 2 Q
000·128 = A
3147·686 = A + 4 P + 2 Q
5·5 = r'
15738430
15738430
3) 17312·2730

125·000
250·047
157·464
532·511 = Q
2
1065·022 = 2 Q

$$\int y^3 \, dx = 5770·7576 = \overline{A + 4P + 2Q} \times \frac{r'}{3} =$$

summation of the cubes of the ordinates of the load-water section: and the height of the metacentre above the centre of gravity of displacement is expressed by $\frac{2}{3}\int\frac{y^3\,dx}{D}$, in which expression $y^3\,dx =$ 5770·75 and D = 979·1 (p. 477) whence $\frac{2}{3} \times \frac{5770{\cdot}75}{979{\cdot}1} = 3{\cdot}98$ feet is the height of the metacentre above the centre of gravity of the displacement.

RESULTS OF THE CALCULATIONS.

1st Method.

Displacement in cubic feet of space = 979·149.
Displacement in tons of 35 cubic feet of water to a ton.............. = 27·974.
Area of midship section.............. = 41·08 superficial feet.
Area of load-water line or plane at the proposed deepest immersion.. = 401·12 superficial feet.
Tons to one inch of immersion at that flotation........................ = ·955 tons.
Longitudinal distance of the centre of gravity of displacement from section 1. = 22·22 feet.
Depth of the centre of gravity of displacement below the load-water section.................................. = 1·4904 feet.
Distance of the centre of gravity of the load-water section from vertical section 1........................ = 22·5 feet.
Relative capacity of the after-body in excess of the fore-body in cubic feet of space........................ = 16·09
Per-centage on the whole displacement................................ = 1·06.
Height of the metacentre above the centre of gravity of displacement, estimated from the expression $\frac{2}{3}\int\frac{y^3\,dx}{D}$. = 3·98 feet.

The young naval architect has thus been led through the essential calculations on the immersed portion of a ship considered as a floating body. The term essential has here been used under a knowledge that the table of results might have been swollen to a small volume by a lengthened comparison of the elements of the naval construction, such as the ratio of the area of the midship section to the area of the load-water section, and that of the area of the midship section to the circumscribing parallelogram; data that will always suggest themselves to the mind, and furnish salutary exercise for his judgment, while the introduction of such comparisons into these rudiments might deter the novice from entering

FORE-BODY.

Summation of the Cubes for the Value of $\int y\ 3\ dx$.	00·032		54·000		125·000		432·000		250·047			
	$\frac{1}{2}$		2		1		2		1			
Cubes of the Ordinates of Load-water Section.	00·64		27·000		125·000		216·000		250·047		Functions of Longitudinal Areas.	
Moment for Centre of Gravity of Load-water Line.	000·00		6·00		10·00		36·00		12·60			
	0		1		2		3		4			
		0·20		6·00		5·00		12·00		3·15	26·35	13·17
$\frac{A}{2}$	·4	0·20	3·0	1·50	5·0	2·50	6·0	3·00	6·3	3·15		
		0·17		4·80		4·20		11·20		2·80	23·17	46·34
2 P	·35	0·70	2·4	4·80	4·2	8·40	5·6	11·20	5·6	11·20		
		0·15		3·40		3·20		8·80		2·50	18·05	18.05
Q	·3	0·30	1·7	1·70	3·2	3·20	4·4	4·40	5·0	5·00		
		0·12		2·00		2·20		6·40		1·90	12·62	25·24
2 P	·25	0·50	1·0	2·00	2·2	4·40	3·2	6·40	3·8	7·60		
		0·10		0·80		1·30		4·00		1·20	7·40	3·70
$\frac{A}{2}$	·2	0·10	·4	·20	1·3	·65	2·0	1·00	2·4	1·20		
B Functions of Vertical Areas.	1·80		10·20		19·15		26·00		28·15		Function of the Solid by Longitudinal Areas 106·50.	
Multipliers for Solid.	$\frac{1}{2}$		2		1		2		$\frac{1}{2}$			
C Functions of the Areas for the Solid.	0·90		20·40		19·15		52·00		14·07		106·52	
	0		1		2		3		4			
Moments for Centre of Gravity of Displacement.	000·00		20·40		38·30		156·00		56·28		Function of the Solid by Vertical Areas	

AFTER-BODY.

Moments for Centre of Gravity of Displacement.		C Functions of the Areas for the Solid.	Multipliers for Solid.	B Functions of Vertical Areas.	$\frac{A}{2}$			2 P			Q			2 P			$\frac{A}{2}$				Moment for Centre of Gravity of Load-water Line.	Cubes of the Ordinates of Load-water Section.		Summation of the Cubes for the Value of $\int y^3 dx$.
56·28	4	14·07	½	28·15	2·4	1·20	1·20	3·8	7·60	1·90	5·0	5·00	2·50	5·6	11·20	2·80	6·3	3·15	3·15	4	12·60	250·047	1	
264·50	5	52·90	2	26·45	2·0	1·00	4·00	3·4	6·80	6·80	4·6	4·60	9·20	5·5	11·00	11·00	6·1	3·05	12·20	5	61·00	226·981	2	453·962
122·40	6	20·40	1	20·40	1·4	·70	1·40	2·4	4·80	2·40	3·4	3·40	3·40	4·4	8·80	4·40	5·4	2·70	5·40	6	32·40	157·464	1	157·464
157·50	7	22·50	2	11·25	·6	·30	1·20	1·1	2·20	2·20	1·7	1·70	3·40	2·6	5·20	5·20	3·7	1·85	7·40	7	51·80	50·653	2	191·306
7·20	8	·90	½	1·80	·2	·10	0·10	·25	0·50	0·12	·3	0·30	0·15	·35	·70	0·17	·4	0·20	0·20	8	1·60	00·064	½	00·032
878·86	Function of the Solid.	110·77					7·90			13·42			18·65			23·57			28·35		224·00			1573·843

				Sum of the Functions of Fore and After-bodies.		Moments for the Centre of Gravity of Displacement.	
$\frac{A}{2}$	7·90	½	3·95	3·95 3·70	7·65	4	30·60
2 P	13·42	2	26·84	26·84 25·21	52·08	3	156·24
Q	18·65	1	18·65	18·65 18·05	36·70	2	73·40
2 P	23·57	2	47·14	47·14 46·34	93·48	1	93·48
$\frac{A}{2}$	28·35	½	14·7	14·17 13·17	27·34	0	000·00
			110·75		217·25		353·72

$r = ·92$ feet.

$r' = 5·5$ feet.

on a task that would thence seem to be involved in such voluminous results. For the second method of calculation, the table of ordinates is in two portions, viz. the fore and after-bodies under the division of the load-water section into two equal parts, the length of such section being restricted to the distance from the fore-edge of the rabbet of the stem to the after-edge of the rabbet of the post. The enlarged tables are shown at pages 484 and 485, and the directions for the working of these tables have been given at page 459, observing only that the ordinates have not been herein inserted in red, as it was there suggested, to insure perspicuity and accuracy.

RESULTS FROM THE TABLES.

By modified rule. Area $= \left\{ \frac{A}{2} + 2\,P + Q \right\} \frac{2\,r}{3}$

And solid = areas for ordinates summed by rule $\left.\right\} = \left\{ \frac{A'}{2} + 2\,P' + Q' \right\} + \frac{2\,r'}{3}$

Functions of the areas marked B $= \left\{ \frac{A}{2} + 2\,P + Q \right\}$

Function of the solid equal to B, placed in O'Neill's rules $=$ $A' + 2\,P' + Q' = E$

Whence displacement $= E \times \frac{2\,r}{3} \times \frac{2\,r'}{3}$, in the example $r = \cdot 92$ $r' = 5{\cdot}5$.

Therefore $\frac{1}{2}$ displacement $= E \times \frac{2\,r}{3} \times \frac{2\,r'}{3} = E \times \frac{1{\cdot}84}{3} \times \frac{11}{3} =$ $E \times \frac{20{\cdot}24}{9}$.

VALUE OF E FROM THE TABLES BY VERTICAL SECTIONS.

Table 1...106·50 = submultiple of the fore-body by vertical sections.
Table 2...110·77 = " after-body " "
217·27 = sum of the submultiples = E.

$\frac{1}{2}$ displacement $= E \times \frac{20{\cdot}24}{9} = \frac{217{\cdot}27 \times 20{\cdot}24}{9} = 24{\cdot}14 \times 20{\cdot}24 =$

$$\begin{array}{r l}
488{\cdot}5936 & = \tfrac{1}{2} \text{ solid of displacement by the summation of the} \\
2 & \quad \text{vertical areas given in cubic feet of space.} \\
\hline
5\,)\,977{\cdot}1872 & \\
7\,)\,195{\cdot}4374 & \\
\hline
27{\cdot}92 & = \text{Displacement by vertical sections in tons of 35} \\
& \quad \text{cubic feet of space.}
\end{array}$$

VALUE OF E FROM THE TABLES BY HORIZONTAL SECTIONS.

Table 1...106·50 = submultiple of the fore-body by horizontal sections.
Table 2...110·75 = submultiple of the after-body by horizontal sections.

From whence the same results will be obtained.

AREA OF MIDSHIP SECTION.

From table 1...28·15 = submultiple of the area of Section 5.
1·84 = 2 r

11260
22520
2015

3) 51·7960

17·265 = ½ area of upper space of midship section.
3·275 = ½ area of the lower " " below $d\ d$,

20·540 = ½ area of midship section.
2

41·08 = area of midship section.

AREA OF THE LOAD-WATER LINE.

From table 1...26·35 = submultiple of the area of the fore-body.
From table 2...28·35 = " " after-body.

54·70 = submultiple for ½ area of load-water line.
11 = 2 r'

3) 601·7

200·56 = ½ area = $\overline{\frac{A}{2} + 2P + Q} \times \frac{2r'}{3}$
2

12) 401·12 = area of load-water line.

5) 33·42

7) 6·684

·955 = tons per inch of immersion at the load-water line.

POSITION OF THE CENTRE OF GRAVITY OF DISPLACEMENT.

By table 2...878·86 = moments from Section 1.
and E........217·27 = corresponding function of the displacement.

217·27) 878·86 (·404 intervals of 5·5 feet, giving 4·04 × 5·5 = 22·22 feet as the distance of the centre of gravity of the displacement from Section 1.
869·08

97800
86908

10892

DEPTH OF THE CENTRE OF GRAVITY OF THE DISPLACEMENT BELOW THE LOAD-WATER SECTION.

By table 2...353·72 = moments from load-water line.
and E........217·25 = corresponding function of the displacement.
217·25) 353·72 (1·62 intervals of ·92 feet, giving 1·62 ×
217·25 ·92 = 1·4904 as the distance that
136·470 the centre of gravity of displacement is below the load-water line.
130·350
61200
43450
17750

POSITION OF THE CENTRE OF GRAVITY OF THE LOAD-WATER LINE OF DEEPEST IMMERSION.

From table 1.......26·35 ft. From table 2...224·000 = moments
" 2.......28·35 from 1st section.
Function for area..54·7) 224·0 (4·09 intervals of 5·5 feet, giving
218·8 4·09 × 5·5 = 22·495 feet
5200 as the distance that the
4923 centre of gravity of the
·277 load-water section is from vertical section 1.

RELATIVE CAPACITIES OF THE CALCULATED IMMERSED BODIES CONTAINED UNDER THE FORE AND AFTER-LENGTHS OF EQUAL DIVISION OF THE LOAD-WATER LINE.

	Feet.
From table 1...Function for the fore-solid......	106·50
From table 2...Function for the after-solid......	110·75
	4·25
Sum of the functions......	217·25

The difference, 4·25 feet, expresses the excess in cubic feet of space of the body, displaced under the after half-length of the load-water line, over that under the fore half-length of the same line, and the sum of the functions, 217·25, is equal to 2·1725 hundreds of cubic feet of space; whence, 4·25 feet, or the difference between the functions for the two bodies, being divided by the function 2·1725, or the function for the displacement of the calculated body expressed in terms of hundreds of cubic feet of space, will give the excess for every hundred cubic feet of that displacement:

Function of Displacement. Excess in Cubic Feet of Space.
2·1725) 4·25000 (1·9 ratio of the excess of the after-
2·1725 body of calculation over the
207750 fore-body of the same, de-
195525 noted by a per-centage of the
·12225 displacement calculated by the table of ordinates.

HEIGHT OF THE METACENTRE ABOVE THE CENTRE OF GRAVITY OF DISPLACEMENT.

From table 2...The summation of the functions of the cubes of the ordinates for the value of the $\int y^3\,dx$......... $= 1573{\cdot}843$.

The corresponding function for the solid.......... $= 217{\cdot}25$.

from whence the height of the metacentre above the centre of gravity of displacement, expressed by $\frac{2}{3}\int\frac{y^3\,dx}{D}$ is as follows:

$$\int y^3\,dx = 1573{\cdot}843 \times \frac{2\,r'}{3} \text{ where } r' = 5{\cdot}5 \text{ feet} =$$

$$\frac{1573{\cdot}843 \times 11}{3} = \frac{17312{\cdot}273}{3} = 5770{\cdot}75 \text{ feet.}$$

(Page 485) $217{\cdot}27 \times \frac{2\,r}{3} \times \frac{2\,r'}{3} = \frac{1}{2}$ displacement $= 488{\cdot}5936$ feet,

whence displacement or $D = 977{\cdot}1872$; and thence

$$\frac{2}{3}\int\frac{y^3\,dx}{D} = \frac{2}{3} \times \frac{5770{\cdot}75}{977{\cdot}1872} = \frac{11541{\cdot}53}{2931{\cdot}5616} = 3{\cdot}98 \text{ feet.}$$

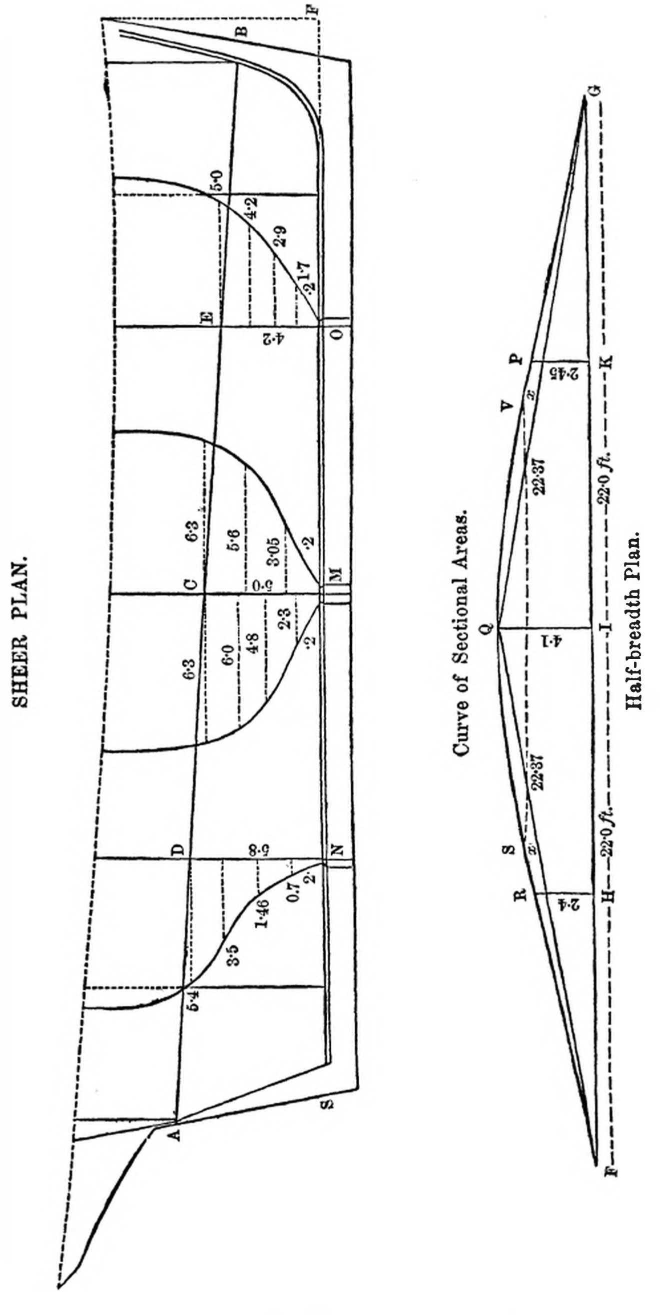
RESULTS OBTAINED UNDER THE TWO METHODS OF CALCULATION CONTRASTED.

	Old Method.	Second Method.
Displacement in cubic feet of space...	979·139	977·187
Displacement in tons of 35 cubic feet of water to a ton.....................	27·985	27·92
	Superficial ft.	Superficial ft.
Area of midship section.................	41·08	41·08
Area of load-water line or plane at the proposed deepest immersion.....	401·12	401·12
Tons to one inch of immersion at line of flotation.............................	·9526 tons.	·955 tons.
Longitudinal distance of the centre of gravity of the displacement from section 1................................	22·22 ft.	22·22 ft.
Depth of the centre of gravity of displacement below the load-water section......................................	1·4812 ft.	1·4904 ft.
Relative capacities of the bodies.......	1·6 per cent.	1·9 per ct.
Height of the metacentre above the centre of gravity of displacement...	3·98 ft.	3·98 ft.

THIRD METHOD OF CALCULATION.

CALCULATIONS ON THE DRAUGHT OF THE YACHT OF 36 TONS USING THE CURVE OF SECTIONAL AREAS.

The load-water line AB, in the sheer plan, is divided into two equal parts at the point C, and those equal parts are again subdivided at the points D and E; at the points C, D, and E,


SHEER PLAN.
F
B
5·0
4·2
2·9
2·17
E
4·2
O
6·3
5·6
3·05
·2
C
0·9
M
6·3
6·0
4·8
2·3
·2
D
5·8
N
1·46
0·7
·2
3·5
5·4
S
A
Curve of Sectional Areas.
G
P
V
x
2·45
K
22·37
22·0 ft.
Q
4·1
I
22·37
x
S
R
2·4
H
22·0 ft.
F
Half-breadth Plan.

Ordinates.

RH = 2·4 feet.	DN = 5·8 feet.	AB = 44 feet.	IG = 22 feet.
QI = 4·1 "	CM = 5·0 "	FG = 44 "	QG = 22·37 "
PK = 2·45 "	EO = 4·2 "	FI = 22 "	FQ = 22·37 "

thus obtained, the transverse vertical sections of the vessel are delineated.

The length of the load-water line from the fore edge of the rabbet of the stem B, to the after edge of the rabbet of the post A, is next drawn below and parallel to the base line SF of the sheer plan; this line, FG, becomes the base line of the curve of the sectional areas. The common sections of the transverse vertical sections of C, D, and E, (which will be straight lines,) with this horizontal and longitudinal plan, are drawn from their respective points of division, H, I, and K, in half-breadth plan. The areas of these transverse vertical sections at D, C, and E, are then calculated, as before, thus:—

$$\text{Area} = \left\{ A + 4P + 2Q \right\} \times \frac{r}{3} = \left\{ \frac{A}{2} + 2P + Q \right\} \times \frac{2r}{3}; \text{or,}$$

$$\text{Area} = \left\{ A + 2P + 3Q \right\} \times \frac{3}{8} r = \left\{ \frac{A}{2} + P + 1{\cdot}5Q \right\} \times \frac{3}{4} r.$$

Half Area of Transverse Vertical Section, at C, by Rule 1,

$$\text{or, } \tfrac{1}{2} \text{ Area} = \left\{ \frac{A}{2} + 2P + Q \right\} \times \frac{2r}{3}.$$

$$\begin{array}{lll}
1\text{st.} \ldots 6{\cdot}3 & 2\text{d} \ldots 6{\cdot}0 & 3\text{d} \ldots 4{\cdot}8 = Q \\
\text{Last} \ldots \ {\cdot}2 & 4\text{th} \ldots 2{\cdot}3 & \\
\hline
2\,)\,6{\cdot}5 & 8{\cdot}3 = P & \\
\hline
3{\cdot}25 = \frac{A}{2} & \quad 2 & \\
\end{array}$$

$$\begin{array}{rl}
16{\cdot}60 &= 2P \\
3{\cdot}25 &= \frac{A}{2} \\
4{\cdot}80 &= Q \\
\hline
24{\cdot}65 &= \frac{A}{2} + 2P + Q \\
{\cdot}83 &= \frac{2r}{3} \\
\hline
7395 & \\
19720 & \\
\hline
20{\cdot}4595 &= \frac{A}{2} + 2P + Q \times \frac{2r}{3} = \tfrac{1}{2} \text{ area}
\end{array}$$

of section C in feet.

CM, or depth $= 5{\cdot}0$ feet, whence $\frac{CM}{4}$, or $\frac{5{\cdot}0}{4} = 1{\cdot}25 = r =$ distance between the ordinates, and $\frac{2r}{3} = \frac{2 \times 1{\cdot}25}{3} = \frac{2{\cdot}5}{3} = {\cdot}83$ feet.

Half Area of Section C, by Rule 2,

$$\text{or, } \tfrac{1}{2} \text{ area} = \left\{\frac{A}{2} + P + 1{\cdot}5\,Q\right\} \times \frac{3}{4}\,r.$$

$$\begin{array}{lcl} \text{1st. } \ldots 6{\cdot}3 & P = 0 & 5{\cdot}6 \text{ 2d.} \\ \text{Last} \ldots {\cdot}2 & & 3{\cdot}05 \text{ 3d.} \\ 2\overline{)6{\cdot}5} & & \overline{8{\cdot}65} = Q. \\ \overline{3{\cdot}25} = \dfrac{A}{2} & & 4{\cdot}32 = \tfrac{1}{2}\,Q. \\ & & \overline{12{\cdot}97} = 1{\cdot}5\,Q. \end{array}$$

$$\begin{array}{rl} \left.\begin{array}{r}3{\cdot}25\\ 12{\cdot}97\end{array}\right\} & = \dfrac{A}{2} + P + 1{\cdot}5\,Q \\ \overline{16{\cdot}22} & \\ 5 & = 3r = CM = 5{\cdot}0 \text{ feet.} \\ 4\overline{)81{\cdot}10} & \\ \overline{20{\cdot}275} & = \tfrac{1}{2} \text{ area} = \overline{\dfrac{A}{2} + P + 1{\cdot}5\,Q} \times \dfrac{3}{4}\,r. \end{array}$$

Half Area of the Transverse Vertical Section at E.

$$\begin{array}{lrl} \text{1st. } \ldots 5{\cdot}0 & \text{2d. } \ldots 4{\cdot}2 & \quad \text{3d. } \ldots 2{\cdot}9 = Q \\ \text{Last} \ldots {\cdot}2 & \text{4th. } \ldots 1{\cdot}7 & \\ 2\overline{)5{\cdot}2} & \overline{5{\cdot}9} & = P \\ \overline{2{\cdot}6} = \dfrac{A}{2} & 2 & \\ & \overline{11{\cdot}8} & = 2\,P \\ & 2{\cdot}6 & = \dfrac{A}{2} \\ & 2{\cdot}9 & = Q \\ & \overline{17{\cdot}3} & = \dfrac{A}{2} + 2\,P + Q \end{array}$$

EO, or depth $= 4{\cdot}2$ feet, whence $\frac{EO}{4} = \frac{4{\cdot}2}{4} = 1{\cdot}05 = r =$ distance between the ordinates, and $\frac{2\,r}{3} = \frac{1{\cdot}05 \times 2}{3} = \frac{2{\cdot}1}{3} = {\cdot}7$ feet; therefore,

$$\text{Area} = \left\{\frac{A}{2} + 2\,P + Q\right\} \times \frac{2\,r}{3} = 17{\cdot}3 \times {\cdot}7 = 12{\cdot}11 = \text{half}$$

area of transverse vertical section at E.

Half Area of the Transverse Vertical Section at D.

$$\begin{array}{lrl} \text{1st. } \ldots 5{\cdot}40 & \text{2d. } \ldots 3{\cdot}5 & \quad \text{3d. } \ldots 1{\cdot}46 = Q \\ \text{Last} \ldots 9{\cdot}2 & \text{4th.} \ldots 0{\cdot}7 & \\ 2\overline{)5{\cdot}6} & \overline{4{\cdot}2} & = P \\ \overline{2{\cdot}8} = \dfrac{A}{2} & 2 & \\ & \overline{8{\cdot}4} & = 2\,P \\ & 2{\cdot}8 & = \dfrac{A}{2} \\ & 1{\cdot}46 & = Q \\ & \overline{12{\cdot}66} & = \dfrac{A}{2} + 2\,P + Q \end{array}$$

DN, or depth = 5·8 feet, whence $\frac{\text{DN}}{4} = \frac{5 \cdot 8}{4} = 1 \cdot 45$ feet = r = distance between the ordinates, and $\frac{2r}{3} = \frac{2 \times 1 \cdot 45}{3} = \frac{2 \cdot 9}{3} =$ ·97 feet; therefore,

Area $= \left\{ \frac{\text{A}}{2} 2 + \text{P} + \text{Q} \right\} \times \frac{2r}{3} = 12 \cdot 66 \times \cdot 97 = 12 \cdot 28$ feet = half area of transverse vertical section at D.

Half Areas of the Transverse Vertical Sections.

$$\begin{array}{ll} & \text{Feet.} \\ \text{At} \left\{ \begin{array}{l} \text{E} = 12 \cdot 11 \\ \text{C} = 20 \cdot 20 \\ \text{D} = 12 \cdot 28 \end{array} \right\} & \begin{array}{l} \text{Divided by 5 as the depth assumed for} \\ \text{the zone, give the ordinates for the curve} \\ \text{of sectional areas, as........................} \end{array} \left\{ \begin{array}{l} \text{Feet.} \\ 2 \cdot 42 \\ 4 \cdot 04 \\ 2 \cdot 45 \end{array} \right. \end{array}$$

of which 2·42 is set off from H as HR, 4·04 feet from I as IQ, and 2·45 feet from K as KP; the curve IRQPG, passing through the extremities P, Q, and R of the ordinates PK, QI, and RH, is the curve bounding the area of a zone, which, to the depth of 5 feet for a solid, will give in cubic feet of space the half displacement of the immersed body, or the displacement of the yacht to the line AB of proposed deepest immersion.

To measure this representative area, and from thence the solid, join the points Q, G, and I by the straight lines QG, QF, dividing the curvilinear area FRQPGF into the two triangles QGI, QFI, and the two areas GPQG, FRQF. The triangles by construction are equal, and the area of each one of them is equivalent to $\frac{\text{GI} \times \text{QI}}{2}$, or the whole area GQFIG $= \frac{\text{GI} \times \text{QI}}{2} \times 2 = \text{GI} \times \text{QI}$ or FI × IQ, FI being equal to IG, each being the half-length of the same element, the load-water line or line of deepest immersion. The areas QPGQ, QRFQ, are bounded by the curve lines QPG, QRF, which are assumed as portions of common parabolas, and under such an assumption their respective areas are equal to $\frac{2}{3}$ of the circumscribing parallelograms, or the area QPGQ = $\frac{2}{3}$ of GQ × x, and the area FRQF = $\frac{2}{3}$ of FQ × x', where x and x' are the greatest perpendiculars that can be drawn from the bases QG and QF to meet the curves QPG, QRF.

DISPLACEMENT.

AB by a scale of parts = 44 feet, whence FI or IG equal $\frac{\text{AB}}{2} = \frac{44}{2}$ feet = 22 feet; ordinate QI of the medial section = 4·04 feet; and QG = FQ, being the respective hypothenuses of the equal triangles QGI, QFI, are each equal to $\sqrt{\text{IG}^2 + \text{QI}^2} = \sqrt{22^2 + 4 \cdot 04^2} = \sqrt{484 + 16 \cdot 32} = \sqrt{500 \cdot 32} = 22 \cdot 37$ feet; and x, by measurement with a scale of parts, = ·6 foot, and x' also ·6 foot, from which the half displacement in cubic feet of space will be obtained as follows:—

		Cubic feet.
Area FQGIF = GI × IQ.		
Solid under the area FQGIF	$= \text{GI} \times \text{IQ} \times 5 = 22 \times 4·1 \times 5 =$	451·00
Area QPGQ = $\frac{2}{3}$ of GQ × x		
Solid under the area QPGQ	$= \frac{2}{3}\text{of GQ} \times x \times 5 = \frac{2}{3} \times 22·37 \times ·6 \times 5 =$	44·74
Area FRQF = $\frac{2}{3}$ of FQ × x'		
Solid under the area FRQF	$= \frac{2}{3}\text{of FQ} \times x' \times 5 = \frac{2}{3} \times = 22·37 \times ·6 \times 5 =$	44·74
		540·48

or area of the triangle QGI + area of the triangle QFI + area of the space QPGQ + area of the space FRQF = to the representative area FRQPG, which being multiplied by the assumed depth of 5 feet for the zone of half displacement gives 540·48 cubic feet of space, which divided by 35, as the number of such cubic feet that are equivalent to one ton of medium water, gives

3) 540·48
7) 108·09
15·44 tons for half displacement,

and that the whole weight of the body is equal to 15·54 × 2 = 30·88 tons = displacement to the line of proposed deepest immersion AB.

RELATIVE CAPACITIES OF THE BODIES IMMERSED UNDER THE FORE AND AFTER HALF-LENGTHS OF THE LOAD-WATER LINE, AS GIVEN BY THE DELINEATED CURVE OF SECTIONAL AREAS.

The triangles QGI and QFI being equal, the relative capacities of the fore and after-bodies will be determined by the proportion that the area QPGI bears to the area QRFI; and as these areas involve two equal terms, or that the base FQ = the base QG, it follows, that the relation of these areas to each other will be expressed by the proportion that the perpendiculars x and x' bear to each other. In the example given, the fore and after-bodies, or the displacements under the fore and after half-lengths of the load-water AB, are equal; as the perpendiculars x and x' taken from the diagram, on a scale of equal parts, are each ·6 of a foot.

The area of the midship section is denoted relatively by the medial ordinate of the curve of sections QI, and the full amount of it is obtained by multiplying the function QI by the depth of the zone M. In the example:

M = 5; QI = 4·04; then half area of medial section = 4·04 × 5
5

Area of midship section......20·20

TABLES OF LOGARITHMS.

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
1000	000000		1060	025306		1120	049218		1180	071882	
1	000434	43	1	025715	41	1	049606	39	1	072250	37
2	000868	86	2	026124	82	2	049993	77	2	072617	73
3	001301	130	3	026533	122	3	050380	116	3	072985	110
4	001734	173	4	026942	163	4	050766	154	4	073352	147
5	002166	216	5	027350	204	5	051152	193	5	073718	183
6	002598	259	6	027757	245	6	051538	232	6	074085	220
7	003029	303	7	028164	286	7	051924	270	7	074451	256
8	003460	346	8	028571	326	8	052309	309	8	074816	293
9	003891	389	9	028978	367	9	052694	347	9	075182	330
1010	004321		1070	029384		1130	053078		1190	075547	
1	004751	43	1	029789	40	1	053463	38	1	675912	36
2	005180	86	2	030195	81	2	053846	77	2	076276	73
3	005609	128	3	030600	121	3	054230	115	3	076640	109
4	006038	171	4	031004	162	4	054613	153	4	077004	145
5	006466	214	5	031408	202	5	054996	191	5	077368	181
6	006894	257	6	031812	242	6	055378	230	6	077731	218
7	007321	300	7	032216	283	7	055760	268	7	078094	254
8	007748	343	8	032619	323	8	056142	306	8	078457	290
9	008174	385	9	033021	364	9	056524	345	9	078819	327
1020	008600		1080	033424		1140	056905		1200	079181	
1	009026	42	1	033826	40	1	057286	38	1	079543	36
2	009451	85	2	034227	80	2	057666	76	2	079904	72
3	009876	127	3	034628	120	3	058046	114	3	080266	108
4	010300	170	4	035029	160	4	058426	152	4	080626	144
5	010724	212	5	035430	200	5	058805	190	5	080987	180
6	011147	254	6	035830	240	6	059185	228	6	081347	216
7	011570	297	7	036229	280	7	059563	266	7	081707	252
8	011993	339	8	036629	321	8	059942	304	8	082067	288
9	012415	382	9	037028	361	9	060320	342	9	082426	324
1030	012837		1090	037426		1150	060698		1210	082785	
1	013259	42	1	037825	40	1	061075	38	1	083144	36
2	013680	84	2	038223	79	2	061452	75	2	083503	71
3	014100	126	3	038620	119	3	061829	113	3	083861	107
4	014520	168	4	039017	159	4	062206	160	4	084219	143
5	014940	210	5	039414	198	5	062582	188	5	084576	179
6	015360	252	6	039811	238	6	062958	226	6	084934	214
7	015779	294	7	040207	278	7	063333	263	7	085291	250
8	016197	336	8	040602	318	8	063709	301	8	085647	286
9	016615	378	9	040998	357	9	064083	338	9	086004	322
1040	017033		1100	041393		1160	064458		1220	086360	
1	017451	42	1	041787	39	1	064832	37	1	086716	35
2	017868	83	2	042182	79	2	065206	75	2	087071	71
3	018284	125	3	042575	118	3	065580	112	3	087426	106
4	018700	166	4	042969	157	4	065953	149	4	087781	142
5	019116	208	5	043362	196	5	066326	186	5	088136	177
6	019532	250	6	043755	236	6	066699	224	6	088490	213
7	019947	291	7	044148	275	7	067071	261	7	088845	248
8	020361	333	8	044540	314	8	067443	298	8	089198	284
9	020775	374	9	044931	354	9	067814	336	9	089552	319
1050	021189		1110	045323		1170	068186		1230	089905	
1	021603	41	1	045714	39	1	068557	37	1	090258	35
2	022016	82	2	046105	78	2	068928	74	2	090611	70
3	022428	124	3	046495	117	3	069298	111	3	090963	106
4	022841	165	4	046885	156	4	069668	148	4	091315	141
5	023252	206	5	047275	195	5	070038	185	5	091667	176
6	023664	247	6	047664	234	6	070407	222	6	092018	211
7	024075	288	7	048053	273	7	070776	259	7	092370	246
8	024486	330	8	048442	312	8	071145	296	8	092721	282
9	024896	371	9	048830	351	9	071514	333	9	093071	317

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
1240	093422		1300	113943		1360	133539		1420	152288	
1	093772	35	1	114277	33	1	133858	32	1	152594	30
2	094122	70	2	114611	67	2	134177	64	2	152900	61
3	094471	105	3	114944	100	3	134496	96	3	153205	91
4	094820	140	4	115278	133	4	134814	127	4	153510	122
5	095169	175	5	115610	167	5	135133	159	5	153815	152
6	095518	210	6	115943	200	6	135451	191	6	154119	183
7	095866	245	7	116276	233	7	135768	223	7	154424	213
8	096215	280	8	116608	267	8	136086	255	8	154728	244
9	096562	315	9	116940	300	9	136403	287	9	155032	274
1250	096910		1310	117271		1370	136721		1430	155336	
1	097257	35	1	117603	33	1	137037	32	1	155640	30
2	097604	69	2	117934	66	2	137354	63	2	155943	60
3	097951	104	3	118265	99	3	137670	94	3	156246	91
4	098297	138	4	118595	132	4	137987	126	4	156549	121
5	098644	173	5	118926	165	5	138303	158	5	156852	151
6	098990	208	6	119256	198	6	138618	189	6	157154	181
7	099335	242	7	119586	231	7	138934	221	7	157457	211
8	099681	277	8	119915	264	8	139249	252	8	157759	242
9	100026	311	9	120245	297	9	139564	284	9	158061	272
1260	100370		1320	120574		1380	139879		1440	158362	
1	100715	34	1	120903	33	1	140194	31	1	158664	30
2	101059	69	2	121231	66	2	140508	63	2	158965	60
3	101403	103	3	121560	98	3	140822	94	3	159266	90
4	101747	137	4	121888	131	4	141136	125	4	159567	120
5	102090	172	5	122216	164	5	141450	157	5	159868	150
6	102434	206	6	122543	197	6	141763	188	6	160168	180
7	102777	240	7	122871	230	7	142076	219	7	160468	210
8	103119	275	8	123198	262	8	142389	251	8	160769	240
9	103462	309	9	123525	295	9	142702	282	9	161068	270
1270	103804		1330	123852		1390	143015		1450	161368	
1	104146	34	1	124178	33	1	143327	31	1	161667	30
2	104487	68	2	124504	65	2	143639	62	2	161967	60
3	104828	102	3	124830	98	3	143951	93	3	162266	89
4	105169	136	4	125156	130	4	144263	125	4	162564	119
5	105510	170	5	125481	163	5	144574	156	5	162863	149
6	105851	204	6	125806	195	6	144885	187	6	163161	179
7	106191	238	7	126131	228	7	145196	218	7	163460	209
8	106531	272	8	126456	260	8	145507	249	8	163757	239
9	106870	306	9	126781	293	9	145818	280	9	164055	269
1280	107210		1340	127105		1400	146128		1460	164353	
1	107549	34	1	127429	32	1	146438	31	1	164650	30
2	107888	67	2	127752	65	2	146748	62	2	164947	59
3	108227	101	3	128076	97	3	147058	93	3	165244	89
4	108565	135	4	128399	129	4	147367	124	4	165541	119
5	108903	169	5	128722	161	5	147676	155	5	165838	148
6	109241	203	6	129045	194	6	147985	186	6	166134	178
7	109578	237	7	129368	226	7	148294	217	7	166430	207
8	109916	270	8	129690	258	8	148603	248	8	166726	237
9	110253	304	9	130012	291	9	148911	279	9	167022	267
1290	110590		1350	130334		1410	149219		1470	167317	
1	110926	34	1	130655	32	1	149527	31	1	167613	29
2	111262	67	2	130977	64	2	149835	61	2	167908	59
3	111598	101	3	131298	96	3	150142	92	3	168203	88
4	111934	134	4	131619	128	4	150449	123	4	168497	118
5	112270	168	5	131939	160	5	150756	154	5	168792	147
6	112605	201	6	132260	192	6	151063	184	6	169086	177
7	112940	235	7	132580	224	7	151370	215	7	169380	206
8	113275	268	8	132900	256	8	151676	246	8	169674	236
9	113609	302	9	133219	288	9	151982	277	9	169968	265

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
1480	170262		1540	187521		1600	204120		1660	220108	
1	170555	29	1	187803	28	1	204391	27	1	220370	26
2	170848	58	2	188084	56	2	204662	54	2	220631	52
3	171141	88	3	188366	84	3	204933	81	3	220892	78
4	171434	117	4	188647	113	4	205204	108	4	221153	104
5	171726	146	5	188928	141	5	205475	135	5	221414	130
6	172019	175	6	189209	169	6	205745	162	6	221675	157
7	172311	204	7	189490	197	7	206016	189	7	221936	183
8	172603	234	8	189771	225	8	206286	216	8	222196	209
9	172895	263	9	190051	253	9	206556	243	9	222456	235
1490	173186		1550	190332		1610	206826		1670	222716	
1	173478	29	1	190612	28	1	207095	27	1	222976	26
2	173769	58	2	190892	56	2	207365	54	2	223236	52
3	174060	87	3	191171	84	3	207634	81	3	223496	78
4	174351	116	4	191451	112	4	207903	108	4	223755	104
5	174641	145	5	191730	140	5	208172	135	5	224015	130
6	174932	175	6	192010	168	6	208441	162	6	224274	156
7	175222	204	7	192289	196	7	208710	188	7	224533	182
8	175512	233	8	192567	224	8	208978	215	8	224792	208
9	175802	261	9	192846	252	9	209247	241	9	225051	234
1500	176091		1560	193125		1620	209515		1680	225309	
1	176381	29	1	193403	28	1	209783	27	1	225568	26
2	176670	58	2	193681	56	2	210051	54	2	225826	52
3	176959	86	3	193959	83	3	210318	80	3	226084	77
4	177248	115	4	194237	111	4	210586	107	4	226342	103
5	177536	144	5	194514	139	5	210853	134	5	226600	129
6	177825	173	6	194792	166	6	211120	161	6	226858	155
7	178113	202	7	195069	194	7	211388	187	7	227115	181
8	178401	231	8	195346	222	8	211654	214	8	227372	206
9	178689	259	9	195623	250	9	211921	240	9	227630	232
1510	178977		1570	195900		1630	212188		1690	227887	
1	179264	29	1	196176	27	1	212454	27	1	228144	26
2	179552	57	2	196452	55	2	212720	53	2	228400	51
3	179839	86	3	196729	83	3	212986	80	3	228657	77
4	180126	115	4	197005	110	4	213252	106	4	228913	102
5	180413	144	5	197281	138	5	213518	133	5	229170	128
6	180699	172	6	197556	166	6	213783	159	6	229426	154
7	180986	201	7	197832	193	7	214049	186	7	229682	179
8	181272	230	8	198107	221	8	214314	212	8	229938	205
9	181558	258	9	198382	248	9	214579	239	9	230193	231
1520	181844		1580	198657		1640	214844		1700	230449	
1	182129	28	1	198932	27	1	215109	26	1	230704	25
2	182415	57	2	199206	55	2	215373	53	2	230960	51
3	182700	86	3	199481	82	3	215638	79	3	231215	76
4	182985	114	4	199755	110	4	215902	106	4	231470	102
5	183270	143	5	200029	137	5	216166	132	5	231724	127
6	183554	171	6	200303	164	6	216430	158	6	231979	153
7	183839	200	7	200577	192	7	216694	185	7	232233	178
8	184123	228	8	200850	219	8	216957	211	8	232488	204
9	184407	256	9	201124	247	9	217221	238	9	232742	229
1530	184691		1590	201397		1650	217484		1710	232996	
1	184975	28	1	201670	27	1	217747	26	1	233250	25
2	185259	57	2	201943	54	2	218010	52	2	233504	51
3	185542	85	3	202216	82	3	218273	79	3	233757	76
4	185825	113	4	202488	109	4	218535	105	4	234011	101
5	186108	142	5	202761	136	5	218798	131	5	234264	127
6	186391	170	6	203033	163	6	219060	157	6	234517	152
7	186674	198	7	203305	191	7	219322	183	7	234770	177
8	186956	227	8	203577	218	8	219584	210	8	235023	202
9	187239	255	9	203848	245	9	219846	236	9	235276	228

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
1720	235528		1780	250420		1840	264818		1900	278754	
1	235781	25	1	250664	24	1	265054	23	1	278982	23
2	236033	50	2	250908	49	2	265290	47	2	279210	45
3	236285	76	3	251151	73	3	265525	70	3	279439	68
4	236537	101	4	251395	97	4	265761	94	4	279667	91
5	236789	126	5	251638	121	5	265996	117	5	279895	114
6	237041	151	6	251881	146	6	266232	141	6	280123	137
7	237292	176	7	252125	171	7	266467	164	7	280351	160
8	237544	202	8	252367	195	8	266702	188	8	280578	182
9	237795	227	9	252610	219	9	266937	211	9	280806	205
1730	238046		1790	252853		1850	267172		1910	281033	
1	238297	25	1	253096	24	1	267406	23	1	281261	23
2	238548	50	2	253338	48	2	267641	47	2	281488	45
3	238799	75	3	253580	73	3	267875	70	3	281715	68
4	239049	100	4	253822	97	4	268110	94	4	281942	91
5	239299	125	5	254064	121	5	268344	117	5	282169	113
6	239550	150	6	254306	145	6	268578	141	6	282395	136
7	239800	175	7	254548	170	7	268812	164	7	282622	159
8	240050	200	8	254790	194	8	269046	188	3	282849	181
9	240300	225	9	255031	218	9	269279	211	9	283075	204
1740	240549		1800	255273		1860	269513		1920	283301	
1	240799	25	1	255514	24	1	269746	23	1	283527	23
2	241048	50	2	255755	48	2	269980	47	2	283753	45
3	241297	75	3	255996	72	3	270213	70	3	283979	68
4	241546	100	4	256236	96	4	270446	93	4	284205	90
5	241795	124	5	256477	120	5	270679	116	5	284431	113
6	242044	149	6	256718	144	6	270912	140	6	284656	135
7	242293	174	7	256958	168	7	271144	163	7	284882	158
8	242541	199	8	257198	192	8	271377	186	8	285107	180
9	242790	223	9	257439	216	9	271609	210	9	285332	203
1750	243038		1810	257679		1870	271842		1930	285557	
1	243286	25	1	257918	24	1	272074	23	1	285782	22
2	243534	50	2	258158	48	2	272306	46	2	286007	45
3	243782	74	3	258398	72	3	272538	70	3	286232	67
4	244030	99	4	258637	96	4	272776	93	4	286456	89
5	244277	124	5	258877	120	5	273001	116	5	286681	112
6	244524	149	6	259116	144	6	273233	139	6	286905	134
7	244772	174	7	259355	167	7	273464	162	7	287130	157
8	245019	198	8	259594	192	8	273696	186	8	287354	179
9	245266	222	9	259833	215	9	273927	209	9	287578	202
1760	245513		1820	260071		1880	274158		1940	287802	
1	245759	25	1	260310	24	1	274389	23	1	288025	22
2	246006	49	2	260548	48	2	274620	46	2	288249	45
3	246252	74	3	260787	71	3	274850	69	3	288473	67
4	246499	98	4	261025	95	4	275081	92	4	288696	89
5	246745	123	5	261263	119	5	275311	115	5	288920	112
6	246991	148	6	261501	143	6	275542	138	6	289143	134
7	247236	173	7	261738	167	7	275772	161	7	289366	156
8	247482	197	8	261976	191	8	276002	184	8	289589	178
9	247728	221	9	262214	214	9	276232	207	9	289812	201
1770	247973		1830	262451		1890	276462		1950	290035	
1	248219	25	1	262688	24	1	276691	23	1	290257	22
2	248464	49	2	262925	47	2	276921	46	2	290480	44
3	248709	74	3	263162	71	3	277151	69	3	290702	67
4	248954	98	4	263399	95	4	277380	92	4	290925	89
5	249198	123	5	263636	118	5	277609	115	5	291147	111
6	249443	147	6	263873	142	6	277838	138	6	291369	133
7	249687	172	7	264109	166	7	278067	161	7	291591	156
8	249932	196	8	264345	190	8	278296	183	8	291813	178
9	250176	220	9	264582	213	9	278525	206	9	292034	200

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
1960	292256		2020	305351		2080	318063		2140	330414	
1	292478	22	1	305566	21	1	318272	21	1	330617	20
2	292699	44	2	305781	43	2	318481	42	2	330819	40
3	292920	66	3	305996	64	3	318689	63	3	331022	61
4	293141	88	4	306211	86	4	318898	83	4	331225	81
5	293363	110	5	306425	107	5	319106	104	5	331427	101
6	293583	133	6	306639	129	6	319314	125	6	331630	121
7	293804	155	7	306854	150	7	319522	146	7	331832	141
8	294025	177	8	307068	172	8	319730	167	8	332034	162
9	294246	199	9	307282	193	9	319938	188	9	332236	182
1970	294466		2030	307496		2090	320146		2150	332438	
1	294687	22	1	307710	21	1	320354	21	1	332640	20
2	294907	44	2	307924	43	2	320562	41	2	332842	40
3	295127	66	3	308137	64	3	320769	62	3	333044	60
4	295347	88	4	308351	85	4	320977	83	4	333246	81
5	295567	110	5	308564	107	5	321184	104	5	333447	101
6	295787	132	6	308778	128	6	321391	125	6	333649	121
7	296007	154	7	308991	149	7	321598	145	7	333850	141
8	296226	176	8	309204	171	8	321805	166	8	334051	161
9	296446	198	9	309417	192	9	322012	187	9	334253	181
1980	296665		2040	309630		2100	322219		2160	334454	
1	296884	22	1	309843	21	1	322426	21	1	334655	20
2	297104	44	2	310056	43	2	322633	41	2	334856	40
3	297323	66	3	310268	64	3	322839	62	3	335056	60
4	297542	88	4	310481	85	4	323046	82	4	335257	80
5	297761	109	5	310693	106	5	323252	103	5	335458	100
6	297979	131	6	310906	127	6	323458	124	6	335658	120
7	298198	153	7	311118	148	7	323665	144	7	335859	140
8	298416	175	8	311330	170	8	323871	165	8	336059	160
9	298635	197	9	311542	191	9	324077	186	9	336260	180
1990	298853		2050	311754		2110	324282		2170	336460	
1	299071	22	1	311966	21	1	324488	21	1	336660	20
2	299289	44	2	312177	42	2	324694	41	2	336860	40
3	299507	65	3	312389	63	3	324899	62	3	337060	60
4	299725	87	4	312600	84	4	325105	82	4	337260	80
5	299943	109	5	312812	106	5	325310	103	5	337459	100
6	300160	131	6	313023	127	6	325516	123	6	337659	120
7	300378	153	7	313234	148	7	325721	144	7	337858	140
8	300595	174	8	313445	160	8	325926	164	8	338058	160
9	300813	196	9	313656	190	9	326131	185	9	338257	180
2000	301030		2060	313867		2120	326336		2180	338456	
1	301247	22	1	314078	21	1	326541	20	1	338656	20
2	301464	43	2	314289	42	2	326745	41	2	338855	40
3	301681	65	3	314499	63	3	326950	61	3	339054	60
4	301898	87	4	314710	84	4	327155	82	4	339253	80
5	302114	108	5	314920	105	5	327359	102	5	339451	100
6	302331	130	6	315130	126	6	327563	123	6	339650	119
7	302547	152	7	315340	147	7	327767	143	7	339849	139
8	302764	173	8	315550	168	8	327972	164	8	340047	159
9	302980	195	9	315760	189	9	328176	184	9	340246	179
2010	303196		2070	315970		2130	328380		2190	340444	
1	303412	22	1	316180	21	1	328583	20	1	340642	20
2	303628	43	2	316390	42	2	328787	41	2	340841	40
3	303844	65	3	316599	63	3	328991	61	3	341039	59
4	304059	86	4	316809	84	4	329194	81	4	341237	79
5	304275	108	5	317018	105	5	329398	102	5	341435	99
6	304490	129	6	317227	126	6	329601	122	6	341632	119
7	304706	151	7	317436	147	7	329805	142	7	341830	139
8	304921	172	8	317645	168	8	330008	163	8	342028	158
9	305136	194	9	317854	189	9	330211	183	9	342225	178

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
2200	342423		2260	354108		2320	365488		2380	376577	
1	342620	20	1	354301	19	1	365675	19	1	376759	18
2	342817	39	2	354493	38	2	365862	37	2	376942	36
3	343014	59	3	354685	58	3	366049	56	3	377124	55
4	343212	79	4	354876	77	4	366236	75	4	377306	73
5	343409	99	5	355068	96	5	366423	93	5	377488	91
6	343606	118	6	355260	115	6	366610	112	6	377670	109
7	343802	138	7	355452	134	7	366796	131	7	377852	127
8	343999	158	8	355643	154	8	366983	150	8	378034	146
9	344196	178	9	355834	173	9	367169	168	9	378216	164
2210	344392		2270	356026		2330	367356		2390	378398	
1	344589	20	1	356217	19	1	367542	19	1	378580	18
2	344785	39	2	356408	38	2	367729	37	2	378761	36
3	344981	59	3	356599	57	3	367915	56	3	378943	55
4	345178	78	4	356790	76	4	368101	75	4	379124	73
5	345374	98	5	356981	95	5	368287	93	5	379306	91
6	345570	118	6	357172	115	6	368473	112	6	379487	109
7	345766	137	7	357363	134	7	368659	130	7	379668	127
8	345962	157	8	357554	153	8	368844	149	8	379849	146
9	346157	176	9	357744	172	9	369030	167	9	380030	164
2220	346353		2280	357935		2340	369216		2400	380211	
1	346549	19	1	358125	19	1	369401	19	1	380392	18
2	346744	39	2	358316	38	2	369587	37	2	380573	36
3	346939	58	3	358506	57	3	369772	56	3	380754	55
4	347135	78	4	358696	76	4	369958	74	4	380934	73
5	347330	97	5	358886	95	5	370143	93	5	381115	91
6	347525	117	6	359076	114	6	370328	111	6	381296	109
7	347720	137	7	359266	133	7	370513	130	7	381476	127
8	347915	156	8	359456	152	8	370698	148	8	381656	145
9	348110	175	9	359646	171	9	370883	167	9	381837	163
2230	848305		2290	359835		2350	371068		2410	382017	
1	818500	19	1	360025	19	1	371253	18	1	382197	18
2	848694	39	2	360215	38	2	371437	37	2	382377	36
3	348889	58	3	360404	57	3	371622	55	3	382557	54
4	349083	78	4	360593	76	4	371806	74	4	382737	72
5	349278	97	5	360783	95	5	371991	92	5	382917	90
6	349472	117	6	360972	114	6	372175	111	6	383097	108
7	349666	137	7	361161	133	7	372360	129	7	383277	126
8	349860	156	8	361350	152	8	372544	148	8	383456	144
9	350054	175	9	361539	171	9	372728	166	9	383636	162
2240	350248		2300	361728		2360	372912		2420	383815	
1	350442	19	1	361917	19	1	373096	18	1	383995	18
2	350636	39	2	362105	38	2	373280	37	2	384174	36
3	350829	58	3	362294	56	3	373464	55	3	384353	54
4	351023	77	4	362482	75	4	373647	74	4	384533	72
5	351216	97	5	362671	94	5	373831	92	5	384712	90
6	351410	116	6	362859	113	6	374015	110	6	384891	108
7	351603	135	7	363048	132	7	374198	129	7	385070	126
8	351796	155	8	363236	151	8	374382	147	8	385249	144
9	351989	174	9	363424	170	9	374565	166	9	385428	162
2250	352182		2310	363612		2370	374748		2430	385606	
1	352375	19	1	363800	19	1	374932	18	1	385785	18
2	352568	38	2	363988	37	2	375115	37	2	385964	35
3	352761	58	3	364176	56	3	375298	55	3	386142	53
4	352954	77	4	364363	75	4	375481	73	4	386321	71
5	353147	96	5	364551	94	5	375664	92	5	386499	89
6	353339	115	6	364739	112	6	375846	110	6	386677	107
7	353532	134	7	364926	131	7	376029	128	7	386856	125
8	353724	154	8	365113	150	8	376212	147	8	387034	143
9	353916	173.	9	365301	169	9	376394	165	9	387212	161

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
2440	387390		2500	397940		2560	408240		2620	418301	
1	387568	18	1	398114	17	1	408410	17	1	418467	17
2	387746	36	2	398287	35	2	408579	34	2	418633	33
3	387923	53	3	398461	53	3	408749	51	3	418798	50
4	388101	71	4	398634	69	4	408918	68	4	418964	66
5	388279	89	5	398808	87	5	409087	85	5	419129	83
6	388456	107	6	398981	104	6	409257	102	6	419295	99
7	388634	125	7	399154	121	7	409426	119	7	419460	116
8	388811	142	8	399327	138	8	409595	136	8	419625	132
9	388989	160	9	399501	156	9	409764	153	9	419791	149
2450	389166		2510	399674		2570	409933		2630	419956	
1	389343	18	1	399847	17	1	410102	17	1	420121	16
2	389520	36	2	400020	35	2	410271	34	2	420286	33
3	389697	53	3	400192	53	3	410440	50	3	420451	49
4	389875	71	4	400365	69	4	410608	67	4	420616	66
5	390051	89	5	400538	87	5	410777	84	5	420781	82
6	390228	107	6	400711	104	6	410946	101	6	420945	99
7	390405	125	7	400883	121	7	411114	118	7	421110	115
8	390582	142	8	401056	138	8	411283	135	8	421275	132
9	390759	160	9	401228	156	9	411451	152	9	421439	148
2460	390935		2520	401400		2580	411620		2640	421604	
1	391112	18	1	401573	17	1	411788	17	1	421768	16
2	391288	35	2	401745	34	2	411956	34	2	421933	33
3	391464	53	3	401917	52	3	412124	50	3	422097	49
4	391641	70	4	402089	69	4	412292	67	4	422261	66
5	391817	88	5	402261	86	5	412460	84	5	422426	82
6	391993	106	6	402433	103	6	412628	101	6	422590	99
7	392169	123	7	402605	120	7	412796	118	7	422754	115
8	392345	141	8	402777	138	8	412964	135	8	422918	132
9	392521	158	9	402949	155	9	413132	152	9	423082	148
2470	392697		2530	403120		2590	413300		2650	423246	
1	392873	18	1	403292	17	1	413467	17	1	423410	16
2	393048	35	2	403464	34	2	413635	33	2	423573	33
3	393224	53	3	403635	52	3	413802	50	3	423737	49
4	393400	70	4	403807	69	4	413970	67	4	423001	65
5	393575	88	5	403978	86	5	414137	84	5	424064	81
6	393751	106	6	404149	103	6	414305	101	6	424228	98
7	393926	123	7	404320	120	7	414472	117	7	424392	114
8	394101	141	8	404492	137	8	414639	134	8	424555	131
9	394276	158	9	404663	154	9	414806	151	9	424718	147
2480	394452		2540	404834		2600	414973		2660	424882	
1	394627	17	1	405005	17	1	415140	17	1	425045	16
2	394802	35	2	405175	34	2	415307	33	2	425208	33
3	394977	53	3	405346	51	3	415474	50	3	425371	49
4	395152	70	4	405517	68	4	415641	67	4	425534	65
5	395326	87	5	405688	85	5	415808	84	5	425697	81
6	395501	104	6	405858	102	6	415974	101	6	425860	98
7	395676	122	7	406029	119	7	416141	117	7	426023	114
8	395850	139	8	406199	136	8	416308	134	8	426186	130
9	396025	157	9	406370	153	9	416474	150	9	426349	147
2490	396199		2550	406540		2610	416640		2670	426511	
1	396374	17	1	406710	17	1	416807	17	1	426674	16
2	396548	35	2	406881	34	2	416973	33	2	426836	33
3	396722	53	3	407051	51	3	417139	50	3	426999	49
4	396896	70	4	407221	68	4	417306	66	4	427161	65
5	397070	87	5	407391	85	5	417472	83	5	427324	81
6	397245	104	6	407561	102	6	417638	100	6	427486	98
7	397418	122	7	407731	119	7	417804	116	7	427648	114
8	397592	139	8	407900	136	8	417970	133	8	427811	130
9	397766	157	9	408070	153	9	418135	149	9	427973	147

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
2680	428135		2740	437751		2800	447158		2860	456366	
1	428297	16	1	437909	16	1	447313	15	1	456518	15
2	428459	32	2	438067	32	2	447468	31	2	456670	30
3	428621	48	3	438226	47	3	447623	46	3	456821	46
4	428782	65	4	438384	63	4	447778	62	4	456973	61
5	428944	81	5	438542	79	5	447933	77	5	457125	76
6	429106	97	6	438700	95	6	448088	93	6	457276	91
7	429268	113	7	438859	111	7	448242	108	7	457428	106
8	429429	129	8	439017	127	8	448397	124	8	457579	122
9	429591	145	9	439175	143	9	448552	139	9	457730	137
2690	429752		2750	439333		2810	448706		2870	457882	
1	429914	16	1	439491	16	1	448861	15	1	458033	15
2	430075	32	2	439648	32	2	449015	31	2	458184	30
3	430236	48	3	439806	47	3	449170	46	3	458336	45
4	430398	65	4	439964	63	4	449324	62	4	458487	61
5	430559	81	5	440122	79	5	449478	77	5	458638	76
6	430720	97	6	440279	95	6	449633	92	6	458789	91
7	430881	113	7	440437	111	7	449787	108	7	458940	106
8	431042	129	8	440594	126	8	449941	123	8	459091	121
9	431203	145	9	440752	142	9	450095	139	9	459242	136
2700	431364		2760	440909		2820	450249		2880	459392	
1	431525	16	1	441066	16	1	450403	15	1	459543	15
2	431685	32	2	441224	31	2	450557	31	2	459694	30
3	431846	48	3	441381	47	3	450711	46	3	459845	45
4	432007	64	4	441538	63	4	450865	62	4	459995	61
5	432167	80	5	441695	78	5	451018	77	5	460146	76
6	432328	96	6	441852	94	6	451172	92	6	460296	91
7	432488	112	7	442009	110	7	451326	108	7	460447	106
8	432649	128	8	442166	126	8	451479	123	8	460597	121
9	432809	144	9	442323	141	9	451633	139	9	460747	136
2710	432969		2770	442480		2830	451786		2890	460898	
1	433129	16	1	442636	16	1	451940	15	1	461048	15
2	433290	32	2	442793	31	2	452093	31	2	461198	30
3	433450	48	3	442950	47	3	452247	46	3	461348	45
4	433610	64	4	443106	63	4	452400	61	4	461498	60
5	433770	80	5	443263	78	5	452553	77	5	461649	75
6	433930	96	6	443419	94	6	452706	92	6	461799	90
7	434090	112	7	443576	110	7	452859	107	7	461948	105
8	434249	128	8	443732	126	8	453012	123	8	462098	120
9	434409	144	9	443888	141	9	453165	138	9	462248	135
2720	434569		2780	444045		2840	453318		2900	462398	
1	434728	16	1	444201	16	1	453471	15	1	462548	15
2	434888	32	2	444357	31	2	453624	31	2	462697	30
3	435048	48	3	444513	47	3	453777	46	3	462847	45
4	435207	64	4	444669	62	4	453930	61	4	462997	60
5	435366	80	5	444825	78	5	454082	77	5	463146	75
6	435526	96	6	444981	94	6	454235	92	6	463296	90
7	435685	112	7	445137	109	7	454387	107	7	463445	105
8	435844	128	8	445293	125	8	454540	123	8	463594	120
9	436003	144	9	445448	140	9	454692	138	9	463744	135
2730	436163		2790	445604		2850	454845		2910	463893	
1	436322	16	1	445760	16	1	454997	15	1	464042	15
2	436481	32	2	445915	31	2	455149	30	2	464191	30
3	436640	47	3	446071	47	3	455302	46	3	464340	45
4	436798	63	4	446226	62	4	455454	61	4	464489	60
5	436957	79	5	446382	78	5	455606	76	5	464639	75
6	437116	95	6	446537	94	6	455758	91	6	464787	90
7	437275	111	7	446692	109	7	455910	106	7	464936	105
8	437433	127	8	446848	125	8	456062	122	8	465085	120
9	437592	143	9	447003	140	9	456214	137	9	465234	135

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
2920	465383		2980	474216		3040	482874		3100	491362	
1	465532	15	1	474362	15	1	483016	14	1	491502	14
2	465680	30	2	474508	29	2	483159	28	2	491642	28
3	465829	44	3	474653	44	3	483302	43	3	491782	42
4	465977	59	4	474799	58	4	483445	57	4	491922	56
5	466126	74	5	474944	73	5	483587	71	5	492062	70
6	466274	89	6	475090	88	6	483730	85	6	492201	84
7	466423	104	7	475235	102	7	483872	99	7	492341	98
8	466571	118	8	475381	117	8	484015	114	8	492481	112
9	466719	133	9	475526	131	9	484157	128	9	492621	126
2930	466868		2990	475671		3050	484300		3110	492760	
1	467016	15	1	475816	15	1	484442	14	1	492900	14
2	467164	30	2	475962	29	2	484584	28	2	493040	28
3	467312	44	3	476107	43	3	484727	43	3	493179	42
4	467460	59	4	476252	58	4	484869	57	4	493319	56
5	467608	74	5	476397	72	5	485011	71	5	493458	70
6	467756	89	6	476542	87	6	485153	85	6	493597	84
7	467904	104	7	476687	101	7	485295	99	7	493737	98
8	468052	118	8	476832	116	8	485437	114	8	493876	142
9	468200	133	9	476976	130	9	485579	128	9	494015	126
2940	468347		3000	477121		3060	485721		3120	494155	
1	468495	15	1	477266	14	1	485863	14	1	494294	14
2	468643	30	2	477411	29	2	486005	28	2	494433	28
3	468790	44	3	477555	43	3	486147	43	3	494572	41
4	868938	59	4	477700	58	4	486289	57	4	494711	56
5	469085	74	5	477844	72	5	486430	71	5	494850	69
6	469233	89	6	477989	87	6	486572	85	6	494989	83
7	469380	104	7	478133	101	7	486714	99	7	495128	97
8	469527	118	8	478278	116	8	486855	114	8	495267	111
9	469675	133	9	478422	130	9	486997	128	9	495406	125
2950	469822		3010	478566		3070	487138		3130	495544	
1	469969	15	1	478711	14	1	487280	14	1	495683	14
2	470116	29	2	478855	29	2	487421	28	2	495822	28
3	470263	44	3	478999	43	3	487563	42	3	495960	41
4	470410	59	4	479143	58	4	487704	57	4	496099	56
5	470557	74	5	479287	72	5	487845	71	5	496237	69
6	470704	88	6	479431	86	6	487986	85	6	496376	83
7	470851	103	7	479575	101	7	488127	99	7	496514	97
8	470998	118	8	479719	115	8	488269	113	8	496653	111
9	471145	132	9	479863	130	9	488410	127	9	496791	125
2960	471292		3020	480007		3080	488551		3140	496930	
1	471438	15	1	480151	14	1	488692	14	1	497068	14
2	471585	29	2	480294	29	2	488833	28	2	497206	28
3	471732	44	3	480438	43	3	488973	42	3	497344	41
4	471878	59	4	480582	58	4	489114	56	4	497482	55
5	472025	73	5	480725	72	5	489255	70	5	497621	69
6	472171	88	6	480869	86	6	489396	84	6	497759	83
7	472317	102	7	481012	101	7	489537	98	7	497897	97
8	472464	117	8	481156	115	8	489677	112	8	498035	110
9	472610	132	9	481299	130	9	489818	126	9	498173	124
2970	472756		3030	481443		3090	489958		3150	498311	
1	472903	15	1	481586	14	1	490099	14	1	498448	14
2	473049	29	2	481729	29	2	490239	28	2	498586	28
3	473195	44	3	481872	43	3	490380	42	3	498724	41
4	473341	59	4	482016	57	4	490520	56	4	498862	55
5	473487	73	5	482159	71	5	490661	70	5	498999	69
6	473633	88	6	482302	86	6	490801	84	6	499137	83
7	473779	102	7	482445	100	7	490941	98	7	499275	97
8	473925	117	8	482588	114	8	491081	112	8	499412	110
9	474070	132	9	482731	129	9	491222	126	9	499550	124

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
3160	499687		3220	507856		3280	515874		3340	523746	
1	499824	14	1	507991	13	1	516006	13	1	523876	13
2	499962	27	2	508125	27	2	516139	26	2	524006	26
3	500099	41	3	508260	40	3	516271	40	3	524136	39
4	500236	55	4	508395	54	4	516403	53	4	524266	52
5	500374	68	5	508530	67	5	516535	66	5	524396	65
6	500511	82	6	508664	81	6	516668	79	6	524526	78
7	500648	96	7	508799	94	7	516800	92	7	524656	91
8	500785	110	8	508933	108	8	516932	106	8	524785	104
9	500922	123	9	509068	121	9	517064	119	9	524915	117
3170	501059		3230	509202		3290	517196		3350	525045	
1	501196	14	1	509337	13	1	517328	13	1	525174	13
2	501333	27	2	509471	27	2	517460	26	2	525304	26
3	501470	41	3	509606	40	3	517592	40	3	525434	39
4	501607	55	4	509740	54	4	517724	53	4	525563	52
5	501744	68	5	509874	67	5	517855	66	5	525693	65
6	501880	82	6	510008	81	6	517987	79	6	525822	78
7	502017	96	7	510143	94	7	518119	92	7	525951	91
8	502154	110	8	510277	108	8	518251	106	8	526081	104
9	502290	123	9	510411	121	9	518382	119	9	526210	117
3180	502427		3240	510545		3300	518514		3360	526339	
1	502564	14	1	510679	13	1	518645	13	1	526468	13
2	502700	27	2	510813	27	2	518777	26	2	526598	26
3	502837	41	3	510947	40	3	518909	39	3	526727	39
4	502973	54	4	511081	54	4	519040	52	4	526856	52
5	503109	68	5	511215	67	5	519171	66	5	526985	65
6	503246	82	6	511348	80	6	519303	79	6	527114	78
7	503382	95	7	511482	94	7	519434	92	7	527243	91
8	503518	109	8	511616	107	8	519565	105	8	527372	104
9	503654	123	9	511750	121	9	519697	118	9	527501	117
3190	503791		3250	511883		3310	519828		3370	527630	
1	503927	14	1	512017	13	1	519959	13	1	527759	13
2	504063	27	2	512150	27	2	520090	26	2	527888	26
3	504199	41	3	512284	40	3	520221	39	3	528016	38
4	504335	54	4	512417	53	4	520352	52	4	528145	51
5	504471	68	5	512551	67	5	520483	66	5	528274	64
6	504607	82	6	512684	80	6	520614	79	6	528402	77
7	504743	95	7	512818	93	7	520745	92	7	528531	90
8	504878	109	8	512951	107	8	520876	105	8	528660	103
9	505014	122	9	513084	120	9	521007	118	9	528788	116
3200	505150		3260	513218		3320	521138		3380	528917	
1	505286	14	1	513351	13	1	521269	13	1	529045	13
2	505421	27	2	513484	27	2	521400	26	2	529174	26
3	505557	41	3	513617	40	3	521530	39	3	529302	38
4	505692	54	4	513750	53	4	521661	52	4	529430	51
5	505828	68	5	513883	66	5	521792	65	5	529559	64
6	505963	82	6	514016	80	6	521922	78	6	529687	77
7	506099	95	7	514149	93	7	522053	97	7	529815	90
8	506234	109	8	514282	106	8	522183	104	8	529943	103
9	506370	122	9	514415	120	9	522314	117	9	530072	116
3210	506505		3270	514548		3330	522444		3390	530200	
1	506640	13	1	514680	13	1	522575	13	1	530328	13
2	506775	27	2	514813	27	2	522705	26	2	530456	26
3	506911	40	3	514946	40	3	522835	39	3	530584	38
4	507046	54	4	515079	53	4	522966	52	4	530712	51
5	507181	67	5	515211	66	5	523096	65	5	530840	64
6	507316	81	6	515344	80	6	523226	78	6	530968	77
7	507451	94	7	515476	93	7	523356	97	7	531095	90
8	507586	108	8	515609	106	8	523486	104	8	531223	102
9	507721	121	9	515741	120	9	523616	117	9	531351	115

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
3400	531479		3460	539076		3520	546543		3580	553883	
1	531607	13	1	539202	13	1	546666	12	1	554004	12
2	531734	25	2	539327	25	2	546789	25	2	554126	24
3	531862	38	3	539452	38	3	546913	37	3	554247	36
4	531990	51	4	539578	50	4	547036	49	4	554368	49
5	532117	63	5	539703	63	5	547159	62	5	554489	61
6	532245	76	6	539829	75	6	547282	74	6	554610	73
7	532372	89	7	539954	88	7	547405	86	7	554731	85
8	532500	102	8	540079	100	8	547529	99	8	554852	97
9	532627	114	9	540204	113	9	547652	111	9	554973	109
3410	532754		3470	540329		3530	547775		3590	555094	
1	532882	13	1	540455	12	1	547898	12	1	555215	12
2	533009	25	2	540580	25	2	548021	25	2	555336	24
3	533136	38	3	540705	37	3	548144	37	3	555457	36
4	533263	51	4	540830	50	4	548266	49	4	555578	48
5	533391	63	5	540955	62	5	548389	61	5	555699	60
6	533518	76	6	541080	75	6	548512	74	6	555820	72
7	533645	89	7	541205	87	7	548635	86	7	555940	84
8	533772	102	8	541330	100	8	548758	98	8	556061	96
9	533899	114	9	541454	112	9	548881	111	9	556182	108
3420	534026		3480	541579		3540	549003		3600	556302	
1	534153	13	1	541704	12	1	549126	12	1	556423	12
2	534280	25	2	541829	25	2	549249	25	2	556544	24
3	534407	38	3	541953	37	3	549371	37	3	556664	36
4	534534	51	4	542078	50	4	549494	49	4	556785	48
5	534661	63	5	542203	62	5	549616	61	5	556905	60
6	534787	76	6	542327	75	6	549739	74	6	557026	72
7	534914	89	7	542452	87	7	549861	86	7	557146	84
8	535041	102	8	542576	100	8	549984	98	8	557267	96
9	535167	114	9	542701	112	9	550106	111	9	557387	108
3430	535294		3490	542825		3550	550228		3610	557507	
1	535421	13	1	542950	12	1	550351	12	1	557627	12
2	535547	25	2	543074	25	2	550473	24	2	557748	24
3	535674	38	3	543199	37	3	550595	37	3	557868	36
4	535800	50	4	543323	50	4	550717	49	4	557988	48
5	535927	63	5	543447	62	5	550840	61	5	558108	60
6	536053	76	6	543571	75	6	550962	73	6	558228	72
7	536179	88	7	543696	87	7	551084	86	7	558348	84
8	536306	101	8	543820	100	8	551206	98	8	558469	96
9	536432	114	9	543944	112	9	551328	110	9	558589	108
3440	536558		3500	544068		3560	551450		3620	558709	
1	536685	13	1	544192	12	1	551572	12	1	558829	12
2	536811	25	2	544316	25	2	551694	24	2	558948	24
3	536937	38	3	544440	37	3	551816	37	3	559068	36
4	537063	50	4	544564	50	4	551938	49	4	559188	48
5	537189	63	5	544688	62	5	552059	61	5	559308	60
6	537315	76	6	544812	74	6	552181	73	6	559428	72
7	537441	88	7	544936	87	7	552303	86	7	559548	84
8	537567	101	8	545060	99	8	552425	98	8	559667	96
9	537693	114	9	545183	112	9	552546	110	9	559787	108
3450	537819		3510	545307		3570	552668		3630	559907	
1	537945	13	1	545431	12	1	552790	12	1	560026	12
2	538071	25	2	545554	25	2	552911	24	2	560146	24
3	538197	38	3	545678	37	3	553033	36	3	560265	36
4	538322	50	4	545802	49	4	553154	49	4	560385	48
5	538448	63	5	545925	62	5	553276	61	5	560504	60
6	538574	76	6	546049	74	6	553398	73	6	560624	72
7	538699	88	7	546172	86	7	553519	85	7	560743	84
8	538825	101	8	546296	99	8	553640	97	8	560863	96
9	538951	114	9	546419	111	9	553762	109	9	560982	108

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
3640	561101		3700	568202		3760	575188		3820	582063	
1	561221	12	1	568319	12	1	575303	12	1	582177	11
2	561340	24	2	568436	23	2	575419	23	2	582291	23
3	561459	36	3	568554	35	3	575534	35	3	582404	34
4	561578	48	4	568671	47	4	575650	46	4	582518	45
5	561698	60	5	568788	58	5	575765	58	5	582631	56
6	561817	72	6	568905	70	6	575880	69	6	582745	68
7	561936	84	7	569023	82	7	575996	80	7	582858	79
8	562055	96	8	569140	94	8	576111	92	8	582972	90
9	562174	108	9	569257	106	9	576226	104	9	583085	102
3650	562293		3710	569374		3770	576341		3830	583199	
1	562412	12	1	569491	12	1	576457	12	1	583312	11
2	562531	24	2	569608	23	2	576572	23	2	583426	23
3	562650	36	3	569725	35	3	576687	35	3	583539	34
4	562768	48	4	569842	47	4	576802	46	4	583652	45
5	562887	60	5	569959	58	5	576917	58	5	583765	56
6	563006	71	6	570076	70	6	577032	69	6	583879	68
7	563125	83	7	570193	82	7	577147	80	7	583992	79
8	563244	95	8	570309	94	8	577262	92	8	584105	90
9	563362	107	9	570426	106	9	577377	104	9	584218	102
3660	563481		3720	570543		3780	577492		3840	584331	
1	563600	12	1	570660	12	1	577607	11	1	584444	11
2	563718	24	2	570776	23	2	577721	23	2	584557	23
3	563837	36	3	570893	35	3	577836	34	3	584670	34
4	563955	48	4	571010	47	4	577951	46	4	584783	45
5	564074	60	5	571126	58	5	578066	57	5	584896	56
6	564192	71	6	571243	70	6	578181	68	6	585009	68
7	564311	83	7	571359	81	7	578295	80	7	585122	79
8	564429	95	8	571476	93	8	578410	91	8	585235	90
9	564548	107	9	571592	105	9	578525	103	9	585348	102
3670	564666		3730	571709		3790	578639		3850	585461	
1	564784	12	1	571825	12	1	578754	11	1	585574	11
2	564903	24	2	571942	23	2	578868	23	2	585686	22
3	565021	36	3	572058	35	3	578983	34	3	585799	34
4	565139	47	4	572174	47	4	579097	46	4	585912	45
5	565257	59	5	572291	58	5	579212	57	5	586024	56
6	565376	71	6	572407	70	6	579326	68	6	586137	67
7	565494	83	7	572523	81	7	579441	80	7	586250	78
8	565612	95	8	572639	93	8	579555	91	8	586362	90
9	565730	107	9	572755	105	9	579669	103	9	586475	101
3680	565848		3740	572872		3800	579784		3860	586587	
1	565966	12	1	572988	12	1	579898	11	1	586700	11
2	566084	24	2	573104	23	2	580012	23	2	586812	22
3	566202	35	3	573220	35	3	580126	34	3	586925	34
4	566320	47	4	573336	46	4	580240	46	4	587037	45
5	566437	59	5	573452	58	5	580355	57	5	587149	56
6	566555	71	6	573568	70	6	580469	68	6	587262	67
7	566673	83	7	573684	81	7	580583	80	7	587374	78
8	566791	94	8	573800	93	8	580697	91	8	587486	90
9	566909	106	9	573915	104	9	580811	103	9	587599	101
3690	567026		3750	574031		3810	580925		3870	587711	
1	567144	12	1	574147	12	1	581039	11	1	587823	11
2	567262	24	2	574263	23	2	581153	23	2	587935	22
3	567379	35	3	574379	35	3	581267	34	3	588047	34
4	567497	47	4	574494	46	4	581381	46	4	588160	45
5	567614	59	5	574610	58	5	581495	57	5	588272	56
6	567732	71	6	574726	70	6	581608	68	6	588384	67
7	567849	83	7	574841	81	7	581722	80	7	588496	78
8	567967	94	8	574957	93	8	581836	91	8	588608	90
9	568084	106	9	575072	104	9	581950	103	9	588720	101

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
3880	588832		3940	595496		4000	602060		4060	608526	
1	588944	11	1	595606	11	1	602169	11	1	608633	11
2	589056	22	2	595717	22	2	602277	22	2	608740	21
3	589167	33	3	595827	33	3	602386	33	3	608847	32
4	589279	44	4	595937	44	4	602494	43	4	608954	43
5	589391	56	5	596047	55	5	602603	54	5	609061	53
6	589503	67	6	596157	66	6	602711	65	6	609167	64
7	589615	78	7	596267	77	7	602819	76	7	609274	75
8	589726	89	8	596377	88	8	602928	87	8	609381	86
9	589838	100	9	596487	99	9	603036	98	9	609488	96
3890	589950		3950	596597		4010	603144		4070	609594	
1	590061	11	1	596707	11	1	603253	11	1	609701	11
2	590173	22	2	596817	22	2	603361	22	2	609808	21
3	590284	33	3	596927	33	3	603469	33	3	609914	32
4	590396	44	4	597037	44	4	603577	43	4	610021	43
5	590507	56	5	597146	55	5	603686	54	5	610128	53
6	590619	67	6	597256	66	6	603794	65	6	610234	64
7	590730	78	7	597366	77	7	603902	76	7	610341	75
8	590842	89	8	597476	88	8	604010	87	8	610447	86
9	590953	100	9	597585	99	9	604118	98	9	610554	96
3900	591065		3960	597695		4020	604226		4080	610660	
1	591176	11	1	597805	11	1	604334	11	1	610767	11
2	591287	22	2	597914	22	2	604442	22	2	610873	21
3	591399	33	3	598024	33	3	604550	32	3	610979	32
4	591510	44	4	598134	44	4	604658	43	4	611086	42
5	591621	56	5	598243	55	5	604766	54	5	611192	53
6	591732	67	6	598353	66	6	604874	65	6	611298	64
7	591843	78	7	598462	77	7	604982	76	7	611405	74
8	591955	89	8	598572	88	8	605089	86	8	611511	85
9	592066	100	9	598681	99	9	605197	97	9	611617	95
3910	592177		3970	598790		4030	605305		4090	611723	
1	592288	11	1	598900	11	1	605413	11	1	611829	11
2	592399	22	2	599009	22	2	605521	22	2	611936	21
3	592510	33	3	599119	33	3	605628	32	3	612042	32
4	592621	44	4	599228	44	4	605736	43	4	612148	42
5	592732	55	5	599337	55	5	605844	54	5	612254	53
6	592843	67	6	599446	66	6	605951	65	6	612360	64
7	592954	78	7	599556	77	7	606059	76	7	612466	74
8	593064	89	8	599665	88	8	606166	86	8	612572	85
9	593175	100	9	599774	99	9	606274	97	9	612678	95
3920	593286		3980	599883		4040	606381		4100	612784	
1	593397	11	1	599992	11	1	606489	11	1	612890	11
2	593508	22	2	600101	22	2	606596	21	2	612996	21
3	593618	33	3	600210	33	3	606704	32	3	613101	32
4	593729	44	4	600319	44	4	606811	43	4	613207	42
5	593840	55	5	600428	54	5	606919	54	5	613313	53
6	593950	66	6	600537	65	6	607026	64	6	613419	64
7	594061	77	7	600646	76	7	607133	75	7	613525	74
8	594171	88	8	600755	87	8	607241	86	8	613630	85
9	594282	99	9	600864	95	9	607348	96	9	613736	95
3930	594393		3990	600973		4050	607455		4110	613842	
1	594503	11	1	601082	11	1	607562	11	1	613947	11
2	594613	22	2	601190	22	2	607669	21	2	614053	21
3	594724	33	3	601299	33	3	607777	32	3	614159	32
4	594834	44	4	601408	44	4	607884	43	4	614264	42
5	594945	55	5	601517	54	5	607991	54	5	614370	53
6	595055	66	6	601625	65	6	608098	64	6	614475	63
7	595165	77	7	601734	76	7	608205	75	7	614581	74
8	595276	88	8	601843	87	8	608312	86	8	614686	84
9	595386	99	9	601951	98	9	608419	96	9	614792	95

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
4120	614897		4180	621176		4240	627366		4300	633468	
1	615003	11	1	621280	10	1	627468	10	1	633569	10
2	615108	21	2	621384	21	2	627571	20	2	633670	20
3	615213	31	3	621488	31	3	627673	31	3	633771	30
4	615319	42	4	621592	42	4	627775	41	4	633872	40
5	615424	52	5	621695	52	5	627878	51	5	633973	50
6	615529	63	6	621799	62	6	627980	61	6	634074	61
7	615634	73	7	621903	73	7	628082	72	7	634175	71
8	615740	84	8	622007	83	8	628184	82	8	634276	81
9	615845	95	9	622110	94	9	628287	92	9	634376	91
4130	615950		4190	622214		4250	628389		4310	634477	
1	616055	11	1	622318	10	1	628491	10	1	634578	10
2	616160	21	2	622421	21	2	628593	20	2	634679	20
3	616265	31	3	622525	31	3	628695	31	3	634779	30
4	616370	42	4	622628	41	4	628797	41	4	634880	40
5	616475	52	5	622732	52	5	628900	51	5	634981	50
6	616580	63	6	622835	62	6	629002	61	6	635081	61
7	616685	73	7	622939	72	7	629104	72	7	635182	71
8	616790	84	8	623042	83	8	629206	82	8	635283	81
9	616895	95	9	623146	93	9	629308	92	9	635383	91
4140	617000		4200	623249		4260	629410		4320	635484	
1	617105	10	1	623353	10	1	629511	10	1	635584	10
2	617210	21	2	623456	21	2	629613	20	2	635685	20
3	617315	31	3	623559	31	3	629715	30	3	635785	30
4	617420	42	4	623663	41	4	629817	41	4	635886	40
5	617524	52	5	623766	52	5	629919	51	5	635986	50
6	617629	63	6	623869	62	6	630021	61	6	636086	60
7	617734	73	7	623972	72	7	630123	71	7	636187	70
8	617839	84	8	624076	83	8	630224	81	8	636287	80
9	617943	94	9	624179	93	9	630326	91	9	636388	90
4150	618048		4210	624282		4270	630428		4330	636488	
1	618153	10	1	624385	10	1	630530	10	1	636588	10
2	618257	21	2	624488	21	2	630631	20	2	636688	20
3	618362	31	3	624591	31	3	630733	30	3	636789	30
4	618466	42	4	624694	41	4	630834	41	4	636889	40
5	618571	52	5	624798	51	5	630936	51	5	636989	50
6	618675	62	6	624901	62	6	631038	61	6	637089	60
7	618780	73	7	625004	72	7	631139	71	7	637189	70
8	618884	83	8	625107	82	8	631241	81	8	637289	80
9	618989	94	9	625209	93	9	631342	91	9	637390	90
4160	619093		4220	625312		4280	631444		4340	637490	
1	619198	10	1	625415	10	1	631545	10	1	637590	10
2	619302	21	2	625518	21	2	631647	20	2	637690	20
3	619406	31	3	625621	31	3	631748	30	3	637790	30
4	619511	42	4	625724	41	4	631849	41	4	637890	40
5	619615	52	5	625827	51	5	631951	51	5	637990	50
6	619719	62	6	625929	62	6	632052	61	6	638090	60
7	619823	73	7	626032	72	7	632153	71	7	638190	70
8	619928	83	8	626135	82	8	632255	81	8	638289	80
9	620032	94	9	626238	93	9	632356	91	9	638389	90
4170	620136		4230	626340		4290	632457		4350	638489	
1	620240	10	1	626443	10	1	632558	10	1	638589	10
2	620344	21	2	626546	21	2	632660	20	2	638689	20
3	620448	31	3	626648	31	3	632761	30	3	638789	30
4	620552	42	4	626751	41	4	632862	41	4	638888	40
5	620656	52	5	626853	51	5	632963	51	5	638988	50
6	620760	62	6	626956	62	6	633064	61	6	639088	60
7	620864	73	7	627058	72	7	633165	71	7	639188	70
8	620968	83	8	627161	82	8	633266	81	8	639287	80
9	621072	94	9	627263	93	9	633367	91	9	639387	90

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
4360	639486		4420	645422		4480	651278		4540	657056	
1	639586	10	1	645520	10	1	651375	10	1	657151	10
2	639686	20	2	645619	20	2	651472	19	2	657247	19
3	639785	30	3	645717	30	3	651569	29	3	657343	28
4	639885	40	4	645815	39	4	651666	38	4	657438	38
5	639984	50	5	645913	49	5	651762	48	5	657534	47
6	640084	60	6	646011	59	6	651859	58	6	657629	57
7	640183	70	7	646109	69	7	651956	67	7	657725	67
8	640283	80	8	646208	79	8	652053	77	8	657820	76
9	640382	90	9	646306	89	9	652150	87	9	657916	86
4370	640481		4430	646404		4490	652246		4550	658011	
1	940581	10	1	646502	10	1	652343	10	1	658107	10
2	640680	20	2	646600	20	2	652440	19	2	658202	19
3	640779	30	3	646698	29	3	652536	29	3	658298	28
4	640879	40	4	646796	39	4	652633	38	4	658393	38
5	640978	50	5	646894	49	5	652730	48	5	658488	47
6	641077	60	6	646991	59	6	652826	58	6	658584	57
7	641176	70	7	647089	69	7	652923	67	7	658679	67
8	641276	80	8	647187	78	8	653019	77	8	658774	76
9	641375	90	9	647285	88	9	653116	87	9	658870	86
4380	641474		4440	647383		4500	653213		4560	658965	
1	641573	10	1	647481	10	1	653309	10	1	659060	10
2	641672	20	2	647579	20	2	653405	19	2	659155	19
3	641771	30	3	647676	29	3	653502	29	3	659250	28
4	641870	40	4	647774	39	4	653598	38	4	659346	38
5	641970	50	5	647872	49	5	653695	48	5	659441	47
6	642069	59	6	647969	59	6	653791	58	6	659536	57
7	642168	69	7	648067	69	7	653888	67	7	659631	67
8	642267	79	8	648165	78	8	653984	77	8	659726	76
9	642366	89	9	648262	88	9	654080	87	9	659821	86
4390	642464		4450	648360		4510	654176		4570	659916	
1	642563	10	1	648458	10	1	654273	10	1	660011	10
2	642662	20	2	648555	19	2	654369	19	2	660106	19
3	642761	30	3	648653	29	3	654465	29	3	660201	28
4	642860	40	4	648750	39	4	654562	38	4	660296	38
5	642959	49	5	648848	49	5	654558	48	5	660391	47
6	643058	59	6	648945	58	6	654754	58	6	660486	57
7	643156	69	7	649043	68	7	654850	67	7	660581	67
8	643255	79	8	649140	78	8	654946	77	8	660676	76
9	643354	89	9	649237	88	9	655042	86	9	660771	86
4400	643453		4460	649335		4520	655138		4580	660865	
1	643551	10	1	649432	10	1	655234	10	1	660960	9
2	643650	20	2	649530	19	2	655331	19	2	661055	19
3	643749	30	3	649627	29	3	655427	29	3	661150	28
4	643847	39	4	649724	39	4	655523	38	4	661245	38
5	643946	49	5	649821	49	5	655619	48	5	661339	47
6	644044	59	6	649919	58	6	655714	58	6	661434	57
7	644143	69	7	650016	68	7	655810	67	7	661529	66
8	644242	79	8	650113	78	8	655906	77	8	661623	76
9	644340	89	9	650210	88	9	656002	86	9	661718	85
4410	644439		4470	650307		4530	656098		4590	661813	
1	644537	10	1	650405	10	1	656194	10	1	661907	9
2	644635	20	2	650502	19	2	656290	19	2	662002	19
3	644734	30	3	650599	29	3	656386	29	3	662096	28
4	644832	39	4	650696	39	4	656481	38	4	662191	38
5	644931	49	5	650793	49	5	656577	48	5	662285	47
6	645029	59	6	650890	58	6	656673	58	6	662380	57
7	645127	69	7	650987	68	7	656769	67	7	662474	66
8	645226	79	8	651084	78	8	656864	77	8	662569	76
9	645324	89	9	651181	88	9	656960	86	9	662663	85

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
4600	662758		4660	668386		4720	673942		4780	679428	
1	662852	9	1	668479	9	1	674034	9	1	679519	9
2	662947	19	2	668572	19	2	674126	18	2	679610	18
3	663041	28	3	668665	28	3	674218	28	3	679700	27
4	663135	38	4	668758	37	4	674310	37	4	679791	36
5	663230	47	5	668852	47	5	674402	46	5	679882	45
6	663324	57	6	668945	56	6	674494	55	6	679973	55
7	663418	66	7	669038	65	7	674586	64	7	680063	64
8	663512	76	8	669131	74	8	674677	74	8	680154	73
9	663607	85	9	669224	84	9	674769	83	9	680245	82
4610	663701		4670	669317		4730	674861		4790	680335	
1	663795	9	1	669410	9	1	674953	9	1	680426	9
2	663889	19	2	669503	19	2	675045	18	2	680517	18
3	663983	28	3	669596	28	3	675136	28	3	680607	27
4	664078	38	4	669689	37	4	675228	37	4	680698	36
5	664172	47	5	669782	47	5	675320	46	5	680789	45
6	664266	56	6	669875	56	6	675412	55	6	680879	55
7	664360	66	7	669967	65	7	675503	64	7	680970	64
8	664454	75	8	670060	74	8	675595	74	8	681060	73
9	664548	85	9	670153	84	9	675687	83	9	681151	82
4620	664642		4680	670246		4740	675778		4800	681241	
1	664736	9	1	670339	9	1	675870	9	1	681332	9
2	664830	19	2	670431	18	2	675962	18	2	681422	18
3	664924	28	3	670524	28	3	676053	27	3	681513	27
4	665018	38	4	670617	37	4	676145	36	4	681603	36
5	665112	47	5	670710	46	5	676236	46	5	681693	45
6	665206	56	6	670802	55	6	676328	55	6	681784	54
7	665299	66	7	670895	64	7	676419	64	7	681874	63
8	665393	75	8	670988	74	8	676511	73	8	681964	72
9	665487	85	9	671080	83	9	676602	82	9	682055	81
4630	665581		4690	671173		4750	676694		4810	682145	
1	665675	9	1	671265	9	1	676785	9	1	682235	9
2	665769	19	2	671358	18	2	676876	18	2	682326	18
3	665862	28	3	671451	28	3	676968	27	3	682416	27
4	665956	38	4	671543	37	4	677059	36	4	682506	36
5	666050	47	5	671636	46	5	677151	46	5	682596	45
6	666143	56	6	671728	55	6	677242	55	6	682686	54
7	666237	66	7	671821	64	7	677333	64	7	682777	63
8	666331	75	8	671913	74	8	677424	73	8	682867	72
9	666424	85	9	672005	83	9	677516	82	9	682957	81
4640	666518		4700	672098		4760	677607		4820	683047	
1	666612	9	1	672190	9	1	677698	9	1	683137	9
2	666705	19	2	672283	18	2	677789	18	2	683227	18
3	666799	28	3	672375	28	3	677881	27	3	683317	27
4	666892	37	4	672467	37	4	677972	36	4	683407	36
5	666986	47	5	672560	46	5	678063	45	5	683497	45
6	667079	56	6	672652	55	6	678154	55	6	683587	54
7	667173	65	7	672744	64	7	678245	64	7	683677	63
8	667266	74	8	672836	74	8	678336	73	8	683767	72
9	667359	84	9	672929	83	9	678427	82	9	683857	81
4650	667453		4710	673021		4770	678518		4830	683947	
1	667546	9	1	673113	9	1	678609	9	1	684037	9
2	667640	19	2	673205	18	2	678700	18	2	684127	18
3	667733	28	3	673297	28	3	678791	27	3	684217	27
4	667826	37	4	673390	37	4	678882	36	4	684307	36
5	667920	47	5	673482	46	5	678973	45	5	684396	45
6	668013	56	6	673574	55	6	679064	55	6	684486	54
7	668106	65	7	673666	64	7	679155	64	7	684576	63
8	668199	74	8	673758	74	8	679246	73	8	684666	72
9	668293	84	9	673850	83	9	679337	82	9	684756	81

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
4840	684845		4900	690196		4960	695482		5020	700704	
1	684935	9	1	690285	9	1	695569	9	1	700790	9
2	685025	18	2	690373	18	2	695657	17	2	700877	17
3	685114	27	3	690462	27	3	695744	26	3	700963	26
4	685204	36	4	690550	35	4	695832	35	4	701050	35
5	685294	45	5	690639	44	5	695919	44	5	701136	43
6	685383	54	6	690727	53	6	696007	52	6	701222	52
7	685473	63	7	690816	62	7	696094	61	7	701309	61
8	685563	72	8	690905	71	8	696182	70	8	701395	70
9	685652	81	9	690993	80	9	696269	79	9	701482	78
4850	685742		4910	691081		4970	696356		5030	701568	
1	685831	9	1	691170	9	1	696444	9	1	701654	9
2	685921	18	2	691258	18	2	696531	17	2	701741	17
3	686010	27	3	691347	27	3	696618	26	3	701827	26
4	686100	36	4	691435	35	4	696706	35	4	701913	35
5	686189	45	5	691524	44	5	696793	44	5	701999	43
6	686279	54	6	691612	53	6	696880	52	6	702086	52
7	686368	63	7	691700	62	7	696968	61	7	702172	61
8	686457	72	8	691789	71	8	697055	70	8	702258	70
9	686547	81	9	691877	80	9	697142	79	9	702344	78
4860	686636		4920	691965		4980	697229		5040	702430	
1	686726	9	1	692053	9	1	697317	9	1	702517	9
2	686815	18	2	692142	18	2	697404	17	2	702603	17
3	686904	27	3	692230	27	3	697491	26	3	702689	26
4	686994	36	4	692318	35	4	697578	35	4	702775	34
5	687083	45	5	692406	44	5	697665	44	5	702861	43
6	687172	54	6	692494	53	6	697752	52	6	702947	52
7	687261	63	7	692583	62	7	697839	61	7	703033	60
8	687351	72	8	692671	71	8	697926	70	8	703119	69
9	687440	81	9	692759	80	9	698013	79	9	703205	77
4870	687529		4930	692847		4990	698100		5050	703291	
1	687618	9	1	692935	9	1	698188	9	1	703377	9
2	687707	18	2	693023	18	2	698275	17	2	703463	17
3	687796	27	3	693111	26	3	698362	26	3	703549	26
4	687886	36	4	693199	35	4	698448	35	4	703635	34
5	687975	45	5	693287	44	5	698535	44	5	703721	43
6	688064	54	6	693375	53	6	698622	52	6	703807	52
7	688153	62	7	693463	62	7	698709	61	7	703893	60
8	688242	72	8	693551	70	8	698796	70	8	703979	69
9	688331	80	9	693639	79	9	698883	79	9	704065	77
4880	688420		4940	693727		5000	698970		5060	704150	
1	688509	9	1	693815	9	1	699057	9	1	704236	9
2	688598	18	2	693903	18	2	699144	17	2	704322	17
3	688687	27	3	693991	26	3	699231	26	3	704408	26
4	688776	36	4	694078	35	4	699317	35	4	704494	34
5	688865	45	5	694166	44	5	699404	43	5	704579	43
6	688953	54	6	694254	53	6	699491	52	6	704665	52
7	689042	62	7	694342	62	7	699578	61	7	704751	60
8	689131	72	8	694430	70	8	699664	70	8	704837	69
9	689220	80	9	694517	79	9	699751	78	9	704922	77
4890	689309		4950	694605		5010	699838		5070	705008	
1	689398	9	1	694693	9	1	699924	9	1	705094	9
2	689486	18	2	694781	18	2	700011	17	2	705179	17
3	689575	27	3	694868	26	3	700098	26	3	705265	26
4	689664	36	4	694956	35	4	700184	35	4	705350	34
5	689753	45	5	695044	44	5	700271	43	5	705436	43
6	689841	54	6	695131	53	6	700358	52	6	705522	52
7	689930	62	7	695219	62	7	700444	61	7	705607	60
8	690019	72	8	695307	70	8	700531	70	8	705693	69
9	690107	80	9	695394	79	9	700617	78	9	705778	77

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
5080	705864		5140	710963		5200	716003		5260	720986	
1	705949	9	1	711048	8	1	716087	8	1	721068	8
2	706035	17	2	711132	17	2	716170	17	2	721151	16
3	706120	26	3	711216	25	3	716254	25	3	721233	25
4	706206	34	4	711301	34	4	716337	34	4	721316	33
5	706291	43	5	711385	42	5	716421	42	5	721398	41
6	706376	51	6	711470	51	6	716504	50	6	721481	49
7	706462	60	7	711554	59	7	716588	59	7	721563	58
8	706547	68	8	711638	68	8	716671	67	8	721646	66
9	706632	77	9	711723	76	9	716754	76	9	721728	74
5090	706718		5150	711807		5210	716838		5270	721811	
1	706803	9	1	711892	8	1	716921	8	1	721893	8
2	706888	17	2	711976	17	2	717004	17	2	721975	16
3	706974	26	3	712060	25	3	717088	25	3	722058	25
4	707059	34	4	712144	34	4	717171	33	4	722140	33
5	707144	43	5	712229	42	5	717254	42	5	722222	41
6	707229	51	6	712313	51	6	717338	50	6	722305	49
7	707315	60	7	712397	59	7	717421	58	7	722387	58
8	707400	68	8	712481	68	8	717504	66	8	722469	66
9	707485	77	9	712566	76	9	717587	75	9	722552	74
5100	707570		5160	712650		5220	717671		5280	722634	
1	707655	9	1	712734	8	1	717754	8	1	722716	8
2	707740	17	2	712818	17	2	717837	17	2	722798	16
3	707826	26	3	712902	25	3	717920	25	3	722881	25
4	707911	34	4	712986	34	4	718003	33	4	722963	33
5	707996	43	5	713070	42	5	718086	42	5	723045	41
6	708081	51	6	713154	50	6	718169	50	6	723127	49
7	708166	60	7	713238	59	7	718253	58	7	723209	58
8	708251	68	8	713322	67	8	718336	66	8	723291	66
9	708336	77	9	713406	76	9	718419	75	9	723374	74
5110	708421		5170	713490		5230	718502		5290	723456	
1	708506	9	1	713574	8	1	718585	8	1	723538	8
2	708591	17	2	713658	17	2	718668	17	2	723620	16
3	708676	26	3	713742	25	3	718751	25	3	723702	25
4	708761	34	4	713826	34	4	718834	33	4	723784	33
5	708846	43	5	713910	42	5	718917	42	5	723866	41
6	708931	51	6	713994	50	6	719000	50	6	723948	49
7	709015	60	7	714078	59	7	719083	58	7	724030	57
8	709100	68	8	714162	67	8	719165	66	8	724112	66
9	709185	77	9	714246	76	9	719248	75	9	724194	74
5120	709270		5180	714330		5240	719331		5300	724276	
1	709355	8	1	714414	8	1	719414	8	1	724358	8
2	709440	17	2	714497	17	2	719497	17	2	724440	16
3	709524	25	3	714581	25	3	719580	25	3	724522	25
4	709609	34	4	714665	34	4	719663	33	4	724603	33
5	709694	42	5	714749	42	5	719745	41	5	724685	41
6	709779	51	6	714832	50	6	719828	50	6	724767	49
7	709863	59	7	714916	59	7	719911	58	7	724849	57
8	709948	68	8	715000	67	8	719994	66	8	724931	66
9	710033	76	9	715084	76	9	720077	75	9	725013	74
5130	710117		5190	715167		5250	720159		5310	725095	
1	710202	8	1	715251	8	1	720242	8	1	725176	8
2	710287	17	2	715335	17	2	720325	17	2	725258	16
3	710371	25	3	715418	25	3	720407	25	3	725340	25
4	710456	34	4	715502	34	4	720490	33	4	725422	33
5	710540	42	5	715586	42	5	720573	41	5	725503	41
6	710625	51	6	715669	50	6	720655	50	6	725585	49
7	710710	59	7	715753	59	7	720738	58	7	725667	57
8	710794	67	8	715836	67	8	720821	66	8	725748	66
9	710879	76	9	715920	76	9	720903	75	9	725830	74

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
5320	725912		5380	730782		5440	735599		5500	740363	
1	725993	8	1	730863	8	1	735679	8	1	740442	8
2	726075	16	2	730944	16	2	735759	16	2	740521	16
3	726156	24	3	731024	24	3	735838	24	3	740599	24
4	726238	33	4	731105	32	4	735918	32	4	740678	32
5	726320	41	5	731186	40	5	735998	40	5	740757	40
6	726401	49	6	731266	49	6	736078	48	6	740836	47
7	726483	57	7	731347	57	7	736157	56	7	740915	55
8	726564	65	8	731428	65	8	736237	64	8	740994	63
9	726646	73	9	731508	73	9	736317	72	9	741073	71
5330	726727		5390	731589		5450	736396		5510	741152	
1	726809	8	1	731669	8	1	736476	8	1	741230	8
2	726890	16	2	731750	16	2	736556	16	2	741309	16
3	726972	24	3	731830	24	3	736635	24	3	741388	24
4	727053	33	4	731911	32	4	736715	32	4	741467	32
5	727134	41	5	731991	40	5	736795	40	5	741546	40
6	727216	49	6	732072	48	6	736874	48	6	741624	47
7	727297	57	7	732152	56	7	736954	56	7	741703	55
8	727379	65	8	732233	64	8	737034	64	8	741782	63
9	727460	73	9	732313	72	9	737113	72	9	741860	71
5340	727541		5400	732394		5460	737193		5520	741939	
1	727623	8	1	732474	8	1	737272	8	1	742018	8
2	727704	16	2	732555	16	2	737352	16	2	742096	16
3	727785	24	3	732635	24	3	737431	24	3	742175	23
4	727866	33	4	732715	32	4	737511	32	4	742254	31
5	727948	41	5	732796	40	5	737590	40	5	742332	39
6	728029	49	6	732876	48	6	737670	48	6	742411	47
7	728110	57	7	732956	56	7	737749	56	7	742489	55
8	728191	65	8	733037	64	8	737829	64	8	742568	63
9	728273	73	9	733117	72	9	737908	72	9	742647	71
5350	728354		5410	733197		5470	737987		5530	742725	
1	728435	8	1	733278	8	1	738067	8	1	742804	8
2	728516	16	2	733358	16	2	738146	16	2	742882	16
3	728597	24	3	733438	24	3	738225	24	3	742961	23
4	728678	33	4	733518	32	4	738305	32	4	743039	31
5	728759	41	5	733598	40	5	738384	40	5	743118	39
6	728841	49	6	733679	48	6	738463	48	6	743196	47
7	728922	57	7	733759	56	7	738543	56	7	743275	55
8	729003	65	8	733839	64	8	738622	64	8	743353	63
9	729084	73	9	733919	72	9	738701	72	9	743431	71
5360	729165		5420	733999		5480	738781		5540	743510	
1	729246	8	1	734079	8	1	738860	8	1	743588	8
2	729327	16	2	734159	16	2	738939	16	2	743667	16
3	729408	24	3	734240	24	3	739018	24	3	743745	23
4	729489	32	4	734320	32	4	739097	32	4	743823	31
5	729570	41	5	734400	40	5	739177	40	5	743902	39
6	729651	49	6	734480	48	6	739256	47	6	743980	47
7	729732	57	7	734560	56	7	739335	55	7	744058	55
8	729813	65	8	734640	64	8	739414	63	8	744136	63
9	729893	73	9	734720	72	9	739493	71	9	744215	71
5370	729974		5430	734800		5490	739572		5550	744293	
1	730055	8	1	734880	8	1	739651	8	1	744371	8
2	730136	16	2	734960	16	2	739730	16	2	744449	16
3	730217	24	3	735040	24	3	739810	24	3	744528	23
4	730298	32	4	735120	32	4	739889	32	4	744606	31
5	730378	40	5	735200	40	5	739968	40	5	744684	39
6	730459	49	6	735279	48	6	740047	47	6	744762	47
7	730540	57	7	735359	56	7	740126	55	7	744840	55
8	730621	65	8	735439	64	8	740205	63	8	744919	63
9	730702	73	9	735519	72	9	740284	71	9	744997	71

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
5560	745075		5620	749736		5680	754348		5740	758912	
1	745153	8	1	749814	8	1	754425	8	1	758988	8
2	745231	16	2	749891	16	2	754501	15	2	759063	15
3	745309	23	3	749968	23	3	754578	23	3	759139	23
4	745387	31	4	750045	31	4	754654	30	4	759214	30
5	745465	39	5	750123	39	5	754730	38	5	759290	38
6	745543	47	6	750200	47	6	754807	46	6	759366	45
7	745621	55	7	750277	54	7	754883	53	7	759441	53
8	745699	62	8	750354	62	8	754960	61	8	759517	60
9	745777	70	9	750431	70	9	755036	69	9	759592	68
5570	745855		5630	750508		5690	755112		5750	759668	
1	745933	8	1	750586	8	1	755189	8	1	759743	8
2	746011	16	2	750663	16	2	755265	15	2	759819	15
3	746089	23	3	750740	23	3	755341	23	3	759894	23
4	746167	31	4	750817	31	4	755417	30	4	759970	30
5	746245	39	5	750894	39	5	755494	38	5	760045	38
6	746323	47	6	750971	47	6	755570	46	6	760121	45
7	746401	55	7	751048	54	7	755646	53	7	760196	53
8	746479	62	8	751125	62	8	755722	61	8	760272	60
9	746556	70	9	751202	70	9	755799	69	9	760347	68
5580	746634		5640	751279		5700	755875		5760	760422	
1	746712	8	1	751356	8	1	755951	8	1	760498	8
2	746790	16	2	751433	15	2	756027	15	2	760573	15
3	746868	23	3	751510	23	3	756103	23	3	760649	23
4	746945	31	4	751587	30	4	756180	30	4	760724	30
5	747023	39	5	751664	38	5	756256	38	5	760799	38
6	747101	47	6	751741	46	6	756332	46	6	760875	45
7	747179	55	7	751818	54	7	756408	53	7	760950	53
8	747256	62	8	751895	62	8	756484	61	8	761025	60
9	747334	70	9	751972	70	9	756560	69	9	761100	68
5590	747412		5650	752048		5710	756636		5770	761176	
1	747489	8	1	752125	8	1	756712	8	1	761251	8
2	747567	16	2	752202	15	2	756788	15	2	761326	15
3	747645	23	3	752279	23	3	756864	23	3	761402	23
4	747722	31	4	752356	30	4	756940	30	4	761477	30
5	747800	39	5	752433	38	5	757016	38	5	761552	38
6	747878	47	6	752509	46	6	757092	46	6	761627	45
7	747955	54	7	752586	54	7	757168	53	7	761702	53
8	748033	62	8	752663	62	8	757244	61	8	761778	60
9	748110	70	9	752740	70	9	757320	69	9	761853	68
5600	748188		5660	752816		5720	757396		5780	761928	
1	748266	8	1	752893	8	1	757472	8	1	762003	8
2	748343	16	2	752970	15	2	757548	15	2	763078	15
3	748421	23	3	753047	23	3	757624	23	3	762153	22
4	748498	31	4	753123	30	4	757700	30	4	762228	30
5	748576	39	5	753200	38	5	757775	38	5	762303	38
6	748653	47	6	753277	46	6	757851	46	6	762378	45
7	748731	54	7	753353	54	7	757927	53	7	762453	52
8	748808	62	8	753430	62	8	758003	61	8	762529	60
9	748885	70	9	753506	70	9	758079	68	9	762604	68
5610	748963		5670	753583		5730	758155		5790	762679	
1	749040	8	1	753660	8	1	758230	8	1	762754	8
2	749118	16	2	753736	15	2	758306	15	2	762829	15
3	749195	23	3	753813	23	3	758382	23	3	762904	22
4	749272	31	4	753889	30	4	758458	30	4	762978	30
5	749350	39	5	753966	38	5	758533	38	5	763053	38
6	749427	47	6	754042	46	6	758609	46	6	763128	45
7	749504	54	7	754119	54	7	758685	53	7	763203	52
8	749582	62	8	754195	62	8	758760	61	8	763278	60
9	749659	70	9	754272	70	9	758836	68	9	763353	68

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
5800	763428		5860	767898		5920	772322		5980	776701	
1	763503	7	1	767972	7	1	772395	7	1	776774	7
2	763578	15	2	768046	15	2	772468	15	2	776846	14
3	763653	22	3	768120	22	3	772542	22	3	776919	22
4	763727	30	4	768194	30	4	772615	29	4	776992	29
5	763802	37	5	768268	37	5	772688	37	5	777064	36
6	763877	45	6	768342	45	6	772762	44	6	777137	43
7	763952	52	7	768416	52	7	772835	51	7	777209	51
8	764027	60	8	768490	59	8	772908	59	8	777282	58
9	764101	67	9	768564	67	9	772981	66	9	777354	65
5810	764176		5870	768638		5930	773055		5990	777427	
1	764251	7	1	768712	7	1	773128	7	1	777499	7
2	764326	15	2	768786	15	2	773201	15	2	777572	14
3	764400	22	3	768860	22	3	773274	22	3	777644	22
4	764475	30	4	768934	30	4	773348	29	4	777717	29
5	764550	37	5	769008	37	5	773421	37	5	777789	36
6	764624	45	6	769082	45	6	773494	44	6	777862	43
7	764699	52	7	769156	52	7	773567	51	7	777934	51
8	764774	60	8	769230	59	8	773640	59	8	778006	58
9	764848	67	9	769303	67	9	773713	66	9	778079	65
5820	764923		5880	769377		5940	773786		6000	778151	
1	764998	7	1	769451	7	1	773860	7	1	778224	7
2	765072	15	2	769525	15	2	773933	15	2	778296	14
3	765147	22	3	769599	22	3	774006	22	3	778368	22
4	765221	30	4	769673	30	4	774079	29	4	778441	29
5	765296	37	5	769746	37	5	774152	37	5	778513	36
6	765370	45	6	769820	45	6	774225	44	6	778585	43
7	765445	52	7	769894	52	7	774298	51	7	778658	51
8	765520	60	8	769968	59	8	774371	59	8	778730	58
9	765594	67	9	770042	67	9	774444	66	9	778802	65
5830	765669		5890	770115		5950	774517		6010	778874	
1	765743	7	1	770189	7	1	774590	7	1	778947	7
2	765818	15	2	770263	15	2	774663	15	2	779019	14
3	765892	22	3	770336	22	3	774736	22	3	779091	22
4	765966	30	4	770410	30	4	774809	29	4	779163	29
5	766041	37	5	770484	37	5	774882	37	5	779236	36
6	766115	45	6	770557	45	6	774955	44	6	779308	43
7	766190	52	7	770631	52	7	775028	51	7	779380	51
8	766264	60	8	770705	59	8	775100	59	8	779452	58
9	766338	67	9	770778	67	9	775173	66	9	779524	65
5840	766413		5900	770852		5960	775246		6020	779596	
1	766487	7	1	770926	7	1	775319	7	1	779669	7
2	766562	15	2	770999	15	2	775392	15	2	779741	14
3	766636	22	3	771073	22	3	775465	22	3	779813	22
4	766710	30	4	771146	30	4	775538	29	4	779885	29
5	766785	37	5	771220	37	5	775610	37	5	779957	36
6	766859	45	6	771293	45	6	775683	44	6	780029	43
7	766933	52	7	771367	52	7	775756	51	7	780101	50
8	767007	60	8	771440	59	8	775829	59	8	780173	58
9	767082	67	9	771514	67	9	775902	66	9	780245	65
5850	767156		5910	771587		5970	775974		6030	780317	
1	767230	7	1	771661	7	1	776047	7	1	780389	7
2	767304	15	2	771734	15	2	776120	15	2	780461	14
3	767379	22	3	771808	22	3	776193	22	3	780533	22
4	767453	30	4	771881	30	4	776265	29	4	780605	29
5	767527	37	5	771955	37	5	776338	37	5	780677	36
6	767601	45	6	772028	44	6	776411	44	6	780749	43
7	767675	52	7	772102	52	7	776483	51	7	780821	50
8	767749	59	8	772175	59	8	776556	59	8	780893	58
9	767823	67	9	772248	67	9	776629	66	9	780965	65

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
6040	781037		6100	785330		6160	789581		6220	793790	
1	781109	7	1	785401	7	1	789651	7	1	793860	7
2	781181	14	2	785472	14	2	789722	14	2	793930	14
3	781253	22	3	785543	21	3	789792	21	3	794000	21
4	781324	29	4	785615	28	4	789863	28	4	794070	28
5	781396	36	5	785686	36	5	789933	35	5	794139	35
6	781468	43	6	785757	43	6	790004	42	6	794209	42
7	781540	50	7	785828	50	7	790074	49	7	794279	49
8	781612	58	8	785899	57	8	790144	56	8	794349	56
9	781684	65	9	785970	64	9	790215	63	9	794418	63
6050	781755		6110	786041		6170	790285		6230	794488	
1	781827	7	1	786112	7	1	790356	7	1	794558	7
2	781899	14	2	786183	14	2	790426	14	2	794627	14
3	781971	22	3	786254	21	3	790496	21	3	794697	21
4	782042	29	4	786325	28	4	790567	28	4	794767	28
5	782114	36	5	786396	36	5	790637	35	5	794836	35
6	782186	43	6	786467	43	6	790707	42	6	794906	42
7	782258	50	7	786538	50	7	790778	49	7	794976	49
8	782329	58	8	786609	57	8	790848	56	8	795045	56
9	782401	65	9	786680	64	9	790918	63	9	795115	63
6060	782473		6120	786751		6180	790988		6240	795185	
1	782544	7	1	786822	7	1	791059	7	1	795254	7
2	782616	14	2	786893	14	2	791129	14	2	795324	14
3	782688	21	3	786964	21	3	791199	21	3	795393	21
4	782759	29	4	787035	28	4	791269	28	4	795463	28
5	782831	36	5	787106	36	5	791340	35	5	795532	35
6	782902	43	6	787177	43	6	791410	42	6	795602	42
7	782974	50	7	787248	50	7	791480	49	7	795671	49
8	783046	57	8	787319	57	8	791550	56	8	795741	56
9	783117	64	9	787390	64	9	791620	63	9	795810	63
6070	783189		6130	787460		6190	791691		6250	795880	
1	783260	7	1	787531	7	1	791761	7	1	795949	7
2	783332	14	2	787602	14	2	791831	14	2	796019	14
3	783403	21	3	787673	21	3	791901	21	3	796088	21
4	783475	29	4	787744	28	4	791971	28	4	796158	28
5	783546	36	5	787815	35	5	792041	35	5	796227	35
6	783618	43	6	787885	42	6	792111	42	6	796297	42
7	783689	50	7	787956	49	7	792181	49	7	796366	49
8	783761	57	8	788027	56	8	792252	56	8	796436	56
9	783832	64	9	788098	63	9	792322	63	9	796505	63
6080	783904		6140	788168		6200	792392		6260	796574	
1	783975	7	1	788239	7	1	792462	7	1	796644	7
2	784046	14	2	788310	14	2	792532	14	2	796713	14
3	784118	21	3	788381	21	3	792602	21	3	796782	21
4	784189	29	4	788451	28	4	792672	28	4	796852	27
5	784261	36	5	788522	35	5	792742	35	5	796921	35
6	784332	43	6	788593	42	6	792812	42	6	796990	42
7	784403	50	7	788663	49	7	792882	49	7	797060	49
8	784475	57	8	788734	56	8	792952	56	8	797129	56
9	784546	64	9	788804	63	9	793022	63	9	797198	62
6090	784617		6150	788875		6210	793092		6270	797268	
1	784689	7	1	788946	7	1	793162	7	1	797337	7
2	784760	14	2	789016	14	2	793231	14	2	797406	14
3	784831	21	3	789087	21	3	793301	21	3	797475	21
4	784902	29	4	789157	28	4	793371	28	4	797545	27
5	784974	36	5	789228	35	5	793441	35	5	797614	35
6	785045	43	6	789299	42	6	793511	42	6	797683	42
7	785116	50	7	789369	49	7	793581	49	7	797752	49
8	785187	57	8	789440	56	8	793651	56	8	797821	56
9	785259	64	9	789510	63	9	793721	63	9	797890	62

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop Part
6280	797960		6340	802089		6400	806180		6460	810233	
1	798029	7	1	802158	7	1	806248	7	1	810300	7
2	798098	14	2	802226	14	2	806316	14	2	810367	13
3	798167	21	3	802295	21	3	806384	20	3	810434	20
4	798236	28	4	802363	27	4	806451	27	4	810501	27
5	798305	34	5	802432	34	5	806519	34	5	810569	33
6	798374	41	6	802500	41	6	806587	41	6	810636	40
7	798443	48	7	802568	48	7	806655	48	7	810703	47
8	798512	55	8	802637	55	8	806723	54	8	810770	54
9	798582	62	9	802705	62	9	806790	61	9	810837	60
6290	798651		6350	802774		6410	806858		6470	810904	
1	798720	7	1	802842	7	1	806926	7	1	810971	7
2	798789	14	2	802910	14	2	806994	14	2	811038	13
3	798858	21	3	802979	21	3	807061	20	3	811106	20
4	798927	28	4	803047	27	4	807129	27	4	811173	27
5	798996	34	5	803116	34	5	807197	34	5	811240	33
6	799065	41	6	803184	41	6	807264	41	6	811307	40
7	799134	48	7	803252	48	7	807332	48	7	811374	47
8	799203	55	8	803320	55	8	807400	54	8	811441	54
9	799272	62	9	803389	62	9	807467	61	9	811508	60
6300	799341		6360	803457		6420	807535		6480	811575	
1	799409	7	1	803525	7	1	807603	7	1	811642	7
2	799478	14	2	803594	14	2	807670	14	2	811709	13
3	799547	21	3	803662	21	3	807738	20	3	811776	20
4	799616	28	4	803730	27	4	807806	27	4	811843	27
5	799685	34	5	803798	34	5	807873	34	5	811910	33
6	799754	41	6	803867	41	6	807941	41	6	811977	40
7	799823	48	7	803935	48	7	808008	48	7	812044	47
8	799892	55	8	804003	55	8	808076	54	8	812111	54
9	799961	62	9	804071	62	9	808143	61	9	812178	60
6310	800029		6370	804139		6430	808211		6490	812245	
1	800098	7	1	804208	7	1	808279	7	1	812312	7
2	800167	14	2	804276	14	2	808346	14	2	812378	13
3	800236	21	3	804344	21	3	808414	20	3	812445	20
4	800305	28	4	804412	27	4	808481	27	4	812512	27
5	800373	34	5	804480	34	5	808549	34	5	812579	33
6	800442	41	6	804548	41	6	808616	41	6	812646	40
7	800511	48	7	804616	48	7	808684	48	7	812713	47
8	800580	55	8	804685	55	8	808751	54	8	812780	54
9	800648	62	9	804753	62	9	808818	61	9	812847	60
6320	800717		6380	804821		6440	808886		6500	812913	
1	800786	7	1	804889	7	1	808953	7	1	812980	7
2	800854	14	2	804957	14	2	809021	13	2	813047	13
3	800923	21	3	805025	20	3	809088	20	3	813114	20
4	800992	28	4	805093	27	4	809156	27	4	813181	27
5	801060	34	5	805161	34	5	809223	34	5	813247	33
6	801129	41	6	805229	41	6	809290	40	6	813314	40
7	801198	48	7	805297	48	7	809358	47	7	813381	47
8	801266	55	8	805365	54	8	809425	54	8	813448	54
9	801335	62	9	805433	61	9	809492	61	9	813514	60
6330	801404		6390	805501		6450	809560		6510	813581	
1	801472	7	1	805569	7	1	809627	7	1	813648	7
2	801541	14	2	805637	14	2	809694	13	2	813714	13
3	801609	21	3	805705	20	3	809762	20	3	813781	20
4	801678	27	4	805773	27	4	809829	27	4	813848	27
5	801747	34	5	805841	34	5	809896	34	5	813914	33
6	801815	41	6	805908	41	6	809964	40	6	813981	40
7	801884	48	7	805976	48	7	810031	47	7	814048	47
8	801952	55	8	806044	54	8	810098	54	8	814114	54
9	802021	62	9	806112	61	9	810165	61	9	814181	60

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
6520	814248		6580	818226		6640	822168		6700	826075	
1	814314	7	1	818292	7	1	822233	7	1	826140	6
2	814381	13	2	818358	13	2	822299	13	2	826204	13
3	814447	20	3	818424	20	3	822364	20	3	826269	19
4	814514	26	4	818490	26	4	822430	26	4	826334	26
5	814581	33	5	818556	33	5	822495	33	5	826399	32
6	814647	40	6	818622	40	6	822560	39	6	826464	39
7	814714	46	7	818688	46	7	822626	46	7	826528	45
8	814780	53	8	818754	53	8	822691	52	8	826593	52
9	814847	60	9	818819	59	9	822756	59	9	826658	58
6530	814913		6590	818885		6650	822823		6710	826722	
1	814980	7	1	818951	7	1	822887	7	1	826787	6
2	815046	13	2	819017	13	2	822952	13	2	826852	13
3	815113	20	3	819083	20	3	823018	20	3	826917	19
4	815179	26	4	819149	26	4	823083	26	4	826981	26
5	815246	33	5	819215	33	5	823148	33	5	827046	32
6	815312	40	6	819281	40	6	823213	39	6	827111	39
7	815378	46	7	819346	46	7	823279	46	7	827175	45
8	815445	53	8	819412	53	8	823344	52	8	827240	52
9	815511	60	9	819478	59	9	823409	59	9	827305	58
6540	815578		6600	819544		6660	823474		6720	827369	
1	815644	7	1	819610	7	1	823539	7	1	827434	6
2	815711	13	2	819675	13	2	823605	13	2	827498	13
3	815777	20	3	819741	20	3	823670	20	3	827563	19
4	815843	26	4	819807	26	4	823735	26	4	827628	26
5	815910	33	5	819873	33	5	823800	33	5	827692	32
6	815976	40	6	819939	40	6	823865	39	6	827757	39
7	816042	46	7	820004	46	7	823930	46	7	827821	45
8	816109	53	8	820070	53	8	823996	52	8	827886	52
9	816175	60	9	820136	59	9	824061	59	9	827951	58
6550	816241		6610	820201		6670	824126		6730	828015	
1	816308	7	1	820267	7	1	824191	6	1	828080	6
2	816374	13	2	820333	13	2	824256	13	2	828144	13
3	816440	20	3	820399	20	3	824321	19	3	828209	19
4	816506	26	4	820464	26	4	824386	26	4	828273	26
5	816573	33	5	820530	33	5	824451	32	5	828338	32
6	816639	40	6	820595	40	6	824516	39	6	828402	39
7	816705	46	7	820661	46	7	824581	45	7	828467	45
8	816771	53	8	820727	53	8	824646	52	8	828531	52
9	816838	60	9	820792	59	9	824711	58	9	828595	58
6560	816904		6620	820858		6680	824776		6740	828660	
1	816970	7	1	820924	7	1	824841	6	1	828724	6
2	817036	13	2	820989	13	2	824906	13	2	828789	13
3	817102	20	3	821055	20	3	824971	19	3	828853	19
4	817169	26	4	821120	26	4	825036	26	4	828918	26
5	817235	33	5	821186	33	5	825101	32	5	828982	32
6	817301	40	6	821251	40	6	825166	39	6	829046	39
7	817367	46	7	821317	46	7	825231	45	7	829111	45
8	817433	53	8	821382	53	8	825296	52	8	829175	52
9	817499	59	9	821448	59	9	825361	58	9	829239	58
6570	817565		6630	821514		6690	825426		6750	829304	
1	817631	7	1	821579	7	1	825491	6	1	829368	6
2	817698	13	2	821644	13	2	825556	13	2	829432	13
3	817764	20	3	821710	20	3	825621	19	3	829497	19
4	817830	26	4	821775	26	4	825686	26	4	829561	26
5	817896	33	5	821841	33	5	825751	32	5	829625	32
6	817962	40	6	821906	39	6	825815	39	6	829690	39
7	818028	46	7	821972	46	7	825880	45	7	829754	45
8	818094	53	8	822037	52	8	825945	52	8	829818	52
9	818160	59	9	822103	59	9	826010	58	9	829882	58

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log	Prop. Part.	No.	Log.	Prop. Part.
6760	829947		6820	833784		6880	837588		6940	841359	
1	830011	6	1	833848	6	1	837652	6	1	841422	6
2	830075	13	2	833912	13	2	837715	13	2	841485	13
3	830139	19	3	833975	19	3	837778	19	3	841547	19
4	830204	26	4	834039	26	4	837841	25	4	841610	25
5	830268	32	5	834103	32	5	837904	32	5	841672	31
6	830332	38	6	834166	38	6	837967	38	6	841735	38
7	830396	45	7	834230	45	7	838030	44	7	841797	44
8	830460	51	8	834293	51	8	838093	50	8	841860	50
9	830525	58	9	834357	58	9	838156	57	9	841922	56
6770	830589		6830	834421		6890	838219		6950	841985	
1	830653	6	1	834484	6	1	838282	6	1	842047	6
2	830717	13	2	834548	13	2	838345	13	2	842110	12
3	830781	19	3	834611	19	3	838408	19	3	842172	19
4	830845	26	4	834675	26	4	838471	25	4	842235	25
5	830909	32	5	834739	32	5	838534	32	5	842297	31
6	830973	38	6	834802	38	6	838597	38	6	842360	37
7	831037	45	7	834866	45	7	838660	44	7	842422	44
8	831102	51	8	834929	51	8	838723	50	8	842484	50
9	831166	58	9	834993	58	9	838786	57	9	842547	56
6780	831230		6840	835056		6900	838849		6960	842609	
1	831294	6	1	835120	6	1	838912	6	1	842672	6
2	831358	13	2	835183	13	2	838975	13	2	842734	12
3	831422	19	3	835247	19	3	839038	19	3	842796	19
4	831486	26	4	835310	26	4	839101	25	4	842859	25
5	831550	32	5	835373	32	5	839164	31	5	842921	31
6	831614	38	6	835437	38	6	839227	38	6	842983	37
7	831678	45	7	835500	45	7	839289	44	7	843046	44
8	831742	51	8	835564	51	8	839352	50	8	843108	50
9	831806	58	9	835627	58	9	839415	57	9	843170	56
6790	831870		6850	835691		6910	839478		6970	843233	
1	831934	6	1	835754	6	1	839541	6	1	843295	6
2	831998	13	2	835817	13	2	839604	13	2	843357	12
3	832062	19	3	835881	19	3	839667	19	3	843420	19
4	832126	26	4	835944	26	4	839729	25	4	843482	25
5	832189	32	5	836007	32	5	839792	31	5	843544	31
6	832253	38	6	836071	38	6	839855	38	6	843606	37
7	832317	45	7	836134	45	7	839918	44	7	843669	43
8	832381	51	8	836197	51	8	839981	50	8	843731	50
9	832445	58	9	836261	58	9	840043	57	9	843793	56
6800	832509		6860	836324		6920	840106		6980	843855	
1	832573	6	1	836387	6	1	840169	6	1	843918	6
2	832637	13	2	836451	13	2	840232	13	2	843980	12
3	832700	19	3	836514	19	3	840294	19	3	844042	19
4	832764	26	4	836577	26	4	840357	25	4	844104	25
5	832828	32	5	836641	32	5	840420	31	5	844166	31
6	832892	38	6	836704	38	6	840482	38	6	844229	37
7	832956	45	7	836767	45	7	840545	44	7	844291	43
8	833020	51	8	836830	51	8	840608	50	8	844353	50
9	833083	58	9	836894	58	9	840671	57	9	844415	56
6810	833147		6870	836957		6930	840733		6990	844477	
1	833211	6	1	837020	6	1	840796	6	1	844539	6
2	833275	13	2	837083	13	2	840859	13	2	844601	12
3	833338	19	3	837146	19	3	840921	19	3	844664	19
4	833402	26	4	837210	25	4	840984	25	4	844726	25
5	833466	32	5	837273	32	5	841046	31	5	844788	31
6	833530	38	6	837336	38	6	841109	38	6	844850	37
7	833593	45	7	837399	44	7	841172	44	7	844912	43
8	833657	51	8	837462	51	8	841234	50	8	844974	50
9	833721	58	9	837525	57	9	841297	56	9	845036	56

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
7000	845098		7060	848805		7120	852480		7180	856124	
1	845160	6	1	848866	6	1	852541	6	1	856185	6
2	845222	12	2	848928	12	2	852602	12	2	856245	12
3	845284	19	3	848989	18	3	852663	18	3	856306	18
4	845346	25	4	849051	25	4	852724	24	4	856366	24
5	845408	31	5	849112	31	5	852785	30	5	856427	30
6	845470	37	6	849174	37	6	852846	37	6	856487	36
7	845532	43	7	849235	43	7	852907	43	7	856548	42
8	845594	50	8	849296	49	8	852968	49	8	856608	48
9	845656	56	9	849358	55	9	853029	55	9	856668	54
7010	845718		7070	849419		7130	853090		7190	856729	
1	845780	6	1	849481	6	1	853150	6	1	856789	6
2	845842	12	2	849542	12	2	853211	12	2	856850	12
3	845904	19	3	849604	18	3	853272	18	3	856910	18
4	845966	25	4	849665	25	4	853333	24	4	856970	24
5	846028	31	5	849726	31	5	853394	30	5	857031	30
6	846090	37	6	849788	37	6	853455	37	6	857091	36
7	846151	43	7	849849	43	7	853516	43	7	857151	42
8	846213	50	8	849911	49	8	853576	49	8	857212	48
9	846275	56	9	849972	55	9	853637	55	9	857272	54
7020	846337		7080	850033		7140	853698		7200	857332	
1	846399	6	1	850095	6	1	853759	6	1	857393	6
2	846461	12	2	850156	12	2	853820	12	2	857453	12
3	846523	19	3	850217	18	3	853881	18	3	857513	18
4	846584	25	4	850279	25	4	853941	24	4	857574	24
5	846646	31	5	850340	31	5	854002	30	5	857634	30
6	846708	37	6	850401	37	6	854063	37	6	857694	36
7	846770	43	7	850462	43	7	854124	43	7	857754	42
8	846832	50	8	850524	49	8	854185	49	8	857815	48
9	846894	56	9	850585	55	9	854245	55	9	857875	54
7030	846955		7090	850646		7150	854306		7210	857935	
1	847017	6	1	850707	6	1	854367	6	1	857995	6
2	847079	12	2	850769	12	2	854427	12	2	858056	12
3	847141	19	3	850830	18	3	854488	18	3	858116	18
4	847202	25	4	850891	25	4	854549	24	4	858176	24
5	847264	31	5	850952	31	5	854610	30	5	858236	30
6	847326	37	6	851014	37	6	854670	36	6	858297	36
7	847388	43	7	851075	43	7	854731	42	7	858357	42
8	847449	50	8	851136	49	8	854792	48	8	858417	48
9	847511	56	9	851197	55	9	854852	54	9	858477	54
7040	847573		7100	851258		7160	854913		7220	858537	
1	847634	6	1	851320	6	1	854974	6	1	858597	6
2	847696	12	2	851381	12	2	855034	12	2	858657	12
3	847758	18	3	851442	18	3	855095	18	3	858718	18
4	847819	25	4	851503	25	4	855156	24	4	858778	24
5	847881	31	5	851564	31	5	855216	30	5	858838	30
6	847943	37	6	851625	37	6	855277	36	6	858898	36
7	848004	43	7	851686	43	7	855337	42	7	858958	42
8	848066	49	8	851747	49	8	855398	48	8	859018	48
9	848127	55	9	851808	55	9	855459	54	9	859078	54
7050	848189		7110	851870		7170	855519		7230	859138	
1	848251	6	1	851931	6	1	855580	6	1	859198	6
2	848312	12	2	851992	12	2	855640	12	2	859258	12
3	848374	18	3	852053	18	3	855701	18	3	859318	18
4	848435	25	4	852114	25	4	855761	24	4	859378	24
5	848497	31	5	852175	31	5	855822	30	5	859438	30
6	848559	37	6	852236	37	6	855882	36	6	859499	36
7	848620	43	7	852297	43	7	855943	42	7	859559	42
8	848682	49	8	852358	49	8	856003	48	8	859619	48
9	848743	55	9	852419	55	9	856064	54	9	859679	54

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
7240	859739		7300	863323		7360	866878		7420	870404	
1	859799	6	1	863382	6	1	866937	6	1	870462	6
2	859858	12	2	863442	12	2	866996	12	2	870521	12
3	859918	18	3	863501	18	3	867055	18	3	870579	18
4	859978	24	4	863561	24	4	867114	24	4	870638	24
5	860038	30	5	863620	30	5	867173	29	5	870696	29
6	860098	36	6	863680	36	6	867232	35	6	870755	35
7	860158	42	7	863739	42	7	867291	41	7	870813	41
8	860218	48	8	863798	48	8	867350	47	8	870872	47
9	860278	54	9	863858	54	9	867409	53	9	870930	53
7250	860338		7310	863917		7370	867467		7430	870989	
1	860398	6	1	863977	6	1	867526	6	1	871047	6
2	860458	12	2	864036	12	2	867585	12	2	871106	12
3	860518	18	3	864096	18	3	867644	18	3	871164	18
4	860578	24	4	864155	24	4	867703	24	4	871223	24
5	860637	30	5	864214	30	5	867762	29	5	871281	29
6	860697	36	6	864274	36	6	867821	35	6	871339	35
7	860757	42	7	864333	42	7	867880	41	7	871398	41
8	860817	48	8	864392	48	8	867939	47	8	871456	47
9	860877	54	9	864452	54	9	867998	53	9	871515	53
7260	860937		7320	864511		7380	868056		7440	871573	
1	860996	6	1	864570	6	1	868115	6	1	871631	6
2	861056	12	2	864630	12	2	868174	12	2	871690	12
3	861116	18	3	864689	18	3	868233	18	3	871748	18
4	861176	24	4	864748	24	4	868292	24	4	871806	23
5	861236	30	5	864808	30	5	868350	29	5	871865	29
6	861295	36	6	864867	36	6	868409	35	6	871923	35
7	861355	42	7	864926	42	7	868468	41	7	871981	41
8	861415	48	8	864985	48	8	868527	47	8	872040	47
9	861475	54	9	865045	54	9	868586	53	9	872098	53
7270	861534		7330	865104		7390	868644		7450	872156	
1	861594	6	1	865163	6	1	868703	6	1	872215	6
2	861654	12	2	865222	12	2	868762	12	2	872273	12
3	861714	18	3	865282	18	3	868821	18	3	872331	18
4	861773	24	4	865341	24	4	868879	24	4	872389	23
5	861833	30	5	865400	30	5	868938	29	5	872448	29
6	861893	36	6	865459	36	6	868997	35	6	872506	35
7	861952	42	7	865518	42	7	869056	41	7	872564	41
8	862012	48	8	865578	48	8	869114	47	8	872622	47
9	862072	54	9	865637	54	9	869173	53	9	872681	53
7280	862131		7340	865696		7400	869232		7460	872739	
1	862191	6	1	865755	6	1	869290	6	1	872797	6
2	862251	12	2	865814	12	2	869349	12	2	872855	12
3	862310	18	3	865874	18	3	869408	18	3	872913	18
4	862370	24	4	865933	24	4	869466	24	4	872972	23
5	862430	30	5	865992	30	5	869525	29	5	873030	29
6	862489	36	6	866051	36	6	869584	35	6	873088	35
7	862549	42	7	866110	42	7	869642	41	7	873146	41
8	862608	48	8	866169	48	8	869701	47	8	873204	47
9	862668	54	9	866228	54	9	869760	53	9	873262	53
7290	862728		7350	866287		7410	869818		7470	873321	
1	862787	6	1	866346	6	1	869877	6	1	873379	6
2	862847	12	2	866405	12	2	869935	12	2	873437	12
3	862906	18	3	866465	18	3	869994	18	3	873495	18
4	862966	24	4	866524	24	4	870053	24	4	873553	23
5	863025	30	5	866583	30	5	870111	29	5	873611	29
6	863085	36	6	866642	35	6	870170	35	6	873669	35
7	863144	42	7	866701	41	7	870228	41	7	873727	41
8	863204	48	8	866760	47	8	870287	47	8	873785	47
9	863263	54	9	866819	53	9	870345	53	9	873844	53

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
7480	873902		7540	877371		7600	880814		7660	884229	
1	873960	6	1	877429	6	1	880871	6	1	884285	6
2	874018	12	2	877486	12	2	880928	11	2	884342	11
3	874076	17	3	877544	17	3	880985	17	3	884399	17
4	874134	23	4	877602	23	4	881042	23	4	884455	23
5	874192	29	5	877659	29	5	881099	28	5	884512	28
6	874250	35	6	877717	34	6	881156	34	6	884569	34
7	874308	41	7	877774	40	7	881213	40	7	884625	40
8	874366	46	8	877832	46	8	881270	46	8	884682	46
9	874424	52	9	877889	52	9	881328	51	9	884739	51
7490	874482		7550	877947		7610	881385		7670	884795	
1	874540	6	1	878004	6	1	881442	6	1	884852	6
2	874598	12	2	878062	12	2	881499	11	2	884909	11
3	874656	17	3	878119	17	3	881556	17	3	884965	17
4	874714	23	4	878177	23	4	881613	23	4	885022	23
5	874772	29	5	878234	29	5	881670	28	5	885078	28
6	874830	35	6	878292	34	6	881727	34	6	885135	34
7	874887	41	7	878349	40	7	881784	40	7	885192	40
8	874945	46	8	878407	46	8	881841	46	8	885248	46
9	875003	52	9	878464	52	9	881898	51	9	885305	51
7500	875061		7560	878522		7620	881955		7680	885361	
1	875119	6	1	878579	6	1	882012	6	1	885418	6
2	875177	12	2	878637	12	2	882069	11	2	885474	11
3	875235	17	3	878694	17	3	882126	17	3	885531	17
4	875293	23	4	878751	23	4	882183	23	4	885587	23
5	875351	29	5	878809	29	5	882240	28	5	885644	28
6	875409	35	6	878866	34	6	882297	34	6	885700	34
7	875466	41	7	878924	40	7	882354	40	7	885757	39
8	875524	46	8	878981	46	8	882411	46	8	885813	45
9	875582	52	9	879038	52	9	882468	51	9	885870	51
7510	875640		7570	879096		7630	882524		7690	885926	
1	875698	6	1	879153	6	1	882581	6	1	885983	6
2	875756	12	2	879211	12	2	882638	11	2	886039	11
3	875813	17	3	879268	17	3	882695	17	3	886096	17
4	875871	23	4	879325	23	4	882752	23	4	886152	23
5	875929	29	5	879383	29	5	882809	28	5	886209	28
6	875987	35	6	879440	34	6	882866	34	6	886265	34
7	876045	41	7	879497	40	7	882923	40	7	886321	39
8	876102	46	8	879555	46	8	882980	46	8	886378	45
9	876160	52	9	879612	52	9	883037	51	9	886434	51
7520	876218		7580	879669		7640	883093		7700	886491	
1	876276	6	1	879726	6	1	883150	6	1	886547	6
2	876333	12	2	879784	11	2	883207	11	2	886604	11
3	876391	17	3	879841	17	3	883264	17	3	886660	17
4	876449	23	4	879898	23	4	883321	23	4	886716	23
5	876507	29	5	879956	28	5	883377	28	5	886773	28
6	876564	34	6	880013	34	6	883434	34	6	886829	34
7	876622	40	7	880070	40	7	883491	40	7	886885	39
8	876680	46	8	880127	46	8	883548	46	8	886942	45
9	876737	52	9	880185	51	9	883605	51	9	886998	51
7530	876795		7590	880242		7650	883661		7710	887054	
1	876853	6	1	880299	6	1	883718	6	1	887111	6
2	876910	12	2	880356	11	2	883775	11	2	887167	11
3	876968	17	3	880413	17	3	883832	17	3	887223	17
4	877026	23	4	880471	23	4	883888	23	4	887280	23
5	877083	29	5	880528	28	5	883945	28	5	887336	28
6	877141	34	6	880585	34	6	884002	34	6	887392	34
7	877198	40	7	880642	40	7	884059	40	7	887449	39
8	877256	46	8	880699	46	8	884115	46	8	887505	45
9	877314	52	9	880756	51	9	884172	51	9	887561	51

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
7720	887617		7780	890980		7840	894316		7900	897627	
1	887674	6	1	891035	6	1	894371	6	1	897682	6
2	887730	11	2	891091	11	2	894427	11	2	897737	11
3	887786	17	3	891147	17	3	894482	17	3	897792	17
4	887842	23	4	891203	22	4	894538	22	4	897847	22
5	887898	28	5	891259	28	5	894593	27	5	897902	27
6	887955	34	6	891314	34	6	894648	33	6	897957	33
7	888011	39	7	891370	39	7	894704	39	7	898012	39
8	888067	45	8	891426	45	8	894759	44	8	898067	44
9	888123	51	9	891482	50	9	894814	50	9	898122	50
7730	888179		7790	891537		7850	894870		7910	898176	
1	888236	6	1	891593	6	1	894925	6	1	898231	6
2	888292	11	2	891649	11	2	894980	11	2	898286	11
3	888348	17	3	891705	17	3	895036	17	3	898341	17
4	888404	22	4	891760	22	4	895091	22	4	898396	22
5	888460	28	5	891816	28	5	895146	27	5	898451	27
6	888516	34	6	891872	33	6	895201	33	6	898506	33
7	888573	39	7	891928	39	7	895257	39	7	898561	39
8	888629	45	8	891983	44	8	895312	44	8	898615	44
9	888685	50	9	892039	50	9	895367	50	9	898670	50
7740	888741		7800	892095		7860	895423		7920	898725	
1	888797	6	1	892150	6	1	895478	6	1	898780	5
2	888853	11	2	892206	11	2	895533	11	2	898835	11
3	888909	17	3	892262	17	3	895588	17	3	898890	17
4	888965	22	4	892317	22	4	895643	22	4	898944	22
5	889021	28	5	892373	28	5	895699	27	5	898999	27
6	889077	34	6	892429	33	6	895754	33	6	899054	33
7	889134	39	7	892484	39	7	895809	39	7	899109	38
8	889190	45	8	892540	44	8	895864	44	8	899164	44
9	889246	50	9	892595	50	9	895920	50	9	899218	50
7750	889302		7810	892651		7870	895975		7930	899273	
1	889358	6	1	892707	6	1	896030	6	1	899328	5
2	889414	11	2	892762	11	2	896085	11	2	899383	11
3	889470	17	3	892818	17	3	896140	17	3	899437	17
4	889526	22	4	892873	22	4	896195	22	4	899492	22
5	889582	28	5	892929	28	5	896251	27	5	899547	27
6	889638	34	6	892985	33	6	896306	33	6	899602	33
7	889694	39	7	893040	39	7	896361	39	7	899656	38
8	889750	45	8	893096	44	8	896416	44	8	899711	44
9	889806	50	9	893151	50	9	896471	50	9	899766	50
7760	889862		7820	893207		7880	896526		7940	899820	
1	889918	6	1	893262	6	1	896581	6	1	899875	5
2	889974	11	2	893318	11	2	896636	11	2	899930	11
3	890030	17	3	893373	17	3	896692	17	3	899985	17
4	890086	22	4	893429	22	4	896747	22	4	900039	22
5	890141	28	5	893484	28	5	896802	27	5	900094	27
6	890197	34	6	893540	33	6	896857	33	6	900149	33
7	890253	39	7	893595	39	7	896912	39	7	900203	38
8	890309	45	8	893651	44	8	896967	44	8	900258	44
9	890365	50	9	893706	50	9	897022	50	9	900312	50
7770	890421		7830	893762		7890	897077		7950	900367	
1	890477	6	1	893817	6	1	897132	6	1	900422	5
2	890533	11	2	893873	11	2	897187	11	2	900476	11
3	890589	17	3	893928	17	3	897242	17	3	900531	17
4	890644	22	4	893984	22	4	897297	22	4	900586	22
5	890700	28	5	894039	28	5	897352	27	5	900640	27
6	890756	34	6	894094	33	6	897407	33	6	900695	33
7	890812	39	7	894150	39	7	897462	39	7	900749	38
8	890868	45	8	894205	44	8	897517	44	8	900804	44
9	890924	50	9	894261	50	9	897572	50	9	900858	50

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
7960	900913		8020	904174		8080	907411		8140	910624	
1	900968	5	1	904228	5	1	907465	5	1	910678	5
2	901022	11	2	904283	11	2	907519	11	2	910731	11
3	901077	16	3	904337	16	3	907573	16	3	910784	16
4	901131	22	4	904391	22	4	907626	22	4	910838	21
5	901186	27	5	904445	27	5	907680	27	5	910891	27
6	901240	33	6	904499	32	6	907734	32	6	910944	32
7	901295	38	7	904553	38	7	907787	38	7	910998	37
8	901349	44	8	904607	43	8	907841	43	8	911051	43
9	901404	49	9	904661	49	9	907895	49	9	911104	48
7970	901458		8030	904715		8090	907948		8150	911158	
1	901513	5	1	904770	5	1	908002	5	1	911211	5
2	901567	11	2	904824	11	2	908056	11	2	911264	11
3	901622	16	3	904878	16	3	908109	16	3	911317	16
4	901676	22	4	904932	22	4	908163	22	4	911371	21
5	901731	27	5	904986	27	5	908217	27	5	911424	27
6	901785	33	6	905040	32	6	908270	32	6	911477	32
7	901840	38	7	905094	38	7	908324	38	7	911530	37
8	901894	44	8	905148	43	8	908378	43	8	911584	42
9	901948	49	9	905202	49	9	908431	49	9	911637	48
7980	902003		8040	905256		8100	908485		8160	911690	
1	902057	5	1	905310	5	1	908539	5	1	911743	5
2	902112	11	2	905364	11	2	908592	11	2	911797	11
3	902166	16	3	905418	16	3	908646	16	3	911850	16
4	902221	22	4	905472	22	4	908699	21	4	911903	21
5	902275	27	5	905526	27	5	908753	27	5	911956	27
6	902329	33	6	905580	32	6	908807	32	6	912009	32
7	902384	38	7	905634	38	7	908860	37	7	912063	37
8	902438	44	8	905688	43	8	908914	43	8	912116	42
9	902492	49	9	905742	49	9	908967	48	9	912169	48
7990	902547		8050	905796		8110	909021		8170	912222	
1	902601	5	1	905850	5	1	909074	5	1	912275	5
2	902655	11	2	905904	11	2	909128	11	2	912328	11
3	902710	16	3	005958	16	3	909181	16	3	912381	16
4	902764	22	4	906012	22	4	909235	21	4	912435	21
5	902818	27	5	906065	27	5	909288	27	5	912488	27
6	902873	33	6	906119	32	6	909342	32	6	912541	32
7	902927	38	7	906173	38	7	909395	37	7	912594	37
8	902981	44	8	906227	43	8	909449	43	8	912647	42
9	903036	49	9	906281	49	9	909502	48	9	912700	48
8000	903090		8060	906335		8120	909556		8180	912753	
1	903144	5	1	906389	5	1	909609	5	1	912806	5
2	903198	11	2	906443	11	2	909663	11	2	912859	11
3	903253	16	3	906497	16	3	909716	16	3	912913	16
4	903307	22	4	906550	22	4	909770	21	4	912966	21
5	903361	27	5	906604	27	5	909823	27	5	913019	27
6	903416	32	6	906658	32	6	909877	32	6	913072	32
7	903470	38	7	906712	38	7	909930	37	7	913125	37
8	903524	43	8	906766	43	8	909984	43	8	913178	42
9	903578	49	9	906820	49	9	910037	48	9	913231	48
8010	903632		8070	906873		8130	910090		8190	913284	
1	903687	5	1	906927	5	1	910144	5	1	913337	5
2	903741	11	2	906981	11	2	910197	11	2	913390	11
3	903795	16	3	907035	16	3	910251	16	3	913443	16
4	903849	22	4	907089	22	4	910304	21	4	913496	21
5	903903	27	5	907142	27	5	910358	27	5	913549	27
6	903958	32	6	907196	32	6	910411	32	6	913602	32
7	904012	38	7	907250	38	7	910464	37	7	913655	37
8	904066	43	8	907304	43	8	910518	43	8	913708	42
9	904120	49	9	907358	49	9	910571	48	9	913761	48

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
8200	913814		8260	916980		8320	920123		8380	923244	
1	913867	5	1	917033	5	1	920175	5	1	923296	5
2	913920	11	2	917085	11	2	920228	10	2	923348	10
3	913973	16	3	917138	16	3	920280	16	3	923399	16
4	914026	21	4	917190	21	4	920332	21	4	923451	21
5	914079	27	5	917243	26	5	920384	26	5	923503	26
6	914131	32	6	917295	31	6	920436	31	6	923555	31
7	914184	37	7	917348	37	7	920489	36	7	923607	36
8	914237	42	8	917400	42	8	920541	42	8	923658	42
9	914290	48	9	917453	47	9	920593	47	9	923710	47
8210	914343		8270	917505		8330	920645		8390	923762	
1	914396	5	1	917558	5	1	920697	5	1	923814	5
2	914449	11	2	917610	11	2	920749	10	2	923865	10
3	914502	16	3	917663	16	3	920801	16	3	923917	16
4	914555	21	4	917715	21	4	920853	21	4	923969	21
5	914608	27	5	917768	26	5	920906	26	5	924021	26
6	914660	32	6	917820	31	6	920958	31	6	924072	31
7	914713	37	7	917873	37	7	921010	36	7	924124	36
8	914766	42	8	917925	42	8	921062	42	8	924176	42
9	914819	48	9	917978	47	9	921114	47	9	924228	47
8220	914872		8280	918030		8340	921166		8400	924279	
1	914925	5	1	918083	5	1	921218	5	1	924331	5
2	914977	11	2	918135	11	2	921270	10	2	924383	10
3	915030	16	3	918188	16	3	921322	16	3	924434	15
4	915083	21	4	918240	21	4	921374	21	4	924486	21
5	915136	27	5	918292	26	5	921426	26	5	924538	26
6	915189	32	6	918345	31	6	921478	31	6	924589	31
7	915241	37	7	918397	37	7	921530	36	7	924641	36
8	915294	42	8	918450	42	8	921582	42	8	924693	41
9	915347	48	9	918502	47	9	921634	47	9	924744	46
8230	915400		8290	918555		8350	921686		8410	924796	
1	915453	5	1	918607	5	1	921738	5	1	924848	5
2	915505	11	2	918659	11	2	921790	10	2	924899	10
3	915558	16	3	918712	16	3	921842	16	3	924951	15
4	915611	21	4	918764	21	4	921894	21	4	925002	21
5	915664	27	5	918816	26	5	921946	26	5	925054	26
6	915716	32	6	918869	31	6	921998	31	6	925106	31
7	915769	37	7	918921	37	7	922050	36	7	925157	36
8	915822	42	8	918973	42	8	922102	42	8	925209	41
9	915874	48	9	919026	47	9	922154	47	9	925260	46
8240	915927		8300	919078		8360	922206		8420	925312	
1	915980	5	1	919130	5	1	922258	5	1	925364	5
2	916033	11	2	919183	11	2	922310	10	2	925415	10
3	916085	16	3	919235	16	3	922362	16	3	925467	15
4	916138	21	4	919287	21	4	922414	21	4	925518	21
5	916191	27	5	919340	26	5	922466	26	5	925570	26
6	916243	32	6	919392	31	6	922518	31	6	925621	31
7	916296	37	7	919444	37	7	922570	36	7	925673	36
8	916349	42	8	919496	42	8	922622	42	8	925724	41
9	916401	48	9	919549	47	9	922674	47	9	925776	46
8250	916454		8310	919601		8370	922725		8430	925828	
1	916507	5	1	919653	5	1	922777	5	1	925879	5
2	916559	11	2	919705	11	2	922829	10	2	925931	10
3	916612	16	3	919758	16	3	922881	16	3	925982	15
4	916664	21	4	919810	21	4	922933	21	4	926034	21
5	916717	26	5	919862	26	5	922985	26	5	926085	26
6	916770	31	6	919914	31	6	923037	31	6	926137	31
7	916822	37	7	919967	37	7	923088	36	7	926188	36
8	916875	42	8	920019	42	8	923140	42	8	926239	41
9	916927	47	9	920071	47	9	923192	47	9	926291	46

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
8440	926342		8500	929419		8560	932474		8620	935507	
1	926394	5	1	929470	5	1	932524	5	1	935558	5
2	926445	10	2	929521	10	2	932575	10	2	935608	10
3	926497	15	3	929572	15	3	932626	15	3	935658	15
4	926548	21	4	929623	20	4	932677	20	4	935709	20
5	926600	26	5	929674	26	5	932727	25	5	935759	25
6	926651	31	6	929725	31	6	932778	30	6	935809	30
7	926702	36	7	929776	36	7	932829	35	7	935860	35
8	926754	41	8	929827	41	8	932879	40	8	935910	40
9	926805	46	9	929878	46	9	932930	45	9	935960	45
8450	926857		8510	929930		8570	932981		8630	936011	
1	926908	5	1	929981	5	1	933031	5	1	936061	5
2	926959	10	2	930032	10	2	933082	10	2	936111	10
3	927011	15	3	930083	15	3	933133	15	3	936162	15
4	927062	21	4	930134	20	4	933183	20	4	936212	20
5	927114	26	5	930185	26	5	933234	25	5	936262	25
6	927165	31	6	930236	31	6	933285	30	6	936313	30
7	927216	36	7	930287	36	7	933335	35	7	936363	35
8	927268	41	8	930338	41	8	933386	40	8	936413	40
9	927319	46	9	930389	46	9	933437	45	9	936463	45
8460	927370		8520	930440		8580	933487		8640	936514	
1	927422	5	1	930491	5	1	933538	5	1	936564	5
2	927473	10	2	930541	10	2	933588	10	2	936614	10
3	927524	15	3	930592	15	3	933639	15	3	936664	15
4	927576	21	4	930643	20	4	933690	20	4	936715	20
5	927627	26	5	930694	25	5	933740	25	5	936765	25
6	927678	31	6	930745	31	6	933791	30	6	936815	30
7	927730	36	7	930796	36	7	933841	35	7	936865	35
8	927781	41	8	930847	41	8	933892	40	8	936916	40
9	927832	46	9	930898	46	9	933943	45	9	936966	45
8470	927883		8530	930949		8590	933993		8650	937016	
1	927935	5	1	931000	5	1	934044	5	1	937066	5
2	927986	10	2	931051	10	2	934094	10	2	937116	10
3	928037	15	3	931102	15	3	934145	15	3	937167	15
4	928088	21	4	931153	20	4	934195	20	4	937217	20
5	928140	26	5	931203	25	5	934246	25	5	937267	25
6	928191	31	6	931254	31	6	934296	30	6	937317	30
7	928242	36	7	931305	36	7	934347	35	7	937367	35
8	928293	41	8	931356	41	8	934397	40	8	937418	40
9	928345	46	9	931407	46	9	934448	45	9	937468	45
8480	928396		8540	931458		8600	934498		8660	937518	
1	928447	5	1	931509	5	1	934549	5	1	937568	5
2	928498	10	2	931560	10	2	934599	10	2	937618	10
3	928549	15	3	931610	15	3	934650	15	3	937668	15
4	928601	21	4	931661	20	4	934700	20	4	937718	20
5	928652	26	5	931712	25	5	934751	25	5	937769	25
6	928703	31	6	931763	31	6	934801	30	6	937819	30
7	928754	36	7	931814	36	7	934852	35	7	937869	35
8	928805	41	8	931864	41	8	934902	40	8	937919	40
9	928856	46	9	931915	46	9	934953	45	9	937969	45
8490	928908		8550	931966		8610	935003		8670	938019	
1	928959	5	1	932017	5	1	935054	5	1	938069	5
2	929010	10	2	932068	10	2	935104	10	2	938119	10
3	929061	15	3	932118	15	3	935154	15	3	938169	15
4	929112	20	4	932169	20	4	935205	20	4	938219	20
5	929163	26	5	932220	25	5	935255	25	5	938269	25
6	929214	31	6	932271	30	6	935306	30	6	938319	30
7	929266	36	7	932321	35	7	935356	35	7	938370	35
8	929317	41	8	932372	40	8	935406	40	8	938420	40
9	929368	46	9	932423	45	9	935457	45	9	938470	45

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
8680	938520		8740	941511		8800	944483		8860	947434	
1	938570	5	1	941561	5	1	944532	5	1	947483	5
2	938620	10	2	941611	10	2	944581	10	2	947532	10
3	938670	15	3	941660	15	3	944631	15	3	947581	15
4	938720	20	4	941710	20	4	944680	20	4	947630	20
5	938770	25	5	941760	25	5	944729	25	5	947679	25
6	938820	30	6	941809	30	6	944779	30	6	947728	29
7	938870	35	7	941859	35	7	944828	35	7	947777	34
8	938920	40	8	941909	40	8	944877	40	8	947826	39
9	938970	45	9	941958	45	9	944927	45	9	947875	44
8690	939020		8750	942008		8810	944976		8870	947924	
1	939070	5	1	942058	5	1	945025	5	1	947973	5
2	939120	10	2	942107	10	2	945074	10	2	948021	10
3	939170	15	3	942157	15	3	945124	15	3	948070	15
4	939220	20	4	942206	20	4	945173	20	4	948119	20
5	939270	25	5	942256	25	5	945222	25	5	948168	25
6	939319	30	6	942306	30	6	945272	30	6	948217	29
7	939369	35	7	942355	35	7	945321	35	7	948266	34
8	939419	40	8	942405	40	8	945370	40	8	948315	39
9	939469	45	9	942454	45	9	945419	45	9	948364	44
8700	939519		8760	942504		8820	945469		8880	948413	
1	939569	5	1	942554	5	1	945518	5	1	948462	5
2	939619	10	2	942603	10	2	945567	10	2	948511	10
3	939669	15	3	942653	15	3	945616	15	3	948560	15
4	939719	20	4	942702	20	4	945665	20	4	948608	20
5	939769	25	5	942752	25	5	945715	25	5	948657	25
6	939819	30	6	942801	30	6	945764	29	6	948706	29
7	939868	35	7	942851	35	7	945813	34	7	948755	34
8	939918	40	8	942900	40	8	945862	39	8	948804	39
9	939968	45	9	942950	45	9	945911	44	9	948853	44
8710	940018		8770	943000		8830	945961		8890	948902	
1	940068	5	1	943049	5	1	946010	5	1	948951	5
2	940118	10	2	943099	10	2	946059	10	2	948999	10
3	940168	15	3	943148	15	3	946108	15	3	949048	15
4	940218	20	4	943198	20	4	946157	20	4	949097	20
5	940267	25	5	943247	25	5	946207	25	5	949146	25
6	940317	30	6	943297	30	6	946256	29	6	949195	29
7	940367	35	7	943346	35	7	946305	34	7	949244	34
8	940417	40	8	943396	40	8	946354	39	8	949292	39
9	940467	45	9	943445	45	9	946403	44	9	949341	44
8720	940516		8780	943494		8840	946452		8900	949390	
1	940566	5	1	943544	5	1	946501	5	1	949439	5
2	940616	10	2	943593	10	2	946550	10	2	949488	10
3	940666	15	3	943643	15	3	946600	15	3	949536	15
4	940716	20	4	943692	20	4	946649	20	4	949585	20
5	940765	25	5	943742	25	5	946698	25	5	949634	25
6	940815	30	6	943791	30	6	946747	29	6	949683	29
7	940865	35	7	943841	35	7	946796	34	7	949731	34
8	940915	40	8	943890	40	8	946845	39	8	949780	39
9	940964	45	9	943939	45	9	946894	44	9	949829	44
8730	941014		8790	943989		8850	946943		8910	949878	
1	941064	5	1	944038	5	1	946992	5	1	949926	5
2	941114	10	2	944088	10	2	947041	10	2	949975	10
3	941163	15	3	944137	15	3	947090	15	3	950024	15
4	941213	20	4	944186	20	4	947139	20	4	950073	20
5	941263	25	5	944236	25	5	947189	25	5	950121	25
6	941313	30	6	944285	30	6	947238	29	6	950170	29
7	941362	35	7	944335	35	7	947287	34	7	950219	34
8	941412	40	8	944384	40	8	947336	39	8	950267	39
9	941462	45	9	944433	45	9	947385	44	9	950316	44

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
8920	950365		8980	953276		9040	956168		9100	959041	
1	950413	5	1	953325	5	1	956216	5	1	959089	5
2	950462	10	2	953373	10	2	956264	10	2	959137	10
3	950511	15	3	953421	15	3	956312	14	3	959184	14
4	950560	19	4	953470	19	4	956361	19	4	959232	19
5	950608	24	5	953518	24	5	956409	24	5	959280	24
6	950657	29	6	953566	29	6	956457	29	6	959328	29
7	950705	34	7	953615	34	7	956505	34	7	959375	34
8	950754	39	8	953663	39	8	956553	38	8	959423	38
9	950803	44	9	953711	44	9	956601	43	9	959471	43
8930	950851		8990	953760		9050	956649		9110	959518	
1	950900	5	1	953808	5	1	956697	5	1	959566	5
2	950949	10	2	953856	10	2	956745	10	2	959614	10
3	950997	15	3	953905	15	3	956792	14	3	959661	14
4	951046	19	4	953953	19	4	956840	19	4	959709	19
5	951095	24	5	954001	24	5	956888	24	5	959757	24
6	951143	29	6	954049	29	6	956936	29	6	959804	29
7	951192	34	7	954098	34	7	956984	34	7	959852	34
8	951240	39	8	954146	39	8	957032	38	8	959900	38
9	951289	44	9	954194	44	9	957080	43	9	959947	43
8940	951337		9000	954242		9060	957128		9120	959995	
1	951386	5	1	954291	5	1	957176	5	1	960042	5
2	951435	10	2	954339	10	2	957224	10	2	960090	10
3	951483	15	3	954387	14	3	957272	14	3	960138	14
4	951532	19	4	954435	19	4	957320	19	4	960185	19
5	951580	24	5	954484	24	5	957368	24	5	960233	24
6	951629	29	6	954532	29	6	957416	29	6	960280	28
7	951677	34	7	954580	34	7	957464	34	7	960328	33
8	951726	39	8	954628	38	8	957511	38	8	960376	38
9	951774	44	9	954677	43	9	957559	43	9	960423	43
8950	951823		9010	954725		9070	957607		9130	960471	
1	951872	5	1	954773	5	1	957655	5	1	960518	5
2	951920	10	2	954821	10	2	957703	10	2	960566	10
3	951969	15	3	954869	14	3	957751	14	3	960613	14
4	952017	19	4	954918	19	4	957799	19	4	960661	19
5	952066	24	5	954966	24	5	957847	24	5	960709	24
6	952114	29	6	955014	29	6	957894	29	6	960756	28
7	952163	34	7	955062	34	7	957942	34	7	960804	33
8	952211	39	8	955110	38	8	957990	38	8	960851	38
9	952259	44	9	955158	43	9	958038	43	9	960899	43
8960	952308		9020	955206		9080	958086		9140	960946	
1	952356	5	1	955255	5	1	958134	5	1	960994	5
2	952405	10	2	955303	10	2	958181	10	2	961041	10
3	952453	15	3	955351	14	3	958229	14	3	961089	14
4	952502	19	4	955399	19	4	958277	19	4	961136	19
5	952550	24	5	955447	24	5	958325	24	5	961184	24
6	952599	29	6	955495	29	6	958373	29	6	961231	28
7	952647	34	7	955543	34	7	958420	34	7	961279	33
8	952696	39	8	955592	38	8	958468	38	8	961326	38
9	952744	44	9	955640	43	9	958516	43	9	961374	43
8970	952792		9030	955688		9090	958564		9150	961421	
1	952841	5	1	955736	5	1	958612	5	1	961469	5
2	952889	10	2	955784	10	2	958659	10	2	961516	10
3	952938	15	3	955832	14	3	958707	14	3	961563	14
4	952986	19	4	955880	19	4	958755	19	4	961611	19
5	953034	24	5	955928	24	5	958803	24	5	961658	24
6	953083	29	6	955976	29	6	958850	29	6	961706	28
7	953131	34	7	956024	34	7	958898	34	7	961753	33
8	953180	39	8	956072	38	8	958946	38	8	961801	38
9	953228	44	9	956120	43	9	958994	43	9	961848	43

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
9160	961895		9220	964731		9280	967548		9340	970347	
1	961943	5	1	964778	5	1	967595	5	1	970393	5
2	961990	10	2	964825	9	2	967642	9	2	970440	9
3	962038	14	3	964872	14	3	967688	14	3	970486	14
4	962085	19	4	964919	19	4	967735	19	4	970533	19
5	962132	24	5	964966	24	5	967782	23	5	970579	23
6	962180	28	6	965013	28	6	967829	28	6	970626	28
7	962227	33	7	965060	33	7	967875	33	7	970672	33
8	962275	38	8	965108	38	8	967922	38	8	970719	37
9	962322	43	9	965155	42	9	967969	42	9	970765	42
9170	962369		9230	965202		9290	968016		9350	970812	
1	962417	5	1	965249	5	1	968062	5	1	970858	5
2	962464	9	2	965296	9	2	968109	9	2	970904	9
3	962511	14	3	965343	14	3	968156	14	3	970951	14
4	962559	19	4	965390	19	4	968203	19	4	970997	19
5	962606	24	5	965437	24	5	968249	23	5	971044	23
6	962653	28	6	965484	28	6	968296	28	6	971090	28
7	962701	33	7	965531	33	7	968343	33	7	971137	33
8	962748	38	8	965578	38	8	968389	38	8	971183	37
9	962795	42	9	965625	42	9	968436	42	9	971229	42
9180	962843		9240	965672		9300	968483		9360	971276	
1	962890	5	1	965719	5	1	968530	5	1	971322	5
2	962937	9	2	965766	9	2	968576	9	2	971369	9
3	962985	14	3	965813	14	3	968623	14	3	971415	14
4	963032	19	4	965860	19	4	968670	19	4	971461	19
5	963079	24	5	965907	24	5	968716	23	5	971508	23
6	963126	28	6	965954	28	6	968763	28	6	971554	28
7	963174	33	7	966001	33	7	968810	33	7	971600	33
8	963221	38	8	966048	38	8	968856	37	8	971647	37
9	963268	42	9	966095	42	9	968903	42	9	971693	42
9190	963315		9250	966142		9310	968950		9370	971740	
1	963363	5	1	966189	5	1	968996	5	1	971786	5
2	963410	9	2	966236	9	2	969043	9	2	971832	9
3	963457	14	3	966283	14	3	969090	14	3	971879	14
4	963504	19	4	966329	19	4	969136	19	4	971925	19
5	963552	24	5	966376	24	5	969183	23	5	971971	23
6	963599	28	6	966423	28	6	969229	28	6	972018	28
7	963646	33	7	966470	33	7	969276	33	7	972064	33
8	963693	38	8	966517	38	8	969323	37	8	972110	37
9	963741	42	9	966564	42	9	969369	42	9	972156	42
9200	963788		9260	966611		9320	969416		9380	972203	
1	963835	5	1	966658	5	1	969462	5	1	972249	5
2	963882	9	2	966705	9	2	969509	9	2	972295	9
3	963929	14	3	966752	14	3	969556	14	3	972342	14
4	963977	19	4	966798	19	4	969602	19	4	972388	18
5	964024	24	5	966845	24	5	969649	23	5	972434	23
6	964071	28	6	966892	28	6	969695	28	6	972480	28
7	964118	33	7	966939	33	7	969742	33	7	972527	32
8	964165	38	8	966986	38	8	969788	37	8	972573	37
9	964212	42	9	967033	42	9	969835	42	9	972619	41
9210	964260		9270	967080		9330	969882		9390	972666	
1	964307	5	1	967127	5	1	969928	5	1	972712	5
2	964354	9	2	967173	9	2	969975	9	2	972758	9
3	964401	14	3	967220	14	3	970021	14	3	972804	14
4	964448	19	4	967267	19	4	970068	19	4	972851	18
5	964495	24	5	967314	24	5	970114	23	5	972897	23
6	964542	28	6	967361	28	6	970161	28	6	972943	28
7	964590	33	7	967408	33	7	970207	33	7	972989	32
8	964637	38	8	967454	38	8	970254	37	8	973035	37
9	964684	42	9	967501	42	9	970300	42	9	973082	41

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
9400	973128		9460	975891		9520	978637		9580	981365	
1	973174	5	1	975937	5	1	978683	5	1	981411	5
2	973220	9	2	975983	9	2	978728	9	2	981456	9
3	973266	14	3	976029	14	3	978774	14	3	981501	14
4	973313	18	4	976075	18	4	978819	18	4	981547	18
5	973359	23	5	976121	23	5	978865	23	5	981592	23
6	973405	28	6	976166	28	6	978911	27	6	981637	27
7	973451	32	7	976212	32	7	978956	32	7	981683	32
8	973497	37	8	976258	37	8	979002	36	8	981728	36
9	973543	41	9	976304	41	9	979047	41	9	981773	41
9410	973590		9470	976350		9530	979093		9590	981819	
1	973636	5	1	976396	5	1	979138	5	1	981864	5
2	973682	9	2	976442	9	2	979184	9	2	981909	9
3	973728	14	3	976487	14	3	979230	14	3	981954	14
4	973774	18	4	976533	18	4	979275	18	4	982000	18
5	973820	23	5	976579	23	5	979321	23	5	982045	23
6	973866	28	6	976625	28	6	979366	27	6	982090	27
7	973913	32	7	976671	32	7	979412	32	7	982135	32
8	973959	37	8	976717	37	8	979457	36	8	982181	36
9	974005	41	9	976762	41	9	979503	41	9	982226	41
9420	974051		9480	976808		9540	979548		9600	982271	
1	974097	5	1	976854	5	1	979594	5	1	982316	5
2	974143	9	2	976900	9	2	979639	9	2	982362	9
3	974189	14	3	976946	14	3	979685	14	3	982407	14
4	974235	18	4	976991	18	4	979730	18	4	982452	18
5	974281	23	5	977037	23	5	979776	23	5	982497	23
6	974327	28	6	977083	27	6	979821	27	6	982543	27
7	974373	32	7	977129	32	7	979867	32	7	982588	32
8	974420	37	8	977175	37	8	979912	36	8	982633	36
9	974466	41	9	977220	41	9	979958	41	9	982678	41
9430	974512		9490	977266		9550	980003		9610	982723	
1	974558	5	1	977312	5	1	980049	5	1	982769	5
2	974604	9	2	977358	9	2	980094	9	2	982814	9
3	974650	14	3	977403	14	3	980140	14	3	982859	14
4	974696	18	4	977449	18	4	980185	18	4	982904	18
5	974742	23	5	977495	23	5	980231	23	5	982949	23
6	974788	28	6	977541	27	6	980276	27	6	982994	27
7	974834	32	7	977586	32	7	980322	32	7	983040	32
8	974880	37	8	977632	37	8	980367	36	8	983085	36
9	974926	41	9	977678	41	9	980412	41	9	983130	41
9440	974972		9500	977724		9560	980458		9620	983175	
1	975018	5	1	977769	5	1	980503	5	1	983220	5
2	975064	9	2	977815	9	2	980549	9	2	983265	9
3	975110	14	8	977861	14	3	980594	14	3	983310	14
4	975156	18	4	977906	18	4	980640	18	4	983356	18
5	975202	23	5	977952	23	5	980685	23	5	983401	23
6	975248	28	6	977998	27	6	980730	27	6	983446	27
7	975294	32	7	978043	32	7	980776	32	7	983491	32
8	975340	37	8	978089	37	8	980821	36	8	983536	36
9	975386	41	9	978135	41	9	980867	41	9	983581	41
9450	975432		9510	978180		9570	980912		9630	983626	
1	975478	5	1	978226	5	1	980957	5	1	983671	5
2	975524	9	2	978272	9	2	981003	9	2	983716	9
3	975570	14	3	978317	14	3	981048	14	3	983762	14
4	975616	18	4	978363	18	4	981093	18	4	983807	18
5	975661	23	5	978409	23	5	981139	23	5	983852	23
6	975707	28	6	978454	27	6	981184	27	6	983897	27
7	975753	32	7	978500	32	7	981229	32	7	983942	32
8	975799	37	8	978546	37	8	981275	36	8	983987	36
9	975845	41	9	978591	41	9	981320	41	9	984032	41

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
9640	984077		9700	986772		9760	989450		9820	992111	
1	984122	5	1	986816	4	1	989494	4	1	992156	4
2	984167	9	2	986861	9	2	989539	9	2	992200	9
3	984212	14	3	986906	13	3	989583	13	3	992244	13
4	984257	18	4	986951	18	4	989628	18	4	992288	18
5	984302	23	5	986995	22	5	989672	22	5	992333	22
6	984347	27	6	987040	27	6	989717	27	6	992377	26
7	984392	32	7	987085	31	7	989761	31	7	992421	31
8	984437	36	8	987130	36	8	989806	36	8	992465	35
9	984482	41	9	987174	40	9	989850	40	9	992509	40
9650	984527		9710	987219		9770	989895		9830	992553	
1	984572	5	1	987264	4	1	989939	4	1	992598	4
2	984617	9	2	987309	9	2	989983	9	2	992642	9
3	984662	14	3	987353	13	3	990028	13	3	992686	13
4	984707	18	4	987398	18	4	990072	18	4	992730	18
5	984752	23	5	987443	22	5	990117	22	5	992774	22
6	984797	27	6	987487	27	6	990161	27	6	992818	26
7	984842	32	7	987532	31	7	990206	31	7	992863	31
8	984887	36	8	987577	36	8	990250	36	8	992907	35
9	984932	41	9	987622	40	9	990294	40	9	992951	40
9660	984977		9720	987666		9780	990339		9840	992995	
1	985022	5	1	987711	4	1	990383	4	1	993039	4
2	985067	9	2	987756	9	2	990428	9	2	993083	9
3	985112	14	3	987800	13	3	990472	13	3	993127	13
4	985157	18	4	987845	18	4	990516	18	4	993172	18
5	985202	23	5	987890	22	5	990561	22	5	993216	22
6	985247	27	6	987934	27	6	990605	27	6	993260	26
7	985292	32	7	987979	31	7	990650	31	7	993304	31
8	985337	36	8	988024	36	8	990694	36	8	993348	35
9	985382	41	9	988068	40	9	990738	40	9	993392	40
9670	985426		9730	988113		9790	990783		9850	993436	
1	985471	4	1	988157	4	1	990827	4	1	993480	4
2	985516	9	2	988202	9	2	990871	9	2	993524	9
3	985561	13	3	988247	13	3	990916	13	3	993568	13
4	985606	18	4	988291	18	4	990960	18	4	993613	18
5	985651	22	5	988336	22	5	991004	22	5	993657	22
6	985696	27	6	988381	27	6	991049	27	6	993701	26
7	985741	31	7	988425	31	7	991093	31	7	993745	31
8	985786	36	8	988470	36	8	991137	36	8	993789	35
9	985830	40	9	988514	40	9	991182	40	9	993833	40
9680	985875		9740	988559		9800	991226		9860	993877	
1	985920	4	1	988603	4	1	991270	4	1	993921	4
2	985965	9	2	988648	9	2	991315	9	2	993965	9
3	986010	13	3	988693	13	3	991359	13	3	994009	13
4	986055	18	4	988737	18	4	991403	18	4	994053	18
5	986100	22	5	988782	22	5	991448	22	5	994097	22
6	986144	27	6	988826	27	6	991492	27	6	994141	26
7	986189	31	7	988871	31	7	991536	31	7	994185	31
8	986234	36	8	988915	36	8	991580	36	8	994229	35
9	986279	40	9	988960	40	9	991625	40	9	994273	40
9690	986324		9750	989005		9810	991669		9870	994317	
1	986369	4	1	989049	4	1	991713	4	1	994361	4
2	986413	9	2	989094	9	2	991757	9	2	994405	9
3	986458	13	3	989138	13	3	991802	13	3	994449	13
4	986503	18	4	989183	18	4	991846	18	4	994493	18
5	986548	22	5	989227	22	5	991890	22	5	994537	22
6	986593	27	6	989272	27	6	991934	27	6	994581	26
7	986637	31	7	989316	31	7	991979	31	7	994625	31
8	986682	36	8	989361	36	8	992023	36	8	994669	35
9	986727	40	9	989405	40	9	992067	40	9	994713	40

No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.	No.	Log.	Prop. Part.
9880	994757		9910	996074		9940	997386		9970	998695	
1	994801	4	1	996117	4	1	997430	4	1	998739	4
2	994845	9	2	996161	9	2	997474	9	2	998782	9
3	994889	13	3	996205	13	3	997517	13	3	998826	13
4	994933	18	4	996249	18	4	997561	17	4	998869	17
5	994977	22	5	996293	22	5	997605	22	5	998913	22
6	995021	26	6	996336	26	6	997648	26	6	998956	26
7	995064	31	7	996380	31	7	997692	30	7	999000	30
8	995108	35	8	996424	35	8	997736	35	8	999043	35
9	995152	40	9	996468	40	9	997779	39	9	999087	39
9890	995196		9920	996512		9950	997823		9980	999130	
1	995240	4	1	996555	4	1	997867	4	1	999174	4
2	995284	9	2	996599	9	2	997910	9	2	999218	9
3	995328	13	3	996643	13	3	997954	13	3	999261	13
4	995372	18	4	996687	18	4	997998	17	4	999305	17
5	995416	22	5	996730	22	5	998041	22	5	999348	22
6	995460	26	6	996774	26	6	998085	26	6	999392	26
7	995504	31	7	996818	31	7	998128	30	7	999435	30
8	995547	35	8	996862	35	8	998172	35	8	999478	35
9	995591	40	9	996905	40	9	998216	39	9	999522	39
9900	995635		9930	996949		9960	998259		9990	999565	
1	995679	4	1	996993	4	1	998303	4	1	999609	4
2	995723	9	2	997037	9	2	998346	9	2	999652	9
3	995767	13	3	997080	13	3	998390	13	3	999696	13
4	995811	18	4	997124	18	4	998434	17	4	999739	17
5	995854	22	5	997168	22	5	998477	22	5	999783	22
6	995898	26	6	997212	26	6	998521	26	6	999826	26
7	995942	31	7	997255	31	7	998564	30	7	999870	30
8	995986	35	8	997299	35	8	998608	35	8	999913	35
9	996030	40	9	997343	39	9	998652	39	9	999957	39

No.	Logarithms to 50 Decimal Places.
1	0·00000000000000000000000000000000000000000000000000
2	0·30102999566398119521373889472449302676818988146211
3	0·47712125471966243729502790325511530920012886419069
4	0·60205999132796239042747778944898605353637976292422
5	0·69897000433601880478626110527550697323181011853789
6	0·77815125038364363250876679797960833596831874565280
7	0·84509804001425683071221625859263619348357239632397
8	0·90308998699194358564121668417347908030456964438633
9	0·95424250943932487459005580651023061840025772838139
10	1·00000000000000000000000000000000000000000000000000
11	1·04139268515822504075019997124302424170670219046645
12	1·07918124604762482772250569270410136273650862711491
13	1·11394335230683676920650515794232843082972918838707
14	1·14612803567823802592595515331712922025176227778607
15	1·17609125905568124208128900853062228243193898272859
16	1·20411998265592478085495557889797210707275952584843
17	1·23044892137827392854016989432833703000756737842505
18	1·25527250510330606980379470123472364516844760984350
19	1·27875360095282896153633347575692931795112933739450
20	1·30102999566398119521373889472449302676818988146211
21	1·32221929473391926800724416184775150268370126051866
22	1·34242268082220623596393886596751726847489207192856
23	1·36172783601759287886777711225118954969751103433610
24	1·38021124171160602293624458742859438950469850857702
25	1·39794000867203760957252221055101394646362023707578

0 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Cosine.	′
0			Infinite.			Infinite.	·000000	10·000000	60
1	6·463726		3·536274	6·463726		13·536274	·000000	10·000000	59
2	6·764756	501717	3·235244	6·764756	501717	13·235244	·000000	10·000000	58
3	6·940847	293485	3·059153	6·940847	293485	13·059153	·000000	10·000000	57
4	7·065786	208231	2·934214	7·065786	208231	12·934214	·000000	10·000000	56
5	7·162696	161517	2·837304	7·162696	161517	12·837304	·000000	10·000000	55
6	7·241877	131968	2·758123	7·241878	131969	12·758122	·000001	9·999999	54
7	7·308824	111578	2·691176	7·308825	111578	12·691175	·000001	9·999999	53
8	7·366816	96653	2·633184	7·366817	96653	12·633183	·000001	9·999999	52
9	7·417968	85254	2·582032	7·417970	85254	12·582030	·000001	9·999999	51
10	7·463726	76262	2·536274	7·463727	76263	12·536273	·000002	9·999998	50
11	7·505118	68988	2·494882	7·505120	68988	12·494880	·000002	9·999998	49
12	7·542906	62981	2·457094	7·542909	62981	12·457091	·000003	9·999997	48
13	7·577668	57936	2·422332	7·577672	57937	12·422328	·000003	9·999997	47
14	7·609853	53641	2·390147	7·609857	53642	12·390143	·000004	9·999996	46
15	7·639816	49938	2·360184	7·639820	49939	12·360180	·000004	9·999996	45
16	7·667845	46714	2·332155	7·667849	46715	12·332151	·000005	9·999995	44
17	7·694173	43881	2·305827	7·694179	43882	12·305821	·000005	9·999995	43
18	7·718997	41372	2·281003	7·719003	41373	12·280997	·000006	9·999994	42
19	7·742478	39135	2·257522	7·742484	39136	12·257516	·000007	9·999993	41
20	7·764754	37127	2·235246	7·764761	37128	12·235239	·000007	9·999993	40
21	7·785943	35315	2·214057	7·785951	35315	12·214049	·000008	9·999992	39
22	7·806146	33672	2·193854	7·806155	33673	12·193845	·000009	9·999991	38
23	7·825451	32175	2·174549	7·825460	32176	12·174540	·000010	9·999990	37
24	7·843934	30805	2·156066	7·843944	30807	12·156056	·000011	9·999989	36
25	7·861662	29547	2·138338	7·861674	29549	12·138326	·000011	9·999989	35
26	7·878695	28388	2·121305	7·878708	28390	12·121292	·000012	9·999988	34
27	7·895085	27317	2·104915	7·895099	27318	12·104901	·000013	9·999987	33
28	7·910879	26323	2·089121	7·910894	26325	12·089106	·000014	9·999986	32
29	7·926119	25399	2·073881	7·926134	25401	12·073866	·000015	9·999985	31
30	7·940842	24538	2·059158	7·940858	24540	12·059142	·000017	9·999983	30
31	7·955082	23733	2·044918	7·955100	23735	12·044900	·000018	9·999982	29
32	7·968870	22980	2·031130	7·968889	22982	12·031111	·000019	9·999981	28
33	7·982233	22273	2·017767	7·982253	22275	12·017747	·000020	9·999980	27
34	7·995198	21608	2·004802	7·995219	21610	12·004781	·000021	9·999979	26
35	8·007787	20981	1·992213	8·007809	20983	11·992191	·000023	9·999977	25
36	8·020021	20390	1·979979	8·020045	20392	11·979955	·000024	9·999976	24
37	8·031919	19831	1·968081	8·031945	19833	11·968055	·000025	9·999975	23
38	8·043501	19302	1·956499	8·043527	19305	11·956473	·000027	9·999973	22
39	8·054781	18801	1·945219	8·054809	18803	11·945191	·000028	9·999972	21
40	8·065776	18325	1·934224	8·065806	18327	11·934194	·000029	9·999971	20
41	8·076500	17872	1·923500	8·076531	17875	11·923469	·000031	9·999969	19
42	8·086965	17441	1·913035	8·086997	17444	11·913003	·000032	9·999968	18
43	8·097183	17031	1·902817	8·097217	17034	11·902783	·000034	9·999966	17
44	8·107167	16639	1·892833	8·107202	16642	11·892798	·000036	9·999964	16
45	8·116926	16265	1·883074	8·116963	16268	11·883037	·000037	9·999963	15
46	8·126471	15908	1·873529	8·126510	15911	11·873490	·000039	9·999961	14
47	8·135810	15566	1·864190	8·135851	15568	11·864149	·000041	9·999959	13
48	8·144953	15238	1·855047	8·144996	15241	11·855004	·000042	9·999958	12
49	8·153907	14924	1·846093	8·153952	14927	11·846048	·000044	9·999956	11
50	8·162681	14622	1·837319	8·162727	14625	11·837273	·000046	9·999954	10
51	8·171280	14333	1·828720	8·171328	14336	11·828672	·000048	9·999952	9
52	8·179713	14054	1·820287	8·179763	14057	11·820237	·000050	9·999950	8
53	8·187985	13786	1·812015	8·188036	13790	11·811964	·000052	9·999948	7
54	8·196102	13529	1·803898	8·196156	13532	11·803844	·000054	9·999946	6
55	8·204070	13280	1·795930	8·204126	13284	11·795874	·000056	9·999944	5
56	8·211895	13041	1·788105	8·211953	13044	11·788047	·000058	9·999942	4
57	8·219581	12810	1·780419	8·219641	12814	11·780359	·000060	9·999940	3
58	8·227134	12587	1·772866	8·227195	12591	11·772805	·000062	9·999938	2
59	8·234557	12372	1·765443	8·234621	12376	11·765379	·000064	9·999936	1
60	8·241855	12164	1·758145	8·241922	12168	11·758078	·000066	9·999934	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.	Sine.	′

89 DEG.

1 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Cosine.	′
0	8·241855		1·758145	8·241921		11·758079	·000066	9·999934	60
1	8·249033	11963	1·750967	8·249102	11967	11·750898	·000068	9·999932	59
2	8·256094	11768	1·743906	8·256165	11772	11·743835	·000071	9·999929	58
3	8·263042	11580	1·736958	8·263115	11584	11·736885	·000073	9·999927	57
4	8·269881	11397	1·730119	8·269956	11402	11·730044	·000075	9·999925	56
5	8·276614	11221	1·723386	8·276691	11225	11·723309	·000078	9·999922	55
6	8·283243	11050	1·716757	8·283323	11054	11·716677	·000080	9·999920	54
7	8·289773	10883	1·710227	8·289856	10887	11·710144	·000082	9·999918	53
8	8·296207	10722	1·703793	8·296292	10726	11·703708	·000085	9·999915	52
9	8·302546	10565	1·697454	8·302634	10570	11·697366	·000087	9·999913	51
10	8·308794	10413	1·691206	8·308884	10418	11·691116	·000090	9·999910	50
11	8·314954	10266	1·685046	8·315046	10270	11·684954	·000093	9·999907	49
12	8·321027	10122	1·678973	8·321122	10126	11·678878	·000095	9·999905	48
13	8·327016	9982	1·672984	8·327114	9987	11·672886	·000098	9·999902	47
14	8·332924	9847	1·667076	8·333025	9851	11·666975	·000101	9·999899	46
15	8·338753	9714	1·661247	8·338856	9719	11·661144	·000103	9·999897	45
16	8·344504	9586	1·655496	8·344610	9590	11·655390	·000106	9·999894	44
17	8·350181	9460	1·649819	8·350289	9465	11·649711	·000109	9·999891	43
18	8·355783	9338	1·644217	8·355895	9343	11·644105	·000112	9·999888	42
19	8·361315	9219	1·638685	8·361430	9224	11·638570	·000115	9·999885	41
20	8·366777	9103	1·633223	8·366895	9108	11·633105	·000118	9·999882	40
21	8·372171	8990	1·627829	8·372292	8995	11·627708	·000121	9·999879	39
22	8·377499	8880	1·622501	8·377622	8885	11·622378	·000124	9·999876	38
23	8·382762	8772	1·617238	8·382889	8777	11·617111	·000127	9·999873	37
24	8·387962	8667	1·612038	8·388092	8672	11·611908	·000130	9·999870	36
25	8·393101	8564	1·606899	8·393234	8570	11·606766	·000133	9·999867	35
26	8·398179	8464	1·601821	8·398315	8470	11·601685	·000136	9·999864	34
27	8·403199	8366	1·596801	8·403338	8371	11·596662	·000139	9·999861	33
28	8·408161	8271	1·591839	8·408304	8276	11·591696	·000142	9·999858	32
29	8·413068	8177	1·586932	8·413213	8182	11·586787	·000146	9·999854	31
30	8·417919	8086	1·582081	8·418068	8091	11·581932	·000149	9·999851	30
31	8·422717	7996	1·577283	8·422869	8002	11·577131	·000152	9·999848	29
32	8·427462	7909	1·572538	8·427618	7914	11·572382	·000156	9·999844	28
33	8·432156	7823	1·567844	8·432315	7829	11·567685	·000159	9·999841	27
34	8·436800	7740	1·563200	8·436962	7745	11·563038	·000162	9·999838	26
35	8·441394	7657	1·558606	8·441560	7663	11·558440	·000166	9·999834	25
36	8·445941	7577	1·554059	8·446110	7583	11·553890	·000169	9·999831	24
37	8·450440	7499	1·549560	8·450613	7505	11·549387	·000173	9·999827	23
38	8·454893	7422	1·545107	8·455070	7428	11·544930	·000176	9·999824	22
39	8·459301	7346	1·540699	8·459481	7352	11·540519	·000180	9·999820	21
40	8·463665	7273	1·536335	8·463849	7279	11·536151	·000184	9·999816	20
41	8·467985	7200	1·532015	8·468172	7206	11·531828	·000187	9·999813	19
42	8·472263	7129	1·527737	8·472454	7135	11·527546	·000191	9·999809	18
43	8·476498	7060	1·523502	8·476693	7066	11·523307	·000195	9·999805	17
44	8·480693	6991	1·519307	8·480892	6998	11·519108	·000199	9·999801	16
45	8·484848	6924	1·515152	8·485050	6931	11·514950	·000203	9·999797	15
46	8·488963	6859	1·511037	8·489170	6865	11·510830	·000206	9·999794	14
47	8·493040	6794	1·506960	8·493250	6801	11·506750	·000210	9·999790	13
48	8·497078	6731	1·502922	8·497293	6738	11·502707	·000214	9·999786	12
49	8·501080	6669	1·498920	8·501298	6676	11·498702	·000218	9·999782	11
50	8·505045	6608	1·494955	8·505267	6615	11·494733	·000222	9·999778	10
51	8·508974	6548	1·491026	8·509200	6555	11·490800	·000226	9·999774	9
52	8·512867	6489	1·487133	8·513098	6496	11·486902	·000231	9·999769	8
53	8·516726	6432	1·483274	8·516961	6439	11·483039	·000235	9·999765	7
54	8·520551	6375	1·479449	8·520790	6382	11·479210	·000239	9·999761	6
55	8·524343	6319	1·475657	8·524586	6326	11·475414	·000243	9·999757	5
56	8·528102	6264	1·471898	8·528349	6272	11·471651	·000247	9·999753	4
57	8·531828	6211	1·468172	8·532080	6218	11·467920	·000252	9·999748	3
58	8·535523	6158	1·464477	8·535779	6165	11·464221	·000256	9·999744	2
59	8·539186	6106	1·460814	8·539447	6113	11·460553	·000260	9·999740	1
60	8·542819	6055	1·457181	8·543084	6062	11·456916	·000265	9·999735	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.	Sine.	′

88 DEG.

2 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	8·542819		1·457181	8·543084		11·456916	·000265		9·999735	60
1	8·546422	6004	1·453578	8·546691	6012	11·453309	·000269	07	9·999731	59
2	8·549995	5955	1·450005	8·550268	5962	11·449732	·000274	07	9·999726	58
3	8·553539	5906	1·446461	8·553817	5914	11·446183	·000278	08	9·999722	57
4	8·557054	5858	1·442946	8·557336	5866	11·442664	·000283	08	9·999717	56
5	8·560540	5811	1·439460	8·560828	5819	11·439172	·000287	07	9·999713	55
6	8·563999	5765	1·436001	8·564291	5773	11·435709	·000292	08	9·999708	54
7	8·567431	5719	1·432569	8·567727	5727	11·432273	·000296	07	9·999704	53
8	8·570836	5674	1·429164	8·571137	5682	11·428863	·000301	08	9·999699	52
9	8·574214	5630	1·425786	8·574520	5638	11·425480	·000306	08	9·999694	51
10	8·577566	5587	1·422434	8·577877	5595	11·422123	·000311	08	9·999689	50
11	8·580892	5544	1·419108	8·581208	5552	11·418792	·000315	07	9·999685	49
12	8·584193	5502	1·415807	8·584514	5510	11·415486	·000320	08	9·999680	48
13	8·587469	5460	1·412531	8·587795	5468	11·412205	·000325	08	9·999675	47
14	8·590721	5419	1·409279	8·591051	5427	11·408949	·000330	08	9·999670	46
15	8·593948	5379	1·406052	8·594283	5387	11·405717	·000335	08	9·999665	45
16	8·597152	5339	1·402848	8·597492	5347	11·402508	·000340	08	9·999660	44
17	8·600332	5300	1·399668	8·600677	5308	11·399323	·000345	08	9·999655	43
18	8·603489	5261	1·396511	8·603839	5270	11·396161	·000350	08	9·999650	42
19	8·606623	5223	1·393377	8·606978	5232	11·393022	·000355	08	9·999645	41
20	8·609734	5186	1·390266	8·610094	5194	11·389906	·000360	08	9·999640	40
21	8·612823	5149	1·387177	8·613189	5158	11·386811	·000365	08	9·999635	39
22	8·615891	5112	1·384109	8·616262	5121	11·383738	·000371	10	9·999629	38
23	8·618937	5076	1·381063	8·619313	5085	11·380687	·000376	08	9·999624	37
24	8·621962	5041	1·378038	8·622343	5050	11·377657	·000381	08	9·999619	36
25	8·624965	5006	1·375035	8·625352	5015	11·374648	·000386	08	9·999614	35
26	8·627948	4972	1·372052	8·628340	4981	11·371660	·000392	10	9·999608	34
27	8·630911	4938	1·369089	8·631308	4947	11·368692	·000397	08	9·999603	33
28	8·633854	4904	1·366146	8·634256	4913	11·365744	·000403	10	9·999597	32
29	8·636776	4871	1·363224	8·637184	4880	11·362816	·000408	08	9·999592	31
30	8·639680	4839	1·360320	8·640093	4848	11·359907	·000414	08	9·999586	30
31	8·642563	4806	1·357437	8·642982	4816	11·357018	·000419	10	9·999581	29
32	8·645428	4775	1·354572	8·645853	4784	11·354147	·000425	10	9·999575	28
33	8·648274	4743	1·351726	8·648704	4753	11·351296	·000430	08	9·999570	27
34	8·651102	4712	1·348898	8·651537	4722	11·348463	·000436	10	9·999564	26
35	8·653911	4682	1·346089	8·654352	4691	11·345648	·000442	10	9·999558	25
36	8·656702	4652	1·343298	8·657149	4661	11·342851	·000447	08	9·999553	24
37	8·659475	4622	1·340525	8·659928	4631	11·340072	·000453	10	9·999547	23
38	8·662230	4592	1·337770	8·662689	4602	11·337311	·000459	10	9·999541	22
39	8·664968	4563	1·335032	8·665433	4573	11·334567	·000465	10	9·999535	21
40	8·667689	4535	1·332311	8·668160	4544	11·331840	·000471	10	9·999529	20
41	8·670393	4506	1·329607	8·670870	4516	11·329130	·000476	08	9·999524	19
42	8·673080	4479	1·326920	8·673563	4488	11·326437	·000482	10	9·999518	18
43	8·675751	4451	1·324249	8·676239	4461	11·323761	·000488	10	9·999512	17
44	8·678405	4424	1·321595	8·678900	4434	11·321100	·000494	10	9·999506	16
45	8·681043	4397	1·318957	8·681544	4407	11·318456	·000500	10	9·999500	15
46	8·683665	4370	1·316335	8·684172	4380	11·315828	·000507	10	9·999493	14
47	8·686272	4344	1·313728	8·686784	4354	11·313216	·000513	12	9·999487	13
48	8·688863	4318	1·311137	8·689381	4328	11·310619	·000519	10	9·999481	12
49	8·691438	4292	1·308562	8·691963	4303	11·308037	·000525	10	9·999475	11
50	8·693998	4267	1·306002	8·694529	4277	11·305471	·000531	10	9·999469	10
51	8·696543	4242	1·303457	8·697081	4252	11·302919	·000537	12	9·999463	9
52	8·699073	4217	1·300927	8·699617	4228	11·300383	·000544	10	9·999456	8
53	8·701589	4192	1·298411	8·702139	4203	11·297861	·000550	10	9·999450	7
54	8·704090	4168	1·295910	8·704646	4179	11·295354	·000557	10	9·999443	6
55	8·706577	4144	1·293423	8·707140	4155	11·292860	·000563	12	9·999437	5
56	8·709049	4121	1·290951	8·709618	4132	11·290382	·000569	10	9·999431	4
57	8·711507	4097	1·288493	8·712083	4108	11·287917	·000576	12	9·999424	3
58	8·713952	4074	1·286048	8·714534	4085	11·285466	·000582	10	9·999418	2
59	8·716383	4052	1·283617	8·716972	4062	11·283028	·000589	12	9·999411	1
60	8·718800	4029	1·281200	8·719396	4040	11·280604	·000596	12	9·999404	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

87 DEG.

3 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	8·718800		1·281200	8·719396		11·280604	·000596		9·999404	60
1	8·721204	4006	1·278796	8·721806	4017	11·278194	·000602	10	9·999398	59
2	8·723595	3984	1·276405	8·724204	3995	11·275796	·000609	12	9·999391	58
3	8·725972	3962	1·274028	8·726588	3974	11·273412	·000616	12	9·999384	57
4	8·728337	3941	1·271663	8·728959	3952	11·271041	·000622	10	9·999378	56
5	8·730688	3919	1·269312	8·731317	3931	11·268683	·000629	12	9·999371	55
6	8·733027	3898	1·266973	8·733663	3909	11·266337	·000636	12	9·999364	54
7	8·735354	3877	1·264646	8·735996	3889	11·264004	·000643	12	9·999357	53
8	8·737667	3857	1·262333	8·738317	3868	11·261683	·000650	12	9·999350	52
9	8·739969	3836	1·260031	8·740626	3848	11·259374	·000657	12	9·999343	51
10	8·742259	3816	1·257741	8·742922	3827	11·257078	·000664	12	9·999336	50
11	8·744536	3796	1·255464	8·745207	3807	11·254793	·000671	12	9·999329	49
12	8·746802	3776	1·253198	8·747479	3787	11·252521	·000678	12	9·999322	48
13	8·749055	3756	1·250945	8·749740	3768	11·250260	·000685	12	9·999315	47
14	8·751297	3737	1·248703	8·751989	3749	11·248011	·000692	12	9·999308	46
15	8·753528	3717	1·246472	8·754227	3729	11·245773	·000699	12	9·999301	45
16	8·755747	3698	1·244253	8·756453	3710	11·243547	·000706	12	9·999294	44
17	8·757955	3679	1·242045	8·758668	3692	11·241332	·000713	13	9·999287	43
18	8·760151	3661	1·239849	8·760872	3673	11·239128	·000721	12	9·999279	42
19	8·762337	3642	1·237663	8·763065	3655	11·236935	·000728	12	9·999272	41
20	8·764511	3624	1·235489	8·765246	3636	11·234754	·000735	12	9·999265	40
21	8·766675	3606	1·233325	8·767417	3618	11·232583	·000743	13	9·999257	39
22	8·768828	3588	1·231172	8·769578	3600	11·230422	·000750	12	9·999250	38
23	8·770970	3570	1·229030	8·771727	3583	11·228273	·000758	13	9·999242	37
24	8·773101	3553	1·226899	8·773866	3565	11·226134	·000765	12	9·999235	36
25	8·775223	3535	1·224777	8·775995	3548	11·224005	·000773	13	9·999227	35
26	8·777333	3518	1·222667	8·778114	3531	11·221886	·000780	12	9·999220	34
27	8·779434	3501	1·220566	8·780222	3514	11·219778	·000788	13	9·999212	33
28	8·781524	3484	1·218476	8·782320	3497	11·217680	·000795	13	9·999205	32
29	8·783605	3467	1·216395	8·784408	3480	11·215592	·000803	13	9·999197	31
30	8·785675	3451	1·214325	8·786486	3464	11·213514	·000811	13	9·999189	30
31	8·787736	3434	1·212264	8·788554	3447	11·211446	·000819	13	9·999181	29
32	8·789787	3418	1·210213	8·790613	3431	11·209387	·000826	12	9·999174	28
33	8·791828	3402	1·208172	8·792662	3415	11·207338	·000834	13	9·999166	27
34	8·793859	3386	1·206141	8·794701	3399	11·205299	·000842	13	9·999158	26
35	8·795881	3370	1·204119	8·796731	3383	11·203269	·000850	13	9·999150	25
36	8·797894	3354	1·202106	8·798752	3368	11·201248	·000858	13	9·999142	24
37	8·799897	3339	1·200103	8·800763	3352	11·199237	·000866	13	9·999134	23
38	8·801892	3323	1·198108	8·802765	3337	11·197235	·000874	13	9·999126	22
39	8·803876	3308	1·196124	8·804758	3322	11·195242	·000882	13	9·999118	21
40	8·805852	3293	1·194148	8·806742	3306	11·193258	·000890	13	9·999110	20
41	8·807819	3278	1·192181	8·808717	3292	11·191283	·000898	13	9·999102	19
42	8·809777	3263	1·190223	8·810683	3277	11·189317	·000906	13	9·999094	18
43	8·811726	3249	1·188274	8·812641	3262	11·187359	·000914	13	9·999086	17
44	8·813667	3234	1·186333	8·814589	3248	11·185411	·000923	15	9·999077	16
45	8·815599	3219	1·184401	8·816529	3233	11·183471	·000931	13	9·999069	15
46	8·817522	3205	1·182478	8·818461	3219	11·181539	·000939	13	9·999061	14
47	8·819436	3191	1·180564	8·820384	3205	11·179616	·000947	15	9·999053	13
48	8·821343	3177	1·178657	8·822298	3191	11·177702	·000956	13	9·999044	12
49	8·823240	3163	1·176760	8·824205	3177	11·175795	·000964	13	9·999036	11
50	8·825130	3149	1·174870	8·826103	3163	11·173897	·000973	15	9·999027	10
51	8·827011	3135	1·172989	8·827992	3150	11·172008	·000981	13	9·999019	9
52	8·828884	3122	1·171116	8·829874	3136	11·170126	·000990	15	9·999010	8
53	8·830749	3108	1·169251	8·831748	3123	11·168252	·000998	13	9·999002	7
54	8·832607	3095	1·167393	8·833613	3109	11·166387	·001007	15	9·998993	6
55	8·834456	3082	1·165544	8·835471	3096	11·164529	·001016	15	9·998984	5
56	8·836297	3069	1·163703	8·837321	3083	11·162679	·001024	13	9·998976	4
57	8·838130	3056	1·161870	8·839163	3070	11·160837	·001033	15	9·998967	3
58	8·839956	3043	1·160044	8·840998	3057	11·159002	·001042	15	9·998958	2
59	8·841774	3030	1·158226	8·842825	3045	11·157175	·001050	13	9·998950	1
60	8·843585	3017	1·156415	8·844644	3032	11·155356	·001059	15	9·998941	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

86 DEG.

4 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	8·843585		1·156415	8·844644		11·155356	·001059		9·998941	60
1	8·845387	3005	1·154613	8·846455	3019	11·153545	·001068	15	9·998932	59
2	8·847183	2992	1·152817	8·848260	3007	11·151740	·001077	15	9·998923	58
3	8·848971	2980	1·151029	8·850057	2995	11·149943	·001086	15	9·998914	57
4	8·850751	2967	1·149249	8·851846	2982	11·148154	·001095	15	9·998905	56
5	8·852525	2955	1·147475	8·853628	2970	11·146372	·001104	15	9·998896	55
6	8·854291	2943	1·145709	8·855403	2958	11·144597	·001113	15	9·998887	54
7	8·856049	2931	1·143951	8·857171	2946	11·142829	·001122	15	9·998878	53
8	8·857801	2919	1·142199	8·858932	2935	11·141068	·001131	15	9·998869	52
9	8·859546	2908	1·140454	8·860686	2923	11·139314	·001140	15	9·998860	51
10	8·861283	2896	1·138717	8·862433	2911	11·137567	·001149	15	9·998851	50
11	8·863014	2884	1·136986	8·864173	2900	11·135827	·001159	17	9·998841	49
12	8·864738	2873	1·135262	8·865906	2888	11·134094	·001168	15	9·998832	48
13	8·866455	2861	1·133545	8·867632	2877	11·132368	·001177	15	9·998823	47
14	8·868165	2850	1·131835	8·869351	2866	11·130649	·001187	17	9·998813	46
15	8·869868	2839	1·130132	8·871064	2854	11·128936	·001196	15	9·998804	45
16	8·871565	2828	1·128435	8·872770	2843	11·127230	·001205	15	9·998795	44
17	8·873255	2817	1·126745	8·874469	2832	11·125531	·001215	17	9·998785	43
18	8·874938	2806	1·125062	8·876162	2821	11·123838	·001224	15	9·998776	42
19	8·876615	2795	1·123385	8·877849	2811	11·122151	·001234	17	9·998766	41
20	8·878285	2784	1·121715	8·879529	2800	11·120471	·001243	15	9·998757	40
21	8·879949	2773	1·120051	8·881202	2789	11·118798	·001253	17	9·998747	39
22	8·881607	2763	1·118393	8·882869	2779	11·117131	·001262	17	9·998738	38
23	8·883258	2752	1·116742	8·884530	2768	11·115470	·001272	15	9·998728	37
24	8·884903	2742	1·115097	8·886185	2758	11·113815	·001282	17	9·998718	36
25	8·886542	2731	1·113458	8·887833	2747	11·112167	·001292	17	9·998708	35
26	8·888174	2721	1·111826	8·889476	2737	11·110524	·001301	15	9·998699	34
27	8·889801	2711	1·110199	8·891112	2727	11·108888	·001311	17	9·998689	33
28	8·891421	2700	1·108579	8·892742	2717	11·107258	·001321	17	9·998679	32
29	8·893035	2690	1·106965	8·894366	2707	11·105634	·001332	17	9·998669	31
30	8·894643	2680	1·105357	8·895984	2697	11·104016	·001341	17	9·998659	30
31	8·896246	2670	1·103754	8·897596	2687	11·102404	·001351	17	9·998649	29
32	8·897842	2660	1·102158	8·899203	2677	11·100797	·001361	17	9·998639	28
33	8·899432	2651	1·100568	8·900803	2667	11·099197	·001371	17	9·998629	27
34	8·901017	2641	1·098983	8·902398	2658	11·097602	·001381	17	9·998619	26
35	8·902596	2631	1·097404	8·903987	2648	11·096013	·001391	17	9·998609	25
36	8·904169	2622	1·095831	8·905570	2638	11·094430	·001401	17	9·998599	24
37	8·905736	2612	1·094264	8·907147	2629	11·092853	·001411	17	9·998589	23
38	8·907297	2603	1·092703	8·908719	2620	11·091281	·001422	18	9·998578	22
39	8·908853	2593	1·091147	8·910285	2610	11·089715	·001432	17	9·998568	21
40	8·910404	2584	1·089596	8·911846	2601	11·088154	·001442	17	9·998558	20
41	8·911949	2575	1·088051	8·913401	2592	11·086599	·001452	17	9·998548	19
42	8·913488	2566	1·086512	8·914951	5283	11·085049	·001463	18	9·998537	18
43	8·915022	2556	1·084978	8·916495	2574	11·083505	·001473	17	9·998527	17
44	8·916550	2547	1·083450	8·918034	2565	11·081966	·001484	18	9·998516	16
45	8·918073	2538	1·081927	8·919568	2556	11·080432	·001494	17	9·998506	15
46	8·919591	2529	1·080409	8·921096	2547	11·078904	·001505	18	9·998495	14
47	8·921103	2520	1·078897	8·922619	2538	11·077381	·001515	17	9·998485	13
48	8·922610	2512	1·077390	8·924136	2530	11·075864	·001526	18	9·998474	12
49	8·924112	2503	1·075888	8·925649	2521	11·074351	·001536	17	9·998464	11
50	8·925609	2494	1·074391	8·927156	2512	11·072844	·001547	18	9·998453	10
51	8·927100	2486	1·072900	8·928658	2503	11·071342	·001558	18	9·998442	9
52	8·928587	2477	1·071413	8·930155	2495	11·069845	·001569	18	9·998431	8
53	8·930068	2469	1·069932	8·931647	2486	11·068353	·001579	17	9·998421	7
54	8·931544	2460	1·068456	8·933134	2478	11·066866	·001590	18	9·998410	6
55	8·933015	2452	1·066985	8·934616	2470	11·065384	·001601	18	9·998399	5
56	8·934481	2443	1·065519	8·936093	2461	11·063907	·001612	18	9·998388	4
57	8·935942	2435	1·064058	8·937565	2453	11·062435	·001623	18	9·998377	3
58	8·937398	2427	1·062602	8·939032	2445	11·060968	·001634	18	9·998366	2
59	8·938850	2419	1·061150	8·940494	2437	11·059506	·001645	18	9·998355	1
60	8·940296	2411	1·059704	8·941952	2429	11·058048	·001656	18	9·998344	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

85 DEG.

5 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	8·940296		1·059704	8·941952		11·058048	·001656		9·998344	60
1	8·941738	2403	1·058262	8·943404	2421	11·056596	·001667	18	9·998333	59
2	8·943174	2394	1·056826	8·944852	2413	11·055148	·001678	18	9·998322	58
3	8·944606	2387	1·055394	8·946295	2405	11·053705	·001689	18	9·998311	57
4	8·946034	2379	1·053966	8·947734	2397	11·052266	·001700	18	9·998300	56
5	8·947456	2371	1·052544	8·949168	2390	11·050832	·001711	18	9·998289	55
6	8·948874	2363	1·051126	8·950597	2382	11·049403	·001723	20	9·998277	54
7	8·950287	2355	1·049713	8·952021	2374	11·047979	·001734	18	9·998266	53
8	8·951696	2348	1·048304	8·953441	2366	11·046559	·001745	18	9·998255	52
9	8·953100	2340	1·046900	8·954856	2359	11·045144	·001757	20	9·998243	51
10	8·954499	2332	1·045501	8·956267	2351	11·043733	·001768	18	9·998232	50
11	8·955894	3325	1·044106	8·957674	2344	11·042326	·001780	20	9·998220	49
12	8·957284	2317	1·042716	8·959075	2336	11·040925	·001791	18	9·998209	48
13	8·958670	2310	1·041330	8·960473	2329	11·039527	·001803	20	9·998197	47
14	8·960052	2302	1·039948	8·961866	2322	11·038134	·001814	18	9·998186	46
15	8·961429	2295	1·038571	8·963255	2314	11·036745	·001826	20	9·998174	45
16	8·962801	2288	1·037199	8·964639	2307	11·035361	·001837	18	9·998163	44
17	8·964170	2280	1·035830	8·966019	2300	11·033981	·001849	20	9·998151	43
18	8·965534	2273	1·034466	8·967394	2293	11·032606	·001861	20	9·998139	42
19	8·966893	2266	1·033107	8·968766	2286	11·031234	·001872	18	9·998128	41
20	8·968249	2259	1·031751	8·970133	2279	11·029867	·001884	20	9·998116	40
21	8·969600	2252	1·030400	8·971496	2271	11·028504	·001896	20	9·998104	39
22	8·970947	2245	1·029053	8·972855	2265	11·027145	·001908	20	9·998092	38
23	8·972289	2238	1·027711	8·974209	2257	11·025791	·001920	20	9·998080	37
24	8·973628	2231	1·026372	8·975560	2251	11·024440	·001932	20	9·998068	36
25	8·974962	2224	1·025038	8·976906	2244	11·023094	·001944	20	9·998056	35
26	8·976293	2217	1·023707	8·978248	2237	11·021752	·001956	20	9·998044	34
27	8·977619	2210	1·022381	8·979586	2230	11·020414	·001968	20	9·998032	33
28	8·978941	2203	1·021059	8·980921	2223	11·019079	·001980	20	9·998020	32
29	8·980259	2197	1·019741	8·982251	2217	11·017749	·001992	20	9·998008	31
30	8·981573	2190	1·018427	8·983577	2210	11·016423	·002004	20	9·997996	30
31	8·982883	2183	1·017117	8·984899	2204	11·015101	·002016	20	9·997984	29
32	8·984189	2177	1·015811	8·986217	2197	11·013783	·002028	20	9·997972	28
33	8·985491	2170	1·014509	8·987532	2191	11·012468	·002041	22	9·997959	27
34	8·986789	2163	1·013211	8·988842	2184	11·011158	·002053	20	9·997947	26
35	8·988083	2157	1·011917	8·990149	2178	11·009851	·002065	20	9·997935	25
36	8·989374	2150	1·010626	8·991451	2171	11·008549	·002078	22	9·997922	24
37	8·990660	2144	1·009340	8·992750	2165	11·007250	·002090	20	9·997910	23
38	8·991943	2138	1·008057	8·994045	2158	11·005955	·002103	22	9·997897	22
39	8·993222	2131	1·006778	8·995337	2152	11·004663	·002115	20	9·997885	21
40	8·994497	2125	1·005503	8·996624	2146	11·003376	·002128	22	9·997872	20
41	8·995768	2119	1·004232	8·997908	2140	11·002092	·002140	20	9·997860	19
42	8·997036	2112	1·002964	8·999188	2134	11·000812	·002153	22	9·997847	18
43	8·998299	2106	1·001701	9·000465	2127	10·999535	·002165	20	9·997835	17
44	8·999560	2100	1·000440	9·001738	2121	10·998262	·002178	22	9·997822	16
45	9·000816	2094	0·999184	9·003007	2115	10·996993	·002191	22	9·997809	15
46	9·002069	2088	0·997931	9·004272	2109	10·995728	·002203	20	9·997797	14
47	9·003318	2082	0·996682	9·005534	2103	10·994466	·002216	22	9·997784	13
48	9·004563	2076	0·995437	9·006792	2097	10·993208	·002229	22	9·997771	12
49	9·005805	2070	0·994195	9·008047	2091	10·991953	·002242	22	9·997758	11
50	9·007044	2064	0·992956	9·009298	2085	10·990702	·002255	22	9·997745	10
51	9·008278	2058	0·991722	9·010546	2079	10·989454	·002268	22	9·997732	9
52	9·009510	2052	0·990490	9·011790	2074	10·988210	·002281	22	9·997719	8
53	9·010737	2046	0·989263	9·013031	2068	10·986969	·002294	22	9·997706	7
54	9·011962	2040	0·988038	9·014268	2062	10·985732	·002307	22	9·997693	6
55	9·013182	2034	0·986818	9·015502	2056	10·984498	·002320	22	9·997680	5
56	9·014400	2029	0·985600	9·016732	2051	10·983268	·002333	22	9·997667	4
57	9·015613	2023	0·984387	9·017959	2045	10·982041	·002346	22	9·997654	3
58	9·016824	2017	0·983176	9·019183	2039	10·980817	·002359	22	9·997641	2
59	9·018031	2012	0·981969	9·020403	2034	10·979597	·002372	22	9·997628	1
60	9·019235	2006	0·980765	9·021620	2028	10·978380	·002386	23	9·997614	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

84 DEG.

6 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·019235		·980765	9·021620		10·978380	·002386		9·997614	60
1	9·020435	2000	·979565	9·022834	2023	10·977166	·002399	22	9·997601	59
2	9·021632	1995	·978368	9·024044	2017	10·975956	·002412	22	9·997588	58
3	9·022825	1989	·977175	9·025251	2011	10·974749	·002426	23	9·997574	57
4	9·024016	1984	·975984	9·026455	2006	10·973545	·002439	22	9·997561	56
5	9·025203	1978	·974797	9·027655	2001	10·972345	·002453	23	9·997547	55
6	9·026386	1973	·973614	9·028852	1995	10·971148	·002466	22	9·997534	54
7	9·027567	1967	·972433	9·030046	1990	10·969954	·002480	23	9·997520	53
8	9·028744	1962	·971256	9·031237	1985	10·968763	·002493	22	9·997507	52
9	9·029918	1957	·970082	9·032425	1979	10·967575	·002507	23	9·997493	51
10	9·031089	1951	·968911	9·033609	1974	10·966391	·002520	22	9·997480	50
11	9·032257	1946	·967743	9·034791	1969	10·965209	·002534	28	9·997466	49
12	9·033421	1941	·966579	9·035969	1964	10·964031	·002548	23	9·997452	48
13	9·034582	1936	·965418	9·037144	1958	10·962856	·002561	22	9·997439	47
14	9·035741	1930	·964259	9·038316	1953	10·961684	·002575	23	9·997425	46
15	9·036896	1925	·963104	9·039485	1948	10·960515	·002589	23	9·997411	45
16	9·038048	1920	·961952	9·040651	1943	10·959349	·002603	23	9·997397	44
17	9·039197	1915	·960803	9·041813	1938	10·958187	·002617	23	9·997383	43
18	9·040342	1910	·959658	9·042973	1933	10·957027	·002631	23	9·997369	42
19	9·041485	1905	·958515	9·044130	1928	10·955870	·002645	23	9·997355	41
20	9·042625	1899	·957375	9·045284	1923	10·954716	·002659	23	9·997341	40
21	9·043762	1895	·956238	9·046434	1918	10·953566	·002673	23	9·997327	39
22	9·044895	1889	·955105	9·047582	1913	10·952418	·002687	23	9·997313	38
23	9·046026	1884	·953974	9·048727	1908	10·951273	·002701	23	9·997299	37
24	9·047154	1879	·952846	9·049869	1903	10·950131	·002715	23	9·997285	36
25	9·048279	1875	·951721	9·051008	1898	10·948992	·002729	23	9·997271	35
26	9·049400	1870	·950600	9·052144	1893	10·947856	·002743	23	9.997257	34
27	9·050519	1865	·949481	9·053277	1889	10·946723	·002758	25	9·997242	33
28	9·051635	1860	·948365	9·054407	1884	10·945593	·002772	23	9·997228	32
29	9·052749	1855	·947251	9·055535	1879	10·944465	·002786	23	9·997214	31
30	9·053859	1850	·946141	9·056659	1874	10·943341	·002801	25	9·997199	30
31	9·054966	1845	·945034	9·057781	1870	10·942219	·002815	23	9·997185	29
32	9·056071	1841	·943929	9·058900	1865	10·941100	·002830	25	9·997170	28
33	9·057172	1836	·942828	9·060016	1860	10·939984	·002844	23	9·997156	27
34	9·058271	1831	·941729	9·061130	1855	10·938870	·002859	25	9·997141	26
35	9·059367	1827	·940633	9·062240	1851	10·937760	·002873	23	9·997127	25
36	9·060460	1822	·939540	9·063348	1846	10·936652	·002888	25	9·997112	24
37	9·061551	1817	·938449	9·064453	1842	10·935547	·002902	23	9·997098	23
38	9·062639	1813	·937361	9·065556	1837	10·934444	·002917	25	9·997083	22
39	9·063724	1808	·936276	9·066655	1833	10·933345	·002932	25	9·997068	21
40	9·064806	1804	·935194	9·067752	1828	10·932248	·002947	25	9·997053	20
41	9·065885	1799	·934115	9·068846	1824	10·931154	·002961	23	9·997039	19
42	9·066962	1794	·933038	9·069938	1819	10·930062	·002976	25	9·997024	18
43	9·068036	1790	·931964	9·071027	1815	10·928973	·002991	25	9·997009	17
44	9·069107	1786	·930893	9·072113	1810	10·927887	·003006	25	9·996994	16
45	9·070176	1781	·929824	9·073197	1806	10·926803	·003021	25	9·996979	15
46	9·071242	1777	·928758	9·074278	1802	10·925722	·003036	25	9·996964	14
47	9·072306	1772	·927694	9·075356	1797	10·024644	·003051	25	9·996949	13
48	9·073366	1768	·926634	9·076432	1793	10·923568	·003066	25	9·996934	12
49	9·074424	1763	·925576	9·077505	1789	10·922495	·003081	25	9·996919	11
50	9·075480	1759	·924520	9.078576	1784	10·921424	·003096	25	9·996904	10
51	9·076533	1755	·923467	9·079644	1780	10·920356	·003111	25	9·996889	9
52	9·077583	1750	·922417	9·080710	1776	10·919290	·003126	25	9·996874	8
53	9·078631	1746	·921369	9·081773	1772	10·918227	·003142	27	9·996858	7
54	9·079676	1742	·920324	9·082833	1767	10·917167	·003157	25	9·996843	6
55	9·080719	1738	·919281	9·083891	1763	10·916109	·003172	25	9·996828	5
56	9·081759	1733	·918241	9·084947	1759	10 915053	·003188	27	9·996812	4
57	9·082797	1729	·917203	9·086000	1755	10·914000	·003203	25	9·996797	3
58	9·083232	1725	·916168	9·087050	1751	10·912950	·003218	25	9·996782	2
59	9·084864	1721	·915136	9·088098	1747	10·911902	·003234	27	9·996766	1
60	9·085894	1717	·914106	9·089144	1743	10·910856	·003249	25	9·996751	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

83 DEG.

7 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·085894		·914106	9·089144		10·910856	·003249		9·996751	60
1	9·086922	1713	·913078	9·090187	1738	10·909813	·003265	27	9·996735	59
2	9·087947	1709	·912053	9·091228	1735	10·908772	·003280	25	9·996720	58
3	9·088970	1704	·911030	9·092266	1731	10·907734	·003296	27	9·996704	57
4	9·089990	1700	·910010	9·093302	1727	10·906698	·003312	27	9·996688	56
5	9·091008	1696	·908992	9·094336	1722	10·905664	·003327	25	9·996673	55
6	9·092024	1692	·907976	9·095367	1719	10·904633	·003343	27	9·996657	54
7	9·093037	1688	·906963	9·096395	1715	10·903605	·003359	27	9·996641	53
8	9·094047	1684	·905953	9·097422	1711	10·902578	·003375	27	9·996625	52
9	9·095056	1680	·904944	9·098446	1707	10·901554	·003390	25	9·996610	51
10	9·096062	1676	·903938	9·099468	1703	10·900532	·003406	27	9·996594	50
11	9·097065	1673	·902935	9·100487	1699	10·899513	·003422	27	9·996578	49
12	9·098066	1668	·901934	9·101504	1695	10·898496	·003438	27	9·996562	48
13	9·099065	1665	·900935	9·102519	1691	10·897481	003454	27	9·996546	47
14	9·100062	1661	·899938	9·103532	1687	10·896468	·003470	27	9·996530	46
15	9·101056	1657	·898944	9·104542	1684	10·895458	·003486	27	9·996514	45
16	9·102048	1653	·897952	9·105550	1680	10·894450	·003502	27	9·996498	44
17	9·103037	1649	·896963	9·106556	1676	10·893444	·003518	27	9·996482	43
18	9·104025	1645	·895975	9·107559	1672	10·892441	·003535	28	9·996465	42
19	9·105010	1642	·894990	9·108560	1669	10·891440	·003551	27	9·996449	41
20	9·105992	1638	·894008	9·109559	1665	10·890441	·003567	27	9·996433	40
21	9·106973	1634	·893027	9·110556	1661	10·889444	·003583	27	9·996417	39
22	9·107951	1630	·892049	9·111551	1658	10·888449	·003600	28	9·996400	38
23	9·108927	1627	·891073	9·112543	1654	10·887457	·003616	27	9·996384	37
24	9·109901	1623	·890099	9·113533	1650	10·886467	·003632	27	9·996368	36
25	9·110873	1619	·889127	9·114521	1647	10·885479	·003649	28	9·996351	35
26	9·111842	1616	·888158	9·115507	1643	10·884493	·003665	27	9·996335	34
27	9·112809	1612	·887191	9·116491	1639	10·883509	·003682	28	9·996318	33
28	9·113774	1608	·886226	9·117472	1636	10·882528	·003698	27	9·996302	32
29	9·114737	1605	·885263	9·118452	1632	10·881548	·003715	28	9·996285	31
30	9·115698	1601	·884302	9·119429	1629	10·880571	·003731	27	9·996269	30
31	9·116656	1597	·883344	9·120404	1625	10·879596	·003748	28	9·996252	29
32	9·117613	1594	·882387	9·121377	1622	10·878623	·003765	28	9·996235	28
33	9·118567	1590	·881433	9·122348	1618	10·877652	·003781	27	9·996219	27
34	9·119519	1587	·880481	9·123317	1615	10·876683	·003798	28	9·996202	26
35	9·120469	1583	·879531	9·124284	1611	10·875716	·003815	28	9·996185	25
36	9·121417	1580	·878583	9·125249	1608	10·874751	·003832	28	9·996168	24
37	9·122362	1576	·877638	9·126211	1604	10·873789	·003849	28	9·996151	23
38	9·123306	1573	·876694	9·127172	1601	10·872828	·003866	28	9·996134	22
39	9·124248	1569	·875752	9·128130	1597	10·871870	·003883	28	9·996117	21
40	9·125187	1566	·874813	9·129087	1594	10·870913	·003900	28	9·996100	20
41	9·126125	1562	·873875	9·130041	1591	10·869959	·003917	28	9·996083	19
42	9·127060	1559	·872940	9·130994	1587	10·869006	·003934	28	9·996066	18
43	9·127993	1556	·872007	9·131944	1584	10·868056	·003951	28	9·996049	17
44	9·128925	1552	·871075	9·132893	1581	10·867107	·003968	28	9·996032	16
45	9·129854	1549	·870146	9·133839	1577	10·866161	·003985	28	9·996015	15
46	9·130781	1545	·869219	9·134784	1574	10·865216	·004002	28	9·995998	14
47	9·131706	1542	·868294	9·135726	1571	10·864274	·004020	30	9·995980	13
48	9·132630	1539	·867370	9·136667	1567	10·863333	·004037	28	9·995963	12
49	9·133551	1535	·866449	9·137605	1564	10·862395	·004054	28	9·995946	11
50	9·134470	1532	·865530	9·138542	1561	10·861458	·004072	30	9·995928	10
51	9·135387	1529	·864613	9·139476	1558	10·860524	·004089	28	9·995911	9
52	9·136303	1525	·863697	9·140409	1555	10·859591	·004106	28	9·995894	8
53	9·137216	1522	·862784	9·141340	1551	10·858660	·004124	30	9·995876	7
54	9·138128	1519	·861872	9·142269	1548	10·857731	·004141	28	9·995859	6
55	9·139037	1516	·860963	9·143196	1545	10·856804	·004159	30	9·995841	5
56	9·139944	1512	·860056	9·144121	1542	10·855879	·004177	30	9·995823	4
57	9·140850	1509	·859150	9·145044	1539	10·854956	·004194	28	9·995806	3
58	9·141754	1506	·858246	9·145966	1535	10·854034	·004212	30	9·995788	2
59	9·142655	1503	·857345	9·146885	1532	10·853115	·004229	28	9·995771	1
60	9·143555	1500	·856445	9·147803	1529	10·852197	·004247	30	9·995753	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

82 DEG.

8 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·143555		·856445	9·147803		10·852197	·004247		9·995753	60
1	9·144453	1496	·855547	9·148718	1526	10·851282	·004265	30	9·995735	59
2	9·145349	1493	·854651	9·149632	1523	10·850368	·004283	30	9·995717	58
3	9·146243	1490	·853757	9·150544	1520	10·849456	·004301	30	9·995699	57
4	9·147136	1487	·852864	9·151454	1517	10·848546	·004319	30	9·995681	56
5	9·148026	1484	·851974	9·152363	1514	10·847637	·004336	28	9·995664	55
6	9·148915	1481	·851085	9·153269	1511	10·846731	·004354	30	9·995646	54
7	9·149802	1478	·850198	9·154174	1508	10·845826	·004372	30	9·995628	53
8	9·150686	1475	·849314	9·155077	1505	10·844923	·004390	30	9·995610	52
9	9·151569	1472	·848431	9·155978	1502	10·844022	·004409	32	9·995591	51
10	9·152451	1469	·847549	9·156877	1499	10·843123	·004427	30	9·995573	50
11	9·153330	1466	·846670	9·157775	1496	10·842225	·004445	30	9·995555	49
12	9·154208	1462	·845792	9·158671	1493	10·841329	·004463	30	9·995537	48
13	9·155083	1460	·844917	9·159565	1490	10·840435	·004481	30	9·995519	47
14	9·155957	1457	·844043	9·160457	1487	10·839543	·004499	30	9·995501	46
15	9·156830	1454	·843170	9·161347	1484	10·838653	·004518	32	9·995482	45
16	9·157700	1451	·842300	9·162236	1481	10·837764	·004536	30	9·995464	44
17	9·158569	1448	·841431	9·163123	1478	10·836877	·004554	30	9·995446	43
18	9·159435	1445	·840565	9·164008	1475	10·835992	·004573	32	9·995427	42
19	9·160301	1442	·839699	9·164892	1473	10·835108	·004591	30	9·995409	41
20	9·161164	1439	·838836	9·165774	1470	10·834226	·004610	32	9·995390	40
21	9·162025	1436	·837975	9·166654	1467	10·833346	·004628	30	9·995372	39
22	9·162885	1433	·837115	9·167532	1464	10·832468	·004647	32	9·995353	38
23	9·163743	1430	·836257	9·168409	1461	10·831591	·004666	32	9·995334	37
24	9·164600	1427	·835400	9·169284	1458	10·830716	·004684	30	9·995316	36
25	9·165454	1424	·834546	9·170157	1455	10·829843	·004703	32	9·995297	35
26	9·166307	1422	·833693	9·171029	1453	10·828971	·004722	32	9·995278	34
27	9·167159	1419	·832841	9·171899	1450	10·828101	·004740	30	9·995260	33
28	9·168008	1416	·831992	9·172767	1447	10·827233	·004759	32	9·995241	32
29	9·168856	1413	·831144	9·173634	1444	10·826366	·004778	32	9·995222	31
30	9·169702	1410	·830298	9·174499	1442	10·825501	·004797	32	9·995203	30
31	9·170547	1407	·829453	9·175362	1439	10·824638	·004816	32	9·995184	29
32	9·171389	1405	·828611	9·176224	1436	10·823776	·004835	32	9·995165	28
33	9·172230	1402	·827770	9·177084	1433	10·822916	·004854	32	9·995146	27
34	9·173070	1399	·826930	9·177942	1431	10·822058	·004873	32	9·995127	26
35	9·173908	1396	·826092	9·178799	1428	10·821201	·004892	32	9·995108	25
36	9·174744	1394	·825256	9·179655	1425	10·820345	·004911	32	9·995089	24
37	9·175578	1391	·824422	9·180508	1423	10·819492	·004930	32	9·995070	23
38	9·176411	1388	·823589	9·181360	1420	10·818640	·004949	32	9·995051	22
39	9·177242	1385	·822758	9·182211	1417	10·817789	·004968	32	9·995032	21
40	9·178072	1383	·821928	9·183059	1415	10·816941	·004987	32	9·995013	20
41	9·178900	1380	·821100	9·183907	1412	10·816093	·005007	33	9·994993	19
42	9·179726	1377	·820274	9·184752	1409	10·815248	·005026	32	9·994974	18
43	9·180551	1374	·819449	9·185597	1407	10·814403	·005045	32	9·994955	17
44	9·181374	1372	·818626	9·186439	1404	10·813561	·005065	33	9·994935	16
45	9·182196	1369	·817804	9·187280	1402	10·812720	·005084	32	9·994916	15
46	9·183016	1367	·816984	9·188120	1399	10·811880	·005104	33	9·994896	14
47	9·183834	1364	·816166	9·188958	1396	10·811042	·005123	32	9·994877	13
48	9·184651	1361	·815349	9·189794	1394	10·810206	·005143	33	9·994857	12
49	9·185466	1359	·814534	9·190629	1391	10·809371	·005162	32	9·994838	11
50	9·186280	1356	·813720	9·191462	1389	10·808538	·005182	33	9·994818	10
51	9·187092	1353	·812908	9·192294	1386	10·807706	·005202	33	9·994798	9
52	9·187903	1351	·812097	9·193124	1384	10·806876	·005221	32	9·994779	8
53	9·188712	1348	·811288	9·193953	1381	10·806047	·005241	33	9·994759	7
54	9·189519	1346	·810481	9·194780	1379	10·805220	·005261	33	9·994739	6
55	9·190325	1343	·809675	9·195606	1376	10·804394	·005281	32	9·994720	5
56	9·191130	1341	·808870	9·196430	1374	10·803570	·005300	33	9·994700	4
57	9·191933	1338	·808067	9·197253	1371	10·802747	·005320	33	9·994680	3
58	9·192734	1336	·807266	9·198074	1369	10·801926	·005340	33	9·994660	2
59	9·193534	1333	·806466	9·198894	1366	10·801106	·005360	33	9·994640	1
60	9·194332	1330	·805668	9·199713	1364	10·800287	·005380	33	9·994620	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

81 DEG.

9 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·194332		·805668	9·199713		10·800287	·005380		9·994620	60
1	9·195129	1328	·804871	9·200529	1361	10·799471	·005400	33	9·994600	59
2	9·195925	1326	·804075	9·201345	1359	10·798655	·005420	33	9·994580	58
3	9·196719	1323	·803281	9·202159	1356	10·797841	·005440	33	9·994560	57
4	9·197511	1321	·802489	9·202971	1354	10·797029	·005460	33	9·994540	56
5	9·198302	1318	·801698	9·203782	1352	10·796218	·005481	35	9·994519	55
6	9·199091	1316	·800909	9·204592	1349	10·795408	·005501	33	9·994499	54
7	9·199879	1313	·800121	9·205400	1347	10·794600	·005521	33	9·994479	53
8	9·200666	1311	·799334	9·206207	1345	10·793793	·005541	33	9·994459	52
9	9·201451	1308	·798549	9·207013	1342	10·792987	·005562	35	9·994438	51
10	9·202234	1306	·797766	9·207817	1340	10·792183	·005582	33	9·994418	50
11	9·203017	1304	·796983	9·208619	1338	10·791381	·005602	33	9·994398	49
12	9·203797	1301	·796203	9·209420	1335	10·790580	·005623	35	9·994377	48
13	9·204577	1299	·795423	9·210220	1333	10·789780	·005643	33	9·994357	47
14	9·205354	1296	·794646	9·211018	1331	10·788982	·005664	35	9·994336	46
15	9·206131	1294	·793869	9·211815	1328	10·788185	·005684	33	9·994316	45
16	9·206906	1292	·793094	9·212611	1326	10·787389	·005705	35	9·994295	44
17	9·207679	1289	·792321	9·213405	1324	10·786595	·005726	35	9·994274	43
18	9·208452	1287	·791548	9·214198	1321	10·785802	·005746	33	9·994254	42
19	9·209222	1285	·790778	9·214989	1319	10·785011	·005767	35	9·994233	41
20	9·209992	1282	·790008	9·215780	1317	10·784220	·005788	35	9·994212	40
21	9·210760	1280	·789240	9·216568	1315	10·783432	·005809	35	9·994191	39
22	9·211526	1278	·788474	9·217356	1312	10·782644	·005829	33	9·994171	38
23	9·212291	1275	·787709	9·218142	1310	10·781858	·005850	35	9·994150	37
24	9·213055	1273	·786945	9·218926	1308	10·781074	·005871	35	9·994129	36
25	9·213818	1271	·786182	9·219710	1305	10·780290	·005892	35	9·994108	35
26	9·214579	1268	·785421	9·220492	1303	10·779508	·005913	35	9·994087	34
27	9·215338	1266	·784662	9·221272	1301	10·778728	·005934	35	9·994066	33
28	9·216097	1264	·783903	9·222052	1299	10·777948	·005955	35	9·994045	32
29	9·216854	1261	·783146	9·222830	1297	10·777170	·005976	35	9·994024	31
30	9·217609	1259	·782391	9·223607	1294	10·776393	·005997	35	9·994003	30
31	9·218363	1257	·781637	9·224382	1292	10·775618	·006018	35	9·993982	29
32	9·219116	1255	·780884	9·225156	1290	10·774844	·006040	37	9·993960	28
33	9·219868	1253	·780132	9·225929	1288	10·774071	·006061	35	9·993939	27
34	9·220618	1250	·779382	9·226700	1286	10·773300	·006082	35	9·993918	26
35	9·221367	1248	·778633	9·227471	1284	10·772529	·006103	35	9·993897	25
36	9·222115	1246	·777885	9·228239	1281	10·771761	·006125	37	9·993875	24
37	9·222861	1244	·777139	9·229007	1279	10·770993	·006146	35	9·993854	23
38	9·223606	1242	·776394	9·229773	1277	10·770227	·006168	37	9·993832	22
39	9·224349	1239	·775651	9·230539	1275	10·769461	·006189	35	9·993811	21
40	9·225092	1237	·774908	9·231302	1273	10·768698	·006211	37	9·993789	20
41	9·225833	1235	·774167	9·232065	1271	10·767935	·006232	35	9·993768	19
42	9·226573	1233	·773427	9·232826	1269	10·767174	·006254	37	9·993746	18
43	9·227311	1231	·772689	9·233586	1267	10·766414	·006275	35	9·993725	17
44	9·228048	1228	·771952	9·234345	1265	10·765655	·006297	37	9·993703	16
45	9·228784	1226	·771216	9·235103	1262	10·764897	·006319	37	9·993681	15
46	9·229518	1224	·770482	9·235859	1260	10·764141	·006340	35	9·993660	14
47	9·230252	1222	·769748	9·236614	1258	10·763386	·006362	37	9·993638	13
48	9·230984	1220	·769016	9·237368	1256	10·762632	·006384	37	9·993616	12
49	9·231715	1218	·768285	9·238120	1254	10·761880	·006406	37	9·993594	11
50	9·232444	1216	·767556	9·238872	1252	10·761128	·006428	37	9·993572	10
51	9·233172	1214	·766828	9·239622	1250	10·760378	·006450	37	9·993550	9
52	9·233899	1212	·766101	9·240371	1248	10·759629	·006472	37	9·993528	8
53	9·234625	1209	·765375	9·241118	1246	10·758882	·006494	37	9·993506	7
54	9·235349	1207	·764651	9·241865	1244	10·758135	·006516	37	9·993484	6
55	9·236073	1205	·763927	9·242610	1242	10·757390	·006538	37	9·993462	5
56	9·236795	1203	·763205	9·243354	1240	10·756646	·006560	37	9·993440	4
57	9·237515	1201	·762485	9·244097	1238	10·755903	·006582	37	9·993418	3
58	9·238235	1199	·761765	9·244839	1236	10·755161	·006604	37	9·993396	2
59	9·238953	1197	·761047	9·245579	1234	10·754421	·006626	37	9·993374	1
60	9·239670	1195	·760330	9·246319	1232	10·753681	·006649	38	9·993351	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

80 DEG.

10 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·239670		·760330	9·246319		10·753681	·006649		9·993351	60
1	9·240386	1193	·759614	9·247057	1230	10·752943	·006671	37	9·993329	59
2	9·241101	1191	·758899	9·247794	1228	10·752206	·006693	37	9·993307	58
3	9·241814	1189	·758181	9·248530	1226	10·751470	·006715	38	9·993285	57
4	9·242526	1187	·757474	9·249264	1224	10·750736	·006738	37	9·993262	56
5	9·243237	1185	·756763	9·249998	1222	10·750002	·006760	37	9·993240	55
6	9·243947	1183	·756053	9·250730	1220	10·749270	·006783	38	9·993217	54
7	9·244656	1181	·755344	9·251461	1218	10·748539	·006805	37	9·993195	53
8	9·245363	1179	·754637	9·252191	1217	10·747809	·006828	38	9·993172	52
9	9·246069	1177	·753931	9·252920	1215	10·747080	·006851	38	9·993149	51
10	9·246775	1175	·753225	9·253648	1213	10·746352	·006873	37	9·993127	50
11	9·247478	1173	·752522	9·254374	1211	10·745626	·006896	38	9·993104	49
12	9·248181	1171	·751819	9·255100	1209	10·744900	·006919	38	9·993081	48
13	9·248883	1169	·751117	9·255824	1207	10·744176	·006941	37	9·993059	47
14	9·249583	1167	·750417	9·256547	1205	10·743453	·006964	38	9·993036	46
15	9·250282	1165	·749718	9·257269	1203	10·742731	·006987	38	9·993013	45
16	9·250980	1163	·749020	9·257990	1201	10·742010	·007010	38	9·992990	44
17	9·251677	1161	·748323	9·258710	1200	10·741290	·007033	38	9·992967	43
18	9·252373	1159	·747627	9·259429	1198	10·740571	·007056	38	9·992944	42
19	9·253067	1158	·746933	9·260146	1196	10·739854	·007079	38	9·992921	41
20	9·253761	1156	·746239	9·260863	1194	10·739137	·007102	38	9·992898	40
21	9·254453	1154	·745547	9·261578	1192	10·738422	·007125	38	9·992875	39
22	9·255144	1152	·744856	9·262292	1190	10·737708	·007148	38	9·992852	38
23	9·255834	1150	·744166	9·263005	1189	10·736995	·007171	38	9·992829	37
24	9·256523	1148	·743477	9·263717	1187	10·736283	·007194	38	9·992806	36
25	9·257211	1146	·742789	9·264428	1185	10·735572	·007217	38	9·992783	35
26	9·257898	1144	·742102	9·265138	1183	10·734862	·007241	40	9·992759	34
27	9·258583	1142	·741417	9·265847	1181	10·734153	·007264	38	9·992736	33
28	9·259268	1141	·740732	9·266555	1179	10·733445	·007287	38	9·992713	32
29	9·259951	1139	·740049	9·267261	1178	10·732739	·007311	38	9·992690	31
30	9·260633	1137	·739367	9·267967	1176	10·732033	·007334	40	9·992666	30
31	9·261314	1135	·738686	9·268671	1174	10·731329	·007357	38	9·992643	29
32	9·261994	1133	·738006	9·269375	1172	10·730625	·007381	40	9·992619	28
33	9·262673	1131	·737327	9·270077	1170	10·729923	·007404	38	9·992596	27
34	9·263351	1130	·736649	9·270779	1169	10·729221	·007428	40	9·992572	26
35	9·264027	1128	·735973	9·271479	1167	10·728521	·007451	38	9·992549	25
36	9·264703	1126	·735297	9·272178	1165	10·727822	·007475	40	9·992525	24
37	9·265377	1124	·734623	9·272876	1164	10·727124	·007499	40	9·992501	23
38	9·266051	1122	·733949	9·273573	1162	10·726427	·007522	38	9·992478	22
39	9·266723	1120	·733277	9·274269	1160	10·725731	·007546	40	9·992454	21
40	9·267395	1119	·732605	9·274964	1158	10·725036	·007570	40	9·992430	20
41	9·268065	1117	·731935	9·275658	1157	10·724342	·007594	40	9·992406	19
42	9·268734	1115	·731266	9·276351	1155	10·723649	·007618	40	9·992382	18
43	9·269402	1113	·730598	9·277043	1153	10·722957	·007642	38	9·992358	17
44	9.270069	1111	·729931	9·277734	1151	10·722266	·007665	40	9·992335	16
45	9·270735	1110	·729265	9·278424	1150	10·721576	·007689	40	9·992311	15
46	9·271400	1108	·728600	9·279113	1148	10·720887	·007713	40	9·992287	14
47	9·272064	1106	·727936	9·279801	1146	10·720199	·007737	40	9·992263	13
48	9·272726	1105	·727274	9·280488	1145	10·719512	·007761	40	9·992239	12
49	9·273388	1103	·726612	9·281174	1143	10·718826	·007786	42	9·992214	11
50	9·274049	1101	·725951	9·281858	1141	10·718142	·007810	40	9·992190	10
51	9.274708	1099	·725292	9·282542	1140	10·717458	·007834	40	9·992166	9
52	9·275367	1098	·724633	9·283225	1138	10·716775	·007858	40	9·992142	8
53	9·276025	1096	·723975	9·283907	1136	10·716093	·007882	40	9·992118	7
54	9·276681	1094	·723319	9·284588	1135	10·715412	·007907	42	9·992093	6
55	9·277337	1092	·722663	9·285268	1133	10·714732	·007931	40	9·992069	5
56	9·277991	1091	·722009	9·285947	1131	10·714053	·007956	42	9·992044	4
57	9·278645	1089	·721355	9·286624	1130	10·713376	·007980	40	9·992020	3
58	9·279297	1087	·720703	9·287301	1128	10·712699	·008004	40	9·991996	2
59	9·279948	1086	·720052	9·287977	1126	10·712023	·008029	42	9·991971	1
60	9·280599	1084	·719401	9·288652	1125	10·711348	·008053	40	9·991947	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

79 DEG.

11 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·280599		·719401	9·288652		10·711348	·008053		9·991947	60
1	9·281248	1082	·718752	9·289326	1123	10·710674	·008078	42	9·991922	59
2	9·281897	1081	·718103	9·289999	1122	10·710001	·008103	42	9·991897	58
3	9·282544	1079	·717456	9·290671	1120	10·709329	·008127	40	9·991873	57
4	9·283190	1077	·716810	9·291342	1118	10·708658	·008152	42	9·991848	56
5	9·283836	1076	·716164	9·292013	1117	10·707987	·008177	42	9·991823	55
6	9·284480	1074	·715520	9·292682	1115	10·707318	·008201	40	9·991799	54
7	9·285124	1072	·714876	9·293350	1114	10·706650	·008226	42	9·991774	53
8	9·285766	1071	·714234	9·294017	1112	10·705983	·008251	42	9·991749	52
9	9·286408	1069	·713592	9·294684	1111	10·705316	·008276	42	9·991724	51
10	9·287048	1067	·712952	9·295349	1109	10·704651	·008301	42	9·991699	50
11	9·287688	1066	·712312	9·296013	1107	10·703987	·008326	42	9·991674	49
12	9·288326	1064	·711674	9·296677	1106	10·703323	·008351	42	9·991649	48
13	9·288964	1063	·711036	9·297339	1104	10·702661	·008376	42	9·991624	47
14	9·289600	1061	·710400	9·298001	1103	10·701999	·008401	42	9·991599	46
15	9·290236	1059	·709764	9·298662	1101	10·701338	·008426	42	9·991574	45
16	9·290870	1058	·709130	9·299322	1100	10·700678	·008451	42	9·991549	44
17	9·291504	1056	·708496	9·299980	1098	10·700020	·008476	42	9·991524	43
18	9·292137	1054	·707863	9·300638	1096	10·699362	·008502	43	9·991498	42
19	9·292768	1053	·707232	9·301295	1095	10·698705	·008527	42	9·991473	41
20	9·293399	1051	·706601	9·301951	1093	10·698049	·008552	42	9·991448	40
21	9·294029	1050	·705971	9·302607	1092	10·697393	·008578	43	9·991422	39
22	9·294658	1048	·705342	9·303261	1090	10·696739	·008603	42	9·991397	38
23	9·295286	1046	·704714	9·303914	1089	10·696086	·008628	42	9·991372	37
24	9·295913	1045	·704087	9·304567	1087	10·695433	·008654	43	9·991346	36
25	9·296539	1043	·703461	9·305218	1086	10·694782	·008679	42	9·991321	35
26	9·297164	1042	·702836	9·305869	1084	10·694131	·008705	43	9·991295	34
27	9·297788	1040	·702212	9·306519	1083	10·693481	·008730	42	9·991270	33
28	9·298412	1039	·701588	9·307168	1081	10·692832	·008756	43	9·991244	32
29	9·299034	1037	·700966	9·307815	1080	10·692185	·008782	43	9·991218	31
30	9·299655	1036	·700345	9·308463	1078	10·691537	·008807	42	9·991193	30
31	9·300276	1034	·699724	9·309109	1077	10·690891	·008833	43	9·991167	29
32	9·300895	1032	·699105	9·309754	1075	10·690246	·008859	43	9·991141	28
33	9·301514	1031	·698486	9·310398	1074	10·689602	·008885	43	9·991115	27
34	9·302132	1029	·697868	9·311042	1073	10·688958	·008910	42	9·991090	26
35	9·302748	1028	·697252	9·311685	1071	10·688315	·008936	43	9·991064	25
36	9·303364	1026	·696636	9·312327	1070	10·687673	·008962	43	9·991038	24
37	9·303979	1025	·696021	9·312967	1068	10·687033	·008988	43	9·991012	23
38	9·304593	1023	·695407	9·313608	1067	10·686392	·009014	43	9·990986	22
39	9·305207	1022	·694793	9·314247	1065	10·685753	·009040	43	9·990960	21
40	9·305819	1020	·694181	9·314885	1064	10·685115	·009066	43	9·990934	20
41	9·306430	1019	·693570	9·315523	1062	10·684477	·009092	43	9·990908	19
42	9·307041	1017	·692959	9·316159	1061	10·683841	·009118	43	9·990882	18
43	9·307650	1016	·692350	9·316795	1060	10·683205	·009145	45	9·990855	17
44	9·308259	1014	·691741	9·317430	1058	10·682570	·009171	43	9·990829	16
45	9·308867	1013	·691133	9·318064	1057	10·681936	·009197	43	9·990803	15
46	9·309474	1011	·690526	9·318697	1055	10·681303	·009223	43	9·990777	14
47	9·310080	1010	·689920	9·319329	1054	10·680671	·009250	45	9·990750	13
48	9·310685	1008	·689315	9·319961	1053	10·680039	·009276	43	9·990724	12
49	9·311289	1007	·688711	9·320592	1051	10·679408	·009303	45	9·990697	11
50	9·311893	1006	·688107	9·321222	1050	10·678778	·009329	43	9·990671	10
51	9·312495	1004	·687505	9·321851	1048	10·678149	·009355	43	9·990645	9
52	9·313097	1003	·686903	9·322479	1047	10·677521	·009382	45	9·990618	8
53	9·313698	1001	·686302	9·323106	1045	10·676894	·009409	45	9·990591	7
54	9·314297	1000	·685703	9·323733	1044	10·676267	·009435	43	9·990565	6
55	9·314897	998	·685103	9·324358	1043	10·675642	·009462	45	9·990538	5
56	9·315495	997	·684505	9·324983	1041	10·675017	·009489	45	9·990511	4
57	9·316092	996	·683908	9·325607	1040	10·674393	·009515	43	9·990485	3
58	9·316689	994	·683311	9·326231	1039	10·673769	·009542	45	9·990458	2
59	9·317284	993	·682716	9·326853	1037	10·673147	·009569	45	9·990431	1
60	9·317879	991	·682121	9·327475	1036	10·672525	·009596	45	9·990404	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

78 DEG.

12 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100′	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·317879		·682121	9·327474		10·672526	·009596		9·990404	60
1	9·318473	990	·681527	9·328095	1035	10·671905	·009622	43	9·990378	59
2	9·319066	988	·680934	9·328715	1033	10·671285	·009649	45	9·990351	58
3	9·319658	987	·680342	9·329334	1032	10·670666	·009676	45	9·990324	57
4	9·320249	986	·679751	9·329953	1030	10·670047	·009703	45	9·990297	56
5	9·320840	984	·679160	9·330570	1029	10·669430	·009730	45	9·990270	55
6	9·321430	983	·678570	9·331187	1028	10·668813	·009757	45	9·990243	54
7	9·322019	982	·677981	9·331803	1026	10·668197	·009785	47	9·990215	53
8	9·322607	980	·677393	9·332418	1025	10·667582	·009812	45	9·990188	52
9	9·323194	979	·676806	9·333033	1024	10·666967	·009839	45	9·990161	51
10	9·323780	977	·676220	9·333646	1023	10·666354	·009866	45	9·990134	50
11	9·324366	976	·675634	9·334259	1021	10·665741	·009893	45	9·990107	49
12	9·324950	975	·675050	9·334871	1020	10·665129	·009921	47	9·990079	48
13	9·325534	973	·674466	9·335482	1019	10·664518	·009948	45	9·990052	47
14	9·326117	972	·673883	9·336093	1017	10·663907	·009975	45	9·990025	46
15	9·326700	970	·673300	9·336702	1016	10·663298	·010003	47	9·989997	45
16	9·327281	969	·672719	9·337311	1015	10·662689	·010030	45	9·989970	44
17	9·327862	968	·672138	9·337919	1013	10·662081	·010058	47	9·989942	43
18	9·328442	966	·671558	9·338527	1012	10·661473	·010085	45	9·989915	42
19	9·329021	965	·670979	9·339133	1011	10·660867	·010113	47	9·989887	41
20	9·329599	964	·670401	9·339739	1010	10·660261	·010140	45	9·989860	40
21	9·330176	962	·669824	9·340344	1008	10·659656	·010168	47	9·989832	39
22	9·330753	961	·669247	9·340948	1007	10·659052	·010196	47	9·989804	38
23	9·331329	960	·668671	9·341552	1006	10·658448	·010223	45	9·989777	37
24	9·331903	958	·668097	9·342155	1004	10·657845	·010251	47	9·989749	36
25	9·332478	957	·667522	9·342757	1003	10·657243	·010279	47	9·989721	35
26	9·333051	956	·666949	9·343358	1002	10·656642	·010307	47	9·989693	34
27	9·333624	954	·666376	9·343958	1001	10·656042	·010335	47	9·989665	33
28	9·334195	953	·665805	9·344558	999	10·655442	·010363	47	9·989637	32
29	9·334767	952	·665233	9·345157	998	10·654843	·010390	45	9·989610	31
30	9·335337	950	·664663	9·345755	997	10·654245	·010418	47	9·989582	30
31	9·335906	949	·664094	9·346353	996	10·653647	·010447	48	9·989553	29
32	9·336475	948	·663525	9·346949	994	10·653051	·010475	47	9·989525	28
33	9·337043	946	·662957	9·347545	993	10·652455	·010503	47	9·989497	27
34	9·337610	945	·662390	9·348141	992	10·651859	·010531	47	9·989469	26
35	9·338176	944	·661824	9·348735	991	10·651265	·010559	47	9·989441	25
36	9·338742	943	·661258	9·349329	990	10·650671	·010587	47	9·989413	24
37	9·339307	941	·660693	9·349922	988	10·650078	·010615	47	9·989385	23
38	9·339871	940	·660129	9·350514	987	10·649486	·010644	48	9·989356	22
39	9·340434	939	·659566	9·351106	986	10·648894	·010672	47	9·989328	21
40	9·340996	937	·659004	9·351697	985	10·648303	·010700	47	9·989300	20
41	9·341558	936	·658442	9·352287	983	10·647713	·010729	48	9·989271	19
42	9·342119	935	·657881	9·352876	982	10·647124	·010757	47	9·989243	18
43	9·342679	934	·657321	9·353465	981	10·646535	·010786	48	9·989214	17
44	9·343239	932	·656761	9·354053	980	10·645947	·010814	47	9·989186	16
45	9·343797	931	·656203	9·354640	979	10·645360	·010843	48	9·989157	15
46	9·344355	930	·655645	9·355227	977	10·644773	·010872	48	9·989128	14
47	9·344912	929	·655088	9·355813	976	10·644187	·010900	47	9·989100	13
48	9·345469	927	·654531	9·356398	975	10·643602	·010929	48	9·989071	12
49	9·346024	926	·653976	9·356982	974	10·643018	·010958	48	9·989042	11
50	9·346579	925	·653421	9·357566	973	10·642434	·010986	47	9·989014	10
51	9·347134	924	·652866	9·358149	971	10·641851	·011015	48	9·988985	9
52	9·347687	922	·652313	9·358731	970	10·641269	·011044	48	9·988956	8
53	9·348240	921	·651760	9·359313	969	10·640687	·011073	48	9·988927	7
54	9·348792	920	·651208	9·359893	968	10·640107	·011102	48	9·988898	6
55	9·349343	919	·650657	9·360474	967	10·639526	·011131	48	9·988869	5
56	9·349893	917	·650107	9·361053	966	10·638947	·011160	48	9·988840	4
57	9·350443	916	·649557	9·361632	965	10·638368	·011189	48	9·988811	3
58	9·350992	915	·649008	9·362210	963	10·637790	·011218	48	9·988782	2
59	9·351540	914	·648460	9·362787	962	10·637213	·011247	48	9·988753	1
60	9·352088	913	·647912	9·363364	961	10·636636	·011276	48	9·988724	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

77 DEG.

13 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff 100″	Cosine.	′
0	9·352088		·647912	9·363364		10·636636	·011276		9·988724	60
1	9·352635	911	·647365	9·363940	960	10·636060	·011305	48	9·988695	59
2	9·353181	910	·646819	9·364515	959	10·635485	·011334	48	9·988666	58
3	9·353726	909	·646274	9·365090	958	10·634910	·011364	50	9·988636	57
4	9·354271	908	·645729	9·365664	957	10·634336	·011393	48	9·988607	56
5	9·354815	907	·645185	9·366237	955	10·633763	·011422	48	9·988578	55
6	9·355358	905	·644642	9·366810	954	10·633190	·011452	50	9·988548	54
7	9·355901	904	·644099	9·367382	953	10·632618	·011481	48	9·988519	53
8	9·356443	903	·643557	9·367953	952	10·632047	·011511	50	9·988489	52
9	9·356984	902	·643016	9·368524	951	10·631476	·011540	48	9·988460	51
10	9·357524	901	·642476	9·369094	950	10·630906	·011570	50	9·988430	50
11	9·358064	899	·641936	9·369663	949	10·630337	·011599	48	9·988401	49
12	9·358603	898	·641397	9·370232	948	10·629768	·011629	50	9·988371	48
13	9·359141	897	·640859	9·370799	946	10·629201	·011658	48	9·988342	47
14	9·359678	896	·640322	9·371367	945	10·628633	·011688	50	9·988312	46
15	9·360215	895	·639785	9·371933	944	10·628067	·011718	50	9·988282	45
16	9·360752	893	·639248	9·372499	943	10·627501	·011748	50	9·988252	44
17	9·361287	892	·638713	9·373064	942	10·626936	·011777	48	9·988223	43
18	9·361822	891	·638178	9·373629	941	10·626371	·011807	50	9·988193	42
19	9·362356	890	·637644	9·374193	940	10·625807	·011837	50	9·988163	41
20	9·362889	889	·637111	9·374756	939	10·625244	·011867	50	9·988133	40
21	9·363422	888	·636578	9·375319	938	10·624681	·011897	50	9·988103	39
22	9·363954	887	·636046	9·375881	937	10·624119	·011927	50	9·988073	38
23	9·364485	885	·635515	9·376442	935	10·623558	·011957	50	9·988043	37
24	9·365016	884	·634984	9·377003	934	10·622997	·011987	50	9·988013	36
25	9·365546	883	·634454	9·377563	933	10·622437	·012017	50	9·987983	35
26	9·366075	882	·633925	9·378122	932	10·621878	·012047	50	9·987953	34
27	9·366604	881	·633396	9·378681	931	10·621319	·012078	52	9·987922	33
28	9·367131	880	·632869	9·379239	930	10·620761	·012108	50	9·987892	32
29	9·367659	879	·632341	9·379797	929	10·620203	·012138	50	9·987862	31
30	9·368185	878	·631815	9·380354	928	10·619646	·012168	50	9·987832	30
31	9·368711	876	·631289	9·380910	927	10·619090	·012199	52	9·987801	29
32	9·369236	875	·630764	9·381466	926	10·618534	·012229	50	9·987771	28
33	9·369761	874	·630239	9·382020	925	10·617980	·012260	52	9·987740	27
34	9·370285	873	·629715	9·382575	924	10·617425	·012290	50	9·987710	26
35	9·370808	872	·629192	9·383129	923	10·616871	·012321	52	9·987679	25
36	9·371330	871	·628670	9·383682	922	10·616318	·012351	50	9·987649	24
37	9·371852	870	·628148	9·384234	921	10·615766	·012382	52	9·987618	23
38	9·372373	869	·627627	9·384786	920	10·615214	·012412	50	9·987588	22
39	9·372894	867	·627106	9·385337	919	10·614663	·012443	52	9·987557	21
40	9·373414	866	·626586	9·385888	918	10·614112	·012474	52	9·987526	20
41	9·373933	865	·626067	9·386438	917	10·613562	·012504	50	9·987496	19
42	9·374452	864	·625548	9·386987	916	10·613013	·012535	52	9·987465	18
43	9·374970	863	·625030	9·387536	914	10·612464	·012566	52	9·987434	17
44	9·375487	862	·624513	9·388084	913	10·611916	·012597	52	9·987403	16
45	9·376003	861	·623997	9·388631	912	10·611369	·012628	52	9·987372	15
46	9·376519	860	·623481	9·389178	911	10·610822	·012659	52	9·987341	14
47	9·377035	859	·622965	9·389724	910	10·610276	·012690	52	9·987310	13
48	9·377549	858	·622451	9·390270	909	10·609730	·012721	52	9·987279	12
49	9·378063	857	·621937	9·390815	908	10·609185	·012752	52	9·987248	11
50	9·378577	856	·621423	9·391360	907	10·608640	·012783	52	9·987217	10
51	9·379089	854	·620911	9·391903	906	10·608097	·012814	52	9·987186	9
52	9·379601	853	·620399	9·392447	905	10·607553	·012845	52	9·987155	8
53	9·380113	852	·619887	9·392989	904	10·607011	·012876	52	9·987124	7
54	9·380624	851	·619376	9·393531	903	10·606469	·012908	53	9·987092	6
55	9·381134	850	·618866	9·394073	902	10·605927	·012939	52	9·987061	5
56	9·381643	849	·618357	9·394614	901	10·605386	·012970	52	9·987030	4
57	9·382152	848	·617848	9·395154	900	10·604846	·013002	53	9·986998	3
58	9·382661	847	·617339	9·395694	899	10·604306	·013033	52	9·986967	2
59	9·383168	846	·616832	9·396233	898	10·603767	·013064	52	9·986936	1
60	9·383675	845	·616325	9·396771	897	10·603229	·013096	53	9·986904	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

76 DEG.

14 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·383675		·616325	9·396771		10·603229	·013096		9·986904	60
1	9·384182	844	·615818	9·397309	896	10·602691	·013127	52	9·986873	59
2	9·384687	843	·615313	9·397846	896	10·602154	·013159	53	9·986841	58
3	9·385192	842	·614808	9·398383	895	10·601617	·013191	53	9·986809	57
4	9·385697	841	·614303	9·398919	894	10·601081	·013222	52	9·986778	56
5	9·386201	840	·613799	9·399455	893	10·600545	·013254	53	9·986746	55
6	9·386704	839	·613296	9·399990	892	10·600010	·013286	53	9·986714	54
7	9·387207	838	·612793	9·400524	891	10·599476	·013317	52	9·986683	53
8	9·387709	837	·612291	9·401058	890	10·598942	·013349	53	9·986651	52
9	9·388210	836	·611790	9·401591	889	10·598409	·013381	53	9·986619	51
10	9·388711	835	·611289	9·402124	888	10·597876	·013413	53	9·986587	50
11	9·389211	834	·610789	9·402656	887	10·597344	·013445	53	9·986555	49
12	9·389711	833	·610289	9·403187	886	10·596813	·013477	53	9·986523	48
13	9·390210	832	·609790	9·403718	885	10·596282	·013509	53	9·986491	47
14	9·390708	831	·609292	9·404249	884	10·595751	·013541	53	9·986459	46
15	9·391206	830	·608794	9·404778	883	10·595222	·013573	53	9·986427	45
16	9·391703	828	·608297	9·405308	882	10·594692	·013605	53	9·986395	44
17	9·392199	827	·607801	9·405836	881	10·594164	·013637	53	9·986363	43
18	9·392695	826	·607305	9·406364	880	10·593636	·013669	53	9·986331	42
19	9·393191	825	·606810	9·406892	879	10·593108	·013701	53	9·986299	41
20	9·393685	824	·606315	9·407419	878	10·592581	·013734	55	9·986266	40
21	9·394179	823	·605821	9·407945	877	10·592055	·013766	53	9·986234	39
22	9·394673	822	·605327	9·408471	876	10·591529	·013798	53	9·986202	38
23	9·395166	821	·604834	9·408997	875	10·591003	·013831	55	9·986169	37
24	9·395658	820	·604342	9·409521	874	10·590479	·013863	53	9·986137	36
25	9·396150	819	·603850	9·410045	874	10·589955	·013896	55	9·986104	35
26	9·396641	818	·603359	9·410569	873	10·589431	·013928	53	9·986072	34
27	9·397132	817	·602868	9·411092	872	10·588908	·013961	55	9·986039	33
28	9·397621	817	·602379	9·411615	871	10·588385	·013993	53	9·986007	32
29	9·398111	816	·601889	9·412137	870	10·587863	·014026	55	9·985974	31
30	9·398600	815	·601400	9·412658	869	10·587342	·014058	53	9·985942	30
31	9·399088	814	·600912	9·413179	868	10·586821	·014091	55	9·985909	29
32	9·399575	813	·600425	9·413699	867	10·586301	·014124	55	9·985876	28
33	9·400062	812	·599938	9·414219	866	10·585781	·014157	55	9·985843	27
34	9·400549	811	·599451	9·414738	865	10·585262	·014189	53	9·985811	26
35	9·401035	810	·598965	9·415257	864	10·584743	·014222	55	9·985778	25
36	9·401520	809	·598480	9·415775	864	10·584225	·014255	55	9·985745	24
37	9·402005	808	·597995	9·416293	863	10·583707	·014288	55	9·985712	23
38	9·402489	807	·597511	9·416810	862	10·583190	·014321	55	9·985679	22
39	9·402972	806	·597028	9·417326	861	10·582674	·014354	55	9·985646	21
40	9·403455	805	·596545	9·417842	860	10·582158	·014387	55	9·985613	20
41	9·403938	804	·596062	9·418358	859	10·581642	·014420	55	9·985580	19
42	9·404420	803	·595580	9·418873	858	10·581127	·014453	55	9·985547	18
43	9·404901	802	·595099	9·419387	857	10·580613	·014486	55	9·985514	17
44	9·405382	801	·594618	9·419901	856	10·580099	·014520	57	9·985480	16
45	9·405862	800	·594138	9·420415	855	10·579585	·014553	55	9·985447	15
46	9·406341	799	·593659	9·420927	855	10·579073	·014586	55	9·985414	14
47	9·406820	798	·593180	9·421440	854	10·578560	·014619	55	9·985381	13
48	9·407299	797	·592701	9·421952	853	10·578048	·014653	57	9·985347	12
49	9·407777	796	·592223	9·422463	852	10·577537	·014686	55	9·985314	11
50	9·408254	795	·591746	9·422974	851	10·577026	·014720	57	9·985280	10
51	9·408731	794	·591269	9·423484	850	10·576516	·014753	55	9·985247	9
52	9·409207	794	·590793	9·423993	849	10·576007	·014787	57	9·985213	8
53	9·409682	793	·590318	9·424503	848	10·575497	·014820	55	9·985180	7
54	9·410157	792	·589843	9·425011	848	10·574989	·014854	57	9·985146	6
55	9·410632	791	·589368	9·425519	847	10·574481	·014887	55	9·985113	5
56	9·411106	790	·588894	9·426027	846	10·573973	·014921	57	9·985079	4
57	9·411579	789	·588421	9·426534	845	10·573466	·014955	57	9·985045	3
58	9·412052	788	·587948	9·427041	844	10·572959	·014989	57	9·985011	2
59	9·412524	787	·587476	9·427547	843	10·572453	·015022	55	9·984978	1
60	9·412996	786	·587004	9·428052	843	10·571948	·015056	57	9·984944	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

75 DEG.

15 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·412996		·587004	9·428052		10·571948	·015056		9·984944	60
1	9·413467	785	·586533	9·428557	842	10·571443	·015090	57	9·984910	59
2	9·413938	784	·586062	9·429062	841	10·570938	·015124	57	9·984876	58
3	9·414408	783	·585592	9·429566	840	10·570434	·015158	57	9·984842	57
4	9·414878	783	·585122	9·430070	839	10·569930	·015192	57	9·984808	56
5	9·415347	782	·584653	9·430573	838	10·569427	·015226	57	9·984774	55
6	9·415815	781	·584185	9·431075	838	10·568925	·015260	57	9·984740	54
7	9·416283	780	·583717	9·431577	837	10·568423	·015294	57	9·984706	53
8	9·416751	779	·583249	9·432079	836	10·567921	·015328	57	9·984672	52
9	9·417217	778	·582783	9·432580	865	10·567420	·015362	57	9·984638	51
10	9·417684	777	·582316	9·433080	834	10·566920	·015397	58	9·984603	50
11	9·418150	776	·581850	9·433580	833	10·566420	·015431	57	9·984569	49
12	9·418615	775	·581385	9·434080	832	10·565920	·015465	57	9·984535	48
13	9·419079	774	·580921	9·434579	832	10·565421	·015500	58	9·984500	47
14	9·419544	773	·580456	9·435078	831	10·564922	·015534	57	9·984466	46
15	9·420007	773	·579993	9·435576	830	10·564424	·015568	57	9·984432	45
16	9·420470	772	·579530	9·436073	829	10·563927	·015603	58	9·984397	44
17	9·420933	771	·579067	9·436570	828	10·563430	·015637	57	9·984363	43
18	9·421395	770	·578605	9·437067	828	10·562933	·015672	58	9·984328	42
19	9·421857	769	·578143	9·437563	827	10·562437	·015706	58	9·984294	41
20	9·422318	768	·577682	9·438059	826	10·561941	·015741	57	9·984259	40
21	9·422778	767	·577222	9·438554	825	10·561446	·015776	58	9·984224	39
22	9·423238	767	·576762	9·439048	824	10·560952	·015810	57	9·984190	38
23	9·423697	766	·576303	9·439543	823	10·560457	·015845	58	9·984155	37
24	9·424156	765	·575844	9·440036	823	10·559964	·015880	58	9·984120	36
25	9·424615	764	·575385	9·440529	822	10·559471	·015915	58	9·984085	35
26	9·425073	763	·574927	9·441022	821	10·558978	·015950	58	9·984050	34
27	9·425530	762	·574470	9·441514	820	10·558486	·015985	58	9·984015	33
28	9·425987	761	·574013	9·442006	819	10·557994	·016019	57	9·983981	32
29	9·426443	760	·573557	9·442497	819	10·557503	·016054	58	9·983946	31
30	9·426899	760	·573101	9·442988	818	10·557012	·016089	58	9·983911	30
31	9·427354	759	·572646	9·443479	817	10·556521	·016125	60	9·983875	29
32	9·427809	758	·572191	9·443968	816	10·556032	·016160	58	9·983840	28
33	9·428263	757	·571737	9·444458	816	10·555542	·016195	58	9·983805	27
34	9·428717	756	·571283	9·444947	815	10·555053	·016230	58	9·983770	26
35	9·429170	755	·570830	9·445435	814	10·554565	·016265	58	9·983735	25
36	9·429623	754	·570377	9·445923	813	10·554077	·016300	58	9·983700	24
37	9·430075	753	·569925	9·446411	812	10·553589	·016336	60	9·983664	23
38	9·430527	752	·569473	9·446898	812	10·553102	·016371	58	9·983629	22
39	9·430978	752	·569022	9·447384	811	10·552616	·016406	58	9·983594	21
40	9·431429	751	·568571	9·447870	810	10·552130	·016442	60	9·983558	20
41	9·431879	750	·568121	9·448356	809	10·551644	·016477	58	9·983523	19
42	9·432329	749	·567671	9·448841	809	10·551159	·016513	60	9·983487	18
43	9·432778	749	·567222	9·449326	808	10·550674	·016548	58	9·983452	17
44	9·433226	748	·566774	9·449810	807	10·550190	·016584	60	9·983416	16
45	9·433675	747	·566325	9·450294	806	10·549706	·016619	58	9·983381	15
46	9·434122	746	·565878	9·450777	806	10·549223	·016655	60	9·983345	14
47	9·434569	745	·565431	9·451260	805	10·548740	·016691	60	9·983309	13
48	9·435016	744	·564984	9·451743	804	10·548257	·016727	60	9·983273	12
49	9·435462	744	·564538	9·452225	803	10·547775	·016762	58	9·983238	11
50	9·435908	743	·564092	9·452706	802	10·547294	·016798	60	9·983202	10
51	9·436353	742	·563647	9·453187	802	10·546813	·016834	60	9·983166	9
52	9·436798	741	·563202	9·453668	801	10·546332	·016870	60	9·983130	8
53	9·437242	740	·562758	9·454148	800	10·545852	·016906	60	9·983094	7
54	9·437686	740	·562314	9·454628	799	10·545372	·016942	60	9·983058	6
55	9·438129	739	·561871	9·455107	799	10·544893	·016978	60	9·983022	5
56	9·438572	738	·561428	9·455586	798	10·544414	·017014	60	9·982986	4
57	9·439014	737	·560986	9·456064	797	10·543936	·017050	60	9·982950	3
58	9·439456	736	·560544	9·456542	796	10·543458	·017086	60	9·982914	2
59	9·439897	736	·560103	9·457019	796	10·542981	·017122	60	9·982878	1
60	9·440338	735	·559662	9·457496	795	10·542504	·017158	60	9·982842	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

74 DEG.

16 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·440338		·559662	9·457496		10·542504	·017158		9·982842	60
1	9·440778	734	·559222	9·457973	794	10·542027	·017195	62	9·982805	59
2	9·441218	733	·558782	9·458449	793	10·541551	·017231	60	9·982769	58
3	9·441658	732	·558342	9·458925	793	10·541075	·017267	60	9·982733	57
4	9·442096	731	·557904	9·459400	792	10·540600	·017304	62	9·982696	56
5	9·442535	731	·557465	9·459875	791	10·540125	·017340	60	9·982660	55
6	9·442973	730	·557027	9·460349	790	10·539651	·017376	69	9·982624	54
7	9·443410	729	·556590	9·460823	790	10·539177	·017413	62	9·982587	53
8	9·443847	728	·556153	9·461297	789	10·538703	·017449	60	9·982551	52
9	9·444284	727	·555716	9·461770	788	10·538230	·017486	62	9·982514	51
10	9·444720	727	·555280	9·462242	788	10·537758	·017523	62	9·982477	50
11	9·445155	726	·554845	9·462714	787	10·537286	·017559	60	9·982441	49
12	9·445590	725	·554410	9·463186	786	10·536814	·017596	62	9·982404	48
13	9·446025	724	·553975	9·463658	785	10·536342	·017633	62	9·982367	47
14	9·446459	723	·553541	9·464128	785	10·535872	·017669	60	9·982331	46
15	9·446893	723	·553107	9·464599	784	10·535401	·017706	62	9·982294	45
16	9·447326	722	·552674	9·465069	783	10·534931	·017743	62	9·982257	44
17	9·447759	721	·552241	9·465539	783	10·534461	·017780	62	9·982220	43
18	9·448191	720	·551809	9·466008	782	10·533992	·017817	62	9·982183	42
19	9·448623	720	·551377	9·466476	781	10·533524	·017854	62	9·982146	41
20	9·449054	719	·550946	9·466945	780	10·533055	·017891	62	9·982109	40
21	9·449485	718	·550515	9·467413	780	10·532587	·017928	62	9·982072	39
22	9·449915	717	·550085	9·467880	779	10·532120	·017965	62	9·982035	38
23	9·450345	716	·549655	9·468347	778	10·531653	·018002	62	9·981998	37
24	9·450775	716	·549225	9·468814	778	10·531186	·018039	62	9·981961	36
25	9·451204	715	·548796	9·469280	777	10·530720	·018076	62	9·981924	35
26	9·451632	714	·548368	9·469746	776	10·530254	·018114	63	9·981886	34
27	9·452060	713	·547940	9·470211	775	10·529789	·018151	62	9·981849	33
28	9·452488	713	·547512	9·470676	775	10·529324	·018188	62	9·981812	32
29	9·452915	712	·547085	9·471141	774	10·528859	·018226	63	9·981774	31
30	9·453342	711	·546658	9·471605	773	10·528395	·018263	62	9·981737	30
31	9·453768	710	·546232	9·472068	773	10·527932	·018300	62	9·981700	29
32	9·454194	710	·545806	9·472532	772	10·527468	·018338	63	9·981662	28
33	9·454619	709	·545381	9·472995	771	10·527005	·018375	62	9·981625	27
34	9·455044	708	·544956	9·473457	771	10·526543	·018413	63	9·981587	26
35	9·455469	707	·544531	9·473919	770	10·526081	·018451	63	9·981549	25
36	9·455893	707	·544107	9·474381	769	10·525619	·018488	62	9·981512	24
37	9·456316	706	·543684	9·474842	769	10·525158	·018526	63	9·981474	23
38	9·456739	705	·543261	9·475303	768	10·524697	·018564	63	9·981436	22
39	9·457162	704	·542838	9·475763	767	10·524237	·018601	62	9·981399	21
40	9·457584	704	·542416	9·476223	767	10·523777	·018639	63	9·981361	20
41	9·458006	703	·541994	9·476683	766	10·523317	·018677	63	9·981323	19
42	9·458427	702	·541573	9·477142	765	10·522858	·018715	63	9·981285	18
43	9·458848	701	·541152	9·477601	765	10·522399	·018753	63	9·981247	17
44	9·459268	701	·540732	9·478059	764	10·521941	·018791	63	9·981209	16
45	9·459688	700	·540312	9·478517	763	10·521483	·018829	63	9·981171	15
46	9·460108	699	·539892	9·478975	763	10·521025	·018867	63	9·981133	14
47	9·460527	698	·539473	9·479432	762	10·520568	·018905	63	9·981095	13
48	9·460946	698	·539054	9·479889	761	10·520111	·018943	63	9·981057	12
49	9·461364	697	·538636	9·480345	761	10·519655	·018981	63	9·981019	11
50	9·461782	696	·538218	9·480801	760	10·519199	·019019	63	9·980981	10
51	9·462199	695	·537801	9·481257	759	10·518743	·019058	65	9·980942	9
52	9·462616	695	·537384	9·481712	759	10·518288	·019096	63	9·980904	8
53	9·463032	694	·536968	9·482167	758	10·517833	·019134	63	9·980866	7
54	9·463448	693	·536552	9 482621	757	10·517379	·019173	65	9·980827	6
55	9·463864	693	·536136	9·483075	757	10·516925	·019211	63	9·980789	5
56	9·464279	692	·535721	9·483529	756	10·516471	·019250	65	9·980750	4
57	9·464694	691	·535306	9·483982	755	10·516018	·019288	63	9·980712	3
58	9·465108	690	·534892	9·484435	755	10·515565	·019327	65	9·980673	2
59	9·465522	690	·534478	9·484887	754	10·515113	·019365	63	9·980635	1
60	9·465935	689	·534065	9·485339	753	10·514661	·019404	65	9·980596	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

73 DEG.

17 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·465935		·534065	9·485339		10·514661	·019404		9·980596	60
1	9·466348	688	·533652	9·485791	753	10·514209	·019442	63	9·980558	59
2	9·466761	688	·533239	9·486242	752	10·513758	·019481	65	9·980519	58
3	9·467173	687	·532827	9·486693	751	10·513307	·019520	65	9·980480	57
4	9·467585	686	·532415	9·487143	751	10·512857	·019558	63	9·980442	56
5	9·467996	685	·532004	9·487593	750	10·512407	·019597	65	9·980403	55
6	9·468407	685	·531593	9·488043	749	10·511957	·019636	65	9·980364	54
7	9·468817	684	·531183	9·488492	749	10·511508	·019675	65	9·980325	53
8	9·469227	683	·530773	9·488941	748	10·511059	·019714	65	9·980286	52
9	9·469637	683	·530363	9·489390	747	10·510610	·019753	65	9·980247	51
10	9·470046	682	·529954	9·489838	747	10·510162	·019792	65	9·980208	50
11	9·470455	681	·529545	9·490286	746	10·509714	·019831	65	9·980169	49
12	9·470863	680	·529137	9·490733	746	10·509267	·019870	65	9·980130	48
13	9·471271	680	·528729	9·491180	745	10·508820	·019909	65	9·980091	47
14	9·471679	679	·528321	9·491627	744	10·508373	·019948	65	9·980052	46
15	9·472086	678	·527914	9·492073	744	10·507927	·019988	67	9·980012	45
16	9·472492	678	·527508	9·492519	743	10·507481	·020027	65	9·979973	44
17	9·472898	677	·527102	9·492965	743	10·507035	·020066	65	9·979934	43
18	9·473304	676	·526696	9·493410	742	10·506590	·020105	65	9·979895	42
19	9·473710	676	·526290	9·493854	741	10·506146	·020145	67	9·979855	41
20	9·474115	675	·525885	9·494299	740	10·505701	·020184	65	9·979816	40
21	9·474519	674	·525481	9·494743	740	10·505257	·020224	67	9·979776	39
22	9·474923	674	·525077	9·495186	739	10·504814	·020263	65	9·979737	38
23	9·475327	673	·524673	9·495630	739	10·504370	·020303	67	9·979697	37
24	9·475730	672	·524270	9·496073	738	10·503927	·020342	65	9·979658	36
25	9·476133	672	·523867	9·496515	737	10·503485	·020382	67	9·979618	35
26	9·476536	671	·523464	9·496957	737	10·503043	·020421	65	9·979579	34
27	9·476938	670	·523062	9·497399	736	10·502601	·020461	67	9·979539	33
28	9·477340	669	·522660	9·497841	736	10·502159	·020501	67	9·979499	32
29	9·477741	669	·522259	9·498282	735	10·501718	·020541	67	9·979459	31
30	9·478142	668	·521858	9·498722	734	10·501278	·020580	65	9·979420	30
31	9·478542	667	·521458	9·499163	734	10·500837	·020620	67	9·979380	29
32	9·478942	667	·521058	9·499603	733	10·500397	·020660	67	9·979340	28
33	9·479342	666	·520658	9·500042	733	10·499958	·020700	67	9·979300	27
34	9·479741	665	·520259	9·500481	732	10·499519	·020740	67	9·979260	26
35	9·480140	665	·519860	9·500920	731	10·499080	·020780	67	9·979220	25
36	9·480539	664	·519461	9·501359	731	10·498641	·020820	67	9·979180	24
37	9·480937	663	·519063	9·501797	730	10·498203	·020860	67	9·979140	23
38	9·481334	663	·518666	9·502235	730	10·497765	·020900	67	9·979100	22
39	9·481731	662	·518269	9·502672	729	10·497328	·020941	68	9·979059	21
40	9·482128	661	·517872	9·503109	728	10·496891	·020981	67	9·979019	20
41	9·482525	661	·517475	9·503546	728	10·496454	·021021	67	9·978979	19
42	9·482921	660	·517079	9·503982	727	10·496018	·021061	67	9·978939	18
43	9·483316	659	·516684	9·504418	727	10·495582	·021102	68	9·978898	17
44	9·483712	659	·516288	9·504854	726	10·495146	·021142	67	9·978858	16
45	9·484107	658	·515893	9·505289	725	10·494711	·021183	68	9·978817	15
46	9·484501	657	·515499	9·505724	725	10·494276	·021223	67	9·978777	14
47	9·484895	657	·515105	9·506159	724	10·493841	·021263	67	9·978737	13
48	9·485289	656	·514711	9·506593	724	10·493407	·021304	68	9·978696	12
49	9·485682	655	·514318	9·507027	723	10·492973	·021345	68	9·978655	11
50	9·486075	655	·513925	9·507460	722	10·492540	·021385	67	9·978615	10
51	9·486467	654	·513533	9·507893	722	10·492107	·021426	68	9·978574	9
52	9·486860	653	·513140	9·508326	721	10·491674	·021467	68	9·978533	8
53	9·487251	653	·512749	9·508759	721	10·491241	·021507	67	9·978493	7
54	9·487643	652	·512357	9·509191	720	10·490809	·021548	68	9·978452	6
55	9·488034	651	·511966	9·509622	719	10·490378	·021589	68	9·978411	5
56	9·488424	651	·511576	9·510054	719	10·489946	·021630	68	9·978370	4
57	9·488814	650	·511186	9·510485	718	10·489515	·021671	68	9·978329	3
58	9·489204	650	·510796	9·510916	718	10·489084	·021712	68	9·978288	2
59	9·489593	649	·510407	9·511346	717	10·488654	·021753	68	9·978247	1
60	9·489982	648	·510018	9·511776	717	10·488224	·021794	68	9·978206	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

72 DEG.

18 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·489982		·510018	9·511776		10·488224	·021794		9·978206	60
1	9·490371	648	·509629	9·512206	716	10·487794	·021835	68	9·978165	59
2	9·490759	648	·509241	9·512635	716	10·487365	·021876	68	9·978124	58
3	9·491147	647	·508853	9·513064	715	10·486936	·021917	68	9·978083	57
4	9·491535	646	·508465	9·513493	714	10·486507	·021958	69	9·978042	56
5	9·491922	646	·508078	9·513921	714	10·486079	·021999	69	9·978001	55
6	9·492308	645	·507692	9·514349	713	10·485651	·022041	69	9·977959	54
7	9·492695	644	·507305	9·514777	713	10·485223	·022082	69	9·977918	53
8	9·493081	644	·506919	9·515204	712	10·484796	·022123	69	9·977877	52
9	9·493466	643	·506534	9·515631	712	10·484369	·022165	69	9·977835	51
10	9·493851	642	·506149	9·516057	711	10·483943	·022206	69	9·977794	50
11	9·494236	642	·505764	9·516484	710	10·483516	·022248	69	9·977752	49
12	9·494621	641	·505379	9·516910	710	10·483090	·022289	69	9·977711	48
13	9·495005	641	·504995	9·517335	709	10·482665	·022331	69	9·977669	47
14	9·495388	640	·504612	9·517761	709	10·482239	·022372	69	9·977628	46
15	9·495772	639	·504228	9·518185	708	10·481815	·022414	69	9·977586	45
16	9·496154	639	·503846	9·518610	708	10·481390	·022456	69	9·977544	44
17	9·496537	638	·503463	9·519034	707	10·480966	·022497	70	9·977503	43
18	9·496919	637	·503081	9·519458	706	10·480542	·022539	70	9·977461	42
19	9·497301	637	·502699	9·519882	706	10·480118	·022581	70	9·977419	41
20	9·497682	636	·502318	9·520305	705	10·479695	·022623	70	9·977377	40
21	9·498064	636	·501936	9·520728	705	10·479272	·022665	70	9·977335	39
22	9·498444	635	·501556	9·521151	704	10·478849	·022707	70	9·977293	38
23	9·498825	634	·501175	9·521573	704	10·478427	·022749	70	9·977251	37
24	9·499204	634	·500796	9·521995	703	10·478005	·022791	70	9·977209	36
25	9·499584	633	·500416	9·522417	703	10·477583	·022833	70	9·977167	35
26	9·499963	632	·500037	9·522838	702	10·477162	·022875	70	9·977125	34
27	9·500342	632	·499658	9·523259	702	10·476741	·022917	70	9·977083	33
28	9·500721	631	·499279	9·523680	701	10·476320	·022959	70	9·977041	32
29	9·501099	631	·498901	9·524100	701	10·475900	·023001	70	9·976999	31
30	9·501476	630	·498524	9·524520	700	10·475480	·023043	70	9·976957	30
31	9·501854	629	·498146	9·524939	699	10·475061	·023086	70	9·976914	29
32	9·502231	629	·497769	9·525359	699	10·474641	·023128	70	9·976872	28
33	9·502607	628	·497393	9·525778	698	10·474222	·023170	70	9·976830	27
34	9·502984	628	·497016	9·526197	698	10·473803	·023213	71	9·976787	26
35	9·503360	627	·496640	9·526615	697	10·473385	·023255	71	9·976745	25
36	9·503735	626	·496265	9·527033	697	10·472967	·023298	71	9·976702	24
37	9·504110	626	·495890	9·527451	696	10·472549	·023340	71	9·976660	23
38	9·504485	625	·495515	9·527868	696	10·472132	·023383	71	9·976617	22
39	9·504860	625	·495140	9·528285	695	10·471715	·023426	71	9·976574	21
40	9·505234	624	·494766	9·528702	695	10·471298	·023468	71	9·976532	20
41	9·505608	623	·494392	9·529119	694	10·470881	·023511	71	9·976489	19
42	9·505981	623	·494019	9·529535	694	10·470465	·023554	71	9·976446	18
43	9·506354	622	·493646	9·529950	693	10·470050	·023596	71	9·976404	17
44	9·506727	622	·493273	9·530366	693	10·469634	·023639	71	9·976361	16
45	9·507099	621	·492901	9·530781	692	10·469219	·023682	71	9·976318	15
46	9·507471	620	·492529	9·531196	691	10·468804	·023725	71	9·976275	14
47	9·507843	620	·492157	9·531611	691	10·468389	·023768	71	9·976232	13
48	9·508214	619	·491786	9·532025	690	10·467975	·023811	72	9·976189	12
49	9·508585	619	·491415	9·532439	690	10·467561	·023854	72	9·976146	11
50	9·508956	618	·491044	9·532853	689	10·467147	·023897	72	9·976103	10
51	9·509326	618	·490674	9·533266	689	10 466734	·023940	72	9·976060	9
52	9·509696	617	·490304	9·533679	688	10·466321	·023983	72	9·976017	8
53	9·510065	616	·489935	9·534092	688	10·465908	·024026	72	9·975974	7
54	9·510434	616	·489566	9·534504	687	10·465496	·024070	72	9·975930	6
55	9·510803	615	·489197	9·534916	687	10·465084	·024113	72	9·975887	5
56	9·511172	615	·488828	9·535328	686	10·464672	·024156	72	9·975844	4
57	9·511540	614	·488460	9·535739	686	10·464261	·024200	72	9·975800	3
58	9·511907	613	·488093	9·536150	685	10·463850	·024243	72	9·975757	2
59	9·512275	613	·487725	9·536561	685	10·463439	·024286	72	9·975714	1
60	9·512642	612	·487358	9·536972	684	10·463028	·024330	72	9·975670	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

71 DEG.

19 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·512642		·487358	9·536972		10·463028	·024330		9·975670	60
1	9·513009	612	·486991	9·537382	684	10·462618	·024373	72	9·975627	59
2	9·513375	611	·486625	9·537792	683	10·462208	·024417	73	9·975583	58
3	9·513741	611	·486259	9·538202	683	10·461798	·024461	73	9·975539	57
4	9·514107	610	·485893	9·538611	682	10·461389	·024504	73	9·975496	56
5	9·514472	609	·485528	9·539020	682	10·460980	·024548	73	9·975452	55
6	9·514837	609	·485163	9·539429	681	10·460571	·024592	73	9·975408	54
7	9·515202	608	·484798	9·539837	681	10·460163	·024635	73	9·975365	53
8	9·515566	608	·484434	9·540245	680	10·459755	·024679	73	9·975321	52
9	9·515930	607	·484070	9·540653	680	10·459347	·024723	73	9·975277	51
10	9·516294	607	·483706	9·541061	679	10·458939	·024767	73	9·975233	50
11	9·516657	606	·483343	9·541468	679	10·458532	·024811	73	9·975189	49
12	9·517020	605	·482980	9·541875	678	10·458125	·024855	73	9·975145	48
13	9·517382	605	·482618	9·542281	678	10·457719	·024899	73	9·975101	47
14	9·517745	604	·482255	9·542688	677	10·457312	·024943	73	9·975057	46
15	9·518107	604	·481893	9·543094	677	10·456906	·024987	73	9·975013	45
16	9·518468	603	·481532	9·543499	676	10·456501	·025031	73	9·974969	44
17	9·518829	603	·481171	9·543905	676	10·456095	·025075	74	9·974925	43
18	9·519190	602	·480810	9·544310	675	10·455690	·025120	74	9·974880	42
19	9·519551	601	·480449	9·544715	675	10·455285	·025164	74	9·974836	41
20	9·519911	601	·480089	9·545119	674	10·454881	·025208	74	9·974792	40
21	9·520271	600	·479729	9·545524	674	10·454476	·025252	74	9·974748	39
22	9·520631	600	·479369	9·545928	673	10·454072	·025297	74	9·974703	38
23	9·520990	599	·479010	9·546331	673	10·453669	·025341	74	9·974659	37
24	9·521349	599	·478651	9·546735	672	10·453265	·025386	74	9·974614	36
25	9·521707	598	·478293	9·547138	672	10·452862	·025430	74	9·974570	35
26	9·522066	598	·477934	9·547540	671	10·452460	·025475	74	9·974525	34
27	9·522424	597	·477576	9·547943	671	10·452057	·025519	74	9·974481	33
28	9·522781	596	·477219	9·548345	670	10·451655	·025564	74	9·974436	32
29	9·523138	596	·476862	9·548747	670	10·451253	·025609	74	9·974391	31
30	9·523495	595	·476505	9·549149	669	10·450851	·025653	74	9·974347	30
31	9·523852	595	·476148	9·549550	669	10·450450	·025698	75	9·974302	29
32	9·524208	594	·475792	9·549951	668	10·450049	·025743	75	9·974257	28
33	9·524564	594	·475436	9·550352	668	10·449648	·025788	75	9·974212	27
34	9·524920	593	·475080	9·550752	667	10·449248	·025833	75	9·974167	26
35	9·525275	593	·474725	9·551152	667	10·448848	·025878	75	9·974122	25
36	9·525630	592	·474370	9·551552	666	10·448448	·025923	75	9·974077	24
37	9·525984	591	·474016	9·551952	666	10·448048	·025968	75	9·974032	23
38	9·526339	591	·473661	9·552351	665	10·447649	·026013	75	9·973987	22
39	9·526693	590	·473307	9·552750	665	10·447250	·026058	75	9·973942	21
40	9·527046	590	·472954	9·553149	665	10·446851	·026103	75	9·973897	20
41	9·527400	589	·472600	9·553548	664	10·446452	·026148	75	9·973852	19
42	9·527753	589	·472247	9·553946	664	10·446054	·026193	75	9·973807	18
43	9·528105	588	·471895	9·554344	663	10·445656	·026239	75	9·973761	17
44	9·528458	588	·471542	9·554741	663	10·445259	·026284	75	9·973716	16
45	9·528810	587	·471190	9·555139	662	10·444861	·026329	76	9·973671	15
46	9·529161	587	·470839	9·555536	662	10·444464	·026375	76	9·973625	14
47	9·529513	586	·470487	9·555933	661	10·444067	·026420	76	9·973580	13
48	9·529864	586	·470136	9·556329	661	10·443671	·026465	76	9·973535	12
49	9·530215	585	·469785	9·556725	660	10·443275	·026511	76	9·973489	11
50	9·530565	585	·469435	9·557121	660	10·442879	·026556	76	9·973444	10
51	9·530915	584	·469085	9·557517	659	10·442483	·026602	76	9·973398	9
52	9·531265	584	·468735	9·557913	659	10·442087	·026648	76	9·973352	8
53	9·531614	583	·468386	9·558308	659	10·441692	·026693	76	9·973307	7
54	9·531963	582	·468037	9·558702	658	10·441298	·026739	76	9·973261	6
55	9·532312	582	·467688	9·559097	658	10·440903	·026785	76	9·973215	5
56	9·532661	581	·467339	9·559491	657	10·440509	·026831	76	9·973169	4
57	9·533009	581	·466991	9·559885	657	10·440115	·026876	76	9·973124	3
58	9·533357	580	·466643	9·560279	656	10·439721	·026922	76	9·973078	2
59	9·533704	580	·466296	9·560673	656	10·439327	·026968	76	9·973032	1
60	9·534052	579	·465948	9·561066	655	10·438934	·027014	76	9·972986	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

70 DEG.

20 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·534052		·465948	9·561066		10·438934	·027014		9·972986	60
1	9·534399	578	·465601	9·561459	655	10·438541	·027060	77	9·972940	59
2	9·534745	577	·465255	9·561851	654	10·438149	·027106	77	9·972894	58
3	9·535092	577	·464908	9·562244	654	10·437756	·027152	77	9·972848	57
4	9·535438	577	·464562	9·562636	653	10·437364	·027198	77	9·972802	56
5	9·535783	576	·464217	9·563028	653	10·436972	·027245	78	9·972755	55
6	9·536129	576	·463871	9·563419	653	10·436581	·027291	77	9·972709	54
7	9·536474	575	·463526	9·563811	652	10·436189	·027337	77	9·972663	53
8	9·536818	574	·463182	9·564202	652	10·435798	·027383	77	9·972617	52
9	9·537163	574	·462837	9·564592	651	10·435408	·027430	78	9·972570	51
10	9·537507	573	·462493	9·564983	651	10·435017	·027476	77	9·972524	50
11	9·537851	573	·462149	9·565373	650	10·434627	·027522	77	9·972478	49
12	9·538194	572	·461806	9·565763	650	10·434237	·027569	78	9·972431	48
13	9·538538	572	·461462	9·566153	649	10·433847	·027615	77	9·972385	47
14	9·538880	571	·461120	9·566542	649	10·433458	·027662	78	9·972338	46
15	9·539223	571	·460777	9·566932	649	10·433068	·027709	78	9·972291	45
16	9·539565	570	·460435	9·567320	648	10·432680	·027755	77	9·972245	44
17	9·539907	570	·460093	9·567709	648	10·432291	·027802	78	9·972198	43
18	9·540249	569	·459751	9·568098	647	10·431902	·027849	78	9·972151	42
19	9·540590	569	·459410	9·568486	647	10·431514	·027895	77	9·972105	41
20	9·540931	568	·459069	9·568873	646	10·431127	·027942	78	9·972058	40
21	9·541272	568	·458728	9·569261	646	10·430739	·027989	78	9·972011	39
22	9·541613	567	·458387	9·569648	645	10·430352	·028036	78	9·971964	38
23	9·541953	567	·458047	9·570035	645	10·429965	·028083	78	9·971917	37
24	9·542293	566	·457707	9·570422	645	10·429578	·028130	78	9·971870	36
25	9·542632	566	·457368	9·570809	644	10·429191	·028177	78	9·971823	35
26	9·542971	565	·457029	9·571195	644	10·428805	·028224	78	9·971776	34
27	9·543310	565	·456690	9·571581	643	10·428419	·028271	78	9·971729	33
28	9·543649	564	·456351	9·571967	643	10·428033	·028318	78	9·971682	32
29	9·543987	564	·456013	9·572352	642	10·427648	·028365	78	9·971635	31
30	9·544325	563	·455675	9·572738	642	10·427262	·028412	78	9·971588	30
31	9·544663	563	·455337	9·573123	642	10·426877	·028460	80	9·971540	29
32	9·545000	562	·455000	9·573507	641	10·426493	·028507	78	9·971493	28
33	9·545338	562	·454662	9·573892	641	10·426108	·028554	78	9·971446	27
34	9·545674	561	·454326	9·574276	640	10·425724	·028602	80	9·971398	26
35	9·546011	561	·453989	9·574660	640	10·425340	·028649	78	9·971351	25
36	9·546347	560	·453653	9·575044	639	10·424956	·028697	80	9·971303	24
37	9·546683	560	·453317	9·575427	639	10·424573	·028744	78	9·971256	23
38	9·547019	559	·452981	9·575810	639	10·424190	·028792	80	9·971208	22
39	9·547354	559	·452646	9·576193	638	10·423807	·028839	78	9·971161	21
40	9·547689	558	·452311	9·576576	638	10·423424	·028887	80	9·971113	20
41	9·548024	558	·451976	9·576958	637	10·423042	·028934	78	9·971066	19
42	9·548359	557	·451641	9·577341	637	10·422659	·028982	80	9·971018	18
43	9·548693	557	·451307	9·577723	636	10·422277	·029030	80	9·970970	17
44	9·549027	556	·450973	9·578104	636	10·421896	·029078	80	9·970922	16
45	9·549360	556	·450640	9·578486	636	10·421514	·029126	80	9·970874	15
46	9·549693	555	·450307	9·578867	635	10·421133	·029173	78	9·970827	14
47	9·550026	555	·449974	9·579248	635	10·420752	·029221	80	9·970779	13
48	9·550359	554	·449641	9·579629	634	10·420371	·029269	80	9·970731	12
49	9·550692	554	·449308	9·580009	634	10·419991	·029317	80	9·970683	11
50	9·551024	553	·448976	9·580389	634	10·419611	·029365	80	9·970635	10
51	9·551356	553	·448644	9·580769	633	10·419231	·029414	82	9·970586	9
52	9·551687	552	·448313	9·581149	633	10·418851	·029462	80	9·970538	8
53	9·552018	552	·447982	9·581528	632	10·418472	·029510	80	9·970490	7
54	9·552349	552	·447651	9·581907	632	10·418093	·029558	80	9·970442	6
55	9·552680	551	·447320	9·582286	632	10·417714	·029606	80	9·970394	5
56	9·553010	551	·446990	9·582665	631	10·417335	·029655	82	9·970345	4
57	9·553341	550	·446659	9·583043	631	10·416957	·029703	80	9·970297	3
58	9·553670	550	·446330	9·583422	630	10·416578	·029751	80	9·970249	2
59	9·554000	549	·446000	9·583800	630	10·416200	·029800	82	9·970200	1
60	9·554329	549	·445671	9·584177	629	10·415823	·029848	80	9·970152	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

69 DEG.

21 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·554329		·445671	9·584177		10·415823	·029848		9·970152	60
1	9·554658	548	·445342	9·584555	629	10·415445	·029897	81	9·970103	59
2	9·554987	548	·445013	9·584932	629	10·415068	·029945	81	9·970055	58
3	9·555315	547	·444685	9·585309	628	10·414691	·029994	81	9·970006	57
4	9·555643	547	·444357	9·585686	628	10·414314	·030043	81	9·969957	56
5	9·555971	546	·444029	9·586062	627	10·413938	·030091	81	9·969909	55
6	9·556299	546	·443701	9·586439	627	10·413561	·030140	81	9·969860	54
7	9·556626	545	·443374	9·586815	627	10·413185	·030189	81	9·969811	53
8	9·556953	545	·443047	9·587190	626	10·412810	·030238	81	9·969762	52
9	9·557280	544	·442720	9·587566	626	10·412434	·030286	81	9·969714	51
10	9·557606	544	·442394	9·587941	625	10·412059	·030335	81	9·969665	50
11	9·557932	543	·442068	9·588316	625	10·411684	·030384	81	9·969616	49
12	9·558258	543	·441742	9·588691	625	10·411309	·030433	82	9·969567	48
13	9·558583	543	·441417	9·589066	624	10·410934	·030482	82	9·969518	47
14	9·558909	542	·441091	9·589440	624	10·410560	·030531	82	9·969469	46
15	9·559234	542	·440766	9·589814	623	10·410186	·030580	82	9·969420	45
16	9·559558	541	·440442	9·590188	623	10·409812	·030630	82	9·969370	44
17	9·559883	541	·440117	9·590562	623	10·409438	·030679	82	9·969321	43
18	9·560207	540	·439793	9·590935	622	10·409065	·030728	82	9·969272	42
19	9·560531	540	·439469	9·591308	622	10·408692	·030777	82	9·969223	41
20	9·560855	539	·439145	9·591681	622	10·408319	·030827	82	9·969173	40
21	9·561178	539	·438822	9·592054	621	10·407946	·030876	82	9·969124	39
22	9·561501	538	·438499	9·592426	621	10·407574	·030925	82	9·969075	38
23	9·561824	538	·438176	9·592798	620	10·407202	·030975	82	9·969025	37
24	9·562146	537	·437854	9·593171	620	10·406829	·031024	82	9·968976	36
25	9·562468	537	·437532	9·593542	620	10·406458	·031074	82	9·968926	35
26	9·562790	536	·437210	9·593914	619	10·406086	·031123	83	9·968877	34
27	9·563112	536	·436888	9·594285	619	10·405715	·031173	83	9·968827	33
28	9·563433	536	·436567	9·594656	618	10·405344	·031223	83	9·968777	32
29	9·563755	535	·436245	9·595027	618	10·404973	·031272	83	9·968728	31
30	9·564075	535	·435925	9·595398	618	10·404602	·031322	83	9·968678	30
31	9·564396	534	·435604	9·595768	617	10·404232	·031372	83	9·968628	29
32	9·564716	534	·435284	9·596138	617	10·403862	·031422	83	9·968578	28
33	9·565036	533	·434964	9·596508	616	10·403492	·031472	83	9·968528	27
34	9·565356	533	·434644	9·596878	616	10·403122	·031521	83	9·968479	26
35	9·565676	532	·434324	9·597247	616	10·402753	·031571	83	9·968429	25
36	9·565995	532	·434005	9·597616	615	10·402384	·031621	83	9·968379	24
37	9·566314	531	·433686	9·597985	615	10·402015	·031671	83	9·968329	23
38	9·566632	531	·433368	9·598354	615	10·401646	·031722	83	9·968278	22
39	9·566951	531	·433049	9·598722	614	10·401278	·031772	83	9·968228	21
40	9·567269	530	·432731	9·599091	614	10·400909	·031822	84	9·968178	20
41	9·567587	530	·432413	9·599459	613	10·400541	·031872	84	9·968128	19
42	9·567904	529	·432096	9·599827	613	10·400173	·031922	84	9·968078	18
43	9·568222	529	·431778	9·600194	613	10·399806	·031973	84	9·968027	17
44	9·568539	528	·431461	9·600562	612	10·399438	·032023	84	9·967977	16
45	9·568856	528	·431144	9·600929	612	10·399071	·032073	84	9·967927	15
46	9·569172	528	·430828	9·601296	611	10·398704	·032124	84	9·967876	14
47	9·569488	527	·430512	9·601662	611	10·398338	·032174	84	9·967826	13
48	9·569804	527	·430196	9·602029	611	10·397971	·032225	84	9·967775	12
49	9·570120	526	·429880	9·602395	610	10·397605	·032275	84	9·967725	11
50	9·570435	526	·429565	9·602761	610	10·397239	·032326	84	9·967674	10
51	9·570751	525	·429249	9·603127	610	10·396873	·032376	84	9·967624	9
52	9·571066	525	·428934	9·603493	609	10·396507	·032427	84	9·967573	8
53	9·571380	524	·428620	9·603858	609	10·396142	·032478	84	9·967522	7
54	9·571695	524	·428305	9·604223	609	10·395777	·032529	85	9·967471	6
55	9·572009	523	·427991	9·604588	608	10·395412	·032579	85	9·967421	5
56	9·572323	523	·427677	9·604953	608	10·395047	·032630	85	9·967370	4
57	9·572636	523	·427364	9·605317	607	10·394683	·032681	85	9·967319	3
58	9·572950	522	·427050	9·605682	607	10·394318	·032732	85	9·967268	2
59	9·573263	522	·426737	9·606046	607	10·393954	·032783	85	9·967217	1
60	9·573575	521	·426425	9·606410	606	10·393590	·032834	85	9·967166	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

68 DEG.

22 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·573575		·426425	9·606410		10·393590	·032834		9·967166	60
1	9·573888	521	·426112	9·606773	606	10·393227	·032885	85	9·967115	59
2	9·574200	520	·425800	9·607137	606	10·392863	·032936	85	9·967064	58
3	9·574512	520	·425488	9·607500	605	10·392500	·032987	85	9·967013	57
4	9·574824	519	·425176	9·607863	605	10·392137	·033039	85	9·966961	56
5	9·575136	519	·424864	9·608225	604	10·391775	·033090	85	9·966910	55
6	9·575447	519	·424553	9·608588	604	10·391412	·033141	85	9·966859	54
7	9·575758	518	·424242	9·608950	604	10·391050	·033192	85	9·966808	53
8	9·576069	518	·423931	9·609312	603	10·390688	·033244	85	9·966756	52
9	9·576379	517	·423621	9·609674	603	10·390326	·033295	86	9·966705	51
10	9·576689	517	·423311	9·610036	603	10·389964	·033347	86	9·966653	50
11	9·576999	516	·423001	9·610397	602	10·389603	·033398	86	9·966602	49
12	9·577309	516	·422691	9·610759	602	10·389211	·033450	86	9·966550	48
13	9·577618	516	·422382	9·611120	602	10·388880	·033501	86	9·966499	47
14	9·577927	515	·422073	9·611480	601	10·388520	·033553	86	9·966447	46
15	9·578236	515	·421764	9·611841	601	10·388159	·033605	86	9·966395	45
16	9·578545	514	·421455	9·612201	601	10·387799	·033656	86	9·966344	44
17	9·578853	514	·421147	9·612561	600	10·387439	·033708	86	9·966292	43
18	9·579162	513	·420838	9·612921	600	10·387079	·033760	86	9·966240	42
19	9·579470	513	·420530	9·613281	600	10·386719	·033812	86	9·966188	41
20	9·579777	513	·420223	9·613641	599	10·386359	·033864	86	9·966136	40
21	9·580085	512	·419915	9·614000	599	10·386000	·033915	86	9·966085	39
22	9·580392	512	·419608	9·614359	598	10·385641	·033967	87	9·966033	38
23	9·580699	511	·419301	9·614718	598	10·385282	·034019	87	9·965981	37
24	9·581005	511	·418995	9·615077	598	10·384923	·034071	87	9·965929	36
25	9·581312	511	·418688	9·615435	597	10·384565	·034124	87	9·965876	35
26	9·581618	510	·418382	9·615793	597	10·384207	·034176	87	9·965824	34
27	9·581924	510	·418076	9·616151	597	10·383849	·034228	87	9·965772	33
28	9·582229	509	·417771	9·616509	596	10·383491	·034280	87	9·965720	32
29	9·582535	509	·417465	9·616867	596	10·383133	·034332	87	9·965668	31
30	9·582840	509	·417160	9·617224	596	10·382776	·034385	87	9·965615	30
31	9·583145	508	·416855	9·617582	595	10·382418	·034437	87	9·965563	29
32	9·583449	508	·416551	9·617939	595	10·382061	·034489	87	9·965511	28
33	9·583754	507	·416246	9·618295	595	10·381705	·034542	87	9·965458	27
34	9·584058	507	·415942	9·618652	594	10·381348	·034594	87	9·965406	26
35	9·584361	506	·415639	9·619008	594	10·380992	·034647	87	9·965353	25
36	9·584665	506	·415335	9·619364	594	10·380636	·034699	88	9·965301	24
37	9·584968	506	·415032	9·619721	593	10·380279	·034752	88	9·965248	23
38	9·585272	505	·414728	9·620076	593	10·379924	·034805	88	9·965195	22
39	9·585574	505	·414426	9·620432	593	10·379568	·034857	88	9·965143	21
40	9·585877	504	·414123	9·620787	592	10·379213	·034910	88	9·965090	20
41	9·586179	504	·413821	9·621142	592	10·378858	·034963	88	9·965037	19
42	9·586482	503	·413518	9·621497	592	10·378503	·035016	88	9·964984	18
43	9·586783	503	·413217	9·621852	591	10·378148	·035069	88	9·964931	17
44	9·587085	503	·412915	9·622207	591	10·377793	·035121	88	9·964879	16
45	9·587386	502	·412614	9·622561	590	10·377439	·035174	88	9·964826	15
46	9·587688	502	·412312	9·622915	590	10·377085	·035227	88	9·964773	14
47	9·587989	501	·412011	9·623269	590	10·376731	·035280	88	9·964720	13
48	9·588289	501	·411711	9·623623	589	10·376377	·035334	88	9·964666	12
49	9·588590	501	·411410	9·623976	589	10·376024	·035387	89	9·964613	11
50	9·588890	500	·411110	9·624330	589	10·375670	·035440	89	9·964560	10
51	9·589190	500	·410810	9·624683	588	10·375317	·035493	89	9·964507	9
52	9·589489	499	·410511	9·625036	588	10·374964	·035546	89	9·964454	8
53	9·589789	499	·410211	9·625388	588	10·374612	·035600	89	9·964400	7
54	9·590088	499	·409912	9·625741	587	10·374259	·035653	89	9·964347	6
55	9·590387	498	·409613	9·626093	587	10·373907	·035706	89	9·964294	5
56	9·590686	498	·409314	9·626445	587	10·373555	·035760	89	9·964240	4
57	9·590984	497	·409016	9·626797	586	10·373203	·035813	89	9·964187	3
58	9·591282	497	·408718	9·627149	586	10·372851	·035867	89	9·964133	2
59	9·591580	497	·408420	9·627501	586	10·372499	·035920	89	9·964080	1
60	9·591878	496	·408122	9·627852	585	10·372148	·035974	89	9·964026	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

67 DEG.

23 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·591878		·408122	9·627852		10·372148	·035974		9·964026	60
1	9·592176	496	·407824	9·628203	585	10·371797	·036028	89	9·963972	59
2	9·592473	495	·407527	9·628554	585	10·371446	·036081	89	9·963919	58
3	9·592770	495	·407230	9·628905	585	10·371095	·036135	89	9·963865	57
4	9·593067	495	·406933	9·629255	584	10·370745	·036189	90	9·963811	56
5	9·593363	494	·406637	9·629606	584	10·370394	·036243	90	9·963757	55
6	9·593659	494	·406341	9·629956	583	10·370044	·036296	90	9·963704	54
7	9·593955	493	·406045	9·630306	583	10·369694	·036350	90	9·963650	53
8	9·594251	493	·405749	9·630656	583	10·369344	·036404	90	9·963596	52
9	9·594547	493	·405453	9·631005	583	10·368995	·036458	90	9·963542	51
10	9·594842	492	·405158	9·631355	582	10·368645	·036512	90	9·963488	50
11	9·595137	492	·404863	9·631704	582	10·368296	·036566	90	9·963434	49
12	9·595432	491	·404568	9·632053	582	10·367947	·036621	90	9·963379	48
13	9·595727	491	·404273	9·632401	581	10·367599	·036675	90	9·963325	47
14	9·596021	491	·403979	9·632750	581	10·367250	·036729	90	9·963271	46
15	9·596315	490	·403685	9·633098	581	10·366902	·036783	90	9·963217	45
16	9·596609	490	·403391	9·633447	580	10·366553	·036837	90	9·963163	44
17	9·596903	489	·403097	9·633795	580	10·366205	·036892	90	9·963108	43
18	9·597196	489	·402804	9·634143	580	10·365857	·036946	91	9·963054	42
19	9·597490	489	·402510	9·634490	579	10·365510	·037001	91	9·962999	41
20	9·597783	488	·402217	9·634838	579	10·365162	·037055	91	9·962945	40
21	9·598075	488	·401925	9·635185	579	10·364815	·037110	91	9·962890	39
22	9·598368	487	·401632	9·635532	578	10·364468	·037164	91	9·962836	38
23	9·598660	487	·401340	9·635879	578	10·364121	·037219	91	9·962781	37
24	9·598952	487	·401048	9·636226	578	10·363774	·037273	91	9·962727	36
25	9·599244	486	·400756	9·636572	577	10·363428	·037328	91	9·962672	35
26	9·599536	486	·400464	9·636919	577	10·363081	·037383	91	9·962617	34
27	9·599827	485	·400173	9·637265	577	10·362735	·037438	91	9·962562	33
28	9·600118	485	·399882	9·637611	577	10·362389	·037492	91	9·962508	32
29	9·600409	485	·399591	9·637956	576	10·362044	·037547	91	9·962453	31
30	9·600700	484	·399300	9·638302	576	10·361698	·037602	91	9·962398	30
31	9·600990	484	·399010	9·638647	576	10·361353	·037657	92	9·962343	29
32	9·601280	484	·398720	9·638992	575	10·361008	·037712	92	9·962288	28
33	9·601570	483	·398430	9·639337	575	10·360663	·037767	92	9·962233	27
34	9·601860	483	·398140	9·639682	575	10·360318	·037822	92	9·962178	26
35	9·602150	482	·397850	9·640027	574	10·359973	·037877	92	9·962123	25
36	9·602439	482	·397561	9·640371	574	10·359629	·037933	92	9·962067	24
37	9·602728	482	·397272	9·640716	574	10·359284	·037988	92	9·962012	23
38	9·603017	481	·396983	9·641060	573	10·358940	·038043	92	9·961957	22
39	9·603305	481	·396695	9·641404	573	10·358596	·038098	92	9·961902	21
40	9·603594	481	·396406	9·641747	573	10·358253	·038154	92	9·961846	20
41	9·603882	480	·396118	9·642091	572	10·357909	·038209	92	9·961791	19
42	9·604170	480	·395830	9·642434	572	10·357566	·038265	92	9·961735	18
43	9·604457	479	·395543	9·642777	572	10·357223	·038320	92	9·961680	17
44	9·604745	479	·395255	9·643120	572	10·356880	·038376	92	9·961624	16
45	9·605032	479	·394968	9·643463	571	10·356537	·038431	93	9·961569	15
46	9·605319	478	·394681	9·643806	571	10·356194	·038487	93	9·961513	14
47	9·605606	478	·394394	9·644148	571	10·355852	·038542	93	9·961458	13
48	9·605892	478	·394108	9·644490	570	10·355510	·038598	93	9·961402	12
49	9·606179	477	·393821	9·644832	570	10·355168	·038654	93	9·961346	11
50	9·606465	477	·393535	9·645174	570	10·354826	·038710	93	9·961290	10
51	9·606751	476	·393249	9·645516	570	10·354484	·038765	93	9·961235	9
52	9·607036	476	·392964	9·645857	569	10·354143	·038821	93	9·961179	8
53	9·607322	476	·392678	9·646199	569	10·353801	·038877	93	9·961123	7
54	9·607607	475	·392393	9·646540	569	10·353460	·038933	93	9·961067	6
55	9·607892	475	·392108	9·646881	568	10·353119	·038989	93	9·961011	5
56	9·608177	474	·391823	9·647222	568	10·352778	·039045	93	9·960955	4
57	9·608461	474	·391539	9·647562	568	10·352438	·039101	93	9·960899	3
58	9·608745	474	·391255	9·647903	567	10·352097	·039157	93	9·960843	2
59	9·609029	473	·390971	9·648243	567	10·351757	·039214	94	9·960786	1
60	9·609313	473	·390687	9·648583	567	10·351417	·039270	94	9·960730	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

66 DEG.

24 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff 100″	Cosine.	′
0	9·609313		·390687	9·648583		10·351417	·039270		9·960730	60
1	9·609597	473	·390403	9·648923	566	10·351077	·039326	94	9·960674	59
2	9·609880	472	·390120	9·649263	566	10·350737	·039382	94	9·960618	58
3	9·610164	472	·389836	9·649602	566	10·350398	·039439	94	9·960561	57
4	9·610447	472	·389553	9·649942	566	10·350058	·039495	94	9·960505	56
5	9·610729	471	·389271	9·650281	565	10·349719	·039552	94	9·960448	55
6	9·611012	471	·388988	9·650620	565	10·349380	·039608	94	9·960392	54
7	9·611294	470	·388706	9·650959	565	10·349041	·039665	94	9·960335	53
8	9·611576	470	·388424	9·651297	564	10·348703	·039721	94	9·960279	52
9	9·611858	470	·388142	9·651636	564	10·348364	·039778	94	9·960222	51
10	9·612140	469	·387860	9·651974	564	10·348026	·039835	94	9·960165	50
11	9·612421	469	·387579	9·652312	563	10·347688	·039891	94	9·960109	49
12	9·612702	469	·387298	9·652650	563	10·347350	·039948	95	9·960052	48
13	9·612983	468	·387017	9·652988	563	10·347012	·040005	95	9·959995	47
14	9·613264	468	·386736	9·653326	563	10·346674	·040062	95	9·959938	46
15	9·613545	467	·386455	9·653663	562	10·346337	·040118	95	9·959882	45
16	9·613825	467	·386175	9·654000	562	10·346000	·040175	95	9·959825	44
17	9·614105	467	·385895	9·654337	562	10·345663	·040232	95	9·959768	43
18	9·614385	466	·385615	9·654674	561	10·345326	·040289	95	9·959711	42
19	9·614665	466	·385335	9·655011	561	10·344989	·040346	95	9·959654	41
20	9·614944	466	·385056	9·655348	561	10·344652	·040404	95	9·959596	40
21	9·615223	465	·384777	9·655684	561	10·344316	·040461	95	9·959539	39
22	9·615502	465	·384498	9·656020	560	10·343980	·040518	95	9·959482	38
23	9·615781	465	·384219	9·656356	560	10·343644	·040575	95	9·959425	37
24	9·616060	464	·383940	9·656692	560	10·343308	·040632	95	9·959368	36
25	9·616338	464	·383662	9·657028	559	10·342972	·040690	95	9·959310	35
26	9·616616	464	·383384	9·657364	559	10·342636	·040747	96	9·959253	34
27	9·616894	463	·383106	9·657699	559	10·342301	·040805	96	9·959195	33
28	9·617172	463	·382828	9·658034	559	10·341966	·040862	96	9·959138	32
29	9·617450	462	·382550	9·658369	558	10·341631	·040919	96	9·959080	31
30	9·617727	462	·382273	9·658704	558	10·341296	·040977	96	9·959023	30
31	9·618004	462	·381996	9·659039	558	10·340961	·041035	96	9·958965	29
32	9·618281	461	·381719	9·659373	558	10·340627	·041092	96	9·958908	28
33	9·618558	461	·381442	9·659708	557	10·340292	·041150	96	9·958850	27
34	9·618834	461	·381166	9·660042	557	10·339958	·041208	96	9·958792	26
35	9·619110	460	·380890	9·660376	557	10·339624	·041266	96	9·958734	25
36	9·619386	460	·380614	9·660710	556	10·339290	·041323	96	9·958677	24
37	9·619662	460	·380338	9·661043	556	10·338957	·041381	96	9·958619	23
38	9·619938	459	·380062	9·661377	556	10·338623	·041439	96	9·958561	22
39	9·620213	459	·379787	9·661710	556	10·338290	·041497	96	9·958503	21
40	9·620488	459	·379512	9·662043	555	10·337957	·041555	97	9·958445	20
41	9·620763	458	·379237	9·662376	555	10·337624	·041613	97	9·958387	19
42	9·621038	458	·378962	9·662709	555	10·337291	·041671	97	9·958329	18
43	9·621313	457	·378687	9·663042	554	10·336958	·041729	97	9·958271	17
44	9·621587	457	·378413	9·663375	554	10·336625	·041787	97	9·958213	16
45	9·621861	457	·378139	9·663707	554	10·336293	·041846	97	9·958154	15
46	9·622135	456	·377865	9·664039	554	10·335961	·041904	97	9·958096	14
47	9·622409	456	·377591	9·664371	553	10·335629	·041962	97	9·958038	13
48	9·622682	456	·377318	9·664703	553	10·335297	·042021	97	9·957979	12
49	9·622956	455	·377044	9·665035	553	10·334965	·042079	97	9·957921	11
50	9·623229	455	·376771	9·665366	553	10·334634	·042137	97	9·957863	10
51	9·623502	455	·376498	9·665697	552	10·334303	·042196	97	9·957804	9
52	9·623774	454	·376226	9·666029	552	10·333971	·042254	97	9·957746	8
53	9·624047	454	·375953	9·666360	552	10·333640	·042313	98	9·957687	7
54	9·624319	454	·375681	9·666691	551	10·333309	·042372	98	9·957628	6
55	9·624591	453	·375409	9·667021	551	10·332979	·042430	98	9·957570	5
56	9·624863	453	·375137	9·667352	551	10·332648	·042489	98	9·957511	4
57	9·625135	453	·374865	9·667682	551	10·332318	·042548	98	9·957452	3
58	9·625406	452	·374594	9·668013	550	10·331987	·042607	98	9·957393	2
59	9·625677	452	·374323	9·668343	550	10·331657	·042665	98	9·957335	1
60	9·625948	452	·374052	9·668672	550	10·331328	·042724	98	9·957276	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

65 DEG.

25 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·625948		·374052	9·668673		10·331327	·042724		9·957276	60
1	9·626219	451	·373781	9·669002	550	10·330998	·042783	98	9·957217	59
2	9·626490	451	·373510	9·669332	549	10·330668	·042842	98	9·957158	58
3	9·626760	451	·373240	9·669661	549	10·330339	·042901	98	9·957099	57
4	9·627030	450	·372970	9·669991	549	10·330009	·042960	98	9·957040	56
5	9·627300	450	·372700	9·670320	548	10·329680	·043019	98	9·956981	55
6	9·627570	450	·372430	9·670649	548	10·329351	·043079	98	9·956921	54
7	9·627840	449	·372160	9·670977	548	10·329023	·043138	99	9·956862	53
8	9·628109	449	·371891	9·671306	548	10·328694	·043197	99	9·956803	52
9	9·628348	449	·371622	9·671634	547	10·328366	·043256	99	9·956744	51
10	9·628647	448	·371353	9·671963	547	10·328037	·043316	99	9·956684	50
11	9·628916	448	·371084	9·672291	547	10·327709	·043375	99	9·956625	49
12	9·629185	447	·370815	9·672619	547	10·327381	·043434	99	9·956566	48
13	9·629453	447	·370547	9·672947	546	10·327053	·043494	99	9·956506	47
14	9·629721	447	·370279	9·673274	546	10·326726	·043553	99	9·956447	46
15	9·629989	446	·370011	9·673602	546	10·326398	·043613	99	9·956387	45
16	9·630257	446	·369743	9·673929	546	10·326071	·043673	99	9·956327	44
17	9·630524	446	·369476	9·674257	545	10·325743	·043732	99	9·956268	43
18	9·630792	446	·369208	9·674584	545	10·325416	·043792	99	9·956208	42
19	9·631059	445	·368941	9·674910	545	10·325090	·043852	100	9·956148	41
20	9·631326	445	·368674	9·675237	544	10·324763	·043911	100	9·956089	40
21	9·631593	445	·368407	9·675564	544	10·324436	·043971	100	9·956029	39
22	9·631859	444	·368141	9·675890	544	10·324110	·044031	100	9·955969	38
23	9·632125	444	·367875	9·676217	544	10·323783	·044091	100	9·955909	37
24	9·632392	444	·367608	9·676543	543	10·323457	·044151	100	9·955849	36
25	9·632658	443	·367342	9·676869	543	10·323131	·044211	100	9·955789	35
26	9·632923	443	·367077	9·677194	543	10·322806	·044271	100	9·955729	34
27	9·633189	443	·366811	9·677520	543	10·322480	·044331	100	9·955669	33
28	9·633454	442	·366546	9·677846	542	10·322154	·044391	100	9·955609	32
29	9·633719	442	·366281	9·678170	542	10·321829	·044452	100	9·955548	31
30	9·633984	442	·366016	9·678496	542	10·321504	·044512	100	9·955488	30
31	9·634249	441	·365751	9·678821	542	10·321179	·044572	100	9·955428	29
32	9·634514	441	·365486	9·679146	541	10·320854	·044632	101	9·955368	28
33	9·634778	440	·365222	9·679471	541	10·320529	·044693	101	9·955307	27
34	9·635042	440	·364958	9·679795	541	10·320205	·044753	101	9·955247	26
35	9·635306	440	·364694	9·680120	541	10·319880	·044814	101	9·955186	25
36	9·635570	439	·364430	9·680444	540	10·319556	·044874	101	9·955126	24
37	9·635834	439	·364166	9·680768	540	10·319232	·044935	101	9·955065	23
38	9·636097	439	·363903	9·681092	540	10·318908	·044995	101	9·955005	22
39	9·636360	438	·363640	9·681416	540	10·318584	·045056	101	9·954944	21
40	9·636623	438	·363377	9·681740	539	10·318260	·045117	101	9·954883	20
41	9·636886	438	·363114	9·682063	539	10·317937	·045177	101	9·954823	19
42	9·637148	437	·362852	9·682387	539	10·317613	·045238	101	9·954762	18
43	9·637411	437	·362589	9·682710	539	10·317290	·045299	101	9·954701	17
44	9·637673	437	·362327	9·683033	538	10·316967	·045360	101	9·954640	16
45	9·637935	437	·362065	9·683356	538	10·316644	·045421	101	9·954579	15
46	9·638197	436	·361803	9·683679	538	10·316321	·045482	101	9·954518	14
47	9·638458	436	·361542	9·684001	538	10·315999	·045543	102	9·954457	13
48	9·638720	436	·361280	9·684324	537	10·315676	·045604	102	9·954396	12
49	9·638981	435	·361019	9·684646	537	10·315354	·045665	102	9·954335	11
50	9·639242	435	·360758	9·684968	537	10·315032	·045726	102	9·954274	10
51	9·639503	435	·360497	9·685290	537	10·314710	·045787	102	9·954213	9
52	9·639764	434	·360236	9·685612	536	10·314388	·045848	102	9·954152	8
53	9·640024	434	·359976	9·685934	536	10·314066	·045910	102	9·954090	7
54	9·640284	434	·359716	9·686255	536	10·313745	·045971	102	9·954029	6
55	9·640544	433	·359456	9·686577	536	10·313423	·046032	102	9·953968	5
56	9·640804	433	·359196	9·686898	535	10·313102	·046094	102	9·953906	4
57	9·641064	433	·358936	9·687219	535	10·312781	·046155	102	9·953845	3
58	9·641324	432	·358676	9·687540	535	10·312460	·046217	102	9·953783	2
59	9·641583	432	·358417	9·687861	535	10·312139	·046278	102	9·953722	1
60	9·641842	432	·358158	9·688182	534	10·311818	·046340	103	9·953660	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

64 DEG.

26 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·641842		·358158	9·688182		10·311818	·046340		9·953660	60
1	9·642101	431	·357899	9·688502	534	10·311498	·046401	103	9·953599	59
2	9·642360	431	·357640	9·688823	534	10·311177	·046463	103	9·953537	58
3	9·642618	431	·357382	9·689143	534	10·310857	·046525	103	9·953475	57
4	9·642877	430	·357123	9·689463	533	10·310537	·046587	103	9·953413	56
5	9·643135	430	·356865	9·689788	533	10·310217	·046648	103	9·953352	55
6	9·643393	430	·356607	9·690103	533	10·309897	·046710	103	9·953290	54
7	9·643650	430	·356350	9·690423	533	10·309577	·046772	103	9·953228	53
8	9·643908	429	·356092	9·690742	533	10·309258	·046834	103	9·953166	52
9	9·644165	429	·355835	9·691062	532	10·308938	·046896	103	9·953104	51
10	9·644423	429	·355577	9·691381	532	10·308619	·046958	103	9·953042	50
11	9·644680	428	·355320	9·691700	532	10·308300	·047020	103	9·952980	49
12	9·644936	428	·355064	9·692019	531	10·307981	·047082	104	9·952918	48
13	9·645193	428	·354807	9·692338	531	10·307662	·047145	104	9·952855	47
14	9·645450	427	·354550	9·692656	531	10·307344	·047207	104	9·952793	46
15	9·645706	427	·354294	9·692975	531	10·307025	·047269	104	9·952731	45
16	9·645962	427	·354038	9·693293	531	10·306707	·047331	104	9·952669	44
17	9·646218	426	·353782	9·693612	530	10·306388	·047394	104	9·952606	43
18	9·646474	426	·353526	9·693930	530	10·306070	·047456	104	9·952544	42
19	9·646729	426	·353271	9·694248	530	10·305752	·047519	104	9·952481	41
20	9·646984	425	·353016	9·694566	530	10·305434	·047581	104	9·952419	40
21	9·647240	425	·352760	9·694883	529	10·305117	·047644	104	9·952356	39
22	9·647494	425	·352506	9·695201	529	10·304799	·047706	104	9·952294	38
23	9·647749	424	·352251	9·695518	529	10·304482	·047769	104	9·952231	37
24	9·648004	424	·351996	9·695836	529	10·304164	·047832	104	9·952168	36
25	9·648258	424	·351742	9·696153	529	10·303847	·047894	105	9·952106	35
26	9·648512	424	·351488	9·696470	528	10·303530	·047957	105	9·952043	34
27	9·648766	423	·351234	9·696787	528	10·303213	·048020	105	9·951980	33
28	9·649020	423	·350980	9·697103	528	10·302897	·048083	105	9·951917	32
29	9·649274	423	·350726	9·697420	528	10·302580	·048146	105	9·951854	31
30	9·649527	422	·350473	9·697736	527	10·302264	·048209	105	9·951791	30
31	9·649781	422	·350219	9·698053	527	10·301947	·048272	105	9·951728	29
32	9·650034	422	·349966	9·698369	527	10·301631	·048335	105	9·951665	28
33	9·650287	422	·349713	9·698685	527	10·301315	·048398	105	9·951602	27
34	9·650539	421	·349461	9·699001	526	10·300999	·048461	105	9·951539	26
35	9·650792	421	·349208	9·699316	526	10·300684	·048524	105	9·951476	25
36	9·651044	421	·348956	9·699632	526	10·300368	·048588	105	9·951412	24
37	9·651297	420	·348703	9·699947	526	10·300053	·048651	105	9·951349	23
38	9·651549	420	·348451	9·700263	526	10·299737	·048714	106	9·951286	22
39	9·651800	420	·348200	9·700578	525	10·299422	·048778	106	9·951222	21
40	9·652052	419	·347948	9·700893	525	10·299107	·048841	106	9·951159	20
41	9·652304	419	·347696	9·701208	525	10·298792	·048904	106	9·951096	19
42	9·652555	419	·347445	9·701523	525	10·298477	·048968	106	9·951032	18
43	9·652806	418	·347194	9·701837	524	10·298163	·049032	106	9·950968	17
44	9·653057	418	·346943	9·702152	524	10·297848	·049095	106	9·950905	16
45	9·653308	418	·346692	9·702466	524	10·297534	·049159	106	9·950841	15
46	9·653558	418	·346442	9·702780	524	10·297220	·049222	106	9·950778	14
47	9·653808	417	·346192	9·703095	523	10·296905	·049286	106	9·950714	13
48	9·654059	417	·345941	9·703409	523	10·296591	·049350	106	9·950650	12
49	9·654309	417	·345691	9·703723	523	10·296277	·049414	106	9·950586	11
50	9·654558	416	·345442	9·704036	523	10·295964	·049478	106	9·950522	10
51	9·654808	416	·345192	9·704350	523	10·295650	·049542	107	9·950458	9
52	9·655058	416	·344942	9·704663	522	10·295337	·049606	107	9·950394	8
53	9·655307	415	·344693	9·704977	522	10·295023	·049670	107	9·950330	7
54	9·655556	415	·344444	9·705290	522	10·294710	·049734	107	9·950266	6
55	9·655805	415	·344195	9·705603	522	10·294397	·049798	107	9·950202	5
56	9·656054	415	·343946	9·705916	521	10·294084	·049862	107	9·950138	4
57	9·656302	414	·343698	9·706228	521	10·293772	·049926	107	9·950074	3
58	9·656551	414	·343449	9·706541	521	10·293459	·049990	107	9·950010	2
59	9·656799	414	·343201	9·706854	521	10·293146	·050055	107	9·949945	1
60	9·657047	413	·342953	9·707166	521	10·292834	·050119	107	9·949881	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

63 DEG.

27 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·657047		·342953	9·707166		10·292834	·050119		9·949881	60
1	9·657295	413	·342705	9·707478	520	10·292522	·050184	107	9·949816	59
2	9·657542	413	·342458	9·707790	520	10·292210	·050248	107	9·949752	58
3	9·657790	412	·342210	9·708102	520	10·291898	·050312	107	9·949688	57
4	9·658037	412	·341963	9·708414	520	10·291586	·050377	108	9·949623	56
5	9·658284	412	·341716	9·708726	519	10·291274	·050442	108	9·949558	55
6	9·658531	412	·341469	9·709037	519	10·290963	·050506	108	9·949494	54
7	9·658778	411	·341222	9·709349	519	10·290651	·050571	108	9·949429	53
8	9·659025	411	·340975	9·709660	519	10·290340	·050636	108	9·949364	52
9	9·659271	411	·340729	9·709971	519	10·290029	·050700	108	9·949300	51
10	9·659517	410	·340483	9·710282	518	10·289718	·050765	108	9·949235	50
11	9·659763	410	·340237	9·710593	518	10·289407	·050830	108	9·949170	49
12	9·660009	410	·339991	9·710904	518	10·289096	·050895	108	9·949105	48
13	9·660255	409	·339745	9·711215	518	10·288785	·050960	108	9·949040	47
14	9·660501	409	·339499	9·711525	518	10·288475	·051025	108	9·948975	46
15	9·660746	409	·339254	9·711836	517	10·288164	·051090	108	9·948910	45
16	9·660991	409	·339009	9·712146	517	10·287854	·051155	108	9·948845	44
17	9·661236	408	·338764	9·712456	517	10·287544	·051220	108	9·948780	43
18	9·661481	408	·338519	9·712766	517	10·287234	·051285	109	9·948715	42
19	9·661726	408	·338274	9·713076	516	10·286924	·051350	109	9·948650	41
20	9·661970	407	·338030	9·713386	516	10·286614	·051416	109	9·948584	40
21	9·662214	407	·337786	9·713696	516	10·286304	·051481	109	9·948519	39
22	9·662459	407	·337541	9·714005	516	10·285995	·051546	109	9·948454	38
23	9·662703	407	·337297	9·714314	516	10·285686	·051612	109	9·948388	37
24	9·662946	406	·337054	9·714624	515	10·285376	·051677	109	9·948323	36
25	9·663190	406	·336810	9·714933	515	10·285067	·051743	109	9·948257	35
26	9·663433	406	·336567	9·715242	515	10·284758	·051808	109	9·948192	34
27	9·663677	405	·336323	9·715551	515	10·284449	·051874	109	9·948126	33
28	9·663920	405	·336080	9·715860	514	10·284140	·051940	109	9·948060	32
29	9·664163	405	·335837	9·716168	514	10·283832	·052005	109	9·947995	31
30	9·664406	405	·335594	9·716477	514	10·283523	·052071	110	9·947929	30
31	9·664648	404	·335352	9·716785	514	10·283215	·052137	110	9·947863	29
32	9·664891	404	·335109	9·717093	514	10·282907	·052203	110	9·947797	28
33	9·665133	404	·334867	9·717401	513	10·282599	·052269	110	9·947731	27
34	9·665375	403	·334625	9·717709	513	10·282291	·052335	110	9·947665	26
35	9·665617	403	·334383	9·718017	513	10·281983	·052400	110	9·947600	25
36	9·665859	403	·334141	9·718325	513	10·281675	·052467	110	9·947533	24
37	9·666100	402	·333900	9·718633	513	10·281367	·052533	110	9·947467	23
38	9·666342	402	·333658	9·718940	512	10·281060	·052599	110	9·947401	22
39	9·666583	402	·333417	9·719248	512	10·280752	·052665	110	9·947335	21
40	9·666824	402	·333176	9·719555	512	10·280445	·052731	110	9·947269	20
41	9·667065	401	·332935	9·719862	512	10·280138	·052797	110	9·947203	19
42	9·667305	401	·332695	9·720169	512	10·279831	·052864	110	9·947136	18
43	9·667546	401	·332454	9·720476	511	10·279524	·052930	111	9·947070	17
44	9·667786	401	·332214	9·720783	511	10·279217	·052996	111	9·947004	16
45	9·668027	400	·331973	9·721089	511	10·278911	·053063	111	9·946937	15
46	9·668267	400	·331733	9·721396	511	10·278604	·053129	111	9·946871	14
47	9·668506	400	·331494	9·721702	511	10·278298	·053196	111	9·946804	13
48	9·668746	399	·331254	9·722009	510	10·277991	·053262	111	9·946738	12
49	9·668986	399	·331014	9·722315	510	10·277685	·053329	111	9·946671	11
50	9·669225	399	·330775	9·722621	510	10·277379	·053396	111	9·946604	10
51	9·669464	399	·330536	9·722927	510	10·277073	·053462	111	9·946538	9
52	9·669703	398	·330297	9·723232	510	10·276768	·053529	111	9·946471	8
53	9·669942	398	·330058	9·723538	509	10·276462	·053596	111	9·946404	7
54	9·670181	398	·329819	9·723844	509	10·276156	·053663	111	9·946337	6
55	9·670419	397	·329581	9·724149	509	10·275851	·053730	111	9·946270	5
56	9·670658	397	·329342	9·724454	509	10·275546	·053797	112	9·946203	4
57	9·670896	397	·329104	9·724759	509	10·275241	·053864	112	9·946136	3
58	9·671134	397	·328866	9·725065	508	10·274935	·053931	112	9·946069	2
59	9·671372	396	·328628	9·725369	508	10·274631	·053998	112	9·946002	1
60	9·671609	396	·328391	9·725674	508	10·274326	·054065	112	9·945935	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

62 DEG.

28 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·671609		·328391	9·725674		10·274326	·054065		9·945935	60
1	9·671847	396	·328153	9·725979	508	10·274021	·054132	112	9·945868	59
2	9·672084	395	·327916	9·726284	508	10·273716	·054200	112	9·945800	58
3	9·672321	395	·327679	9·726588	507	10·273412	·054267	112	9·945733	57
4	9·672558	395	·327442	9·726892	507	10·273108	·054334	112	9·945666	56
5	9·672795	395	·327205	9·727197	507	10·272803	·054402	112	9·945598	55
6	9·673032	394	·326968	9·727501	507	10·272499	·054469	112	9·945531	54
7	9·673268	394	·326732	9·727805	507	10·272195	·054536	112	9·945464	53
8	9·673505	394	·326495	9·728109	506	10·271891	·054604	113	9·945396	52
9	9·673741	394	·326259	9·728412	506	10·271588	·054672	113	9·945328	51
10	9·673977	393	·326023	9·728716	506	10·271284	·054739	113	9·945261	50
11	9·674213	393	·325787	9·729020	506	10·270980	·054807	113	9·945193	49
12	9·674448	393	·325552	9·729323	506	10·270677	·054875	113	9·945125	48
13	9·674684	392	·325316	9·729626	505	10·270374	·054942	113	9·945058	47
14	9·674919	392	·325081	9·729929	505	10·270071	·055010	113	9·944990	46
15	9·675155	392	·324845	9·730233	505	10·269767	·055078	113	9·944922	45
16	9·675390	392	·324610	9·730535	505	10·269465	·055146	113	9·944854	44
17	9·675624	391	·324376	9·730838	505	10·269162	·055214	113	9·944786	43
18	9·675859	391	·324141	9·731141	504	10·268859	·055282	113	9·944718	42
19	9·676094	391	·323906	9·731444	504	10·268556	·055350	113	9·944650	41
20	9·676328	391	·323672	9·731746	504	10·268254	·055418	113	9·944582	40
21	9·676562	390	·323438	9·732048	504	10·267952	·055486	114	9·944514	39
22	9·676796	390	·323204	9·732351	504	10·267649	·055554	114	9·944446	38
23	9·677030	390	·322970	9·732653	503	10·267347	·055623	114	9·944377	37
24	9·677264	390	·322736	9·732955	503	10·267045	·055691	114	9·944309	36
25	9·677498	389	·322502	9·733257	503	10·266743	·055759	114	9·944241	35
26	9·677731	389	·322269	9·733558	503	10·266442	·055828	114	9·944172	34
27	9·677964	389	·322036	9·733860	503	10·266140	·055896	114	9·944104	33
28	9·678197	388	·321803	9·734162	503	10·265838	·055964	114	9·944036	32
29	9·678430	388	·321570	9·734463	502	10·265537	·056033	114	9·943967	31
30	9·678663	388	·321337	9·734764	502	10·265236	·056102	114	9·943899	30
31	9·678895	388	·321105	9·735066	502	10·264934	·056170	114	9·943830	29
32	9·679128	387	·320872	9·735367	502	10·264633	·056239	114	9·943761	28
33	9·679360	387	·320640	9·735668	502	10·264332	·056307	114	9·943693	27
34	9·679592	387	·320408	9·735969	501	10·264031	·056376	115	9·943624	26
35	9·679824	387	·320176	9·736269	501	10·263731	·056445	115	9·943555	25
36	9·680056	386	·319944	9·736570	501	10·263430	·056514	115	9·943486	24
37	9·680288	386	·319712	9·736871	501	10·263129	·056583	115	9·943417	23
38	9·680519	386	·319481	9·737171	501	10·262829	·056652	115	9·943348	22
39	9·680750	385	·319250	9·737471	500	10·262529	·056721	115	9·943279	21
40	9·680982	385	·319018	9·737771	500	10·262229	·056790	115	9·943210	20
41	9·681213	385	·318787	9·738071	500	10·261929	·056859	115	9·943141	19
42	9·681443	385	·318557	9·738371	500	10·261629	·056928	115	9·943072	18
43	9·681674	384	·318326	9·738671	500	10·261329	·056997	115	9·943003	17
44	9·681905	384	·318095	9·738971	500	10·261029	·057066	115	9·942934	16
45	9·682135	384	·317865	9·739271	499	10·260729	·057136	115	9·942864	15
46	9·682365	384	·317635	9·739570	499	10·260430	·057205	115	9·942795	14
47	9·682595	383	·317405	9·739870	499	10·260130	·057274	116	9·942726	13
48	9·682825	383	·317175	9·740169	499	10·259831	·057344	116	9·942656	12
49	9·683055	383	·316945	9·740468	499	10·259532	·057413	116	9·942587	11
50	9·683284	383	·316716	9·740767	498	10·259233	·057483	116	9·942517	10
51	9·683514	382	·316486	9·741066	498	10·258934	·057552	116	9·942448	9
52	9·683743	382	·316257	9·741365	498	10·258635	·057622	116	9·942378	8
53	9·683972	382	·316028	9·741664	498	10·258336	·057692	116	9·942308	7
54	9·684201	382	·315799	9·741962	498	10·258038	·057761	116	9·942239	6
55	9·684430	381	·315570	9·742261	498	10·257739	·057831	116	9·942169	5
56	9·684658	381	·315342	9·742559	497	10·257441	·057901	116	9·942099	4
57	9·684887	381	·315113	9·742858	497	10·257142	·057971	116	9·942029	3
58	9·685115	380	·314885	9·743156	497	10·256844	·058041	116	9·941959	2
59	9·685343	380	·314657	9·743454	497	10·256546	·058111	116	9·941889	1
60	9·685571	380	·314429	9·743752	497	10·256248	·058181	117	9·941819	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

61 DEG.

29 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff 100″	Cosine.	′
0	9·685571		·314429	9·743752		10·256248	·058181		9·941819	60
1	9·685799	380	·314201	9·744050	496	10·255950	·058251	117	9·941749	59
2	9·686027	379	·313973	9·744348	496	10·255652	·058321	117	9·941679	58
3	9·686254	379	·313746	9·744645	496	10·255355	·058391	117	9·941609	57
4	9·686482	379	·313518	9·744943	496	10·255057	·058461	117	9·941539	56
5	9·686709	379	·313291	9·745240	496	10·254760	·058531	117	9·941469	55
6	9·686936	378	·313064	9·745538	496	10·254462	·058602	117	9·941398	54
7	9·687163	378	·312837	9·745835	495	10·254165	·058672	117	9·941328	53
8	9·687389	378	·312611	9·746132	495	10·253868	·058742	117	9·941258	52
9	9·687616	378	·312384	9·746429	495	10·253571	·058813	117	9·941187	51
10	9·687843	377	·312157	9·746726	495	10·253274	·058883	117	9·941117	50
11	9·688069	377	·311931	9·747023	495	10·252977	·058954	117	9·941046	49
12	9·688295	377	·311705	9·747319	494	10·252681	·059025	118	9·940975	48
13	9·688521	377	·311479	9·747616	494	10·252384	·059095	118	9·940905	47
14	9·688747	376	·311253	9·747913	494	10·252087	·059166	118	9·940834	46
15	9·688972	376	·311028	9·748209	494	10·251791	·059237	118	9·940763	45
16	9·689198	376	·310802	9·748505	494	10·251495	·059367	118	9·940693	44
17	9·689423	376	·310577	9·748801	494	10·251199	·059378	118	9·940622	43
18	9·689648	375	·310352	9·749097	493	10·250903	·059449	118	9·940551	42
19	9·689873	375	·310127	9·749393	493	10·250607	·059520	118	9·940480	41
20	9·690098	375	·309902	9·749689	493	10·250311	·059591	118	9·940409	40
21	9·690323	375	·309677	9·749985	493	10·250015	·059662	118	9·940338	39
22	9·690548	374	·309452	9·750281	493	10·249719	·059733	118	9·940267	38
23	9·690772	374	·309228	9·750576	493	10·249424	·059804	118	9·940196	37
24	9·690996	374	·309004	9·750872	492	10·249128	·059875	118	9·940125	36
25	9·691220	374	·308780	9·751167	492	10·248833	·059946	119	9·940054	35
26	9·691444	373	·308556	9·751462	492	10·248538	·060018	119	9·939982	34
27	9·691668	373	·308332	9·751757	492	10·248243	·060089	119	9·939911	33
28	9·691892	373	·308108	9·752052	492	10·247948	·060160	119	9·939840	32
29	9·692115	373	·307885	9·752347	491	10·247653	·060232	119	9·939768	31
30	9·692339	372	·307661	9·752642	491	10·247358	·060303	119	9·939697	30
31	9·692562	372	·307438	9·752937	491	10·247063	·060375	119	9·939625	29
32	9·692785	372	·307215	9·753231	491	10·246769	·060446	119	9·939554	28
33	9·693008	371	·306992	9·753526	491	10·246474	·060518	119	9·939482	27
34	9·693231	371	·306769	9·753820	491	10·246180	·060590	119	9·939410	26
35	9·693453	371	·306547	9·754115	490	10·245885	·060661	119	9·939339	25
36	9·693676	371	·306324	9·754409	490	10·245591	·060733	119	9·939267	24
37	9·693898	370	·306102	9·754703	490	10·245297	·060805	120	9·939195	23
38	9·694120	370	·305880	9·754997	490	10·245003	·060877	120	9·939123	22
39	9·694342	370	·305658	9·755291	490	10·244709	·060948	120	9·939052	21
40	9·694564	370	·305436	9·755585	490	10·244415	·061020	120	9·938980	20
41	9·694786	369	·305214	9·755878	489	10·244122	·061092	120	9·938908	19
42	9·695007	369	·304993	9·756172	489	10·243828	·061164	120	9·938836	18
43	9·695229	369	·304771	9·756465	489	10·243535	·061237	120	9·938763	17
44	9·695450	369	·304550	9·756759	489	10·243241	·061309	120	9·938691	16
45	9·695671	368	·304329	9·757052	489	10·242948	·061381	120	9·938619	15
46	9·695892	368	·304108	9·757345	489	10·242655	·061453	120	9·938547	14
47	9·696113	368	·303887	9·757638	488	10·242362	·061525	120	9·938475	13
48	9·696334	368	·303666	9·757931	488	10·242069	·061598	120	9·938402	12
49	9·696554	367	·303446	9·758224	488	10·241776	·061670	121	9·938330	11
50	9·696775	367	·303225	9·758517	488	10·241483	·061742	121	9·938258	10
51	9·696995	367	·303005	9·758810	488	10·241190	·061815	121	9·938185	9
52	9·697215	367	·302785	9·759102	488	10·240898	·061887	121	9·938113	8
53	9·697435	366	·302565	9·759395	487	10·240605	·061960	121	9·938040	7
54	9·697654	366	·302346	9·759687	487	10·240313	·062033	121	9·937967	6
55	9·697874	366	·302126	9·759979	487	10·240021	·062105	121	9·937895	5
56	9·698094	366	·301906	9·760272	487	10·239728	·062178	121	9·937822	4
57	9·698313	365	·301687	9·760564	487	10·239436	·062251	121	9·937749	3
58	9·698532	365	·301468	9·760856	487	10·239144	·062324	121	9·937676	2
59	9·698751	365	·301249	9·761148	486	10·238852	·062396	121	9·937604	1
60	9·698970	365	·301030	9·761439	486	10·238561	·062469	121	9·937531	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

60 DEG.

30 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·698970		·301030	9·761439		10·238561	·062469		9·937531	60
1	9·699189	364	·300811	9·761731	486	10·238269	·062542	121	9·937458	59
2	9·699407	364	·300593	9·762023	486	10·237977	·062615	122	9·937385	58
3	9·699626	364	·300374	9·762314	486	10·237686	·062688	122	9·937312	57
4	9·699844	364	·300156	9·762606	486	10·237394	·062762	122	9·937238	56
5	9·700062	363	·299938	9·762897	485	10·237103	·062835	122	9·937165	55
6	9·700280	363	·299520	9·763188	485	10·236812	·062908	122	9·937092	54
7	9·700498	363	·299702	9·763479	485	10·236521	·062981	122	9·937019	53
8	9·700716	363	·299284	9·763770	485	10·236230	·063054	122	9·936946	52
9	9·700933	363	·299067	9·764061	485	10·235939	·063128	122	9·936872	51
10	9·701151	362	·298849	9·764352	485	10·235648	·063201	122	9·936799	50
11	9·701368	362	·298632	9·764643	485	10·235357	·063275	122	9·936725	49
12	9·701585	362	·298415	9·764933	484	10·235067	·063348	122	9·936652	48
13	9·701802	362	·298198	9·765224	484	10·234776	·063422	123	9·936578	47
14	9·702019	361	·297981	9·765514	484	10·234486	·063495	123	9·936505	46
15	9·702236	361	·297764	9·765805	484	10·234195	·063569	123	9·936431	45
16	9·702452	361	·297548	9·766095	484	10·233905	·063643	123	9·936357	44
17	9·702669	361	·297331	9·766385	484	10·233615	·063716	123	9·936284	43
18	9·702885	360	·297115	9·766675	483	10·233325	·063790	123	9·936210	42
19	9·703101	360	·296899	9·766965	483	10·233035	·063864	123	9·936136	41
20	9·703317	360	·296683	9·767255	483	10·232745	·063938	123	9·936062	40
21	9·703533	360	·296467	9·767545	483	10·232455	·064012	123	9·935988	39
22	9·703749	359	·296251	9·767834	483	10·232166	·064086	123	9·935914	38
23	9·703964	359	·296036	9·768124	483	10·231876	·064160	123	9·935840	37
24	9·704179	359	·295821	9·768414	482	10·231586	·064234	123	9·935766	36
25	9·704395	359	·295605	9·768703	482	10·231297	·064308	124	9·935692	35
26	9·704610	359	·295390	9·768992	482	10·231008	·064382	124	9·935618	34
27	9·704825	358	·295175	9·769281	482	10·230719	·064457	124	9·935543	33
28	9·705040	358	·294960	9·769570	482	10·230430	·064531	124	9·935469	32
29	9·705254	358	·294746	9·769860	482	10·230140	·064605	124	9·935395	31
30	9·705469	358	·294531	9·770148	481	10·229852	·064680	124	9·935320	30
31	9·705683	357	·294317	9·770437	481	10·229563	·064754	124	9·935246	29
32	9·705898	357	·294102	9·770726	481	10·229274	·064829	124	9·935171	28
33	9·706112	357	·293888	9·771015	481	10·228985	·064903	124	9·935097	27
34	9·706326	357	·293674	9·771303	481	10·228697	·064978	124	9·935022	26
35	9·706539	356	·293461	9·771592	481	10·228408	·065052	124	9·934948	25
36	9·706753	356	·293247	9·771880	481	10·228120	·065127	124	9·934873	24
37	9·706967	356	·293033	9·772168	480	10·227832	·065202	124	9·934798	23
38	9·707180	356	·292820	9·772457	480	10·227543	·065277	125	9·934723	22
39	9·707393	355	·292607	9·772745	480	10·227255	·065351	125	9·934649	21
40	9·707606	355	·292394	9·773033	480	10·226967	·065426	125	9·934574	20
41	9·707819	355	·292181	9·773321	480	10·226679	·065501	125	9·934499	19
42	9·708032	355	·291968	9·773608	480	10·226392	·065576	125	9·934424	18
43	9·708245	354	·291755	9·773896	480	10·226104	·065651	125	9·934349	17
44	9·708458	354	·291542	9·774184	479	10·225816	·065726	125	9·934274	16
45	9·708670	354	·291330	9·774471	479	10·225529	·065801	125	9·934199	15
46	9·708882	354	·291118	9·774759	479	10·225241	·065877	125	9·934123	14
47	9·709094	353	·290906	9·775046	479	10·224954	·065952	125	9·934048	13
48	9·709306	353	·290694	9·775333	479	10·224667	·066027	125	9·933973	12
49	9·709518	353	·290482	9·775621	479	10·224379	·066102	125	9·933898	11
50	9·709730	353	·290270	9·775908	478	10·224092	·066178	126	9·933822	10
51	9·709941	353	·290059	9·776195	478	10·223805	·066253	126	9·933747	9
52	9·710153	352	·289847	9·776482	478	10·223518	·066329	126	9·933671	8
53	9·710364	352	·289636	9·776769	478	10·223231	·066404	126	9·933596	7
54	9·710575	352	·289425	9·777055	478	10·222945	·066480	126	9·933520	6
55	9·710786	352	·289214	9·777342	478	10·222658	·066555	126	9·933445	5
56	9·710997	351	·289003	9·777628	478	10·222372	·066631	126	9·933369	4
57	9·711208	351	·288792	9·777915	477	10·222085	·066707	126	9·933293	3
58	9·711419	351	·288581	9·778201	477	10·221799	·066783	126	9·933217	2
59	9·711629	351	·288371	9·778487	477	10·221513	·066859	126	9·933141	1
60	9·711839	350	·288161	9·778774	477	10·221226	·066934	126	9·933066	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

59 DEG.

31 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·711839		·288161	9·778774		10·221226	·066934		9·933066	60
1	9·712050	350	·287950	9·779060	477	10·220940	·067010	126	9·932990	59
2	9·712260	350	·287740	9·779346	477	10·220654	·067086	127	9·932914	58
3	9·712469	350	·287531	9·779632	477	10·220368	·067162	127	9·932838	57
4	9·712679	349	·287321	9·779918	476	10·220082	·067238	127	9·932762	56
5	9·712889	349	·287111	9·780203	476	10·219797	·067315	127	9·932685	55
6	9·713098	349	·286902	9·780489	476	10·219511	·067391	127	9·932609	54
7	9·713308	349	·286692	9·780775	476	10·219225	·067467	127	9·932533	53
8	9·713517	349	·286483	9·781060	476	10·218940	·067543	127	9·932457	52
9	9·713726	348	·286274	9·781346	476	10·218654	·067620	127	9·932380	51
10	9·713935	348	·286065	9·781631	476	10·218369	·067696	127	9·932304	50
11	9·714144	348	·285856	9·781916	475	10·218084	·067772	127	9·932228	49
12	9·714352	348	·285648	9·782201	475	10·217799	·067849	127	9·932151	48
13	9·714561	347	·285439	9·782486	475	10·217514	·067925	127	9·932075	47
14	9·714769	347	·285231	9·782771	475	10·217229	·068002	128	9·931998	46
15	9·714978	347	·285022	9·783056	475	10·216944	·068079	128	9·931921	45
16	9·715186	347	·284814	9·783341	475	10·216659	·068155	128	9·931845	44
17	9·715394	347	·284606	9·783626	475	10·216374	·068232	128	9·931768	43
18	9·715602	346	·284398	9·783910	474	10·216090	·068309	128	9·931691	42
19	9·715809	346	·284191	9·784195	474	10·215805	·068386	128	9·931614	41
20	9·716017	346	·283983	9·784479	474	10·215521	·068463	128	9·931537	40
21	9·716224	346	·283776	9·784764	474	10·215236	·068540	128	9·931460	39
22	9·716432	345	·283568	9·785048	474	10·214952	·068617	128	9·931383	38
23	9·716639	345	·283361	9·785332	474	10·214668	·068694	128	9·931306	37
24	9·716846	345	·283154	9·785616	474	10·214384	·068771	128	9·931229	36
25	9·717053	345	·282947	9·785900	473	10·214100	·068848	129	9·931152	35
26	9·717259	345	·282741	9·786184	473	10·213816	·068925	129	9·931075	34
27	9·717466	344	·282534	9·786468	473	10·213532	·069002	129	9·930998	33
28	9·717673	344	·282327	9·786752	473	10·213248	·069079	129	9·930921	32
29	9·717879	344	·282121	9·787036	473	10·212964	·069157	129	9·930843	31
30	9·718085	344	·281915	9·787319	473	10·212681	·069234	129	9·930766	30
31	9·718291	343	·281709	9·787603	473	10·212397	·069312	129	9·930688	29
32	9·718497	343	·281503	9·787886	472	10·212114	·069389	129	9·930611	28
33	9·718703	343	·281297	9·788170	472	10·211830	·069467	129	9·930533	27
34	9·718909	343	·281091	9·788453	472	10·211547	·069544	129	9·930456	26
35	9·719114	343	·280886	9·788736	472	10·211264	·069622	129	9·930378	25
36	9·719320	342	·280680	9·789019	472	10·210981	·069700	129	9·930300	24
37	9·719525	342	·280475	9·789302	472	10·210698	·069777	130	9·930223	23
38	9·719730	342	·280270	9·789585	472	10·210415	·069855	130	9·930145	22
39	9·719935	342	·280065	9·789868	471	10·210132	·069933	130	9·930067	21
40	9·720140	341	·279860	9·790151	471	10·209849	·070011	130	9·929989	20
41	9·720345	341	·279655	9·790433	471	10·209567	·070089	130	9·929911	19
42	9·720549	341	·279451	9·790716	471	10·209284	·070167	130	9·929833	18
43	9·720754	341	·279246	9·790999	471	10·209001	·070245	130	9·929755	17
44	9·720958	340	·279042	9·791281	471	10·208719	·070323	130	9·929677	16
45	9·721162	340	·278838	9·791563	471	10·208437	·070401	130	9·929599	15
46	9·721366	340	·278634	9·791846	470	10·208154	·070479	130	9·929521	14
47	9·721570	340	·278430	9·792128	470	10·207872	·070558	130	9·929442	13
48	9·721774	340	·278226	9·792410	470	10·207590	·070636	130	9·929364	12
49	9·721978	339	·278022	9·792692	470	10·207308	·070714	131	9·929286	11
50	9·722181	339	·277819	9·792974	470	10·207026	·070793	131	9·929207	10
51	9·722385	339	·277615	9·793256	470	10·206744	·070871	131	9·929129	9
52	9·722588	339	·277412	9·793538	470	10·206462	·070950	131	9·929050	8
53	9·722791	339	·277209	9·793819	469	10·206181	·071028	131	9·928972	7
54	9·722994	338	·277006	9·794101	469	10·205899	·071107	131	9·928893	6
55	9·723197	338	·276803	9·794383	469	10·205617	·071185	131	9·928815	5
56	9·723400	338	·276600	9·794664	469	10·205336	·071264	131	9·928736	4
57	9·723603	338	·276397	9·794945	469	10·205055	·071343	131	9·928657	3
58	9·723805	337	·276195	9·795227	469	10·204773	·071422	131	9·928578	2
59	9·724007	337	·275993	9·795508	469	10·204492	·071501	131	9·928499	1
60	9·724210	337	·275790	9·795789	468	10·204211	·071580	131	9·928420	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

58 DEG.

32 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·724210		·275790	9·795789		10·204211	·071580		9·928420	60
1	9·724412	337	·275588	9·796070	468	10·203930	·071658	132	9·928342	59
2	9·724614	337	·275386	9·796351	468	10·203649	·071737	132	9·928263	58
3	9·724816	336	·275184	9·796632	468	10·203368	·071817	132	9·928183	57
4	9·725017	336	·274983	9·796913	468	10·203087	·071896	132	9·928104	56
5	9·725219	336	·274781	9·797194	468	10·202806	·071975	132	9·928025	55
6	9·725420	336	·274580	9·797475	468	10·202525	·072054	132	9·927946	54
7	9·725622	335	·274378	9·797755	468	10·202245	·072133	132	9·927867	53
8	9·725823	335	·274177	9·798036	467	10·201964	·072213	132	9·927787	52
9	9·726024	335	·273976	9·798316	467	10·201684	·072292	132	9·927708	51
10	9·726225	335	·273775	9·798596	467	10·201404	·072371	132	9·927629	50
11	9·726426	335	·273574	9·798877	467	10·201123	·072451	132	9·927549	49
12	9·726626	334	·273374	9·799157	467	10·200843	·072530	132	9·927470	48
13	9·726827	334	·273173	9·799437	467	10·200563	·072610	133	9·927390	47
14	9·727027	334	·272973	9·799717	467	10·200283	·072690	133	9·927310	46
15	9·727228	334	·272772	9·799997	467	10·200003	·072769	133	9·927231	45
16	9·727428	334	·272572	9·800277	466	10·199723	·072849	133	9·927151	44
17	9·727628	333	·272372	9·800557	466	10·199443	·072929	133	9·927071	43
18	9·727828	333	·272172	9·800836	466	10·199164	·073009	133	9·926991	42
19	9·728027	333	·271973	9·801116	466	10·198884	·073089	133	9·926911	41
20	9·728227	333	·271773	9·801396	466	10·198604	·073169	133	9·926831	40
21	9·728427	333	·271573	9·801675	466	10·198325	·073249	133	9·926751	39
22	9·728626	332	·271374	9·801955	466	10·198045	·073329	133	9·926671	38
23	9·728825	332	·271175	9·802234	466	10·197766	·073409	133	9·926591	37
24	9·729024	332	·270976	9·802513	465	10·197487	·073489	133	9·926511	36
25	9·729223	332	·270777	9·802792	465	10·197208	·073569	134	9·926431	35
26	9·729422	331	·270578	9·803072	465	10·196928	·073649	134	9·926351	34
27	9·729621	331	·270379	9·803351	465	10·196649	·073730	134	9·926270	33
28	9·729820	331	·270180	9·803630	465	10·196370	·073810	134	9·926190	32
29	9·730018	331	·269982	9·803908	465	10·196092	·073890	134	9·926110	31
30	9·730217	330	·269783	9·804187	465	10·195813	·073971	134	9·926029	30
31	9·730415	330	·269585	9·804466	465	10·195534	·074051	134	9·925949	29
32	9·730613	330	·269387	9·804745	464	10·195255	·074132	134	9·925868	28
33	9·730811	330	·269189	9·805023	464	10·194977	·074212	134	9·925788	27
34	9·731009	330	·268991	9·805302	464	10·194698	·074293	134	9·925707	26
35	9·731206	329	·268794	9·805580	464	10·194420	·074374	134	9·925626	25
36	9·731404	329	·268596	9·805859	464	10·194141	·074455	134	9·925545	24
37	9·731602	329	·268398	9·806137	464	10·193863	·074535	135	9·925465	23
38	9·731799	329	·268201	9·806415	464	10·193585	·074616	135	9·925384	22
39	9·731996	329	·268004	9·806693	464	10·193307	·074697	135	9·925303	21
40	9·732193	328	·267807	9·806971	463	10·193029	·074778	135	9·925222	20
41	9·732390	328	·267610	9·807249	463	10·192751	·074859	135	9·925141	19
42	9·732587	328	·267413	9·807527	463	10·192473	·074940	135	9·925060	18
43	9·732784	328	·267216	9·807805	463	10·192195	·075021	135	9·924979	17
44	9·732980	328	·267020	9·808083	463	10·191917	·075103	135	9·924897	16
45	9·733177	327	·266823	9·808361	463	10·191639	·075184	135	9·924816	15
46	9·733373	327	·266627	9·808638	463	10·191362	·075265	135	9·924735	14
47	9·733569	327	·266431	9·808916	463	10·191084	·075346	136	9·924654	13
48	9·733765	327	·266235	9·809193	462	10·190807	·075428	136	9·924572	12
49	9·733961	327	·266039	9·809471	462	10·190529	·075509	136	9·924491	11
50	9·734157	326	·265843	9·809748	462	10·190252	·075591	136	9·924409	10
51	9·734353	326	·265647	9·810025	462	10·189975	·075672	136	9·924328	9
52	9·734549	326	·265451	9·810302	462	10·189698	·075754	136	9·924246	8
53	9·734744	326	·265256	9·810580	462	10·189420	·075836	136	9·924164	7
54	9·734939	325	·265061	9·810857	462	10·189143	·075917	136	9·924083	6
55	9·735135	325	·264865	9·811134	462	10·188866	·075999	136	9·924001	5
56	9·735330	325	·264670	9·811410	461	10·188590	·076081	136	9·923919	4
57	9·735525	325	·264475	9·811687	461	10·188313	·076163	136	9·923837	3
58	9·735719	325	·264281	9·811964	461	10·188036	·076245	136	9·923755	2
59	9·735914	324	·264086	9·812241	461	10·187759	·076327	137	9·923673	1
60	9·736109	324	·263891	9·812517	461	10·187483	·076409	137	9·923591	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

57 DEG.

33 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	D.ff. 100″	Cosine.	′
0	9·736109		·263891	9·812517		10·187483	·076409		9·923591	60
1	9·736303	324	·263697	9·812794	461	10·187206	·076491	137	9·923509	59
2	9·736498	324	·263502	9·813070	461	10·186930	·076573	137	9·923427	58
3	9·736692	324	·263308	9·813347	461	10·186653	·076655	137	9·923345	57
4	9·736886	323	·263114	9·813623	460	10·186377	·076737	137	9·923263	56
5	9·737080	323	·262920	9·813899	460	10·186101	·076819	137	9·923181	55
6	9·737274	323	·262726	9·814175	460	10·185825	·076902	137	9·923098	54
7	9·737467	323	·262533	9·814452	460	10·185548	·076984	137	9·923016	53
8	9·737661	323	·262339	9·814728	460	10·185272	·077067	137	9·922933	52
9	9·737855	322	·262145	9·815004	460	10·184996	·077149	137	9·922851	51
10	9·738048	322	·261952	9·815279	460	10·184721	·077232	137	9·922768	50
11	9·738241	322	·261759	9·815555	460	10·184445	·077314	138	9·922686	49
12	9·738434	322	·261566	9·815831	460	10·184169	·077397	138	9·922603	48
13	9·738627	322	·261373	9·816107	459	10·183893	·077480	138	9·922520	47
14	9·738820	321	·261180	9·816382	459	10·183618	·077562	138	9·922438	46
15	9·739013	321	·260987	9·816658	459	10·183342	·077645	138	9·922355	45
16	9·739206	321	·260794	9·816933	459	10·183067	·077728	138	9·922272	44
17	9·739398	321	·260602	9·817209	459	10·182791	·077811	138	9·922189	43
18	9·739590	321	·260410	9·817484	459	10·182516	·077894	138	9·922106	42
19	9·739783	320	·260217	9·817759	459	10·182241	·077977	138	9·922023	41
20	9·739975	320	·260025	9·818035	459	10·181965	·078060	138	9·921940	40
21	9·740167	320	·259833	9·818310	459	10·181690	·078143	138	9·921857	39
22	9·740359	320	·259641	9·818585	458	10·181415	·078226	139	9·921774	38
23	9·740550	320	·259450	9·818860	458	10·181140	·078309	139	9·921691	37
24	9·740742	319	·259258	9·819135	458	10·180865	·078393	139	9·921607	36
25	9·740934	319	·259066	9·819410	458	10·180590	·078476	139	9·921524	35
26	9·741125	319	·258875	9·819684	458	10·180316	·078559	139	9·921441	34
27	9·741316	319	·258684	9·819959	458	10·180041	·078643	139	9·921357	33
28	9·741508	319	·258492	9·820234	458	10·179766	·078726	139	9·921274	32
29	9·741699	318	·258301	9·820508	458	10·179492	·078810	139	9·921190	31
30	9·741889	318	·258111	9·820783	458	10·179217	·078893	139	9·921107	30
31	9·742080	318	·257920	9·821057	457	10·178943	·078977	139	9·921023	29
32	9·742271	318	·257729	9·821332	457	10·178668	·079061	139	9·920939	28
33	9·742462	318	·257538	9·821606	457	10·178394	·079144	139	9·920856	27
34	9·742652	317	·257348	9·821880	457	10·178120	·079228	140	9·920772	26
35	9·742842	317	·257158	9·822154	457	10·177846	·079312	140	9·920688	25
36	9·743033	317	·256967	9·822429	457	10·177571	·079396	140	9·920604	24
37	9·743223	317	·256777	9·822703	457	10·177297	·079480	140	9·920520	23
38	9·743413	317	·256587	9·822977	457	10·177023	·079564	140	9·920436	22
39	9·743602	316	·256398	9·823250	457	10·176750	·079648	140	9·920352	21
40	9·743792	316	·256208	9·823524	456	10·176476	·079732	140	9·920268	20
41	9·743982	316	·256018	9·823798	456	10·176202	·079816	140	9·920184	19
42	9·744171	316	·255829	9·824072	456	10·175928	·079901	140	9·920099	18
43	9·744361	316	·255639	9·824345	456	10·175655	·079985	140	9·920015	17
44	9·744550	315	·255450	9·824619	456	10·175381	·080069	140	9·919931	16
45	9·744739	315	·255261	9·824893	456	10·175107	·080154	141	9·919846	15
46	9·744928	315	·255072	9·825166	456	10·174834	·080238	141	9·919762	14
47	9·745117	315	·254883	9·825439	456	10·174561	·080323	141	9·919677	13
48	9·745306	315	·254694	9·825713	456	10·174287	·080407	141	9·919593	12
49	9·745494	314	·254506	9·825986	455	10·174014	·080492	141	9·919508	11
50	9·745683	314	·254317	9·826259	455	10·173741	·080576	141	9·919424	10
51	9·745871	314	·254129	9·826532	455	10·173468	·080661	141	9·919339	9
52	9·746060	314	·253940	9·826805	455	10·173195	·080746	141	9·919254	8
53	9·746248	314	·253752	9·827078	455	10·172922	·080831	141	9·919169	7
54	9·746436	313	·253564	9·827351	455	10·172649	·080915	141	9·919085	6
55	9·746624	313	·253376	9·827624	455	10·172376	·081000	141	9·919000	5
56	9·746812	313	·253188	9·827897	455	10·172103	·081085	142	9·918915	4
57	9·746999	313	·253001	9·828170	455	10·171830	81170	142	9·918830	3
58	9·747187	313	·252813	9·828442	454	10·171558	·081255	142	9·918745	2
59	9·747374	312	·252626	9·828715	454	10·171285	·081341	142	9·918659	1
60	9·747562	312	·252438	9·828987	454	10·171013	·081426	142	9·918574	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	

56 DEG.

34 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100′	Cosine.	′
0	9·747562		·252438	9·828987		10·171013	·081426		9·918574	60
1	9·747749	312	·252251	9·829260	454	10·170740	·081511	142	9·918489	59
2	9·747936	312	·252064	9·829532	454	10·170468	·081596	142	9·918404	58
3	9·748123	312	·251877	9·829805	454	10·170195	·081682	142	9·918318	57
4	9·748310	311	·251690	9·830077	454	10·169923	·081767	142	9·918233	56
5	9·748497	311	·251503	9·830349	454	10·169651	·081853	142	9·918147	55
6	9·748683	311	·251317	9·830621	454	10·169379	·081938	142	9·918062	54
7	9·748870	311	·251130	9·830893	453	10·169107	·082024	143	9·917976	53
8	9·749056	311	·250944	9·831165	453	10·168835	·082109	143	9·917891	52
9	9·749243	310	·250757	9·831437	453	10·168563	·082195	143	9·917805	51
10	9·749429	310	·250571	9·831709	453	10·168291	·082281	143	9·917719	50
11	9·749615	310	·250385	9·831981	453	10·168019	·082366	143	9·917634	49
12	9·749801	310	·250199	9·832253	453	10·167747	·082452	143	9·917548	48
13	9·749987	310	·250013	9·832525	453	10·167475	·082538	143	9·917462	47
14	9·750172	309	·249828	9·832796	453	10·167204	·082624	143	9·917376	46
15	9·750358	309	·249642	9·833068	453	10·166932	·082710	143	9·917290	45
16	9·750543	309	·249457	9·833339	453	10·166661	·082796	143	9·917204	44
17	9·750729	309	·249271	9·833611	452	10·166389	·082882	143	9·917118	43
18	9·750914	309	·249086	9·833882	452	10·166118	·082968	144	9·917032	42
19	9·751099	308	·248901	9·834154	452	10·165846	·083054	144	9·916946	41
20	9·751284	308	·248716	9·834425	452	10·165575	·083141	144	9·916859	40
21	9·751469	308	·248531	9·834696	452	10·165304	·083227	144	9·916773	39
22	9·751654	308	·248346	9·834967	452	10·165033	·083313	144	9·916687	38
23	9·751839	308	·248161	9·835238	452	10·164762	·083400	144	9·916600	37
24	9·752023	308	·247977	9·835509	452	10·164491	·083486	144	9·916514	36
25	9·752208	307	·247792	9·835780	452	10·164220	·083573	144	9·916427	35
26	9·752392	307	·247608	9·836051	452	10·163949	·083659	144	9·916341	34
27	9·752576	307	·247424	9·836322	451	10·163678	·083746	144	9·916254	33
28	9·752760	307	·247240	9·836593	451	10·163407	·083833	144	9·916167	32
29	9·752944	307	·247056	9·836864	451	10·163136	·083919	145	9·916081	31
30	9·753128	306	·246872	9·837134	451	10·162866	·084006	145	9·915994	30
31	9·753312	306	·246688	9·837405	451	10·162595	·084093	145	9·915907	29
32	9·753495	306	·246505	9·837675	451	10·162325	·084180	145	9·915820	28
33	9·753679	306	·246321	9·837946	451	10·162054	·084267	145	9·915733	27
34	9·753862	306	·246138	9·838216	451	10·161784	·084354	145	9·915646	26
35	9·754046	305	·245954	9·838487	451	10·161513	·084441	145	9·915559	25
36	9·754229	305	·245771	9·838757	451	10·161243	·084528	145	9·915472	24
37	9·754412	305	·245588	9·839027	450	10·160973	·084615	145	9·915385	23
38	9·754595	305	·245405	9·839297	450	10·160703	·084703	145	9·915297	22
39	9·754778	305	·245222	9·839568	450	10·160432	·084790	145	9·915210	21
40	9·754960	304	·245040	9·839838	450	10·160162	·084877	145	9·915123	20
41	9·755143	304	·244857	9·840108	450	10·159892	·084965	146	9·915035	19
42	9·755326	304	·244674	9·840378	450	10·159622	·085052	146	9·914948	18
43	9·755508	304	·244492	9·840647	450	10·159353	·085140	146	9·914860	17
44	9·755690	304	·244310	9·840917	450	10·159083	·085227	146	9·914773	16
45	9·755872	304	·244128	9·841187	450	10·158813	·085315	146	9·914685	15
46	9·756054	303	·243946	9·841457	450	10·158543	·085402	146	9·914598	14
47	9·756236	303	·243764	9·841726	449	10·158274	·085490	146	9·914510	13
48	9·756418	303	·243582	9·841996	449	10·158004	·085578	146	9·914422	12
49	9·756600	303	·243400	9·842266	449	10·157734	·085666	146	9·914334	11
50	9·756782	303	·243218	9·842535	449	10·157465	·085754	146	9·914246	10
51	9·756963	302	·243037	9·842805	449	10·157195	·085842	147	9·914158	9
52	9·757144	302	·242856	9·843074	449	10·156926	·085930	147	9·914070	8
53	9·757326	302	·242674	9·843343	449	10·156657	·086018	147	9·913982	7
54	9·757507	302	·242493	9·843612	449	10·156388	·086106	147	9·913894	6
55	9·757688	302	·242312	9·843882	449	10·156118	·086194	147	9·913806	5
56	9·757869	301	·242131	9·844151	449	10·155849	·086282	147	9·913718	4
57	9·758050	301	·241950	9·844420	448	10·155580	·086370	147	9·913630	3
58	9·758230	301	·241770	9·844689	448	10·155311	·086459	147	9·913541	2
59	9·758411	301	·241589	9·844958	448	10·155042	·086547	147	9·913453	1
60	9·758591	301	·241409	9·845227	448	10·154773	·086635	147	9·913365	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

55 DEG.

35 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·758591		·241409	9·845227		10·154773	·086635		9·913365	60
1	9·758772	301	·241228	9·845496	448	10·154504	·086724	147	9·913276	59
2	9·758952	300	·241048	9·845764	448	10·154236	·086813	147	9·913187	58
3	9·759132	300	·240868	9·846033	448	10·153967	·086901	148	9·913099	57
4	9·759312	300	·240688	9·846302	448	10·153698	·086990	148	9·913010	56
5	9·759492	300	·240508	9·846570	448	10·153430	·087078	148	9·912922	55
6	9·759672	300	·240328	9·846839	448	10·153161	·087167	148	9·912833	54
7	9·759852	299	·240148	9·847107	448	10·152893	·087256	148	9·912744	53
8	9·760031	299	·239969	9·847376	447	10·152624	·087345	148	9·912655	52
9	9·760211	299	·239789	9·847644	447	10·152356	·087434	148	9·912566	51
10	9·760390	299	·239610	9·847913	447	10·152087	·087523	148	9·912477	50
11	9·760569	299	·239431	9·848181	447	10·151819	·087612	148	9·912388	49
12	9·760748	298	·239252	9·848449	447	10·151551	·087701	148	9·912299	48
13	9·760927	298	·239073	9·848717	447	10·151283	·087790	149	9·912210	47
14	9·761106	298	·238894	9·848986	447	10·151014	·087879	149	9·912121	46
15	9·761285	298	·238715	9·849254	447	10·150746	·087969	149	9·912031	45
16	9·761464	298	·238536	9·849522	447	10·150478	·088058	149	9·911942	44
17	9·761642	298	·238358	9·849790	447	10·150210	·088147	149	9·911853	43
18	9·761821	297	·238179	9·850058	446	10·149942	·088237	149	9·911763	42
19	9·761999	297	·238001	9·850325	446	10·149675	·088326	149	9·911674	41
20	9·762177	297	·237823	9·850593	446	10·149407	·088416	149	9·911584	40
21	9·762356	297	·237644	9·850861	446	10·149139	·088505	149	9·911495	39
22	9·762534	297	·237466	9·851129	446	10·148871	·088595	149	9·911405	38
23	9·762712	296	·237288	9·851396	446	10·148604	·088685	149	9·911315	37
24	9·762889	296	·237111	9·851664	446	10·148336	·088774	150	9·911226	36
25	9·763067	296	·236933	9·851931	446	10·148069	·088864	150	9·911136	35
26	9·763245	296	·236755	9·852199	446	10·147801	·088954	150	9·911046	34
27	9·763422	296	·236578	9·852466	446	10·147534	·089044	150	9·910956	33
28	9·763600	296	·236400	9·852733	446	10·147267	·089134	150	9·910866	32
29	9·763777	295	·236223	9·853001	445	10·146999	·089224	150	9·910776	31
30	9·763954	295	·236046	9·853268	445	10·146732	·089314	150	9·910686	30
31	9·764131	295	·235869	9·853535	445	10·146465	·089404	150	9·910596	29
32	9·764308	295	·235692	9·853802	445	10·146198	·089494	150	9·910506	28
33	9·764485	295	·235515	9·854069	445	10·145931	·089585	150	9·910415	27
34	9·764662	294	·235338	9·854336	445	10·145664	·089675	150	9·910325	26
35	9·764838	294	·235162	9·854603	445	10·145397	·089765	151	9·910235	25
36	9·765015	294	·234985	9·854870	445	10·145130	·089856	151	9·910144	24
37	9·765191	294	·234809	9·855137	445	10·144863	·089946	151	9·910054	23
38	9·765367	294	·234633	9·855404	445	10·144596	·090037	151	9·909963	22
39	9·765544	294	·234456	9·855671	445	10·144329	·090127	151	9·909873	21
40	9·765720	293	·234280	9·855938	444	10·144062	·090218	151	9·909782	20
41	9·765896	293	·234104	9·856204	444	10·143796	·090309	151	9·909691	19
42	9·766072	293	·233928	9·856471	444	10·143529	·090399	151	9·909601	18
43	9·766247	293	·233753	9·856737	444	10·143263	·090490	151	9·909510	17
44	9·766423	293	·233577	9·857004	444	10·142996	·090581	151	9·909419	16
45	9·766598	293	·233402	9·857270	444	10·142730	·090672	151	9·909328	15
46	9·766774	292	·233226	9·857537	444	10·142463	·090763	152	9·909237	14
47	9·766949	292	·233051	9·857803	444	10·142197	·090854	152	9·909146	13
48	9·767124	292	·232876	9·858069	444	10·141931	·090945	152	9·909055	12
49	9·767300	292	·232700	9·858336	444	10·141664	·091036	152	9·908964	11
50	9·767475	292	·232525	9·858602	444	10·141398	·091127	152	9·908873	10
51	9·767649	291	·232351	9·858868	444	10·141132	·091219	152	9·908781	9
52	9·767824	291	·232176	9·859134	443	10·140866	·091310	152	9·908690	8
53	9·767999	291	·232001	9·859400	443	10·140600	·091401	152	9·908599	7
54	9·768173	291	·231827	9·859666	443	10·140334	·091493	152	9·908507	6
55	9·768348	291	·231652	9·859932	443	10·140068	·091584	152	9·908416	5
56	9·768522	290	·231478	9·860198	443	10·139802	·091676	153	9·908324	4
57	9·768697	290	·231303	9·860464	443	10·139536	·091767	153	9·908233	3
58	9·768871	290	·231129	9·860730	443	10·139270	·091859	153	9·908141	2
59	9·769045	290	·230955	9·860995	443	10·139005	·091951	153	9·908049	1
60	9·769219	290	·230781	9·861261	443	10·138739	·092042	153	9·907958	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

54 DEG.

36 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·769219		·230781	9·861261		10·138739	·092042		9·907958	60
1	9·769393	290	·230607	9·861527	443	10·138473	·092134	153	9·907866	59
2	9·769566	289	·230434	9·861792	443	10·138208	·092226	153	9·907774	58
3	9·769740	289	·230260	9·862058	443	10·137942	·092318	153	9·907682	57
4	9·769913	289	·230087	9·862323	442	10·137677	·092410	153	9·907590	56
5	9·770087	289	·229913	9·862589	442	10·137411	·092502	153	9·907498	55
6	9·770260	289	·229740	9·862854	442	10·137146	·092594	153	9·907406	54
7	9·770433	288	·229567	9·863119	442	10·136881	·092686	153	9·907314	53
8	9·770606	288	·229394	9·863385	442	10·136615	·092778	154	9·907222	52
9	9·770779	288	·229221	9·863650	442	10·136350	·092871	154	9·907129	51
10	9·770952	288	·229048	9·863915	442	10·136085	·092963	154	9·907037	50
11	9·771125	288	·228875	9·864180	442	10·135820	·093055	154	9·906945	49
12	9·771298	288	·228702	9·864445	442	10·135555	·093148	154	9·906852	48
13	9·771470	287	·228530	9·864710	442	10·135290	·093240	154	9·906760	47
14	9·771643	287	·228357	9·864975	442	10·135025	·093333	154	9·906667	46
15	9·771815	287	·228185	9·865240	442	10·134760	·093425	154	9·906575	45
16	9·771987	287	·228013	9·865505	441	10·134495	·093518	154	9·906482	44
17	9·772159	287	·227841	9·865770	441	10·134230	·093611	154	9·906389	43
18	9·772331	287	·227669	9·866035	441	10·133965	·093704	155	9·906296	42
19	9·772503	286	·227497	9·866300	441	10·133700	·093796	155	9·906204	41
20	9·772675	286	·227325	9·866564	441	10·133436	·093889	155	9·906111	40
21	9·772847	286	·227153	9·866829	441	10·133171	·093982	155	9·906018	39
22	9·773018	286	·226982	9·867094	441	10·132906	·094075	155	9·905925	38
23	9·773190	286	·226810	9·867358	441	10·132642	·094168	155	9·905832	37
24	9·773361	286	·226639	9·867623	441	10·132377	·094261	155	9·905739	36
25	9·773533	285	·226467	9·867887	441	10·132113	·094355	155	9·905645	35
26	9·773704	285	·226296	9·868152	441	10·131848	·094448	155	9·905552	34
27	9·773875	285	·226125	9·868416	441	10·131584	·094541	155	9·905459	33
28	9·774046	285	·225954	9·868680	441	10·131320	·094634	155	9·905366	32
29	9·774217	285	·225783	9·868945	440	10·131055	·094728	156	9·905272	31
30	9·774388	285	·225612	9·869209	440	10·130791	·094821	156	9·905179	30
31	9·774558	284	·225442	9·869473	440	10·130527	·094915	156	9·905085	29
32	9·774729	284	·225271	9·869737	440	10·130263	·095008	156	9·904992	28
33	9·774899	284	·225101	9·870001	440	10·129999	·095102	156	9·904898	27
34	9·775070	284	·224930	9·870265	440	10·129735	·095196	156	9·904804	26
35	9·775240	284	·224760	9·870529	440	10·129471	·095289	156	9·904711	25
36	9·775410	284	·224590	9·870793	440	10·129207	·095383	156	9·904617	24
37	9·775580	283	·224420	9·871057	440	10·128943	·095477	156	9·904523	23
38	9·775750	283	·224250	9·871321	440	10·128679	·095571	156	9·904429	22
39	9·775920	283	·224080	9·871585	440	10·128415	·095665	157	9·904335	21
40	9·776090	283	·223910	9·871849	440	10·128151	·095759	157	9·904241	20
41	9·776259	283	·223741	9·872112	440	10·127888	·095853	157	9·904147	19
42	9·776429	283	·223571	9·872376	439	10·127624	·095947	157	9·904053	18
43	9·776598	282	·223402	9·872640	439	10·127360	·096041	157	9·903959	17
44	9·776768	282	·223232	9·872903	439	10·127097	·096136	157	9·903864	16
45	9·776937	282	·223063	9·873167	439	10·126833	·096230	157	9·903770	15
46	9·777106	282	·222894	9·873430	439	10·126570	·096324	157	9·903676	14
47	9·777275	282	·222725	9·873694	439	10·126306	·096419	157	9·903581	13
48	9·777444	281	·222556	9·873957	439	10·126043	·096513	157	9·903487	12
49	9·777613	281	·222387	9·874220	439	10·125780	·096608	157	9·903392	11
50	9·777781	281	·222219	9·874484	439	10·125516	·096702	158	9·903298	10
51	9·777950	281	·222050	9·874747	439	10·125253	·096797	158	9·903203	9
52	9·778119	281	·221881	9·875010	439	10·124990	·096892	158	9·903108	8
53	9·778287	281	·221713	9·875273	439	10·124727	·096986	158	9·903014	7
54	9·778455	280	·221545	9·875536	439	10·124464	·097081	158	9·902919	6
55	9·778624	280	·221376	9·875800	439	10·124200	·097176	158	9·902824	5
56	9·778792	280	·221208	9·876063	438	10·123937	·097271	158	9·902729	4
57	9·778960	280	·221040	9·876326	438	10·123674	·097366	158	9·902634	3
58	9·779128	280	·220872	9·876589	438	10·123411	·097461	158	9·902539	2
59	9·779295	280	·220705	9·876851	438	10·123149	·097556	159	9·902444	1
60	9·779463	279	·220537	9·877114	438	10·122886	·097651	159	9·902349	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

53 DEG.

37 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·779463		·220537	9·877114		10·122886	·097651		9·902349	60
1	9·779631	279	·220369	9·877377	438	10·122623	·097747	159	9·902253	59
2	9·779798	279	·220202	9·877640	438	10·122360	·097842	159	9·902158	58
3	9·779966	279	·220034	9·877903	438	10·122097	·097937	159	9·902063	57
4	9·780133	279	·219867	9·878165	438	10·121835	·098033	159	9·901967	56
5	9·780300	279	·219700	9·878428	438	10·121572	·098128	159	9·901872	55
6	9·780467	278	·219533	9·878691	438	10·121309	·098224	159	9·901776	54
7	9·780634	278	·219366	9·878953	438	10·121047	·098319	159	9·901681	53
8	9·780801	278	·219199	9·879216	437	10·120784	·098415	159	9·901585	52
9	9·780968	278	·219032	9·879478	437	10·120522	·098510	159	9·901490	51
10	9·781134	278	·218866	9·879741	437	10·120259	·098606	159	9·901394	50
11	9·781301	278	·218699	9·880003	437	10·119997	·098702	160	9·901298	49
12	9·781468	277	·218532	9·880265	437	10·119735	·098798	160	9·901202	48
13	9·781634	277	·218366	9·880528	437	10·119472	·098894	160	9·901106	47
14	9·781800	277	·218200	9·880790	437	10·119210	·098990	160	9·901010	46
15	9·781966	277	·218034	9·881052	437	10·118948	·099086	160	9·900914	45
16	9·782132	277	·217868	9·881314	437	10·118686	·099182	160	9·900818	44
17	9·782298	277	·217702	9·881576	437	10·118424	·099278	160	9·900722	43
18	9·782464	276	·217536	9·881839	437	10·118161	·099374	160	9·900626	42
19	9·782630	276	·217370	9·882101	437	10·117899	·099471	160	9·900529	41
20	9·782796	276	·217204	9·882363	437	10·117637	·099567	160	9·900433	40
21	9·782961	276	·217039	9·882625	437	10·117375	·099663	161	9·900337	39
22	9·783127	276	·216873	9·882887	436	10·117113	·099760	161	9·900240	38
23	9·783292	276	·216708	9·883148	436	10·116852	·099856	161	9·900144	37
24	9·783458	275	·216542	9·883410	436	10·116590	·099953	161	9·900047	36
25	9·783623	275	·216377	9·883672	436	10·116328	·100049	161	9·899951	35
26	9·783788	275	·216212	9·883934	436	10·116066	·100146	161	9·899854	34
27	9·783953	275	·216047	9·884196	436	10·115804	·100243	161	9·899757	33
28	9·784118	275	·215882	9·884457	436	10·115543	·100340	161	9·899660	32
29	9·784282	275	·215718	9·884719	436	10·115281	·100436	161	9·899564	31
30	9·784447	274	·215553	9·884980	436	10·115020	·100533	161	9·899467	30
31	9·784612	274	·215388	9·885242	436	10·114758	·100630	162	9·899370	29
32	9·784776	274	·215224	9·885503	436	10·114497	·100727	162	9·899273	28
33	9·784941	274	·215059	9·885765	436	10·114235	·100824	162	9·899176	27
34	9·785105	274	·214895	9·886026	436	10·113974	·100922	162	9·899078	26
35	9·785269	274	·214731	9·886288	436	10·113712	·101019	162	9·898981	25
36	9·785433	273	·214567	9·886549	436	10·113451	·101116	162	9·898884	24
37	9·785597	273	·214403	9·886810	436	10·113190	·101213	162	9·898787	23
38	9·785761	273	·214239	9·887072	435	10·112928	·101311	162	9·898689	22
39	9·785925	273	·214075	9·887333	435	10·112667	·101408	162	9·898592	21
40	9·786089	273	·213911	9·887594	435	10·112406	·101506	162	9·898494	20
41	9·786252	273	·213748	9·887855	435	10·112145	·101603	163	9·898397	19
42	9·786416	272	·213584	9·888116	435	10·111884	·101701	163	9·898299	18
43	9·786579	272	·213421	9·888377	435	10·111623	·101798	163	9·898202	17
44	9·786742	272	·213258	9·888639	435	10·111361	·101896	163	9·898104	16
45	9·786906	272	·213094	9·888900	435	10·111100	·101994	163	9·898006	15
46	9·787069	272	·212931	9·889160	435	10·110840	·102092	163	9·897908	14
47	9·787232	272	·212768	9·889421	435	10·110579	·102190	163	9·897810	13
48	9·787395	271	·212605	9·889682	435	10·110318	·102288	163	9·897712	12
49	9·787557	271	·212443	9·889943	435	10·110057	·102386	163	9·897614	11
50	9·787720	271	·212280	9·890204	435	10·109796	·102484	163	9·897516	10
51	9·787883	271	·212117	9·890465	435	10·109535	·102582	163	9·897418	9
52	9·788045	271	·211955	9·890725	435	10·109275	·102680	164	9·897320	8
53	9·788208	271	·211792	9·890986	434	10·109014	·102778	164	9·897222	7
54	9·788370	271	·211630	9·891247	434	10·108753	·102877	164	9·897123	6
55	9·788532	270	·211468	9·891507	434	10·108493	·102975	164	9·897025	5
56	9·788694	270	·211306	9·891768	434	10·108232	·103074	164	9·896926	4
57	9·788856	270	·211144	9·892028	434	10·107972	·103172	164	9·896828	3
58	9·789018	270	·210982	9·892289	434	10·107711	·103271	164	9·896729	2
59	9·789180	270	·210820	9·892549	434	10·107451	·103369	164	9·896631	1
60	9·789342	270	·210658	9·892810	434	10·107190	·103468	164	9·896532	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

52 DEG.

38 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·789342		·210658	9·892810		10·107190	·103468		9·896532	60
1	9·789504	269	·210496	9·893070	434	10·106930	·103567	164	9·896433	59
2	9·789665	269	·210335	9·893331	434	10·106669	·103665	165	9·896335	58
3	9·789827	269	·210173	9·893591	434	10·106409	·103764	165	9·896236	57
4	9·789988	269	·210012	9·893851	434	10·106149	·103863	165	9·896137	56
5	9·790149	269	·209851	9·894111	434	10·105889	·103962	165	9·896038	55
6	9·790310	269	·209690	9·894371	434	10·105629	·104061	165	9·895939	54
7	9·790471	268	·209529	9·894632	434	10·105368	·104160	165	9·895840	53
8	9·790632	268	·209368	9·894892	434	10·105108	·104259	165	9·895741	52
9	9·790793	268	·209207	9·895152	433	10·104848	·104359	165	9·895641	51
10	9·790954	268	·209046	9·895412	433	10·104588	·104458	165	9·895542	50
11	9·791115	268	·208885	9·895672	433	10·104328	·104557	165	9·895443	49
12	9·791275	268	·208725	9·895932	433	10·104068	·104657	166	9·895343	48
13	9·791436	267	·208564	9·896192	433	10·103808	·104756	166	9·895244	47
14	9·791596	267	·208404	9·896452	433	10·103548	·104855	166	9·895145	46
15	9·791757	267	·208243	9·896712	433	10·103288	·104955	166	9·895045	45
16	9·791917	267	·208083	9·896971	433	10·103029	·105055	166	9·894945	44
17	9·792077	267	·207923	9·897231	433	10·102769	·105154	166	9·894846	43
18	9·792237	267	·207763	9·897491	433	10·102509	·105254	166	9·894746	42
19	9·792397	266	·207603	9·897751	433	10·102249	·105354	166	9·894646	41
20	9·792557	266	·207443	9·898010	433	10·101990	·105454	166	9·894546	40
21	9·792716	266	·207284	9·898270	433	10·101730	·105554	166	9·894446	39
22	9·792876	266	·207124	9·898530	433	10·101470	·105654	167	9·894346	38
23	9·793035	266	·206965	9·898789	433	10·101211	·105754	167	9·894246	37
24	9·793195	266	·206805	9·899049	433	10·100951	·105854	167	9·894146	36
25	9·793354	265	·206646	9·899308	432	10·100692	·105954	167	9·894046	35
26	9·793514	265	·206486	9·899568	432	10·100432	·106054	167	9·893946	34
27	9·793673	265	·206327	9·899827	432	10·100173	·106154	167	9·893846	33
28	9·793832	265	·206168	9·900086	432	10·099914	·106255	167	9·893745	32
29	9·793991	265	·206009	9·900346	432	10·099654	·106355	167	9·893645	31
30	9·794150	265	·205850	9·900605	432	10·099395	·106456	167	9·893544	30
31	9·794308	264	·205692	9·900864	432	10·099136	·106556	167	9·893444	29
32	9·794467	264	·205533	9·901124	432	10·098876	·106657	168	9·893343	28
33	9·794626	264	·205374	9·901383	432	10·098617	·106757	168	9·893243	27
34	9·794784	264	·205216	9·901642	432	10·098358	·106858	168	9·893142	26
35	9·794942	264	·205058	9·901901	432	10·098099	·106959	168	9·893041	25
36	9·795101	264	·204899	9·902160	432	10·097840	·107060	168	9·892940	24
37	9·795259	264	·204741	9·902419	432	10·097581	·107161	168	9·892839	23
38	9·795417	263	·204583	9·902679	432	10·097321	·107261	168	9·892739	22
39	9·795575	263	·204425	9·902938	432	10·097062	·107362	168	9·892638	21
40	9·795733	263	·204267	9·903197	432	10·096803	·107464	168	9·892536	20
41	9·795891	263	·204109	9·903455	432	10·096545	·107565	168	9·892435	19
42	9·796049	263	·203951	9·903714	431	10·096286	·107666	169	9·892334	18
43	9·796206	263	·203794	9·903973	431	10·096027	·107767	169	9·892233	17
44	9·796364	263	·203636	9·904232	431	10·095768	·107868	169	9·892132	16
45	9·796521	262	·203479	9·904491	431	10·095509	·107970	169	9·892030	15
46	9·796679	262	·203321	9·904750	431	10·095250	·108071	169	9·891929	14
47	9·796836	262	·203164	9·905008	431	10·094992	·108173	169	9·891827	13
48	9·796993	262	·203007	9·905267	431	10·094733	·108274	169	9·891726	12
49	9·797150	262	·202850	9·905526	431	10·094474	·108376	169	9·891624	11
50	9·797307	262	·202693	9·905784	431	10·094216	·108477	169	9·891523	10
51	9·797464	261	·202536	9·906043	431	10·093957	·108579	169	9·891421	9
52	9·797621	261	·202379	9·906302	431	10·093698	·108681	170	9·891319	8
53	9·797777	261	·202223	9·906560	431	10·093440	·108783	170	9·891217	7
54	9·797934	261	·202066	9·906819	431	10·093181	·108885	170	9·891115	6
55	9·798091	261	·201909	9·907077	431	10·092923	·108987	170	9·891013	5
56	9·798247	261	·201753	9·907336	431	10·092664	·109089	170	9·890911	4
57	9·798403	261	·201597	9·907594	431	10·092406	·109191	170	9·890809	3
58	9·798560	260	·201440	9·907852	431	10·092148	·109293	170	9·890707	2
59	9·798716	260	·201284	9·908111	431	10·091889	·109395	170	9·890605	1
60	9·798872	260	·201128	9·908369	431	10·091631	·109497	170	9·890503	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

51 DEG.

39 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100	Cosine.	′
0	9·798872		·201128	9·908369		10·091631	·109497		9·890503	60
1	9·799028	260	·200972	9·908628	430	10·091372	·109600	170	9·890400	59
2	9·799184	260	·200816	9·908886	430	10·091114	·109702	171	9·890298	58
3	9·799339	260	·200661	9·909144	430	10·090856	·109805	171	9·890195	57
4	9·799495	259	·200505	9·909402	430	10·090598	·109907	171	9·890093	56
5	9·799651	259	·200349	9·909660	430	10·090340	·110010	171	9·889990	55
6	9·799806	259	·200194	9·909918	430	10·090082	·110112	171	9·889888	54
7	9·799962	259	·200038	9·910177	430	10·089823	·110215	171	9·889785	53
8	9·800117	259	·199883	9·910435	430	10·089565	·110318	171	9·889682	52
9	9·800272	259	·199728	9·910693	430	10·089307	·110421	171	9·889579	51
10	9·800427	258	·199573	9·910951	430	10·089049	·110523	171	9·889477	50
11	9·800582	258	·199418	9·911209	430	10·088791	·110626	171	9·889374	49
12	9·800737	258	·199263	9·911467	430	10·088533	·110729	172	9·889271	48
13	9·800892	258	·199108	9·911724	430	10·088276	·110832	172	9·889168	47
14	9·801047	258	·198953	9·911982	430	10·088018	·110936	172	9·889064	46
15	9·801201	258	·198799	9·912240	430	10·087760	·111039	172	9·888961	45
16	9·801356	258	·198644	9·912498	430	10·087502	·111142	172	9·888858	44
17	9·801511	257	·198489	9·912756	430	10·087244	·111245	172	9·888755	43
18	9·801665	257	·198335	9·913014	430	10·086986	·111349	172	9·888651	42
19	9·801819	257	·198181	9·913271	430	10·086729	·111452	172	9·888548	41
20	9·801973	257	·198027	9·913529	429	10·086471	·111556	172	9·888444	40
21	9·802128	257	·197872	9·913787	429	10·086213	·111659	173	9·888341	39
22	9·802282	257	·197718	9·914044	429	10·085956	·111763	173	9·888237	38
23	9·802436	256	·197564	9·914302	429	10·085698	·111866	173	9·888134	37
24	9·802589	256	·197411	9·914560	429	10·085440	·111970	173	9·888030	36
25	9·802743	256	·197257	9·914817	429	10·085183	·112074	173	9·887926	35
26	9·802897	256	·197103	9·915075	429	10·084925	·112178	173	9·887822	34
27	9·803050	256	·196950	9·915332	429	10·084668	·112282	173	9·887718	33
28	9·803204	256	·196796	9·915590	429	10·084410	·112386	173	9·887614	32
29	9·803357	256	·196643	9·915847	429	10·084153	·112490	173	9·887510	31
30	9·803511	255	·196489	9·916104	429	10·083896	·112594	173	9·887406	30
31	9·803664	255	·196336	9·916362	429	10·083638	·112698	174	9·887302	29
32	9·803817	255	·196183	9·916619	429	10·083381	·112802	174	9·887198	28
33	9·803970	255	·196030	9·916877	429	10·083123	·112907	174	9·887093	27
34	9·804123	255	·195877	9·917134	429	10·082866	·113011	174	9·886989	26
35	9·804276	255	·195724	9·917391	429	10·082609	·113115	174	9·886885	25
36	9·804428	254	·195572	9·917648	429	10·082352	·113220	174	9·886780	24
37	9·804581	254	·195419	9·917905	429	10·082095	·113324	174	9·886676	23
38	9·804734	254	·195266	9·918163	429	10·081837	·113429	174	9·886571	22
39	9·804886	254	·195114	9·918420	429	10·081580	·113534	174	9·886466	21
40	9·805039	254	·194961	9·918677	429	10·081323	·113638	174	9·886362	20
41	9·805191	254	·194809	9·918934	429	10·081066	·113743	175	9·886257	19
42	9·805343	254	·194657	9·919191	428	10·080809	·113848	175	9·886152	18
43	9·805495	253	·194505	9·919448	428	10·080552	·113953	175	9·886047	17
44	9·805647	253	·194353	9·919705	428	10·080295	·114058	175	9·885942	16
45	9·805799	253	·194201	9·919962	428	10·080038	·114163	175	9·885837	15
46	9·805951	253	·194049	9·920219	428	10·079781	·114268	175	9·885732	14
47	9·806103	253	·193897	9·920476	428	10·079524	·114373	175	9·885627	13
48	9·806254	253	·193746	9·920733	428	10·079267	·114478	175	9·885522	12
49	9·806406	253	·193594	9·920990	428	10·079010	·114584	175	9·885416	11
50	9·806557	252	·193443	9·921247	428	10·078753	·114689	175	9·885311	10
51	9·806709	252	·193291	9·921503	428	10·078497	·114795	176	9·885205	9
52	9·806860	252	·193140	9·921760	428	10·078240	·114900	176	9·885100	8
53	9·807011	252	·192989	9·922017	428	10·077983	·115006	176	9·884994	7
54	9·807163	252	·192837	9·922274	428	10·077726	·115111	176	9·884889	6
55	9·807314	252	·192686	9·922530	428	10·077470	·115217	176	9·884783	5
56	9·807465	252	·192535	9·922787	428	10·077213	·115323	176	9·884677	4
57	9·807615	251	·192385	9·923044	428	10·076956	·115428	176	9·884572	3
58	9·807766	251	·192234	9·923300	428	10·076700	·115534	176	9·884466	2
59	9·807917	251	·192083	9·923557	428	10·076443	·115640	176	9·884360	1
60	9·808067	251	·191933	9·923813	428	10·076187	·115746	176	9·884254	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

50 DEG.

40 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·808067		·191933	9·923813		10·076187	·115746		9·884254	60
1	9·808218	251	·191782	9·924070	428	10·075930	·115852	177	9·884148	59
2	9·808368	251	·191632	9·924327	428	10·075673	·115958	177	9·884042	58
3	9·808519	251	·191481	9·924583	428	10·075417	·116064	177	9·883936	57
4	9·808669	250	·191331	9·924840	427	10·075160	·116171	177	9·883829	56
5	9·808819	250	·191181	9·925096	427	10·074904	·116277	177	9·883723	55
6	9·808969	250	·191031	9·925352	427	10·074648	·116383	177	9·883617	54
7	9·809119	250	·190881	9·925609	427	10·074391	·116490	177	9·883510	53
8	9·809269	250	·190731	9·925865	427	10·074135	·116596	177	9·883404	52
9	9·809419	250	·190581	9·926122	427	10·073878	·116703	177	9·883297	51
10	9·809569	249	·190431	9·926378	427	10·073622	·116809	178	9·883191	50
11	9·809718	249	·190282	9·926634	427	10·073366	·116916	178	9·883084	49
12	9·809868	249	·190132	9·926890	427	10·073110	·117023	178	9·882977	48
13	9·810017	249	·189983	9·927147	427	10·072853	·117129	178	9·882871	47
14	9·810167	249	·189833	9·927403	427	10·072597	·117236	178	9·882764	46
15	9·810316	249	·189684	9·927659	427	10·072341	·117343	178	9·882657	45
16	9·810465	248	·189535	9·927915	427	10·072085	·117450	178	9·882550	44
17	9·810614	248	·189386	9·928171	427	10·071829	·117557	178	9·882443	43
18	9·810763	248	·189237	9·928427	427	10·071573	·117664	178	9·882336	42
19	9·810912	248	·189088	9·928683	427	10·071317	·117771	179	9·882229	41
20	9·811061	248	·188939	9·928940	427	10·071060	·117879	179	9·882121	40
21	9·811210	248	·188790	9·929196	427	10·070804	·117986	179	9·882014	39
22	9·811358	248	·188642	9·929452	427	10·070548	·118093	179	9·881907	38
23	9·811507	247	·188493	9·929708	427	10·070292	·118201	179	9·881799	37
24	9·811655	247	·188345	9·929964	427	10·070036	·118308	179	9·881692	36
25	9·811804	247	·188196	9·930220	427	10·069780	·118416	179	9·881584	35
26	9·811952	247	·188048	9·930475	427	10·069525	·118523	179	9·881477	34
27	9·812100	247	·187900	9·930731	427	10·069269	·118631	179	9·881369	33
28	9·812248	247	·187752	9·930987	426	10·069013	·118739	179	9·881261	32
29	9·812396	247	·187604	9·931243	426	10·068757	·118847	180	9·881153	31
30	9·812544	246	·187456	9·931499	426	10·068501	·118954	180	9·881046	30
31	9·812692	246	·187308	9·931755	426	10·068245	·119062	180	9·880938	29
32	9·812840	246	·187160	9·932010	426	10·067990	·119170	180	9·880830	28
33	9·812988	246	·187012	9·932266	426	10·067734	·119178	180	9·880722	27
34	9·813135	246	·186865	9·932522	426	10·067478	·119387	180	9·880613	26
35	9·813283	246	·186717	9·932778	426	10·067222	·119495	180	9·880505	25
36	9·813430	246	·186570	9·933033	426	10·066967	·119603	180	9·880397	24
37	9·813578	245	·186422	9·933289	426	10·066711	·119711	180	9·880289	23
38	9·813725	245	·186275	9·933545	426	10·066455	·119820	181	9·880180	22
39	9·813872	245	·186128	9·933800	426	10·066200	·119928	181	9·880072	21
40	9·814019	245	·185981	9·934056	426	10·065944	·120037	181	9·879963	20
41	9·814166	245	·185834	9·934311	426	10·065689	·120145	181	9·879855	19
42	9·814313	245	·185687	9·934567	426	10·065433	·120254	181	9·879746	18
43	9·814460	245	·185540	9·934823	426	10·065177	·120363	181	9·879637	17
44	9·814607	244	·185393	9·935078	426	10·064922	·120471	181	9·879529	16
45	9·814753	244	·185247	9·935333	426	10·064667	·120580	181	9·879420	15
46	9·814900	244	·185100	9·935589	426	10·064411	·120689	181	9·879311	14
47	9·815046	244	·184954	9·935844	426	10·064156	·120798	181	9·879202	13
48	9·815193	244	·184807	9·936100	426	10·063900	·120907	182	9·879093	12
49.	9·815339	244	·184661	9·936355	426	10·063645	·121016	182	9·878984	11
50	9·815485	244	·184515	9·936610	426	10·063390	·121125	182	9·878875	10
51	9·815632	243	·184368	9·936866	426	10·063134	·121234	182	9·878766	9
52	9·815778	243	·184222	9·937121	426	10·062879	·121344	182	9·878656	8
53	9·815924	243	·184076	9·937376	426	10·062624	·121453	182	9·878547	7
54	9·816069	243	·183931	9·937632	425	10·062368	·121562	182	9·878438	6
55	9·816215	243	·183785	9·937887	425	10·062113	·121672	182	9·878328	5
56	9·816361	243	·183639	9·938142	425	10·061858	·121781	182	9·878219	4
57	9·816507	243	·183493	9·938398	425	10·061602	·121891	183	9·878109	3
58	9·816652	242	·183348	9·938653	425	10·061347	·122001	183	9·877999	2
59	9·816798	242	·183202	9·938908	425	10·061092	·122110	183	9·877890	1
60	9·816943	242	·183057	9·939163	425	10·060837	·122220	183	9·877780	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

49 DEG.

41 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·816943		·183057	9·939163		10·060837	·122220		9·877780	60
1	9·817088	242	·182912	9·939418	425	10·060582	·122330	183	9·877670	59
2	9·817233	242	·182767	9·939673	425	10·060327	·122440	183	9·877560	58
3	9·817379	242	·182621	9·939928	425	10·060072	·122550	183	9·877450	57
4	9·817524	242	·182476	9·940183	425	10·059817	·122660	183	9·877340	56
5	9·817668	241	·182332	9·940438	425	10·059562	·122770	183	9·877230	55
6	9·817813	241	·182187	9·940694	425	10·059306	·122880	184	9·877120	54
7	9·817958	241	·182042	9·940949	425	10·059051	·122990	184	9·877010	53
8	9·818103	241	·181897	9·941204	425	10·058796	·123101	184	9·876899	52
9	9·818247	241	·181753	9·941458	425	10·058542	·123211	184	9·876789	51
10	9·818392	241	·181608	9·941714	425	10·058286	·123322	184	9·876678	50
11	9·818536	241	·181464	9·941968	425	10·058032	·123432	184	9·876568	49
12	9·818681	240	·181319	9·942223	425	10·057777	·123543	184	9·876457	48
13	9·818825	240	·181175	9·942478	425	10·057522	·123653	184	9·876347	47
14	9·818969	240	·181031	9·942733	425	10·057267	·123764	184	9·876236	46
15	9·819113	240	·180887	9·942988	425	10·057012	·123875	185	9·876125	45
16	9·819257	240	·180743	9·943243	425	10·056757	·123986	185	9·876014	44
17	9·819401	240	·180599	9·943498	425	10·056502	·124096	185	9·875904	43
18	9·819545	240	·180455	9·943752	425	10·056248	·124207	185	9·875793	42
19	9·819689	239	·180311	9·944007	425	10·055993	·124318	185	9·875682	41
20	9·819832	239	·180168	9·944262	425	10·055738	·124429	185	9·875571	40
21	9·819976	239	·180024	9·944517	425	10·055483	·124541	185	9·875459	39
22	9·820120	239	·179880	9·944771	425	10·055229	·124652	185	9·875348	38
23	9·820263	239	·179737	9·945026	425	10·054974	·124763	185	9·875237	37
24	9·820406	239	·179594	9·945281	425	10·054719	·124874	185	9·875126	36
25	9·820550	239	·179450	9·945535	425	10·054465	·124986	186	9·875014	35
26	9·820693	238	·179307	9·945790	425	10·054210	·125097	186	9·874903	34
27	9·820836	238	·179164	9·946045	425	10·053955	·125209	186	9·874791	33
28	9·820979	238	·179021	9·946299	425	10·053701	·125320	186	9·874680	32
29	9·821122	238	·178878	9·946554	425	10·053446	·125432	186	9·874568	31
30	9·821265	238	·178735	9·946808	425	10·053192	·125544	186	9·874456	30
31	9·821407	238	·178593	9·947063	425	10·052937	·125656	186	9·874344	29
32	9·821550	238	·178450	9·947318	424	10·052682	·125768	186	9·874232	28
33	9·821693	238	·178307	9·947572	424	10·052428	·125879	186	9·874121	27
34	9·821835	237	·178165	9·947826	424	10·052174	·125991	187	9·874009	26
35	9·821977	237	·178023	9·948081	424	10·051919	·126104	187	9·873896	25
36	9·822120	237	·177880	9·948336	424	10·051664	·126216	187	9·873784	24
37	9·822262	237	·177738	9·948590	424	10·051410	·126328	187	9·873672	23
38	9·822404	237	·177596	9·948844	424	10·051156	·126440	187	9·873560	22
39	9·822546	237	·177454	9·949099	424	10·050901	·126552	187	9·873448	21
40	9·822688	237	·177312	9·949353	424	10·050647	·126665	187	9·873335	20
41	9·822830	236	·177170	9·949607	424	10·050393	·126777	187	9·873223	19
42	9·822972	236	·177028	9·949862	424	10·050138	·126890	187	9·873110	18
43	9·823114	236	·176886	9·950116	424	10·049884	·127002	188	9·872998	17
44	9·823255	236	·176745	9·950370	424	10·049630	·127115	188	9·872885	16
45	9·823397	236	·176603	9·950625	424	10·049375	·127228	188	9·872772	15
46	9·823539	236	·176461	9·950879	424	10·049121	·127341	188	9·872659	14
47	9·823680	236	·176320	9·951133	424	10·048867	·127453	188	9·872547	13
48	9·823821	235	·176179	9·951388	424	10·048612	·127566	188	9·872434	12
49	9·823963	235	·176037	9·951642	424	10·048358	·127679	188	9·872321	11
50	9·824104	235	·175896	9·951896	424	10·048104	·127792	188	9·872208	10
51	9·824245	235	·175755	9·952150	424	10·047850	·127905	188	9·872095	9
52	9·824386	235	·175614	9·952405	424	10·047595	·128019	189	9·871981	8
53	9·824527	235	·175473	9·952659	424	10·047341	·128132	189	9·871868	7
54	9·824668	235	·175332	9·952913	424	10·047087	·128245	189	9·871755	6
55	9·824808	234	·175192	9·953167	424	10·046833	·128359	189	9·871641	5
56	9·824949	234	·175051	9·953421	423	10·046579	·128472	189	9·871528	4
57	9·825090	234	·174910	9·953675	423	10·046325	·128586	189	9·871414	3
58	9·825230	234	·174770	9·953929	423	10·046071	·128699	189	9·871301	2
59	9·825371	234	·174629	9·954183	423	10·045817	·128813	189	9·871187	1
60	9·825511	234	·174489	9·954437	423	10·045563	·128927	189	9·871073	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

48 DEG.

42 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·825511		·174489	9·954437		10·045563	·128927		9·871073	60
1	9·825651	234	·174349	9·954691	423	10·045309	·129040	190	9·870960	59
2	9·825791	233	·174209	9·954945	423	10·045055	·129154	190	9·870846	58
3	9·825931	233	·174069	9·955200	423	10·044800	·129268	190	9·870732	57
4	9·826071	233	·173929	9·955454	423	10·044546	·129382	190	9·870618	56
5	9·826211	233	·173789	9·955707	423	10·044293	·129496	190	9·870504	55
6	9·826351	233	·173649	9·955961	423	10·044039	·129610	190	9·870390	54
7	9·826491	233	·173509	9·956215	423	10·043785	·129724	190	9·870276	53
8	9·826631	233	·173369	9·956469	423	10·043531	·129839	190	9·870161	52
9	9·826770	233	·173230	9·956723	423	10·043277	·129953	190	9·870047	51
10	9·826910	232	·173090	9·956977	423	10·043023	·130067	191	9·869933	50
11	9·827049	232	·172951	9·957231	423	10·042769	·130182	191	9·869818	49
12	9·827189	232	·172811	9·957485	423	10·042515	·130296	191	9·869704	48
13	9·827328	232	·172672	9·957739	423	10·042261	·130411	191	9·869589	47
14	9·827467	232	·172533	9·957993	423	10·042007	·130526	191	9·869474	46
15	9·827606	232	·172394	9·958246	423	10·041754	·130640	191	9·869360	45
16	9·827745	232	·172255	9·958500	423	10·041500	·130755	191	9·869245	44
17	9·827884	232	·172116	9·958754	423	10·041246	·130870	191	9·869130	43
18	9·828023	231	·171977	9·959008	423	10·040992	·130985	191	9·869015	42
19	9·828162	231	·171838	9·959262	423	10·040738	·131100	192	9·868900	41
20	9·828301	231	·171699	9·959516	423	10·040484	·131215	192	9·868785	40
21	9·828439	231	·171561	9·959769	423	10·040231	·131330	192	9·868670	39
22	9·828578	231	·171422	9·960023	423	10·039977	·131445	192	9·868555	38
23	9·828716	231	·171284	9·960277	423	10·039723	·131560	192	9·868440	37
24	9·828855	231	·171145	9·960531	423	10·039469	·131676	192	9·868324	36
25	9·828993	230	·171007	9·960784	423	10·039216	·131791	192	9·868209	35
26	9·829131	230	·170869	9·961038	423	10·038962	·131907	192	9·868093	34
27	9·829269	230	·170731	9·961291	423	10·038709	·132022	192	9·867978	33
28	9·829407	230	·170593	9·961545	423	10·038455	·132138	193	9·867862	32
29	9·829545	230	·170455	9·961799	423	10·038201	·132253	193	9·867747	31
30	9·829683	230	·170317	9·962052	423	10·037948	·132369	193	9·867631	30
31	9·829821	230	·170179	9·962306	423	10·037694	·132485	193	9·867515	29
32	9·829959	229	·170041	9·962560	423	10·037440	·132601	193	9·867399	28
33	9·830097	229	·169903	9·962813	423	10·037187	·132717	193	9·867283	27
34	9·830234	229	·169766	9·963067	423	10·036933	·132833	193	9·867167	26
35	9·830372	229	·169628	9·963320	423	10·036680	·132949	193	9·867051	25
36	9·830509	229	·169491	9·963574	423	10·036426	·133065	193	9·866935	24
37	9·830646	229	·169354	9·963827	423	10·036173	·133181	194	9·866819	23
38	9·830784	229	·169216	9·964081	423	10·035919	·133297	194	9·866703	22
39	9·830921	229	·169079	9·964335	423	10·035665	·133414	194	9·866586	21
40	9·831058	228	·168942	9·964588	423	10·035412	·133530	194	9·866470	20
41	9·831195	228	·168805	9·964842	423	10·035158	·133647	194	9·866353	19
42	9·831332	228	·168668	9·965095	422	10·034905	·133763	194	9·866237	18
43	9·831469	228	·168531	9·965349	422	10·034651	·133880	194	9·866120	17
44	9·831606	228	·168394	9·965602	422	10·034398	·133996	194	9·866004	16
45	9·831742	228	·168258	9·965855	422	10·034145	·134113	195	9·865887	15
46	9·831879	228	·168121	9·966109	422	10·033891	·134230	195	9·865770	14
47	9·832015	228	·167985	9·966362	422	10·033638	·134347	195	9·865653	13
48	9·832152	227	·167848	9·966616	422	10·033384	·134464	195	9·865536	12
49	9·832288	227	·167712	9·966869	422	10·033131	·134581	195	9·865419	11
50	9·832425	227	·167575	9·967123	422	10·032877	·134698	195	9·865302	10
51	9·832561	227	·167439	9·967376	422	10·032624	·134815	195	9·865185	9
52	9·832697	227	·167303	9·967629	422	10·032371	·134932	195	9·865068	8
53	9·832833	227	·167167	9·967883	422	10·032117	·135050	195	9·864950	7
54	9·832969	227	·167031	9·968136	422	10·031864	·135167	195	9·864833	6
55	9·833105	226	·166895	9·968389	422	10·031611	·135284	196	9·864716	5
56	9·833241	226	·166759	9·968643	422	10·031357	·135402	196	9·864598	4
57	9·833377	226	·166623	9·968896	422	10·031104	·135519	196	9·864481	3
58	9·833512	226	·166488	9·969149	422	10·030851	·135637	196	9·864363	2
59	9·833648	226	·166352	9·969403	422	10·030597	·135755	196	9·864245	1
60	9·833783	226	·166217	9·969656	422	10·030344	·135873	196	9·864127	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

47 DEG.

43 DEG.

′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·833783		·166217	9·969656		10·030344	·135873		9·864127	60
1	9·833919	226	·166081	9·969909	422	10·030091	·135990	196	9·864010	59
2	9·834054	225	·165946	9·970162	422	10·029838	·136108	196	9·863892	58
3	9·834189	225	·165811	9·970416	422	10·029584	·136226	197	9·863774	57
4	9·834325	225	·165675	9·970669	422	10·029331	·136344	197	9·863656	56
5	9·834460	225	·165540	9·970922	422	10·029078	·136462	197	9·863538	55
6	9·834595	225	·165405	9·971175	422	10·028825	·136581	197	9·863419	54
7	9·834730	225	·165270	9·971429	422	10·028571	·136699	197	9·863301	53
8	9·834865	225	·165135	9·971682	422	10·028318	·136817	197	9·863183	52
9	9·834999	225	·165001	9·971935	422	10·028065	·136936	197	9·863064	51
10	9·835134	224	·164866	9·972188	422	10·027812	·137054	197	9·862946	50
11	9·835269	224	·164731	9·972441	422	10·027559	·137173	198	9·862827	49
12	9·835403	224	·164597	9·972694	422	10·027306	·137291	198	9·862709	48
13	9·835538	224	·164462	9·972948	422	10·027052	·137410	198	9·862590	47
14	9·835672	224	·164328	9·973201	422	10·026799	·137529	198	9·862471	46
15	9·835807	224	·164193	9·973454	422	10·026546	·137647	198	9·862353	45
16	9·835941	224	·164059	9·973707	422	10·026293	·137766	198	9·862234	44
17	9·836075	224	·163925	9·973960	422	10·026040	·137885	198	9·862115	43
18	9·836209	223	·163791	9·974213	422	10·025787	·138004	198	9·861996	42
19	9·836343	223	·163657	9·974466	422	10·025534	·138123	198	9·861877	41
20	9·836477	223	·163523	9·974719	422	10·025281	·138242	198	9·861758	40
21	9·836611	223	·163389	9·974973	422	10·025027	·138362	199	9·861638	39
22	9·836745	223	·163255	9·975226	422	10·024774	·138481	199	9·861519	38
23	9·836878	223	·163122	9·975479	422	10·024521	·138600	199	9·861400	37
24	9·837012	223	·162988	9·975732	422	10·024268	·138720	199	9·861280	36
25	9·837146	222	·162854	9·975985	422	10·024015	·138839	199	9·861161	35
26	9·837279	222	·162721	9·976238	422	10·023762	·138959	199	9·861041	34
27	9·837412	222	·162588	9·976491	422	10·023509	·139078	199	9·860922	33
28	9·837546	222	·162454	9·976744	422	10·023256	·139198	199	9·860802	32
29	9·837679	222	·162321	9·976997	422	10·023003	·139318	199	9·860682	31
30	9·837812	222	·162188	9·977250	422	10·022750	·139438	200	9·860562	30
31	9·837945	222	·162055	9·977503	422	10·022497	·139558	200	9·860442	29
32	9·838078	222	·161922	9·977756	422	10·022244	·139678	200	9·860322	28
33	9·838211	221	·161789	9·978009	422	10·021991	·139798	200	9·860202	27
34	9·838344	221	·161656	9·978262	422	10·021738	·139918	200	9·860082	26
35	9·838477	221	·161523	9·978515	422	10·021485	·140038	200	9·859962	25
36	9·838610	221	·161390	9·978768	422	10·021232	·140158	200	9·859842	24
37	9·838742	221	·161258	9·979021	422	10·020979	·140279	200	9·859721	23
38	9·838875	221	·161125	9·979274	422	10·020726	·140399	201	9·859601	22
39	9·839007	221	·160993	9·979527	422	10·020473	·140520	201	9·859480	21
40	9·839140	221	·160860	9·979780	422	10·020220	·140640	201	9·859360	20
41	9·839272	220	·160728	9·980033	422	10·019967	·140761	201	9·859239	19
42	9·839404	220	·160596	9·980286	422	10·019714	·140881	201	9·859119	18
43	9·839536	220	·160464	9·980538	422	10·019462	·141002	201	9·858998	17
44	9·839668	220	·160332	9·980791	422	10·019209	·141123	201	9·858877	16
45	9·839800	220	·160200	9·981044	422	10·018956	·141244	201	9·858756	15
46	9·839932	220	·160068	9·981297	422	10·018703	·141365	202	9·858635	14
47	9·840064	220	·159936	9·981550	422	10·018450	·141486	202	9·858514	13
48	9·840196	219	·159804	9·981803	422	10·018197	·141607	202	9·858393	12
49	9·840328	219	·159672	9·982056	422	10·017944	·141728	202	9·858272	11
50	9·840459	219	·159541	9·982309	422	10·017691	·141849	202	9·858151	10
51	9·840591	219	·159409	9·982562	421	10·017438	·141971	202	9·858029	9
52	9·840722	219	·159278	9·982814	421	10·017186	·142092	202	9·857908	8
53	9·840854	219	·159146	9·983067	421	10·016933	·142214	202	9·857786	7
54	9·840985	219	·159015	9·983320	421	10·016680	·142335	202	9·857665	6
55	9·841116	219	·158884	9·983573	421	10·016427	·142457	203	9·857543	5
56	9·841247	218	·158753	9·983826	421	10·016174	·142578	203	9·857422	4
57	9·841378	218	·158622	9·984079	421	10·015921	·142700	203	9·857300	3
58	9·841509	218	·158491	9·984331	421	10·015669	·142822	203	9·857178	2
59	9·841640	218	·158360	9·984584	421	10·015416	·142944	203	9·857056	1
60	9·841771	218	·158229	9·984837	421	10·015163	·143066	203	9·856934	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

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′	Sine.	Diff. 100″	Cosecant.	Tangent.	Diff. 100″	Cotangent.	Secant.	Diff. 100″	Cosine.	′
0	9·841771		·158229	9·984837		10·015163	·143066		9·856934	60
1	9·841902	218	·158098	9·985090	421	10·014910	·143188	203	9·856812	59
2	9·842033	218	·157967	9·985343	421	10·014657	·143310	203	9·856690	58
3	9·842163	218	·157837	9·985596	421	10·014404	·143432	204	9·856568	57
4	9·842294	217	·157706	9·985848	421	10·014152	·143554	204	9·856446	56
5	9·842424	217	·157576	9·986101	421	10·013899	·143677	204	9·856323	55
6	9·842555	217	·157445	9·986354	421	10·013646	·143799	204	9·856201	54
7	9·842685	217	·157315	9·986607	421	10·013393	·143922	204	9·856078	53
8	9·842815	217	·157185	9·986860	421	10·013140	·144044	204	9·855956	52
9	9·842946	217	·157054	9·987112	421	10·012888	·144167	204	9·855833	51
10	9·843076	217	·156924	9·987365	421	10·012635	·144289	204	9·855711	50
11	9·843206	217	·156794	9·987618	421	10·012382	·144412	205	9·855588	49
12	9·843336	216	·156664	9·987871	421	10·012129	·144535	205	9·855465	48
13	9·843466	216	·156534	9·988123	421	10·011877	·144658	205	9·855342	47
14	9·843595	216	·156405	9·988376	421	10·011624	·144781	205	9·855212	46
15	9·843725	216	·156275	9·988629	421	10·011371	·144904	205	9·855096	45
16	9·843855	216	·156145	9·988882	421	10·011118	·145027	205	9·854973	44
17	9·843984	216	·156016	9·989134	421	10·010866	·145150	205	9·854850	43
18	9·844114	216	·155886	9·989387	421	10·010613	·145273	205	9·854727	42
19	9·844243	216	·155757	9·989640	421	10·010360	·145397	206	9·854603	41
20	9·844372	215	·155628	9·989893	421	10·010107	·145520	206	9·854480	40
21	9·844502	215	·155498	9·990145	421	10·009855	·145644	206	9·854356	39
22	9·844631	215	·155369	9·990398	421	10·009602	·145767	206	9·854233	38
23	9·844760	215	·155240	9·990651	421	10·009349	·145891	206	9·854109	37
24	9·844889	215	·155111	9·990903	421	10·009097	·146014	206	9·853986	36
25	9·845018	215	·154982	9·991156	421	10·008844	·146138	206	9·853862	35
26	9·845147	215	·154853	9·991409	421	10·008591	·146262	206	9·853738	34
27	9·845276	215	·154724	9·991662	421	10·008338	·146386	206	9·853614	33
28	9·845405	214	·154595	9·991914	421	10·008086	·146510	207	9·853490	32
29	9·845533	214	·154467	9·992167	421	10·007833	·146634	207	9·853366	31
30	9·845662	214	·154338	9·992420	421	10·007580	·146758	207	9·853242	30
31	9·845790	214	·154210	9·992672	421	10·007328	·146882	207	9·853118	29
32	9·845919	214	·154081	9·992925	421	10·007075	·147006	207	9·852994	28
33	9·846047	214	·153953	9·993178	421	10·006822	·147131	207	9·852869	27
34	9·846175	214	·153825	9·993430	421	10·006570	·147255	207	9·852745	26
35	9·846304	214	·153696	9·993683	421	10·006317	·147380	207	9·852620	25
36	9·846432	214	·153568	9·993936	421	10·006064	·147504	207	9·852496	24
37	9·846560	213	·153440	9·994189	421	10·005811	·147629	208	9·852371	23
38	9·846688	213	·153312	9·994441	421	10·005559	·147753	208	9·852247	22
39	9·846816	213	·153184	9·994694	421	10·005306	·147878	208	9·852122	21
40	9·846944	213	·153056	9·994947	421	10·005053	·148003	208	9·851997	20
41	9·847071	213	·152929	9·995199	421	10·004801	·148128	208	9·851872	19
42	9·847199	213	·152801	9·995452	421	10·004548	·148253	208	9·851747	18
43	9·847327	213	·152673	9·995705	421	10·004295	·148378	208	9·851622	17
44	9·847454	213	·152546	9·995957	421	10·004043	·148503	208	9·851497	16
45	9·847582	212	·152418	9·996210	421	10·003790	·148628	209	9·851372	15
46	9·847709	212	·152291	9·996463	421	10·003537	·148754	209	9·851246	14
47	9·847836	212	·152164	9·996715	421	10·003285	·148879	209	9·851121	13
48	9·847964	212	·152036	9·996968	421	10·003032	·149004	209	9·850996	12
49	9·848091	212	·151909	9·997221	421	10·002779	·149130	209	9·850870	11
50	9·848218	212	·151782	9·997473	421	10·002527	·149255	209	9·850745	10
51	9·848345	212	·151655	9·997726	421	10·002274	·149381	209	9·850619	9
52	9·848472	212	·151528	9·997979	421	10·002021	·149507	209	9·850493	8
53	9·848599	211	·151401	9·998231	421	10·001769	·149632	210	9·850368	7
54	9·848726	211	·151274	9·998484	421	10·001516	·149758	210	9·850242	6
55	9·848852	211	·151148	9·998737	421	10·001263	·149884	210	9·850116	5
56	9·848979	211	·151021	9·998989	421	10·001011	·150010	210	9·849990	4
57	9·849106	211	·150894	9·999242	421	10·000758	·150136	210	9·849864	3
58	9·849232	211	·150768	9·999495	421	10·000505	·150262	210	9·849738	2
59	9·849359	211	·150641	9·999747	421	10·000253	·150389	210	9·849611	1
60	9·849485	211	·150515	10·000000	421	10·000000	·150515	210	9·849485	0
′	Cosine.		Secant.	Cotangent.		Tangent.	Cosecant.		Sine.	′

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